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Stability Analysis of Slopes with Non-Circular Slip Surface and Non-Vertical Slices

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Introduction

For the analysis of stability of slopes numerous methods are available, differing mainly in handling of the degree of indeterminacy of the problem, shape of slip surface and slices. Sometimes, owing to the inherent structural weak planes in the sliding mass or other reasons, it becomes essential to consider slip surface as non-circular and slice as non-vertical and non-parallel. From observations it is known that sliding, depending on the main discontinuities of the rock, may take place along polygonally shaped surface (Fig. 1). For kinematical reasons, sliding on such "external" polygonal surfaces involve failure within the sliding slope mass as well, i.e., a sufficient number of internal slip surfaces will develop (Kovari and Fritz, 1978). Taylor (1948) suggested to replace actual slip surface by circular slip surface but to use actual strength parameters for each slice. Most of the available methods cannot handle both the non-circular slip surface and non-vertical slices. Only Sarma's (1979) and Kovari and Fritz's (1978, 1984) methods can handle such problems but these are very complex especially for beginners and field engineers. Details of these methods are not given as these are used frequently and their computer programs are available (Hoek, 1987). Ramamurthy (1985) developed a variational calculus method of stability analysis and plotted charts for soil and rock slopes for static case. Here, a new method is being presented which makes use of simple equations of statics and which can handle all types of slip surfaces and all shapes of slices, and above all the proposed method is easily comprehensible. Owing to its generality this

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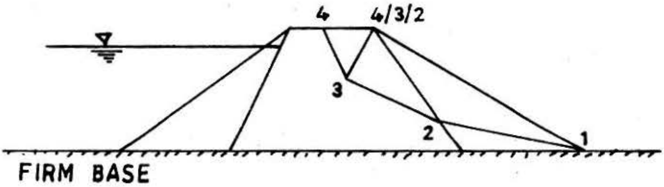
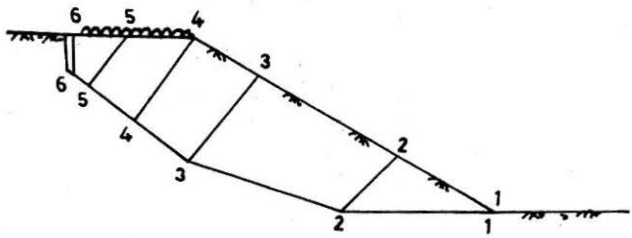
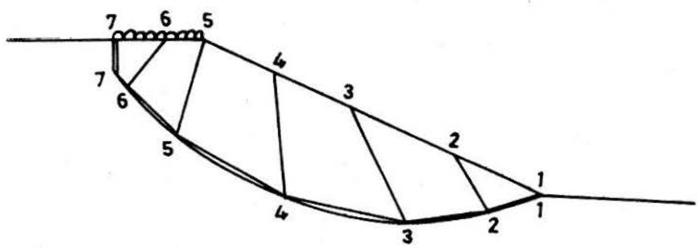


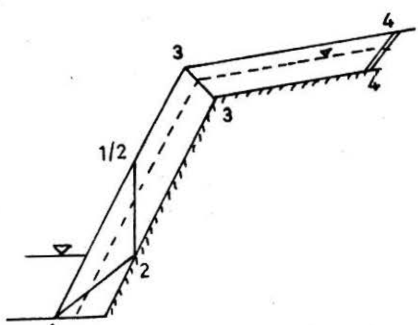
(i) NON-HOMOGENEOUS EARTH DAM WITH SLICES ALONG POTENTIAL PLANES OF WEAKNESS OR FAILURE



(ii) ROCK SLOPE WITH SLICES ALONG PREEXISTING JOINTS



(iii) CIRCULAR FAILURE WITH RADIAL SLICES TO GIVE MINIMUM FACTOR OF SAFETY IN HOMOGENEOUS SLOPES



(iv) DEBRIS SLIDE

FIGURE 1 : Applications of Proposed Method of Stability Analysis of Slope with Non-Circular Slip Surface and Non-Vertical slices

method can be used for analysing both the static and dynamic stability of non-homogenous earth dams, complex talus/debris slides, complex landslides and for almost all types of slope failures due to sliding.

It is assumed in the proposed analysis that overtoppling of the slice does not occur. It fails only in sliding.

Figure 1 shows some cases where the use of non-circular slip surface and non-vertical slice becomes inevitable.

