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Response of a Pile to Cyclic Lateral Loads Using Moment Area Method

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Introduction

For estimating the acceptable deflection at working loads in case of laterally loaded piles, modulus of subgrade reaction approach (Reese and Matlock, 1956) is commonly used. This approach treats laterally loaded pile as a beam on elastic foundation. The soil behaviour is modelled on the basis of Winkler's hypothesis, in which, the pressure, p, and deflection, y, at a point are assumed to be related through a modulus of subgrade reaction, which for horizontal loading, is denoted as K_h. Thus,

$$p = K_h * y \tag{1}$$

where, K_h has the units of force/length^3.

The analysis of pile behaviour using the subgrade reaction requires knowledge of variation of K_h along the pile length, Reese and Matlock employed finite difference scheme for the case of subgrade modulus varying linearly with depth. With reference to this case, the authors have proposed Moment Area Method of analysis, in which, no resort to finite difference equations is required. Using this method, a non-linear analysis of laterally loaded pile subjected to cyclic loading is presented.

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Proposed Method

The pile is assumed to be a thin rectangular strip of width, b (for circular pile, b represent diameter); length, L; flexibility, EI; and is divided into a specified number (n) of elements as shown in Fig. 1(a).

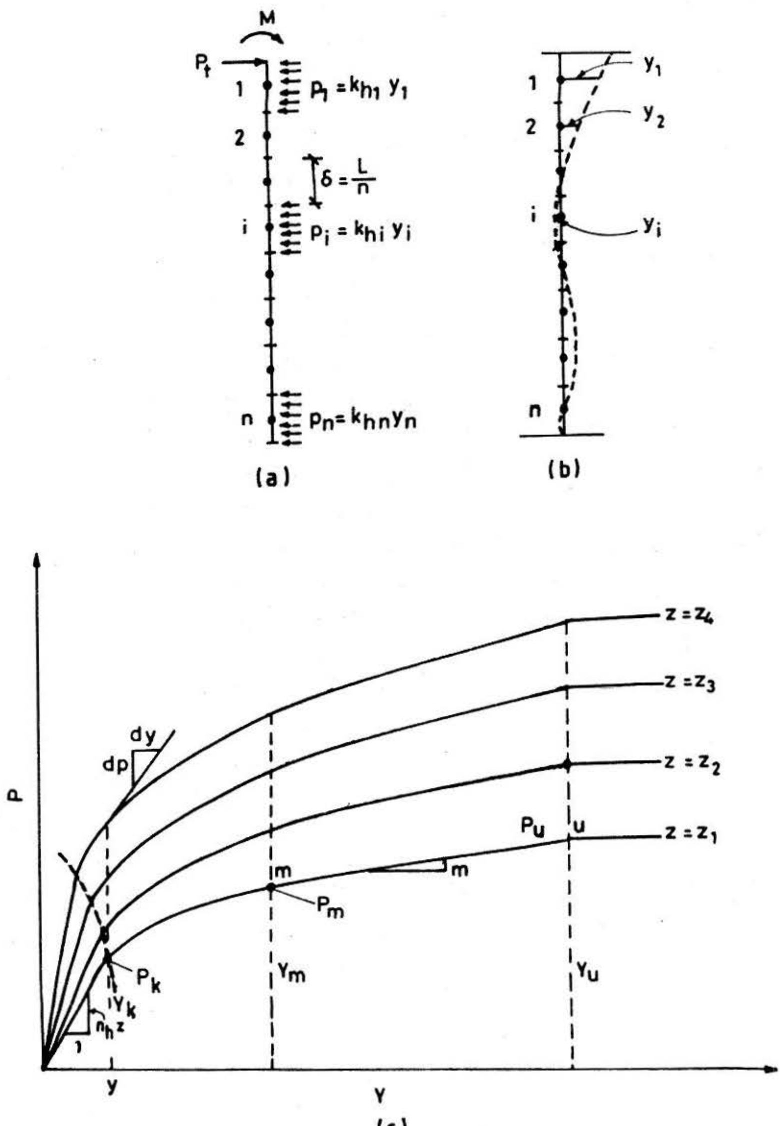

FIGURE 1 : a) Stresses Acting on Pile b) Stresses Acting on Adjacent Soil c) Typical p-y Curves

Any element, i, of the pile is acted upon by a uniform horizontal soil pressure, p_i, which is assumed constant across the pile width. Pressure, p_i, and deflection, y_i, (Fig. 1b) are related through modulus of subgrade reaction, K_{hi}, at that depth.

