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Optimal Capacity of Bent Pile Embedded in Clay

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Introduction

Whenever near surface soils are weak, piles are used to transfer loads from superstructure to the underlying competent strata. Piles are seldom installed perfectly straight. The initial out-of-straightness of piles can be as large as 1.75% to 4.75% of the length of the pile (Johnson, 1962; Hanna, 1968; York, 1971; Chan and Hanna, 1979 and Sovinc, 1981). It is this fact that dictates a flexural analysis for the bent pile problem (Glick, 1948; Gibson, 1952; Broms, 1963 and Mazindrani, 1979, 1994). Elastic buckling loads (eigen values) of piles are therefore unrealistic and are the upper bounds to the actual pile capacity which can be as low as 50% or even lower (Broms, 1986 and Mazindrani, 1996). Further all piles in one particular project may not have the same initial curvature which renders the bent pile problem highly complex and indeterminate.

Theoretical solutions to this highly complex problem were attempted by several investigators (Glick, 1948; Gibson, 1952; Broms, 1963; Mazindrani et al., 1977; Mazindrani, 1979; Rao and Mazindrani, 1981; Rao and Murthy, 1981; Broms, 1986; Rao and Madhav, 1986 and Mazindrani, 1996).

Analysis of the bent pile considering the initial bending stresses was attempted, among others, by Broms (1963) and Mazindrani (1979, 1994). This paper presents some of the results from the latter two papers.

Statement of Problem

Piles during installation in the ground by driving to bear on hard and competent strata develop out-of straightness defects from its straight geometric shape causing initial bending stresses to be locked in. The

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objectives of this paper is to establish the response of such a pile to vertical loads.

Differential Equation

The differential equation governing the behaviour of a bent pile embedded in clay soil is :

$$E\,I\,\frac{d^4y}{dx^4} + P\left(\frac{d^2y}{dx^2} + \frac{d^2y_o}{dx^2}\right) + k_h\,y = 0 \tag{1}$$

where $E\,I$ = flexural rigidity of Pile,

P = axial force in the pile at depth x,

k_h = soil stiffness, constant with depth,

y_o = initial pile deflection at depth x, and

y = further deflection in the pile under the action of applied vertical load as shown in Fig. 1.

Assumptions

(i) The soil behaviour is elastic upto ultimate strength of soil P_{ult} where after its behaviour becomes totally plastic.

(ii) The soil stiffness k_h is constant

(iii) The pile is hinged at both ends although in real life situations the end conditions can range between free-translating to fixed for which results are obtained (Mazindrani, 1996) and will be documented in a subsequent paper.

(iv) Although the exact profile of the out-of-straightness defects may be represented by Fourier series, it is found (Glick, 1948; Gibson, 1952 and Mazindrani, 1979, 1994) that it can be approximated as

$$y_o = a_1 \sin\pi\frac{x}{l} \tag{2}$$

where $a_1 = \frac{1}{400}$ to $\frac{1}{1000}$ times the pile length l.

(v) The axial force variation along the pile is given by (Reddy and Valsangkar, 1970)

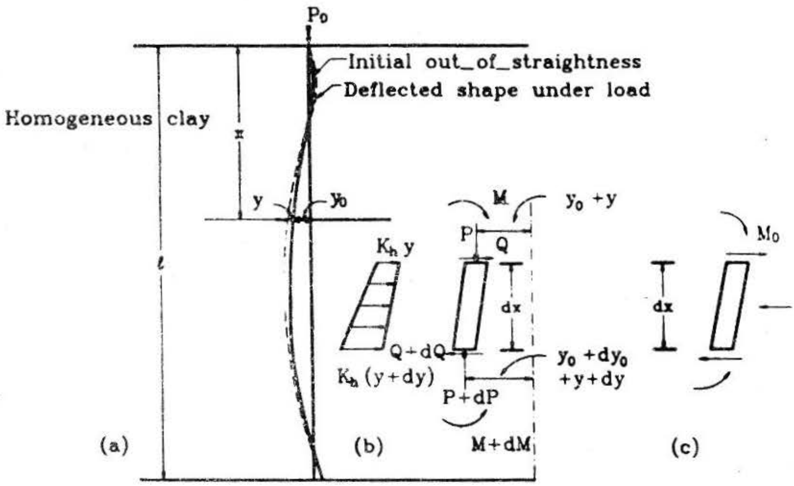

FIGURE 1 : Initially Bent Pile (Imperfections Shown Magnified)

$$P = P_o\left(1-\alpha\frac{x}{l}\right) \tag{3}$$

where P_o = axial force at top of pile,

α = constant of axial force variation.