Figure 1(i) : This is the case of non-homogenous earth dam with firm base or foundation. For the stability of d/s slope, the slip surface 1-2-3-4 should be considered. Here 2-2 is the joint between the outer body and inner core, i.e. junction of non-homogeneity and hence it is the potential plane of sliding. Slip surface 1-2-3-4 is non-circular and the slope can be analysed by considering three slices and trying various inclinations of boundary 1-2-3-4 and finally choosing the one which gives a minimum factor of safety for the system.

Figure 1(ii) : This is the case of rock slope in the rock mass with pre-existing joint sets 1-2, 2-3, 3-5 and 5-6 etc. Here 6-6 is a tension crack. As it will be discussed later that in the proposed method, the interslice boundaries are not merely fictitious lines but they are actual sliding planes. In this case the interslice movement will occur along plane 2-2, 3-3, 4-4 and 5-5. The slip surface will be 1-2-3-4-5-6 (non-circular) and there will be five slices.

Figure 1(iii) : Sarma (1979) has shown that the most critical slice side inclinations are approximately normal to the basal failure surface. Hence in the case of conventional circular slip surface, interslice boundaries should be considered approximately normal to the failure surface rather than by considering them vertical (Hoek, 1987).

Figure 1(iv) : Generally rock surface is undulating and talus or debris is also non-uniform. The slip surface is 1-2-3-4 and interslice boundaries are 2-2, 3-3 and non-vertical tension crack 4-4.

Analysis

(A) Factor of Safety and Interslice Forces :

For the sake of simplicity, here the internal slip surfaces are assumed to be plane, starting from the intersection of the external sliding planes. The potential sliding mass is divided into n slices. Thus there are total n slices and (n + 1) sides. Out of these n slices, the i^{th} slice is considered for the analysis. The various forces acting on the slice are shown in the Fig. 2.

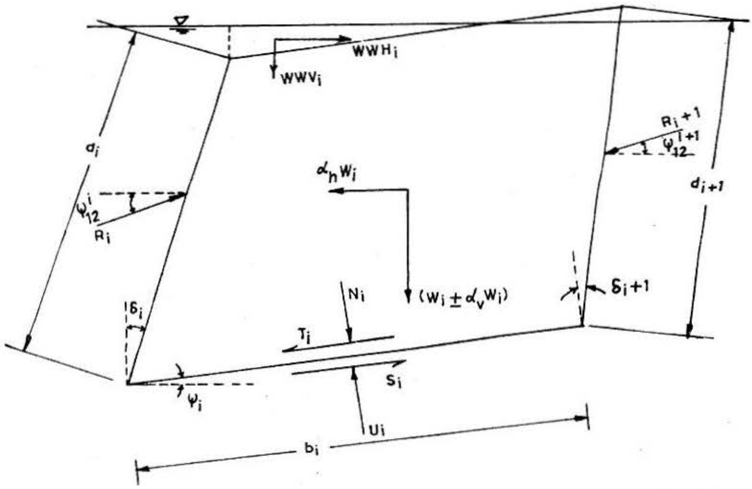


FIGURE 2 : Definition of Various Forces Acting on the i^th Slice

The following notations are used in Fig. 2 :

N_i	=	total normal force on the base of slice
R_i	=	interslice force on side i
$R_{(i+1)}$	=	interslice force on side (i + 1)
S_i	=	shear resistance on the base of slice
T_i	=	shear force on base of slice
U_i	=	total pore water pressure on the base of slice
	=	$u_i b_i$
W_i	=	weight of the i^{th} slice
WWH_i	=	horizontal force due to water above the top of slice
WWV_i	=	vertical force due to water above the top of slice
b_i	=	width of the base of slice
d_i	=	length of side i
$d_{(i+1)}$	=	length of side (i + 1)
u_i	=	mean pore water pressure on the base
α_h	=	coefficient of horizontal earthquake acceleration
α_v	=	coefficient of vertical earthquake acceleration
δ_i	=	inclination of side i from vertical
$\delta_{(i+1)}$	=	inclination of side (i + 1) from vertical

ψ_i = dip (inclination with horizontal) of the base of slice

ψ_{12}^{i} = angle of R_i, from horizontal

$\psi_{12}^{(i+1)}$ = angle of $R_{(i+1)}$, from horizontal

It may be noted that ψ_{12} refers to side of a slice and ψ refers to base of the slice. Resolving the forces along the normal to the base of slice,

$$N_i = (W_i \pm \alpha_v W_i)\cos\psi_i - \alpha_h W_i \sin\psi_i + R_i \sin(\psi_i - \psi_{12^i}) - R_{i+1}\sin(\psi_i - \psi_{12^{i+1}}) + WWV_i \cos\psi_i + WWH_i \sin\psi_i \tag{1}$$

