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Convergence of Slope Stability Computations Using GPS

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Introduction

There are at present two basic lines of approach in the slope stability analysis; namely, the limit equilibrium approach and the stress-strain analysis using the finite element technique. The latter approach is a sophisticated one, but it requires very accurate input data. Otherwise, the results obtained from such analysis become as doubtful as the input data itself (Sarma, 1979). On the other hand, the limit equilibrium approach is relatively simple and has been widely used by the practising engineers. This, in turn, has all along attracted the attention of the researchers. Reservations have been raised against the limit equilibrium approach on the grounds that the factors such as slope deformation, the history of slope formation and initial state of stress are not considered in the analysis. Nevertheless, success in the usage of the limit equilibrium methods has been rated as commendable (Fredlund, 1984).

Over the years, limit equilibrium methods have been extensively refined by various investigators. Perhaps the most remarkable refinement has come in the form of development of methods which do not require any a priori assumption regarding the shape of the slip surface. Some of the widely studied methods in this category are those credited to Janbu (1957, 1973), Morgenstern and Price (1965), and Spencer (1967, 1973). Subsequently, the refinement, which has so far been concentrated only on the method of analysis, has been extended to the search for critical slip surface. It is now well appreciated that limit equilibrium slope stability analysis is a problem of optimization wherein the shape and location of the critical slip surface which

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yields the minimum factor of safety, are found out. The use of powerful and efficient minimization techniques available in the optimization literature has been a topic of increasing interest among the researchers in the area (Basudhar, 1976; Baker, 1980; Martins, 1982; Bhattacharya, 1990). The well known sequential unconstrained minimization technique (SUMT) of non-linear optimization has been adopted in a general purpose slope stability program SUMSTAB developed by the authors. In addition to yielding the critical slip surface and the associated minimum factor of safety, any rigorous limit equilibrium solution has to satisfy some acceptability or admissibility criteria such that the Mohr-Coulomb failure criteria is not violated anywhere within the sliding body, no tension is implied and the direction of forces are kinematically admissible (Morgenstern and Price, 1965; Sarma, 1979).

In the process of finding the critical slip surface it is required to evaluate the factor of safety of a number of trial surfaces. The evaluation of factor of safety of a trial slip surface requires solving non-linear equation or pairs of equations. Some convergence difficulties have been reported in solving such non-linear factor of safety equations. Much concern has been expressed over the solution of Janbu's GPS. One of the suggestions made (Janbu, 1980) to circumvent this difficulty is to reduce the number of slices. Keeping the above in view, in this paper an attempt has been made to investigate the effects of variation of number of slices on the convergence of the iterative schemes employed for solving the non-linear factor of safety equations and, more importantly, to observe how the effect is finally reflected on obtaining acceptable solutions for the least value of the factor of safety and the corresponding critical slip surface for the slope section concerned. Anticipating that such effect is likely to be more pronounced in the case of heterogeneous slopes, two example problems have been selected, namely, a zoned dam section and an embankment on clay foundation. This study has been done with reference to the Janbu's (1973) method and the Spencer's (1973) method. The program SUMSTAB has been utilized for all computations.

Brief Critical Review of the Janbu's And The Spencer's Methods

The difference between the above mentioned limit equilibrium methods relate to the assumptions used to make the problem statically determinate and the particular condition or conditions of equilibrium considered.

In Janbu's generalized method, n (the number of slices into which the sliding mass has been subdivided) assumptions are made concerning the point of application of the normal force N at the slice base and (n − 1) assumptions are made concerning the point of application of the interslice normal forces, E. The total number of assumptions (2n − 1) is, thus, one more than the degree of indeterminacy (2n − 2). Sarma (1979) points out that Janbu's method, therefore, should not be technically called a rigorous solution. However, in the solution one of the assumptions i.e., the position of the last normal force, N, is not used. Had it been used, it would have been found that the moment equilibrium of the last slice is not satisfied, i.e., $M_n \neq 0$. This check was not made by Janbu. Moreover, Madej (1971) points out that Janbu's moment equilibrium equation is incomplete. The moment equilibrium equation that has been used to compute the interslice shear forces can be visualized as generating an interslice force function. The position of the line of thrust has to be assumed in this method.