Introducing,

characteristic length, $T = \sqrt[4]{\frac{EI}{k_h}}$

non-dimensional depth, $z = \frac{x}{T}$

non-dimensional pile length, $z_{max} = \frac{l}{T}$

axial stress at any depth, $\sigma = \frac{P}{A}$

axial stress at pile top, $\sigma_o = \frac{P_o}{A}$

moment of inertia, $I = A\,k^2$

where A = cross-sectional area of pile, and
k = minimum radius of gyration

Eqn. 1 can be written as (Mazindrani, 1979, 1994) :

$$\frac{d^4y}{dz^4} + U\frac{d^2y}{dz^2} + y = V \sin \pi \frac{z}{z_{max}} \tag{4}$$

in which,
$$U = \frac{\sigma}{E}\left(\frac{T}{k}\right)^2, \text{ and} \tag{5}$$

$$V = \frac{\Pi^2}{N}\frac{\sigma}{E}\left(\frac{T}{k}\right)^2 \frac{T}{z_{max}} \tag{6}$$

where
$$\frac{1}{N} = \frac{1}{400} \text{ to } \frac{1}{1000}$$

as defined in Eqn. 2. Equation 4 is solved on a personal computer with the following design criteria (Broms, 1963; Mazindrani, 1979, 1994) :

$$(\sigma_{total})_{max} = \left[\sigma + \left(\frac{M_o + M}{A\,R}\right)\right]_{max} \leq \left(\frac{1}{2} \text{ to } \frac{1}{3}\right)\sigma_{yield}, \text{ and} \tag{7}$$

$$P_{max} \leq \left(\frac{1}{2} \text{ to } \frac{1}{3}\right) P_{ult} \tag{8}$$

where σ_{total} = sum of axial and bending stress,

M_o = moment in the pile due to initial out-of-straightness,

M = moment in the pile due to applied axial load,

R = section modulus of pile divided by area of pile section,

σ_{yield} = yield strength of pile, and

$$P_{max} = k_h\, y_{max} \tag{9}$$

and

$$P_{ult} = 9\, C_u\, B \tag{10}$$

where P_{max} = maximum stress in the surrounding soil due to maximum lateral pile movement y_{max}

C_u = undrained cohesive strength, and

B = pile width

The computer output is plotted in the form of parametric relations in Figs. 2 to 13.

Discussion of Results and Conclusions

Parametric relations, slenderness parameter T/k vs. stress parameter σ_o/σ_{yield} for various combination of z_{max}, α, T/k, T and R are presented in Figs. 2 to 13 (Mazindrani, 1994). It is observed that these relations are not very sensitive to the exact value of yield strength of the pile. Hence the parametric relations can be used over wide range of yield strength values of steel usually encountered in practice (Mazindrani, 1979). An interesting result immediately catching the eye in Figs. 2 to 13 is the optimum T/k value resulting in the highest bent pile capacity. To keep the axial load capacity of the bent pile near to the yield loads, the following design criteria are suggested. For example :

For $5 \leq Z_{max} \leq 7.5$

$$\frac{T}{k} = (50 \text{ to } 60) \tag{11}$$

substituting

$$T = \sqrt[4]{\frac{EI}{k_h}}, \text{ and } \quad I = A k^2$$

Eqn. 11 can be written as :

$$\sqrt[4]{\frac{E}{k_h}} = (50 \text{ to } 60)\sqrt{\frac{k}{\sqrt{A}}} \tag{12}$$

similarly for other ranges of Z_{max}, the following design criteria are obtained.