Hence, Effective normal force on the base

$$\begin{aligned} N_i' &= N_i - U_i = N_i - u_i b_i \\ &= (W_i \pm \alpha_v W_i)\cos\psi_i - \alpha_h W_i \sin\psi_i + R_i \sin(\psi_i - \psi_{12^i}) \\ &\quad - R_{i+1}\sin(\psi_i - \psi_{12^{i+1}}) + WWV_i \cos\psi_i + WWH_i \sin\psi_i - u_i b_i \end{aligned} \tag{2}$$

Resolving the force along the base of the slice

$$T_i = (W_i \pm \alpha_v W_i)\sin\psi_i + \alpha_h W_i \cos\psi_i - R_i \cos(\psi_i - \psi_{12^i}) + R_{i+1}\cos(\psi_i - \psi_{12^{i+1}}) + WWV_i \sin\psi_i - WWH_i \cos\psi_i \tag{3}$$

$$S_i = N_i' \tan\phi_i + b_i c_i \tag{4}$$

where, c_i = *effective cohesion for the base of slice i*

ϕ_i = effective angle of shearing resistance for the base of slice i

The general notations c′ and ϕ' for effective cohesion and effective angle of shearing resistance are replaced by c and ϕ for simplicity.

Therefore, factor of safety, $F_i = \dfrac{S_i}{T_i}$ (5)

and, overall factor of safety, $FS = \sum_i^n S_i \Big/ \sum_i^n T_i$ (6)

Let $\quad Fi = FS,$

$\quad i = n, n-1, \dots\dots\dots 2, 1$

Then from Eqn. 6,

$$FS \,.\, T_i = S_i$$

making use of Eqns. 3 and 4,

$$FS\begin{bmatrix}(W_i \pm \alpha_v W_i)\sin\psi_i + \alpha_h W_i \cos\psi_i - R_i\cos(\psi_i - \psi_{12^i}) \\ + R_{i+1}\cos(\psi_i - \psi_{12^{i+1}}) + WWV_i \sin\psi_i - WWH_i\cos\psi_i\end{bmatrix}$$

$$= b_i\,c_i + \tan\phi_i\begin{bmatrix}(W_i \pm \alpha_v W_i)\cos\psi_i - \alpha_h W_i\sin\psi_i \\ + R_i\sin(\psi_i - \psi_{12^i}) - R_{i+1}\sin(\psi_i - \psi_{12^{i+1}}) \\ + WWV_i\cos\psi_i + WWH_i\sin\psi_i - u_i\,b_i\end{bmatrix}$$

If, $C_4 = \tan\phi_i \sin(\psi_i - \psi_{12^i}) + FS.\cos(\psi_i - \psi_{12^i})$

and

$$C_3 = FS\begin{bmatrix}(W_i \pm \alpha_v W_i)\sin\psi_i + \alpha_h W_i\cos\psi_i \\ + R_{i+1}\cos(\psi_i - \psi_{12^{i+1}}) + WWV_i\sin\psi_i - WWH_i\cos\psi_i\end{bmatrix}$$

$$- b_i\,c_i - \tan\phi_i\begin{bmatrix}(W_i \pm \alpha_v W_i)\cos\psi_i - \alpha_h W_i\sin\psi_i \\ - R_{i+1}\sin(\psi_i - \psi_{12^{i+1}}) - u_i\,b_i \\ + WWV_i\cos\psi_i + WWH_i\sin\psi_i\end{bmatrix}$$

Then, $R_i = \dfrac{C_3}{C_4}$ (7)

(B) Direction of Interslice Forces :

Initially all the wedges have different factors of safety F_i. Since the upper wedge has lower factor of safety, it will exert force R_i on the lower wedge. Since the limit equilibrium condition occurs at the interslice boundaries also, so the angle of interslice force with the normal to the interslice boundary is equal to ϕ_{mi}, i.e. mobilised angle of shearing resistance on side. The detail of interslice boundary is shown in Fig. 3. The components