Spencer's method makes n assumptions for the point of application of the normal force N and $(n - 1)$ assumptions regarding the relationship between the magnitudes of the interslice forces. This method, therefore, satisfies static equilibrium conditions rigorously. Unlike Janbu's method, in this method line of thrust need not be assumed; it comes out as a part of the solution.

The above mentioned deficiencies in the Janbu's GPS is likely to contribute to the convergence difficulties (Fredlund, 1984).

Number Of Slices To Be Considered In The Analysis

In all the methods of slices it is required to decide on the number of slices before computations are carried out. For circular slip surface, Wright (1969) recommends that the number of slices may be taken such that the slice bases have a fixed are length equal to 10% of the slope height. To meet this requirement 10 to 50 slices are needed. Spencer (1967) has reported that for circular slip surfaces an increase in the number of slices from 16 to 32 results in a negligible change of only about 0.3% in the values of factor of safety (an absolute difference of only 0.004).

With respect to slip surfaces of general shape in homogeneous slopes, Spencer (1973) considered 14 to 16 slices to demonstrate his method of analysis. However, Janbu (1973) with reference to the GPS, has observed that 6 to 9 slices are adequate from practical point of view. Janbu (1980) has cautioned against the notion that a larger number of slices gives more accurate results. According to him, this is by no means so since all formulae were originally derived for finite differences, neglecting terms of second order. He has suggested that the ratio between the mean slice height and the slice width should usually be 1 to 2.5 or a maximum 3. He has also warned about another faulty notion that when numerical instability is encountered, the applied procedure must be wrong, while the reason may be one or more of the following :

1. Statically inadmissible slip surface

2. Number of slices being too large
3. Wrong line of thrust (to assume tension crack)
4. Incorrect input data

Bhowmik (1984), based on studies with reference to both Janbu's and Spencer's methods, observed that for homogeneous slopes the number of slices beyond 12 has no significant effect on factor safety values.

In case of slopes in complex heterogeneous soil conditions such as zoned dams the number of slices to be considered in the analysis is likely to be dependent on the method of analysis to a greater extent than that in homogeneous slopes. For the same method of analysis it might as well vary from one problem to another.

The Program SUMSTAB

The program SUMSTAB developed by the authors for slope stability computations is a FORTRAN version of the mathematical programming formulation in which the slip surface co-ordinates are treated as the decision variables and factor of safety functional as the objective function. To ensure a geometrically reasonable and physically acceptable critical slip surface, some restrictions or constraints are imposed on the decision variables such that the acceptability or the admissibility criteria (Morgenstern and Price, 1965; Sarma, 1979) are not violated.

The constrained minimization problem thus formulated has been converted to an unconstrained one by using the penalty function method and carrying out the sequential unconstrained minimization of the composite function. As in most cases it is difficult to find an initial feasible design vector, the extended penalty function method suggested by Kavlie (1971) has been used. For the unconstrained minimization Powell's conjugate direction method for multi-dimensional search and quadratic interpolation method for unidirectional search has been adopted.

Earlier, a program with the same name was developed by Babu (1986) using sequential unconstrained minimization technique and Janbu's GPS. The present program differs significantly from the previous one and can accommodate any type of zoning of the body of the dam and its foundation; furthermore, in addition to Janbu's GPS, a more rigorous method (Spencer, 1973) of factor of safety computation has been adopted in the current version of the program SUMSTAB.

Acceptable Solution

As stated earlier, the obtained solution needs to be checked for acceptability. As discussed by Spencer (1973) and Janbu (1973) and others, a solution may be considered acceptable as discussed below.