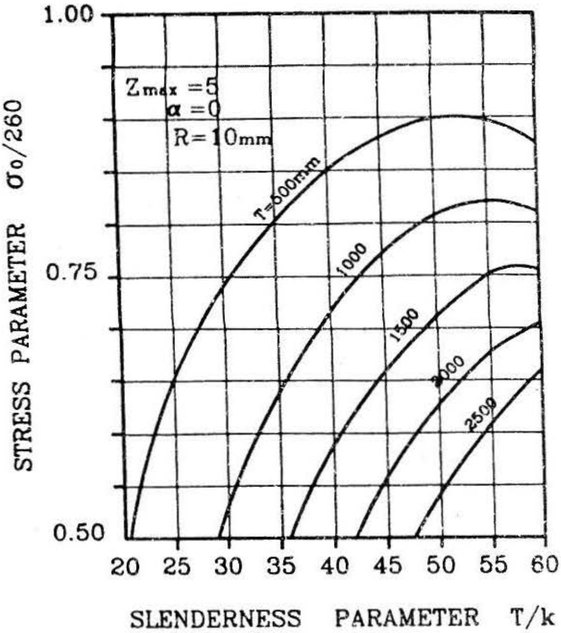

FIGURE 2 : **T/k vs.** $\sigma_o/260$ **for** $Z_{max} = 5$, $\alpha = 0$ **and R = 10 mm**

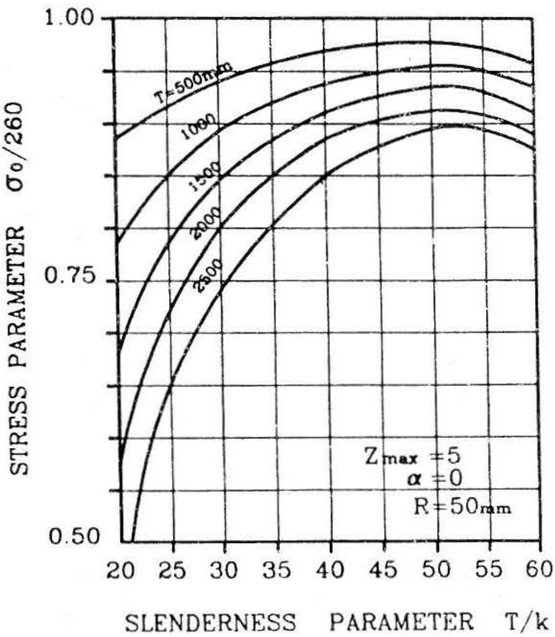

FIGURE 3 : **T/k vs.** $\sigma_o/260$ **for** $Z_{max} = 5$, $\alpha = 0$ **and R = 50 mm**