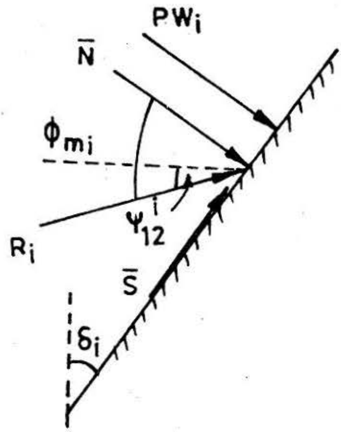


FIGURE 3 : Detail of Interslice Boundary

$\overline{N}$, $\overline{S}$ of R_i must fulfil the failure criterion. By definition the mobilised shear force on side i is given as :

$$\left| R_i \sin \phi_{mi} \right| = \frac{1}{\underline{F_i}}\left[\left(R_i \cos \phi_{mi} - P\,W_i \right) \tan \phi_{si} + c_{si}\, \underline{A_i} \right] \tag{8}$$

Dividing both sides of Eqn. 8 by $R_i \cos\phi_{mi}$, the value of ϕ_{mi} is obtained as :

$$\tan \phi_{mi} = \frac{1}{\underline{F_i}}\left[\tan \phi_{si} + \frac{\underline{c_{si}}\, \underline{A_i}}{R_i \cos \phi_{mi}} \right] \tag{9}$$

where,

$$\underline{F_i} = F_i\ \text{sign}\left(1, R_i \sin \phi_{mi}\right)$$

$$\underline{c_{si}} = c_{si} - \frac{P\,W_i}{\underline{A_i}} \tan \phi_{si}$$

PW_i = Water force on side i

ϕ_{si} = effective angle of shearing resistance on side *i*

c_{si} = effective cohesion on side i

$\underline{c_{si}}$ = mobilised cohesion on side i

F_i = factor of safety for wedge i

$\underline{F_i}$ = factor of safety on interslice boundary of wedge i

$\underline{A_i}$ = area of side i

= d_i (considering unit thickness)

Sign (a_1, a_2) = function with magnitude a_1 and sign of a_2

$\overline{S} = R_i \sin \phi_{mi}$

= tangential component of R_i

$\overline{N} = R_i \cos \phi_{mi}$

= normal component of R_i

Since, Eqn. 9 is an implicit equation, so initially is taken as zero. The Eqn. 9 then reduces to :

$$\tan \phi_{mi} = \frac{\tan \phi_{si}}{\underline{F_i}} \tag{10}$$

Using Eqn. 10 value of ϕ_{mi} is obtained and this value is substituted in the right hand side of Eqn. 9 and new value of ϕ_{mi} is obtained. The iterations on the values of ϕ_{mi} are done till the convergence occurs. Knowing the value of ϕ_{mi}, the value of ψ_{12^i} can be obtained as :

$$\psi_{12^i} = \phi_{mi} - \delta_i \tag{11}$$

(C) Procedure of Calculation :

The calculation for "Factor of Safety" and "Interslice Forces" are done in following steps.

Step 1 : Consider the overall equilibrium and calculate a value of factor of safety from Eqn. 6 assuming $R_i = 0$

Step 2 : Find the direction of interslice forces, as described in Art. B.

Step 3 : Consider the n^{th} slice at the top of the slope. Since values of R_{n+1} (water thrust in tension crack) is known. Then calculate R_n by making use of Eqn. 7.

Step 4 : Consider next $(n-1)^{th}$ slice and calculate R_{n-1} successively, thus calculate all interslice forces. Please note that $R_1 = 0$ as first side of first wedge is a free surface of slope.

Step 5 : Now calculate factor of safety of each wedge by making use of Eqn. 5.

Step 6 : Repeat the above steps (1) to (5), 2n times taking into account the values of interslice forces obtained in step (4). It is observed that in 2n cycles the convergence is achieved (sometimes even in less cycles convergence is achieved).

Step 7 : If effective normal force on the side or base of any slice is tensile, solution is termed unacceptable.

Step 8 : Try another kinematically possible slip surface to obtain lowest factor of safety.

It is to be noted that the foundation load on the slice may be taken into account by proportionately enhanced unit weight of that slice.

(D) Dynamic Settlement and its Calculation :

The assessment of the dynamic stability of slopes and embankments should be based on the dynamic displacement approach rather than the factor of safety approach. During an earthquake, the factor of safety may fall below unity several times (for very small fraction of second), but unless the dynamic displacement becomes considerable the slope should not be considered as unstable.