$$p_i = K_{hi} * y_i \tag{2}$$

In non-linear analysis, reaction between soil pressure, p, and displacement, y, is taken into account, using p-y curves developed by Reese et al. (1974) and in the matrix form, it is expressed as,

$$\{p\} = [V] * \{y_p\} \tag{3}$$

where {p} represents matrix of soil pressure, {yp} represents the matrix of pile displacement and [V] is a diagonal matrix that incorporates the subgrade modulus variation with depth (z), as defined by the following expression :

$$V(I, I) = \frac{dp}{dy} \quad \text{for deflection, y} \tag{4}$$

At y = 0,

$$V(I, I) = \text{initial tangent modulus of p-y curve} \tag{5}$$

Total load on pile is applied in increments. Pile displacements are obtained for each increment and added to previous displacements. After obtaining the displacement at a particular depth, it is entered into the p-y curve for that depth and corresponding pressure is determined. The slope of p-y curve (dp/dy) is then found out and previous value of V(I, I) is replaced by the new value of slope. This new value, V(I, I), is then used for computing the displacement at that depth for the next incremental load. The procedure is repeated for other increments.

Pile displacements are evaluated by moment are area method. Soil pressure, $b * p_i$, acting over the element, i, is shown in Fig. 2. The tangential deviation y_{tA}, of node point, i, with respect to tangent at top point, A, is given by moment of area diagram between A and node point, i, about point, i.

$$\{y_{tA}\} = \frac{b * \delta^4}{EI}[X] * \{p\} + \frac{P_t * \delta^3}{EI}\{B\} \tag{6}$$

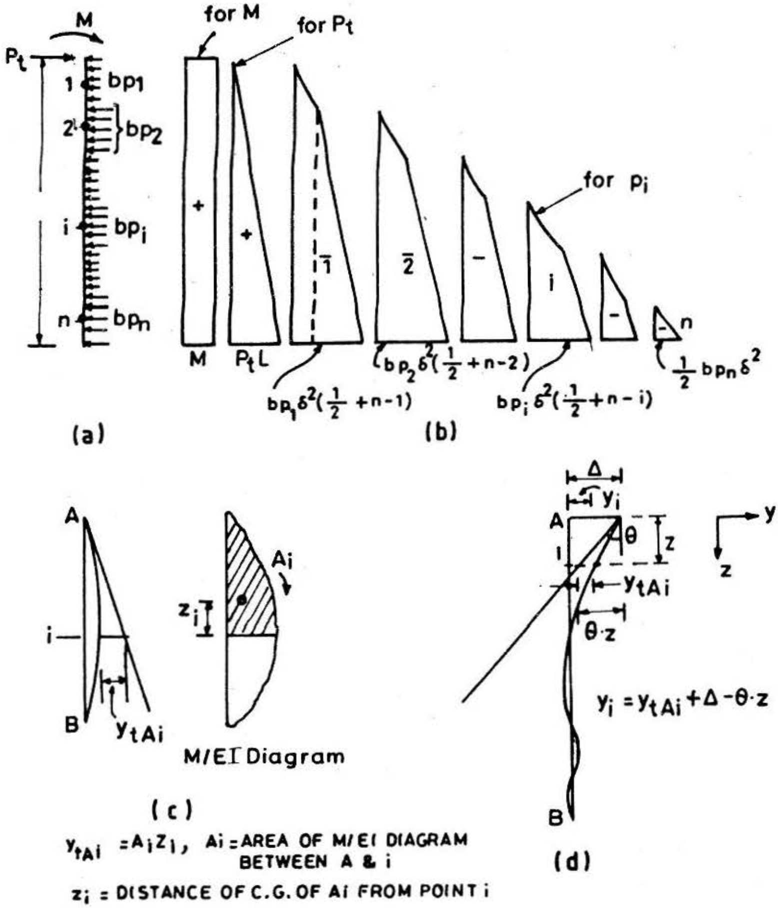

FIGURE 2 : Pile Deflection by Moment Area Method a) Load Acting on Pile b) Bending Moment Diagram by Parts c) Moment Area Concept d) Actual Pile Displacements

where, $\{y_{tA}\}$ represents tangential deviation matrix and [X] and [B] are non-dimensional coefficient matrices for deflection due to soil pressure loading, applied horizontal load (P_t) and moment (M) respectively, with their details given below.

