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Technical Note

A Note on the Fundamental Period of Earth Dams

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Introduction

Earthquakes induce acceleration in the dam which may increase the water pressure on the dam and the stresses within the dam. Therefore, some allowances must be made in the design of dams to be constructed in seismic zones.

For studying the vibration characteristics of a dam subjected to ground motion, a dynamic analysis needs to be performed. In general, considerable time and effort is required for extracting the eigen pairs (eigen values and eigen vectors) of a dynamical system. Of the many frequencies with which the system oscillates, it is found that the lowest frequency (fundamental frequency) contributes greatly to the forced response of the system. Hence this frequency is of greatest interest. In view of its importance, currently it is generally recommended that quasi-static precedure be used for estimating the fundamental period in designing earthquake resistant dams.

The objective of the note is to propound a new and simple procedure for assessing the fundamental period of the dams.

Currently Available Methods

For reckoning the natural period of an earth dam, the following methods are available :

(1) Mononobe's Theory (1936)

(2) Computer Oriented Numerical Procedure (Manickaselvam et at., 1994)

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Mononobe's theory centres around classical theory of elasticity. Based on a rigerous treatment of the partial differential equation, the following formula for the natural period T was evolved :

$$T = 2.60\, H \left(\frac{\rho}{g} \right)^{0.5} \tag{1}$$

where, H = is the height of the dam,

ρ = mass density, and

g = acceleration due to gravity

In Eqn. (1), H, g and ρ must be in consistent units.

Though the procedure is quite involved, it culminates in a simple expression as indicated in Eqn. (1).

Recently a computer oriented numerical procedure was put forward for computing the fundamental period of earth dam as suggested by Manickaselvam et al. (1994). This procedure has certain flexibility over Mononobe's Eqn. (1). Though the computer technique is time consuming and tedious, the answer furnished by simplified method such as the one presented is this note.

Proposed Method

For finding the fundamental period, a versatile method available is the Rayleigh method. It is a general energy comparison procedure which is acceptable to systems vibrating in different deformable modes. While the application of this method is well known for systems vibrating in flexural, torsional and axial mode, its utility to systems vibrating in shearing mode is scantily dealt with in technical literature. Therefore, using the information furnished by Lekha (1995), this note presents a method of obtaining the fundamental frequency of systems vibrating in shear mode in general and in particular to assessing the fundamental period of earth dam.

Rayleigh's General Procedure

Rayleigh's method is based on energy principle. Lekha (1995) derived that the strain energy U, due to shear is given by

$$U = \int \frac{G\,A}{2\,f_s} \left\{ \frac{dw}{dx} \right\}^2 \cdot dx \tag{2}$$

where, w = displacement function,

G = modulus of rigidity,

A = area of cross section, and

f_s = form factor which is equal to 1.2 for a rectangular section

The shearing fundamental mode is described by the dynamic deflection curve $w = w(x)$. The kinetic energy (K.E.) associated with the shearing mode is :

$$K.E. = p^2 T^* \tag{3}$$

where, p = fundamental frequency, and

$$T^* = \int \frac{1}{2} m(x) \{w(x)\}^2 \cdot dx \tag{4}$$

where, m (x) is the function describing the distribution of the mass in the system.

Using Rayleigh's energy principal, equating Eqns. (1) and (2), leads to :

$$p^2 = \frac{\int \frac{GA}{2f_s}\left(\frac{dw}{dx}\right)^2 dx}{\int \frac{1}{2} m(x)\{w(x)\}^2 dx} \tag{5}$$

With the help of Eqn. (5), the fundamental period of any system vibrating in shear mode can be assessed. Meirovitch (1986) has been proved that Rayleigh's solution is an upper bound solution. Secondly, in Eqn. (5) any shape function resembling the mode of vibration and satisfying the minimal geometric boundary conditions may be assumed. The closer the function to the true vibration mode, the better will be the solution.

Application of Eqn. (5) to Earth Dam

Earth dam is a rigid structure possessing negligible bending resistance. However, there is considerable shearing strength present in the dam which enables it to perform oscillations in shearing mode. A typical earth dam is shown in Fig. 1. In formulating the theory, the following assumptions are introduced.