In the present work, use of correlation of Jansen (1990) has been made to calculate the dynamic settlement. First of all "Critical Acceleration" i.e. acceleration for unit factor of safety is calculated.. It is based on the assumption that 1/F varies linearly with α_h. Finally, dynamic displacement is computed using *Eqn. 12 as given below :*

$$S_{dyn} = 5.8\left(0.1M\right)^{8}\left(\alpha_h - \alpha_{cr}\right)/\left(\alpha_{cr}\right)^{0.5} \qquad (12)$$

where, S_{dyn} = dynamic settlement in metres

M = magnitude of design earthquake on Ritcher scale

α_{cr} = Critical coefficient of horizontal earthquake acceleration for dynamic factor of safety of 1.0

The slope is considered to be unstable if dynamic settlement exceeds 0.01 times the slope height or 1 m whichever is less.

Results and Discussion

The various equations (Eqn. 1 to 11) are required to be solved iteratively. Hence, a computer program (SANC.FOR) has been developed by Shekhawat (1993) to solve different slope stability problems. A computer program (SARMA.BAS) written by Hoek (1987), based upon Sarma's (1979) method was also available. Figure 3 shows the slope analysed. The details of slope geometry and material properties are given by Shekhawat (1993) and Hoek (1987). The results obtained by both the programs i.e. SANC.FOR and SARMA.BAS were compared and a close comparison in the results was obtained as shown in Tables 1 and 2.

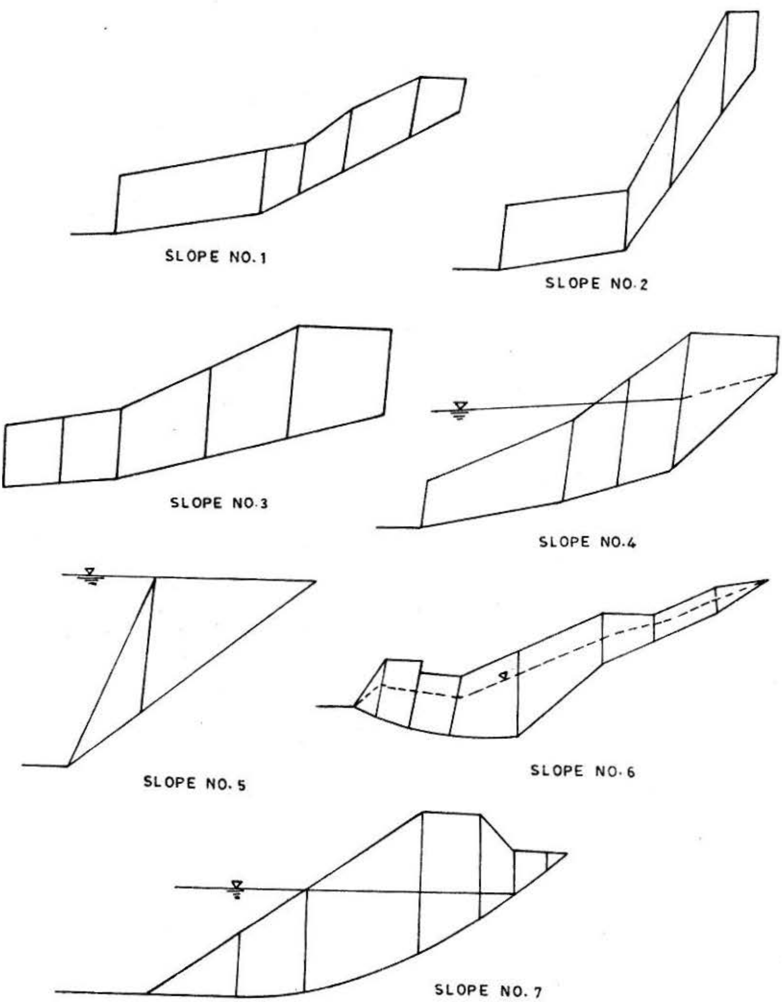


FIGURE 4 : Geometry of Slope Analysed using SANC.FOR and SARMA.BAS (after Hoek, 1987 and Shekhawat, 1993)