The square matrix [X], with diagonal elements, X(I, I), is given as,

$$X(I, I) = -1/384 \qquad \ldots\ldots \ I = 1, n \tag{7}$$

Upper triangular elements of this matrix are zero and lower triangular elements are given as,

$$X(I,J) = \frac{-1}{48}\left[2+4(2I-2J-1)+3(2I-2J-1)^2+(2I-2J-1)^3\right] \tag{8}$$

where J = 1, n − 1 and I = (J + 1), n and,

$$B(I) = \frac{-1}{48}(2I-1)^3+\frac{e}{8\delta}(2I-1)^2 \;\ldots\ldots\; I = 1, n \tag{9}$$

where, e = applied moment $(M)/\text{applied load}(P_t)$

If Δ and θ are displacement and rotation of pile head in case of a free head pile, then following relations can be given for actual displacement, y_i, of node point, i (Fig. 2c)

$$\begin{aligned} y_i &= y_t A_i + \Delta - \theta * z \\ &= y_t A_i + \Delta - L\theta\left(\frac{z}{L}\right) \end{aligned} \tag{10}$$

On the basis of above equation, the matrix, $[y_p]$, of pile displacements can be expressed by the following relation :

$$\{y_p\} = \{y_{tA}\}+\Delta\{U1\}-L\theta\{UL\} \tag{11}$$

In the above relation, {U1} represents column matrix with all elements equal to unity and {UL} is also a column matrix as given below :

$$UL(I), \frac{I-0.5}{n} \qquad I = 1, n \tag{12}$$

Substituting for matrix $\{y_{tA}\}$ for Eqn. 6, in Eqn. 11, the following matrix expression is obtained :

$$\{y_p\} = \frac{b\delta^4}{EI}[X]\{p\}+\frac{P_t\delta^3}{EI}\{B\}+\Delta\{UI\}-L\theta\{UL\} \tag{13}$$

After rearranging the above equation, the following expression is obtained :

$$\left[[I]-\frac{b\delta^4}{EI}[X][V]\right]\{y_p\}-\Delta\{U1\}+L\theta\{UL\} = \frac{P_t\delta^3}{EI}\{B\} \tag{14}$$

in which, $[I]$ represents a unit matrix.

The above relationship provides n equations for (n + 2) unknowns, i.e. y_1 to y_n, Δ and θ. Additional two equations are furnished by equilibrium conditions.

Horizontal equilibrium

$$p_1 + p_2 + p_3 + \cdots\cdots + p_n = [K_h]\{y_p\} = \frac{P_t}{b\delta} \tag{15}$$

where $[K_h]$ is a row matrix for modulus of subgrade reaction as given by following expression :

$$K_h(I) = V(I, I) \tag{16}$$

Substitution of Eqn. 16 into Eqn. 15, gives the following expression :

$$[V] * \{y_p\} = \frac{P_t}{b\delta} \tag{17}$$

Moment equilibrium

$$(n-0.5)p_1 + (n-1.5)p_2 + \cdots\cdots + 0.5 * p_n = \frac{P_t}{b\delta}\left(n + \frac{e}{\delta}\right) \tag{18}$$

Above equation can be finally transformed into the following matrix equation :

$$[K_M] * \{y_p\} = \frac{P_t}{b\delta}\left(n + \frac{e}{\delta}\right) \tag{19}$$

where, $[K_M]$ is a row matrix as given below :

$$K_M(I) = (n - I + 0.5) * V(I, I) \tag{20}$$

Equations 14, 17 and 19 are solved simultaneously to get unknowns y_1 to y_n, Δ and θ, for each increment of load.

After calculation of displacements, pressures, Δ and θ, the moments can subsequently be evaluated.

Cyclic Loading

The prediction of response of piles subjected to cyclic lateral loading is one of the main problems encountered in the analysis and design of offshore structures. The response of pile to cyclic lateral loading depends on the change in the stiffness of soil and pile with applied cycles of load. The changes in the soil and pile stiffness are influenced by the magnitude of load, character of load, initial stiffness of the pile and soil, and the change in the stiffness of pile and soil with number of cycles.

Because of the high non-linearity of the stress-strain soil behaviour, the lateral pile response is also non-linear. Therefore, non-linear p-y curves for static loading are used to account this non-linearity and with modification for cyclic loading.

The soil is assumed to be insensitive to creep, and pile material is assumed to remain elastic. The pile properties and the magnitude of maximum lateral load are assumed to remain unchanged during cyclic loading. Therefore the increase in pile displacement is affected only by a change in the coefficient of soil reaction.

Cyclic lateral loads applied to the head of pile result in deformations that may increase with each cycle. The increase in the pile head deformation with continued cyclic loading may be modelled by reducing the coefficient of soil reaction. The effect of cyclic loading on reduction of soil reaction coefficient (n_h), in respect of cohesionless soil is taken as (Long and Venneste, 1994) :

$$R_H = \frac{n_{hN}}{n_{h1}} = N^{-t} \quad \text{i.e. } n_{hN} = n_{h1} * N^{-t} \tag{21}$$

where n_{h1} and n_{hN} are the coefficients of soil reaction for first and N^{th} cycles of load respectively, t is the degradation parameter and, R_H is cyclic load ratio.