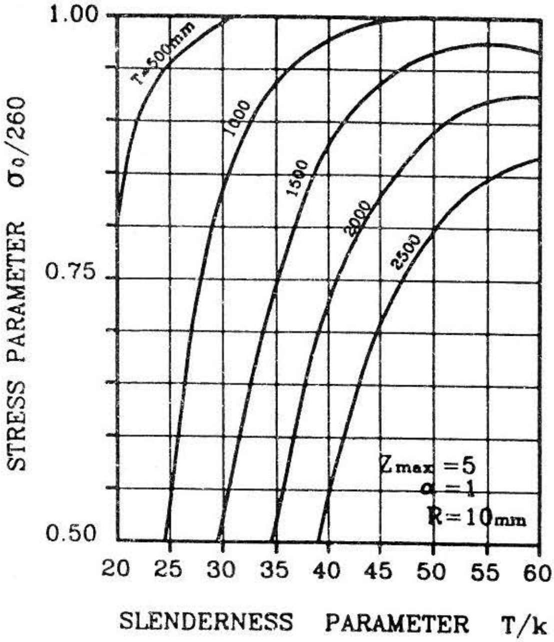

FIGURE 4 : T/k vs. $\sigma_o/260$ for Z_{max} = 5, α = 1 and R = 10 mm

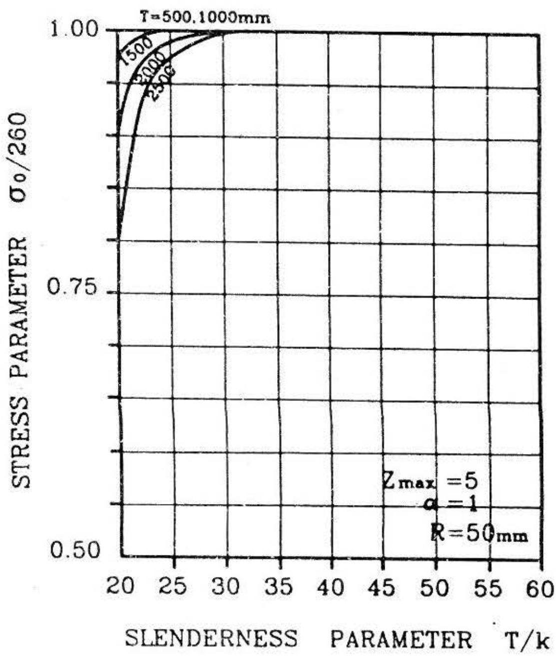

FIGURE 5 : T/k vs. $\sigma_o/260$ for Z_{max} = 5, α = 1 and R = 50 mm

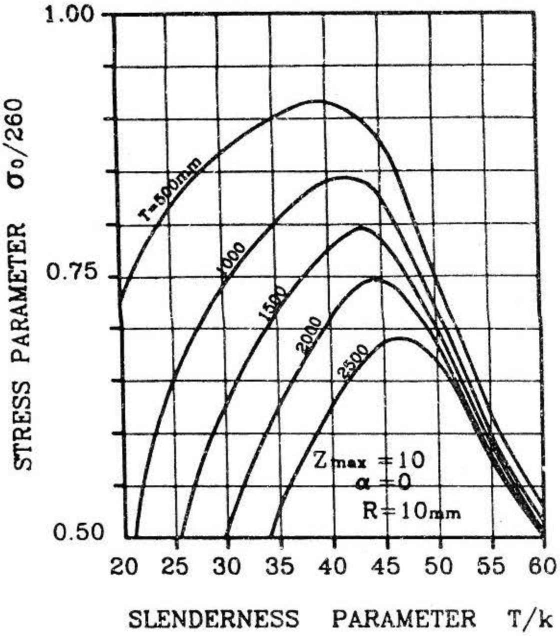

FIGURE 6 : T/k vs. $\sigma_o/260$ for Z_{max} = 10, α = 0 and R = 10 mm

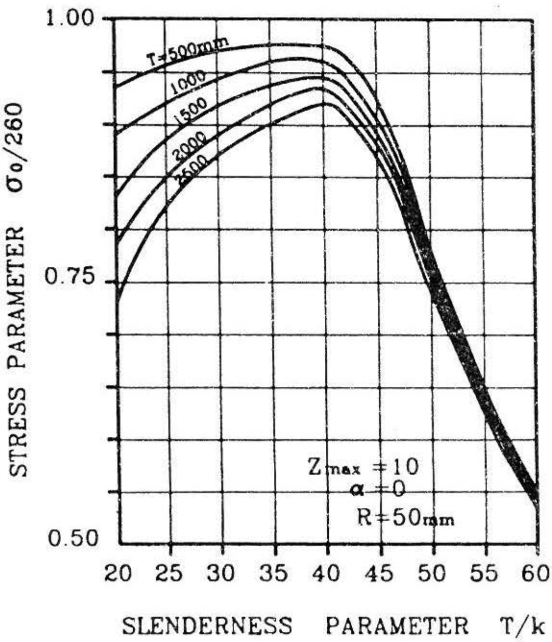

FIGURE 7 : T/k vs. $\sigma_o/260$ for Z_{max} = 10, α = 0 and R = 50 mm

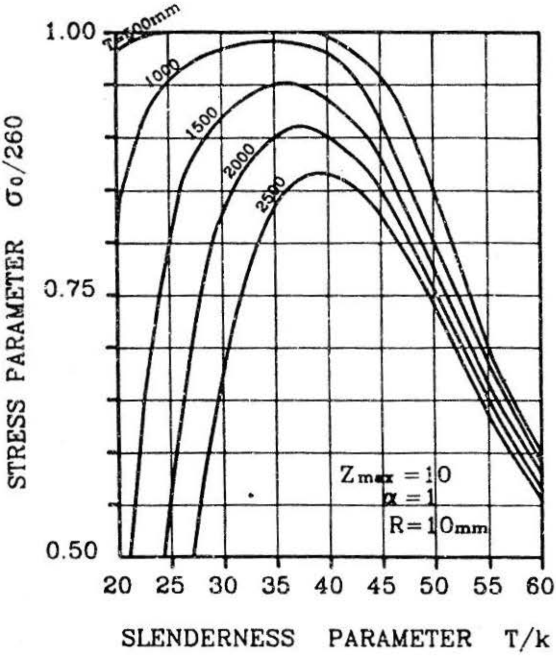

FIGURE 8 : T/k vs. $\sigma_o/260$ for Z_{max} = 10, α = 1 and R = 10 mm

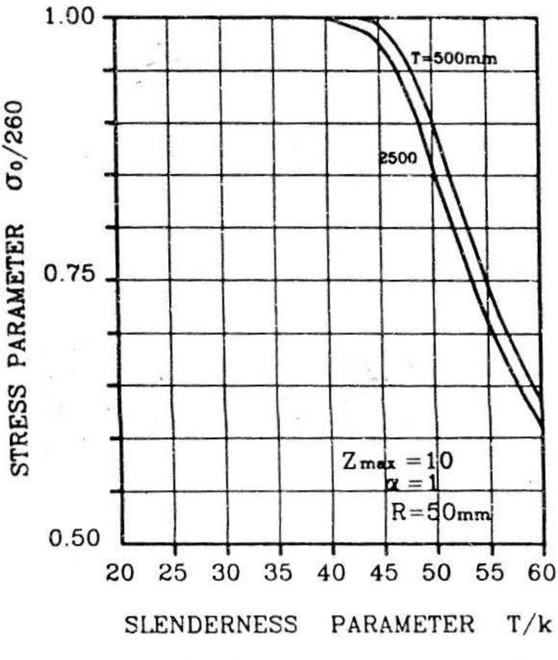

FIGURE 9 : T/k vs. $\sigma_o/260$ for Z_{max} = 10, α = 1 and R = 50 mm

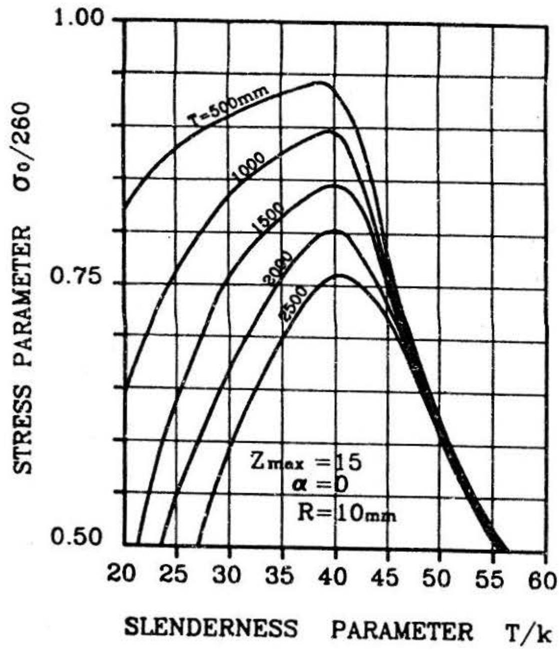

FIGURE 10 : T/k vs. $\sigma_o/260$ for Z_{max} = 15, α = 0 and R = 10 mm

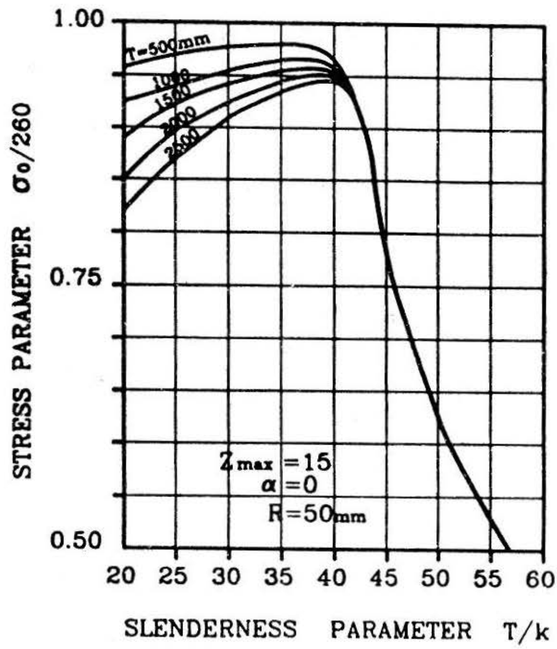