For the Spencer's method of analysis, a solution may be considered acceptable if the associated line of thrust lies entirely within the sliding mass. In addition to this, the associated resultant interslice forces should be all positive, as per the conventions followed in the Spencer's (1973) method.

For the Janbu's method of analysis, a solution may be considered acceptable if the calculated stresses (normal and shear) along the shear surface are all positive. Furthermore, the horizontal component of the interslice forces (E) should be all positive whereas the vertical forces (T) should be all negative as per the conventions followed in the Janbu's method (Janbu, 1973).

Illustrative Examples

Example Problem 1

Figure 1 shows an embankment made of a cohesionless soil fill ($c' = O$) on a saturated clay foundation ($\phi_u = O$). The same problem was earlier attempted by Wright (1969) using both Spencer's (1967) and Janbu's (1967) methods.

Solution Using Spencer (1973) Method

Assuming circular slip surface and Spencer (1967) method, Wright (1969) reports an acceptable solution with F_{min} of 1.199. Taking this as the initial surface in the present investigation, an acceptable solution with F_{min} of 1.182 has been obtained without any numerical difficulty. The corresponding critical slip surface is also shown Fig. 1. 15 slices have been considered in the analysis. Wright (1969), however, took as many an 40 slices for his slip circle analysis.

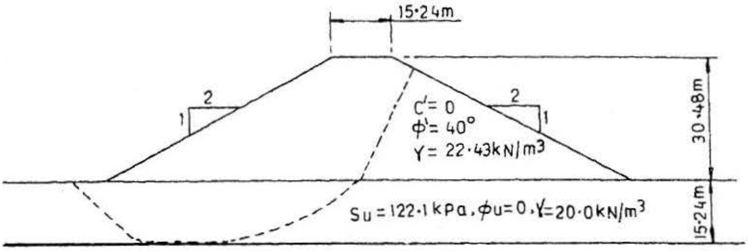


FIGURE 1 : Example Problem 1

Solution using Janbu's GPS :

It is pertinent to note the experience by Wright (1969) in an attempt to solve the same problem using Janbu's GPS. He attempted a number of analyses for the example problem and observed distinct divergence in the solution for any reasonable configuration of the critical shear surface with statically correct entrance and exit angles with and without assumed tension cracks. As such, he could not obtain any reasonable convergent solution.

In the present investigation, as the first step, attempt has been made to analyze a given (trial) slip surface adopting various number of slices. The initial slip surface in the solution by Spencer method has been chosen for the purpose. Number of slices (n) considered for the analysis are 6, 8, 10, 12 and 15. The results are presented in Table 1. The following observations have been made from the obtained results :

1. For analyses with 6, 8 and 10 slices, solution converges to a strict convergence limit of e = 0.001;
2. As n is increased to 12 convergence is not possible with e = 0.001;
3. Solution converges only when e is relaxed to 0.01;
4. When n is increased to 15 there is no convergence even with e = 0.01; it has to be further relaxed to 0.05.

Table 1
Variation of Factor of Safety of a Trial Shear Surface with Number of Slices

S.No.	No. of Slices n	Factor of Safety F	Convergence Limit, ε	(h_m/b)max	Remarks
1	6	1.51	0.001	≤ 4	Maximum Difference in F is 15%
2	8	1.71	0.001	≤ 4	
3	10	1.47	0..001	≤ 4	
4	12	1.49	0.010	> 5	
5	15	1.45	0.050	> 5	