The specific value of t, for a cyclic laterally loaded pile depends on the magnitude of cyclic load ratio, R_H, the installation method, soil density, and if the pile has been precycled. The effects of load direction, installation procedure and soil density are expressed by Long and Venneste, 1994) as :

$$t = 0.17 * F_L * F_I * F_D \tag{22}$$

where F_L, F_I, F_D are the factors based upon details of cyclic load ratio, pile installation, and soil density respectively and their values are reported by Long and Venneste.

Since static p-y curves are non-linear, additional recommendations for the reduction in soil resistance, p, must be provided, or, any combination of decrease in soil resistance, p, and increase in pile deflection, y, could be used to satisfy above equation.

For modifying non-linear p-y curves, decrease in soil reaction modulus is effected by reduction in the soil resistance, p, and an increase in pile deflection, y, and it is done as follows :

$$p_N = p_1 * N^{(\alpha-1)t}$$

and

$$y_N = y_1 * N^{\alpha t} \tag{23}$$

where p_N represents soil resistance for N cycles of load and p_1 represents soil resistance for first cycle of load. Similarly, y_N and y_1 represents displacements for Nth and first cycles respectively.

The factor, α, controls relative contribution of soil resistance and deflection towards decreasing the soil reaction modulus. It varies from 0 (change in p only) to 1 (change in y only). A value of 0.6, for α, is used in the analysis as suggested by Long and Venneste, according to which, the above equation now becomes :

$$p_N = p_1 * N^{-0.4t} \tag{24}$$

and

$$y_N = y_1 * N^{-0.6t} \tag{25}$$

The static p-y curve is thus modified by reducing soil resistance, p, and increasing the pile deflection, y. For computations of displacements and pressures, the whole procedure remains same as that for static loading except that, modified p-y curve is used to obtain the pressure and slope dp/dy is replaced accordingly, and n_h is replaced as per Eqn. 21. After calculation of displacements, pressures, Δ and θ, the moments can subsequently be evaluated.

Results

Reese, Cox and Koop (1974) have reported results of tests conducted on 610 mm (24 in.) diameter steel tube pile with wall thickness of about