FIGURE 11 : T/k vs. $\sigma_o/260$ for Z_{max} = 15, α = 0 and R = 50 mm

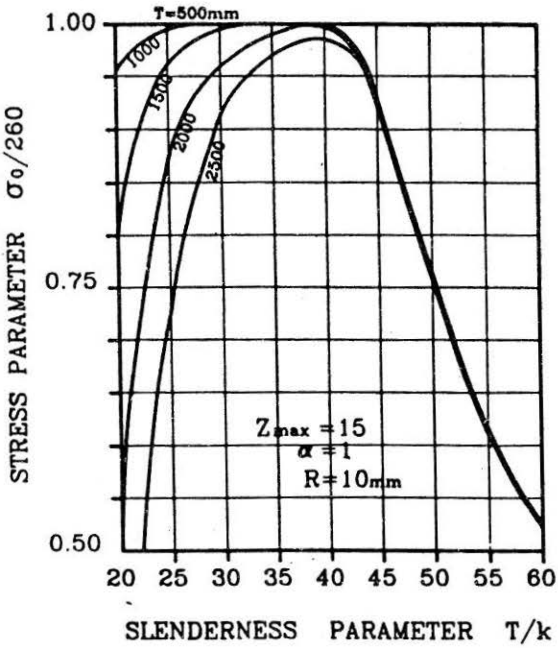

FIGURE 12 : T/k vs. $\sigma_o/260$ for Z_{max} = 15, α = 1 and R = 10 mm

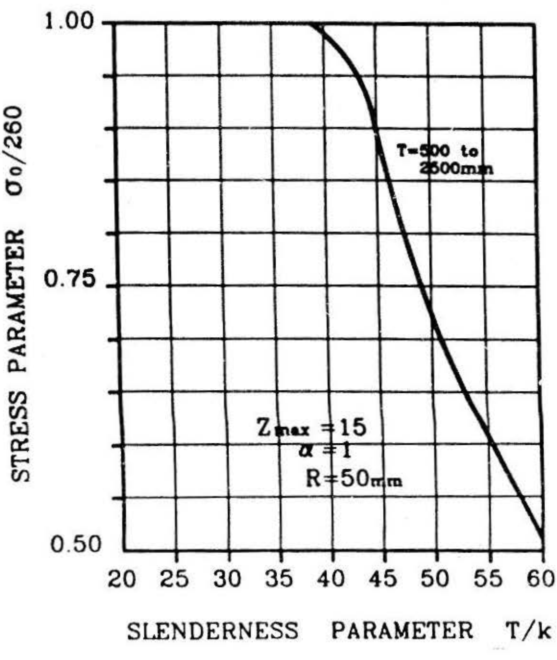

FIGURE 13 : T/k vs. $\sigma_o/260$ for Z_{max} = 15, α = 1 and R = 50 mm

For $7.5 \leq Z_{max} \leq 12.5$

$$\sqrt[4]{\frac{E}{k_h}} = (35 \text{ to } 45)\sqrt{\frac{k}{\sqrt{A}}} \tag{13}$$

For $Z_{max} \geq 12.5$

$$\sqrt[4]{\frac{E}{k_h}} = (30 \text{ to } 40)\sqrt{\frac{k}{\sqrt{A}}} \tag{14}$$

For optimal design of bent pile, using Eqns. 12 to 14, $\sqrt[4]{E/k_h}$ being constant for a given soil-pile system, pile section is chosen such that its k and A values satisfy the above relations. This ensures optimal design of pile and causes considerable saving of piling projects. The other conclusion drawn from the results are set forth.

For a given set of other parameters bent pile capacity reduces with increasing value of T. Increased relative stiffness factor T indicates pile becoming stiffer compared to soil whereby the presence of soil is felt less by the pile which conveys that the soil-pile interaction has become less effective resulting in the reduced capacity for the pile.

Bent pile capacity increases with increasing slenderness parameter T/k upto $(T/k)_{opt}$, where after pile capacity is reduced with further increase of slenderness parameter. Further $(T/k)_{opt}$ decreases with increasing Z_{max} values. These results were first established in Mazindrani (1979).

Capacity of fully shaft bearing piles ($\alpha = 1$) is observed to be larger than fully toe bearing pile ($\alpha = 0$).