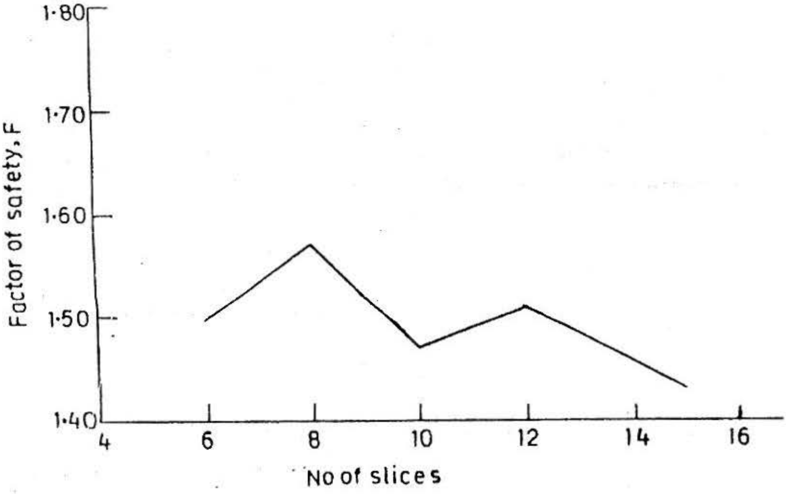


FIGURE 2 : Variation of Factor of Safety with Number of Slices for a Given Shear Surface

It has been observed that as the number of slices increases, the, maximum value of the ratio of mean height to width (slices are all of equal width, h_m/b), increases. Upto n = 10, maximum (h_m/b) remains less than 4. As n becomes greater or equal to 12, maximum (h_m/b) goes above 5.

Thus it is seen that when the number of slices exceeds a certain limit, difficulty in convergence arises and the larger the number of slices greater is the difficulty or poorer is the convergence. This finding corroborates Janbu's suggestion that the ratio (h_m/b) be restricted to about 3 (corresponding to n = 6, this ratio is 3) in order to avoid probable convergence difficulties. Figure 2 shows a plot of F versus n. It is seen that the variation is not monotonic. For the given range of n the maximum difference in the factor of safety value is about 15%.

Variation off F_{min} with n

Starting with the same initial slip surface as analyzed above, a series of attempts have been made to find F_{min} values and the corresponding critical slip surfaces, with the sliding Q_ mass discretized into 6, 8, 10, 12 and 15 slices. Results are presented in Table 2. It has been observed that the acceptable solutions are obtained in the analyses with 6, 8 and 10 slices and the F_{min} values are 1.27, 1.33 and 1.33 respectively. In Table 3 the assumed line of thrust (h_t/z) and the calculated response for 6 slices are presented. Analyses using 12 and 15 slices both yield unacceptable solutions with F_{min}

Table 2
Variation of Minimum Factor of Safety, F_{min} with Number of Slices (Example Problem 1)

S.No.	No. of Slices	F_{min}	Remarks
1	6	1.27	Acceptable solution : Maximum difference in Acceptable Factor of Safety is 5%
2	8	1.33	Acceptable solution : Maximum difference in Acceptable Factor of Safety is 5%
3	10	1.33	Acceptable solution : Maximum difference in Acceptable Factor of Safety is 5%
4	12	1.27	Unacceptable solution : End constraints applied
		1.00	Unacceptable solution : End constraints not applied
5	15	1.19	Unacceptable solution : End constraints applied
		1.00	Unacceptable solution : End constraints not applied

Table 3
Solution by Janbu's GPS with 6 slices (Example Problem 1)

Slice No.	σ (kPa)	τ (kPa)	h_t/z	E (kN/m)	T (kN/m)
1	148.4	98.0			
			0.33	2571	-538
2	638.0	96.2			
			0.34	7292	-1304
3	648.0	96.2			
			0.36	9076	-2337
4	580.0	96.2			
			0.38	7949	-2903
5	588.2	96.2			
			0.43	5390	-918
6	295.4	96.2			

Note :

h_t = distance between shear surface and line of thrust in Janbu's method
z = height of interslice boundary
E = horizontal component of the interslice force
T = vertical component of the interslice force
σ = normal stress at base of slice
τ = shear stress at base of slice.

values of 1.27and 1.19 respectively. The solutions are unacceptable as the directions of some of the internal forces are not admissible and stresses imply development of tension. In Table 4 the assumed line of thrust and the calculated response for 15 slices are presented. It can be observed from the Table that some of the interslice shear forces "T" are positive while one normal stress value, σ, is negative. For acceptable solution these signs should have been opposite.