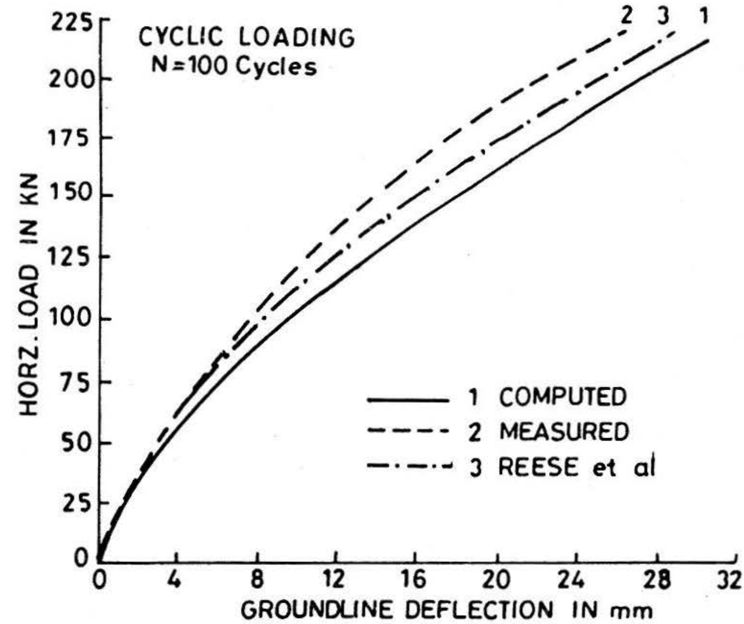

FIGURE 3 : Comparison of Results : Groundline Deflection

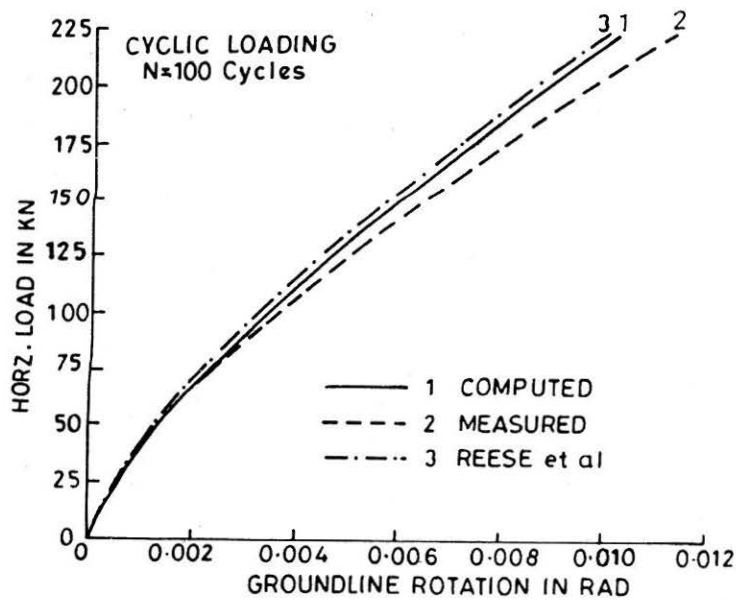

FIGURE 4 : Comparison of Results : Pile Head Rotation

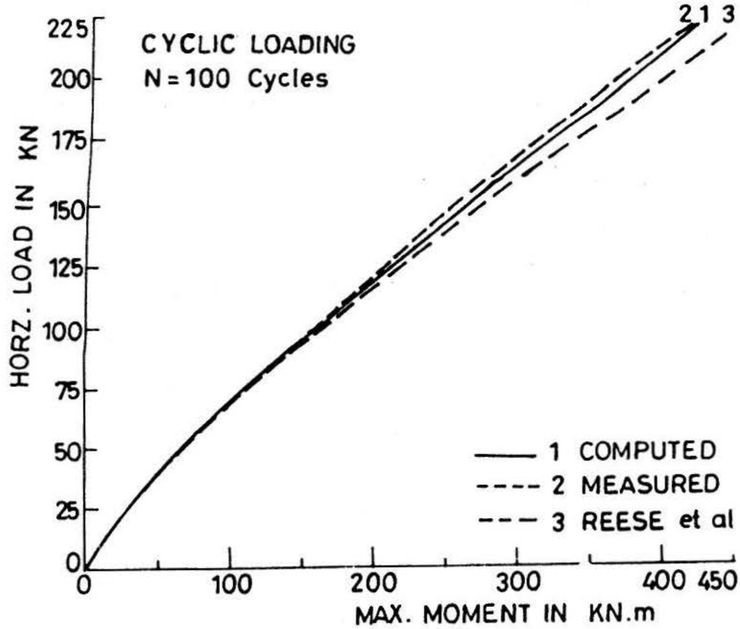

FIGURE 5 : Comparison of Results : Maximum Bending Moment

from clean fine sand to silty fine sand, both having high relative densities. The angle of internal friction, ϕ, was determined to be 39° and submerged unit weight, γ', was 10.37 kN/m^3 (66 lbs/cft). A value of 34013.75 kN/m^3 (125 lb/in^3) was chosen for n_h, on the basis of recommendations made by Terzaghi. For cyclic loading, lateral load was applied such that cyclic load ratio, R_H, became equal to −0.25 corresponding to a maximum load upto 244.651 kN (55 kips) after 100 cycles. Groundline deflection, pile head rotation and maximum moment were computed from measured data.

To facilitate the comparison of results obtained using the proposed method of analysis, with the results of above field tests, p-y curves similar to those shown in Fig. 1(c) were generated for cyclic loading using the above data. The analysis was carried out using these p-y curves and the comparisons are shown in Figs. 3, 4 and 5 respectively for groundline deflection, pile head rotation and maximum bending moment. In the same figures, comparisons are made with the analytical results reported by Reese et al. (1974), in which, linear variation of modulus of subgrade reaction with depth is assumed. It is observed that in respect of groundline deflection analytical results reported by Reese et al. show close agreement with measured field data and in respect of maximum moment and pile head rotation, the proposed method shows better agreement with field data.

It may be noted that in the proposed method, no assumptions are made regarding variation of modulus of subgrade reaction and exact path of p-y curve is followed using latest developments (Long and Venneste, 1994).

Conclusions

A method based primarily on matrix formulation is suggested for the evaluation of soil response to cyclic lateral loads, in case of a pile. The non-linear analysis follows actual path on p-y curve and thus ensures better accuracy. The cyclic loading behaviour is modelled by considering the cyclic load ratio, pile installation procedure and soil density along with the number of cycles.

References

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Notations

b	=	pile diameter
e	=	load eccentricity
N	=	number of cycles of load
Δ	=	groundline displacement of pile
δ	=	length of pile element (L/n)
γ'	=	submerged unit weight of soil
ϕ	=	angle of internal friction
θ	=	pile head rotation