Application

Brandtzaeg and Harboe (1957) conducted loading tests on some of the piles which were driven for under-pinning a church in Trondheim, Norway. The data for one of the piles is as follows :

Type of pile	= Steel H section
Edge to edge distance of flanges, d	= 112.5 mm
width of flanges, b_f	= 117.5 mm
area of pile section, A	= 4200 mm^2

moment of inertia, I_{min}	$= 187 \times 10^4\ mm^4$
pile section modulus, S	$= 31.83 \times 10^3\ mm^3$
modulus of elasticity, E	$= 218000\ N/mm^2$
pile yield strength, σ_{yield}	$= 291\ N/mm^2$
length of pile to bed rock, l	$= 35.4 \times 10^3\ mm$
test failure load	$= 1200\ kN$
horizontal soil stiffness, k_h	$= 0.837\ N/mm^2$
undrained shear strength of clay, C_u	$= 12.5\ kPa$
Yield load of the pile	$= 291 \times \frac{4200}{1000}$
	$= 1222.2\ kN$
Elastic buckling load from Granholm's (1929) formula	$= 1172.4\ kN$

From the soil and pile properties;

$$T = \sqrt[4]{\frac{E\,I}{k_h}} = \sqrt{\frac{2.18 \times 10^5 \times 187 \times 10^4}{0.837}} = 835\ mm$$

$$z_{max} = \frac{35.4 \times 10^3}{8.35 \times 10^2} = 42.4$$

$$\frac{T}{k} = \frac{8.35 \times 10^2}{21.1} = 39.6$$

$$\alpha = 0$$

$$R = \frac{31.83 \times 10^3}{4200} = 7.8\ mm$$

Figure 14 shows the effect of Z_{max} on pile capacity. It can be seen that for values of $Z_{max} > 15$, increase in pile capacity is comparatively small.

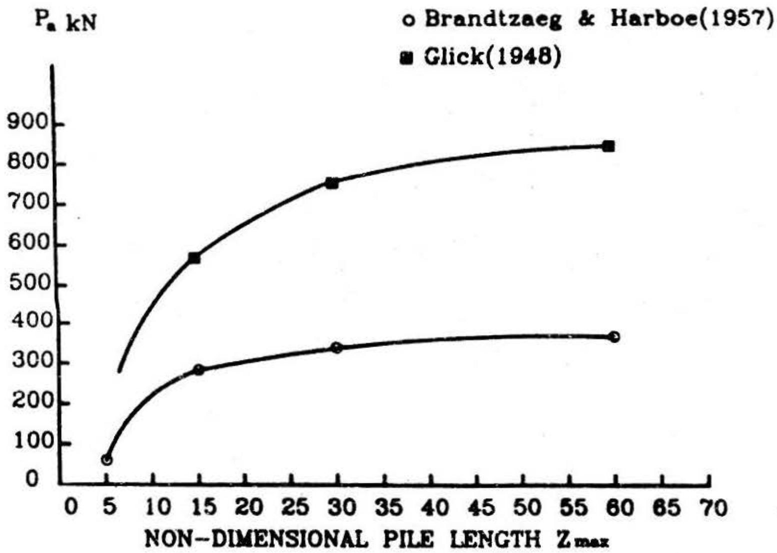

FIGURE 14 : Effect of Z_{max} on Allowable Pile Capacity P_a

Hence from Figs. 10 and 11,

we have, $$\frac{\sigma_o}{291} = 0.9$$

$$\text{the bent pile capacity} = \frac{0.9 \times 291 \times 4200}{1000} = 1100 \text{ kN}$$

with, factor of safety = 3

$$\text{allowable pile load} = \frac{1100}{3} = 366.6 \text{ kN}$$

The reason for the large bent pile capacity is that the soil pile system satisfied the optimal design criteria as shown below. From Eqn. 14,

$$\sqrt[4]{\frac{E}{k_h}} = \sqrt[4]{\frac{2.18 \times 10^5}{0.837}} = 22.59 \text{ , and}$$

$$(30 \text{ to } 40)\sqrt{\frac{k}{\sqrt{A}}} = (30 \text{ to } 40)\sqrt{\frac{21.1}{\sqrt{4200}}} = 17.1 \text{ to } 22.8$$

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