Effect of End Inclination Constraints on the Acceptability of the Solution

According to some investigators (Janbu 1980; Ching and Fredlund, 1984) one of the means to avoid difficulties in convergence is to ensure that active and passive conditions at the scarp and toe of the slope are not

Table 4
Solution by Janbu's GPS with 15 slices (Example Problem 1)

Slice No.	σ (kPa)	τ (kPa)	h_t/z	E (kN/m)	T (kN/m)
1	53.0	35.0			
			0.34	251	67
2	139.7	92.7			
			0.35	1190	36
3	103.7	69.0			
			0.36	2413	-1365
4	173.2	115.8			
			0.38	2380	-3021
5	2106.0	104.0			
			0.38	1203	4798
6	592.7	104.0			
			0.38	7304	4731
7	-930.5	104.0			
			0.38	9501	-3339
8	758.9	104.0			
			0.38	9172	-3045
9	641.4	104.0			
			0.38	8665	-3102
10	640.5	104.0			
			0.38	7826	-2902
11	606.0	104.0			
			0.38	6670	-2561
12	574.7	104.0			
			0.38	5177	-2009
13	514.7	104.0			
			0.38	3441	-1323
14	496.0	104.0			
			0.43	1611	-202
15	222.3	104.0			

violated. But from the present study it has been observed that acceptable solutions can be obtained without imposing the end constraints when the number of slices is small (up to 10) and that in such cases subsequent imposition of the above constraints does not alter the solution significantly. When larger number of slices (12 and 15) are adopted without any imposition of end constraints the obtained solutions are unacceptable; even when such constraints are imposed it is not possible to get any acceptable solution. However, there is large change in the value of the minimum factor of safety. For instance, the F_{min} values of 1.27 and 1.19 reported above for 12 and 15 slices respectively are obtained with the imposition of end constraints; the corresponding values without end constraints are both close to 1.0 (which is the lower limit of F set in the program) .

Chowdhury (1990) has indicated that no such restrictions regarding the end inclination needs to be imposed to achieve convergence; in other words, it is not necessary to discard any shear surface, which does not meet the above requirements, as 'inadmissible'. It may be noted that for the same problem and for the same slip surface acceptable solution has been obtained using the usual number of slices without applying any end constraint.