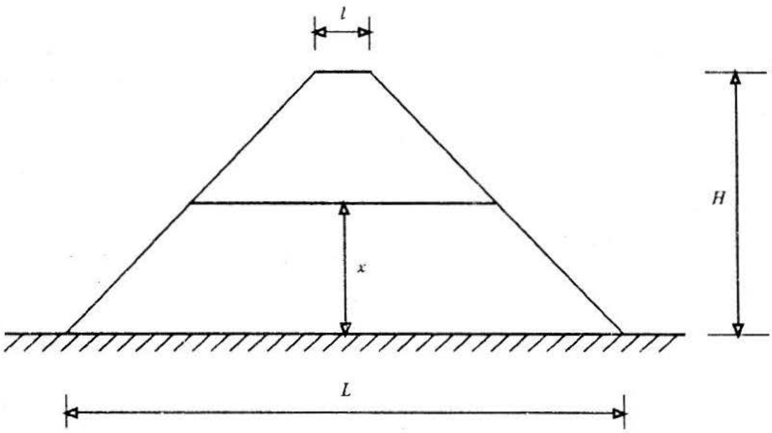

FIGURE 1 : Typical Cross Section of an Earth Dam

(i) There exists perfect fixity between the foundation and the super structure.

(ii) The earth dam behaves as an elastic system.

(iii) The second order effect due to axial shortening is negligible.

(iv) The dam vibrates purely in shearing mode.

(v) The density of the material and the modulus of rigidity are uniform throughout the body of the dam.

With these assumptions, the function m (x) is described as given below. Measuring x from the bottom as shown in Fig. 1,

$$m(x) = \frac{\rho\{L\,H - x(L - l)\}}{H} \tag{6}$$

where ρ = mass density,

L = base width of the dam,

l = top width of the dam, and

H = height of the dam

Assumption of Displacement Function

The success of Rayleigh's Eqn. (5) depends on the proximity of the assumed displacement function to the true mode shape. For earth dam, the shearing displacement function is assumed as

$$w = w(x) = \frac{\delta x}{H} \tag{7}$$

At the end, it is explained that Eqn. (7) fairly represents the true dynamic deflection curve. Now substituting Eqn. (6) and Eqn. (7) in Eqn. (5) and upon simplification, it is found

$$p^2 = \frac{6(L+l)G}{(L+3l)H^2 \rho f_s} \tag{8}$$

Let $$k^2 = \frac{(L+l)}{(L+3l)}$$

and for a rectangular cross section, the form factor f_s is known to be equal to 1.2. Substitution of these values in Eqn. (8) leads to :

$$p = 2.236 \frac{k}{H} \left(\frac{G}{\rho} \right)^{0.5} \tag{9}$$

It is known that

$$T = \frac{2\pi}{p} \tag{10}$$

Substitution of Eqn. (9) in Eqn. (10) gives

$$T = 2.809 \frac{H}{k} \left(\frac{\rho}{G} \right)^{0.5} \tag{11}$$

In earth dams usually L is very large compared with l. For practical purpose l may be assumed to be negligible. This fact makes k assume a value equal to unity. With this simplification Eqn. (11) turns out to be

$$T = 2.809\,H\left(\frac{\rho}{G}\right)^{0.5} \tag{12}$$

It is interseting to note that Eqn. (12) is identical in structure to Eqn. (1), though these two are derived in a distinctly different manner employing altogether different theoratical considerations. In Mononobe's Eqn. (1), the constant has a value of 2.6 whereas in Eqn. (12), it happens to be 2.809.

Discussion

The source of error in the proposed method lies in the assumption of the displacement function w (x). One method of improving the result is to have recourse to the Rayleigh-Ritz method, which aims at producing a shape function closer to the true dynamic deflection curve. Hence, the problem was solved using the Rayleigh-Ritz method with the following equation :

$$w(x) = a\,x + b\,x^2 \tag{13}$$

where a and b are Ritz parameters to be evaluated using the minimization principle. The method yielded the following solution :

$$y = a\left(\frac{x}{H} - \frac{0.05422\,x^2}{H^2}\right) \tag{14}$$

and

$$T = 2.7975\,H\left(\frac{\rho}{G}\right)^{0.5} \tag{15}$$

Comparing Eqn. (15) with Eqn. (12), it is inferred that there is no radical change in the solution. This fact leads to the illation that the monomial assumed in Eqn. (7) fairly represent the true mode shape.

Summary

Dynamic characteristics of a system can be studied to a great extent through the use of the fundamental period and the corresponding eigen vector. For an earth dam this period can be reckoned using Mononobe's Eqn. (1) or through the computer oriented numerical approach.

The proposed method combines the best features of simplicity of Eqn. 1 and the accuracy of computer solution. While Mononobe's theory deals in classical methametics, numerical method involves considerable effort for getting the solution. The Rayleigh concept advanced in this note is conceptually elegant and results in simple equation similer to that of Mononobe.

Dedication

This paper is reverentially dedicated to the memory of Mahakavi C. Subramania Bharathi.

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