TABLE 1 Comparison of Results from SANC.FOR and SARMA.BAS

	Static Factor of Safety	Dynamic Factor of Safety	Critical Acceleration	Dynamic Displacement	Remark about Results
SLOPE NO. 1 : DRY, HOMOGENOUS SLOPE					
SANC.FOR	1.48	1.11	0.146	0	—
SARMA.BAS	1.50	1.12	0.1429	—	—
SLOPE NO. 2 : DRY, HOMOGENOUS SLOPE					
SANC.FOR	1.46	1.09	0.136	0	—
SARMA.BAS	1.48	1.10	0.1426	—	—
SLOPE NO. 3 : DRY, HOMOGENOUS SLOPE					
SANC.FOR	1.26	0.97	0.086	6 mm	—
SARMA.BAS	1.28	0.97	0.0922	—	—
SLOPE NO. 4 : PARTIALLY SUBMERGED, HOMOGENOUS SLOPE					
SANC.FOR	1.25	0.87	0.057	197 mm	—
SARMA.BAS	1.32	0.92	0.0741	—	—
SLOPE NO. 5A : PLANE WEDGE FAILURE (SUBMERGED SLOPE)					
SANC.FOR	1.93	1.18	0.148	0	—
SARMA.BAS	1.97	1.18	0.1445	—	Unacceptable*
SLOPE NO. 5B : SLOPE 5A WITH 50% DRAINAGE					
SANC.FOR	1.71	1.21	0.171	0	—
SARMA.BAS	1.72	1.22	0.1664	—	Unacceptable*
SLOPE NO. 5C : SLOPE 5A WITH 100% DRAINAGE					
SANC.FOR	1.60	1.22	0.193	0	—
SARMA.BAS	1.60	1.22	0.1882	—	—
SLOPE NO. 6A : PARTIALLY SATURATED SLOPE (NON-HOMOGENOUS)					
SANC.FOR	0.96	0.77	−0.015	30.853 m	Unacceptable*
SARMA.BAS	1.17	1.00	0.1008	—	—
SLOPE NO. 6B : SLOPE 6A WITH 50% DRAINAGE					
SANC.FOR	*1.20*	*0.96*	*0.080*	30.853 m	Unacceptable*
SARMA.BAS	1.41	1.15	0.2184	—	—
SLOPE NO. 6C : SLOPE 6A WITH 100% DRAINAGE					
SANC.FOR	1.43	1.16	0.180	0	—
SARMA.BAS	1.65	1.32	0.3361	—	—
SLOPE NO. 7A : PARTIALLY SATURATED SLOPE (NON-HOMOGENOUS)					
SANC.FOR	1.73	1.17	0.151	0	—
SARMA.BAS	1.91	1.32	0.2106	—	—
SLOPE NO. 7B : SLOPE 7A WITH 50% DRAINAGE					
SANC.FOR	1.85	1.36	0.234	0	—
SARMA.BAS	2.02	1.51	0.3131	—	—
SLOPE NO. 7C : SLOPE 7A WITH 100% DRAINAGE					
SANC.FOR	1.92	1.69	0.318	0	—
SARMA.BAS	2.10	1.64	0.4157	—	—

* Results are considered unacceptable when the normal stress at the base or side becomes tensile

TABLE 2 Summary of Results by Proposed and SARMA Methods

S.No.	Type of Slope	Comparison of Results
1.	*Dry Homogenous Slope*	Results from the two programs in close agreement
2.	Sublerged Homogenous	Deviation in results : "0.05 to 0.08" in factors of safety and almost negligible in critical accelerations with results of SANC.FOR on lower side. The deviation is less than 5% on safer side.
3.	Submerged Non-Homogenous	Deviation in results : "0.1 to 0.2" in factor of safety and "0.1" in critical accelerations with results of SANC.FOR on lower side. The deviation is less than 10% on safer side.

Applications

Three sites at Lal Bahadur Shastri Academy of Administration, Mussourie having steeper bedding planes in soft shale at the top terrace and flatter towards the toe of the hill were analysed using SANC program to suggest remedial measures for the stability of buildings. Computer program predicted negligible settlement. Later, no settlement was observed during Uttarkashi earthquake in 1991. The program has been used at many other sites. Unlike the program SARMA.BAS, fortunately SANC did not give the problem of convergence and unrealistic results at all the above three sites and for other slopes (Sarma, 1996). Program SANCG is developed with graphics to display the slope, slice and ground water table on monitor for better insight of the mechanics of slope failure.

Conclusion

On the basis of computations it may be concluded that the results obtained are reasonably accurate (maximum deviation is 10% for submerged non-homogenous slopes) and the method is simple and easy to understand. Hence, propossed program SANC is easy to use for analysing the non-homogeneous earth dams, talus/debris slides, planar rock slides, complex landslides and other types of failures of slopes in seismic area.

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