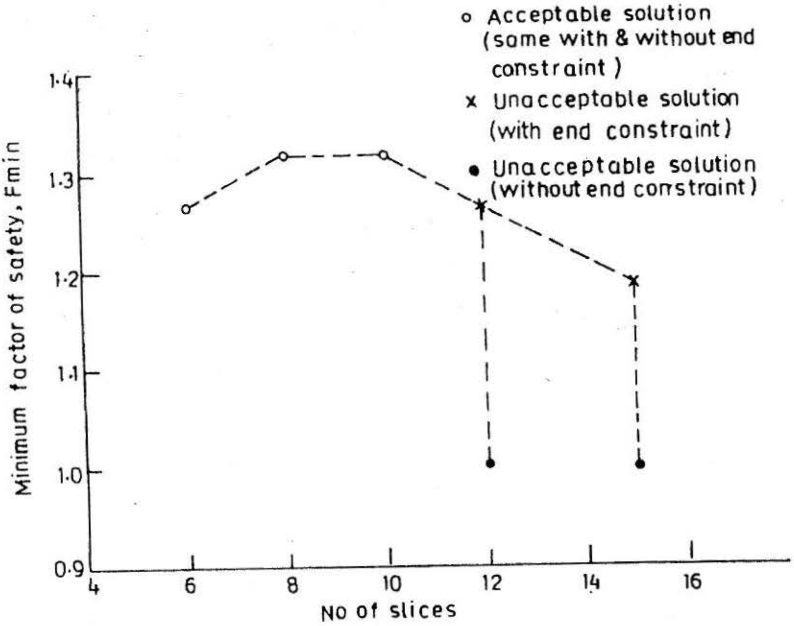


FIGURE 3 : Variation of Minimum Factor of Safety with Number of Slices

Plot of F_{min} vs. n

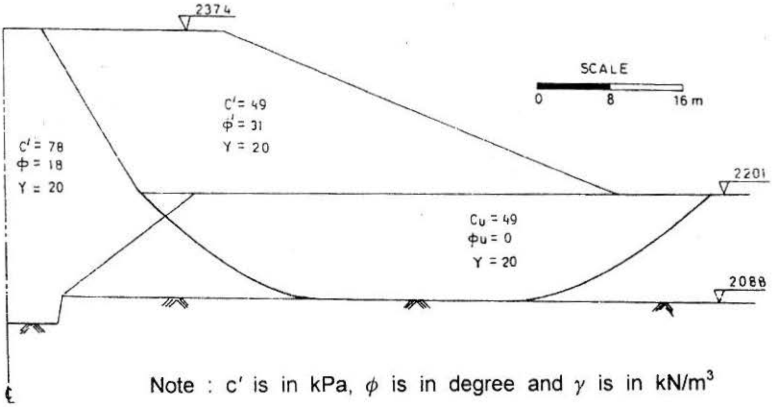
In Fig. 3 the variation of F_{min} with n as obtained through this example problem has been presented. This, however, also includes unacceptable solutions which can be quite misleading. For instance, the unacceptable solutions obtained with 12 and 15 slice (without end constraints) yield F_{min} values of 1.0 indicating that the slope is at critical equilibrium, whereas acceptable solutions (corresponding to 6, 8 and 10 slices) show that the slope is a stable one. Among the acceptable solutions, the difference in F_{min} values is about 5%.

Predominance of Number of Slices on Convergence:

The above observations possibly explain the failure of Wright's attempt (1969) to converge to a 'reasonable solution'. Considering that Wright's analysis took care of the theoretically correct entrance and exit inclinations as well as assumed tension crack the non-convergence can be attributed solely to the fat that as many as 40 slices were adopted. So it may be inferred that out of all possible factors influencing the convergence (or non-convergence) the number of slices may by far be the most dominating factor.

Example Problem 2

Figure 4 shows a section of the Birch Dam in Oklahoma along with the soil profile as described by Celestino and Duncan (1981). Using Spencer's method of analysis in SUMSTAB and 14 slices without the imposition of any constraint on the line of thrust the value of F_{min} has been obtained as 1.22; but the value is not acceptable as it associates with an unreasonable line of



Note : c' is in kPa, ϕ is in degree and γ is in kN/m^3

FIGURE 4 : Example Problem 2

Table 5
Influence of the Number of Slices on the Acceptability of the Solution (Example Problem 2)

(a) Unacceptable Solution
Number of Slices = 14; F_{min} = 1.22

Slice No.	σ (kPa)	τ (kPa)	L/H	$Z/(\gamma b H_t)$
1	69.7	40.30		
			0.51	0.26
2	148.3	40.27		
			0.43	0.69
3	226.3	40.27		
			0.33	1.09
4	290.7	40.27		
			0.28	1.37
5	338.7	40.27		
			0.23	1.53
6	381.0	40.27		
			0.20	1.65
7	423.1	40.27		
			0.17	1.76
8	464.7	40.27		
			0.15	1.87
9	495.4	40.27		
			0.13	1.83
10	479.0	40.27		
			0.06	1.17
11	422.6	40.27		
			-0.17	-0.35
12	228.9	104.30		
			0.60	-0.02
13	109.9	93.21		
			0.49	-0.23
14	-11.2	61.10		

thrust (Table 5a). The obtained critical slip surface is also shown in Fig 4. If constraints are imposed on the line of thrust, it is found that the solution does not converge at all.

In order to study the effect of reducing the number of slices and also in view of the fact that in their study Celestino and Duncan (1981) have considered only five slices, an analysis has been carried out using five slices and without applying any constraint on the line of thrust to avoid any convergence difficulty. Due to the reduction of the number of slices from 14 to 5 an acceptable solution (F_{min} = 1.12) with a reasonable line of thrust and interslice forces (Table 5b) could be obtained.

Table 5
contd.

(b) Apparently Acceptable Solution
Number of Slices = 5; F_{min} = 1.12

Slice No.	σ (kPa)	τ (kPa)	L/H	$Z/(\gamma b H_t)$
1	170.7	43.9		
			0.44	0.54
2	329.5	43.9		
			0.29	0.66
3	420.8	43.9		
			0.22	0.64
4	368.9	43.9		
			0.48	0.02
5	69.3	81.2		

Note :

L = height of interslice force above shear surface (in Spencer's method of analysis
H = height of interslice boundary
Z = resultant interslice force
γ = bulk density of soil
b = width of a force
H_t = height of embankment

Critical Comments

The method of slices is based on the hypothesis that the stress at a point on a slip surface is some component of the weight of the slice directly above the point. So, analysis with too few slices might lead to suppression of the true (with respect to the particular method of slices) stress conditions or, in other words, the limit equilibrium solution would be fraught with the danger of being further off from real stress picture (Spencer, 1968). As such, it is very important to keep the above observation in view while solutions are obtained with too few slices and caution should be exercised in using the obtained factor of safety values in making any engineering decision. While using rigorous methods e.g., the Spencer's method, it would be more prudent to try and obtain a convergent (as well as acceptable) solution taking number of slices in the range of 10 – 16. In those cases where acceptability criteria are violated, acceptable solutions can still be achieved without taking recourse to the reduction of number of slices, by adopting one or more of the following alternative means (Spencer, 1970; Bhattacharya, 1990).

1. by introduction of tension crack;

2. by attempting solution with various side force functions (not restricting to the assumption of parallel side forces, as in Spencer (1967) method).

In using methods which do not strictly satisfy statics, such as the Janbu's GPS, the number of slices may be restricted such that the ratio of the mean height to width for any slice does not exceed a certain limit (about 4) as also suggested by Janbu (1980). It has been further observed that the introduction of end inclination constraint may not help achieving convergence or acceptability. Yet, it is better to use such constraint as this is likely to yield a factor of safety (though the solution might not be strictly acceptable) value which would be close to that corresponding to an acceptable solution. Otherwise, the factor of safety value could be misleading (Example Problem 1).

Conclusions

On the basis of the study reported herein the following conclusions are drawn :

1. In limit equilibrium methods of slices, the number of equally spaced vertical slices to be adopted in the computations is found to have remarkable influence on the convergence as well as the acceptability of the solution.

2. The effect of variation of number of slices is method-dependent; the effect is more pronounced in the case of those methods in which static equilibrium conditions are not rigorously satisfied e.g., Janbu's GPS.

3. Contrary to popular belief, in general the prospect of achieving convergent and acceptable solutions can be improved by decreasing and not increasing the number of slices. However, too few slices might lead to suppression of the true stress state (true within the limitations of the method).

4. The variation of number of slices is found to show less effect on the minimum factor of safety associated with the (acceptable) critical slip surface (5%) than on the factor of safety of a trial slip surface (up to 15%).

5. With respect to Janbu's GPS, in all probability a convergent and acceptable solution can be obtained if the number of slices is restricted such that the ratio of the mean height to width of any slice does not exceed a certain limit (about 4) even without imposing any end

inclination constraint. However, in using Janbu's method, it is better to impose such constraints to avoid misleading (unacceptable) results. The same is not a requirement in the case of analysis by the Spencer's method.

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