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Stiffness and Damping Coefficients of Pile Foundations for Compressor Foundation Design

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Introduction

Novak (1974, 1977, 1979), and Novek and Sharnouby (1983) have given the expressions for single pile undergoing vertical motion. In case of pile groups, there is an Interaction effect amongst the piles which causes a reduction in the pile stiffness and damping. The effect was first expressed in the form of 'interaction factors' by Poulos (1968, 1971) for groups of statically loaded piles which may also be applied to pile groups undergoing steady state vibrations. Later, experiments by Novak and Grigg (1976) and Novak and Sheta (1982) have revealed that the static interaction factors may not be applicable universally and may lead to significant errors in some cases. However, in the case of floating piles the error in using the static interaction factors is negligible (Sharnouby and Novak, 1985). The theory for finding the stiffness and damping of pile groups was revised by Sheta and Novak (1982) for dynamic vibratory loading and a factor called group efficiency ratio (GER) is defined to express the true dynamic pile group stiffness and damping.

The solutions for pure horizontal motion of vertical piles follow the same logic as that for vertical motion (Novak, 1974; Novak and Sharnouby, 1983; and Sharnouby and Novak, 1985). However, the value of shear modulus of soil would be much smaller since the soils very near the surface control the load-deformation properties of the laterally loaded piles. For pile groups undergoing horizontal motion, use of interaction factors for group action proposed by Novak and Sharnouby (1983, 1985) is suggested.

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Both the vertical and lateral motions are most influenced by the dynamic shear modulus of the soil mass, which in turn is significantly affected by the confining pressure and the strain level (Drnevich, Hall and Richart, 1969). The range of strain levels associated with different in-situ and laboratory tests are given by Ishihara (1971). For these and other reasons, the shear modulus of soil determined from various tests shows considerable difference.

Nogami and Novak (1976) have also derived expressions for the stiffness and damping of the pile-soil system which are frequency dependent but these become nearly independent of the frequency of vibration over a wide range of practical frequencies.

Problem Definition

The work presented in this paper is based on the field problem of suggesting dynamic stiffness and damping coefficients of pile foundations for high speed centrifugal compressor units in a gas processing plant. Natural frequencies of vibration of single piles were obtained from field tests on prototype piles under both vertical and lateral excitations. These frequencies were subsequently used to compute stiffness and damping coefficients for single piles in both the modes. The stiffness and damping coefficients so obtained were further modified to yield similar coefficients for pile groups.

Problem Data

Cast-in-situ bored R.C.C. piles, 400 mm in diameter and 16.5 m long have been used, with the bottom of the pile-cap at 2.0 m below the ground surface. The piles have been driven through layers of 1.5 m loose backfill and 4.5 m thick silty clay of high compressibility (CH as per Unified Soil Classification System) followed by 11.0 m thick medium to dense sand below which lies 6.0 m thick hard silty clay. The depth of water table varied from 3.5 m to 4.0 m below the natural ground surface. The sub-soil data at 2.0 m depth is presented in Table 1. The machine and foundation data is given in table.

General Analytical Approach

The stiffness and damping coefficients have been evaluated by two methods. The analysis is carried out first for single piles and subsequently extended to pile groups.

In-Situ Resonance Test

In the resonance tests, the piles were vibrated by using a mechanical oscillator and the amplitude of vibration plotted against the frequency of vibration. The stiffness of the pile-soil system has been worked out from the measured resonant frequency and the computed effective mass. Corrections for confining pressure and the strain level were applied to obtain the design value of single pile stiffness in both the horizontal and vertical directions. Finally, the stiffnesses of pile groups of different compressor units have been obtained using reduction factors for group action given by Major (1980) for the vertical motion.

TABLE 1
Sub-Soil Data

S.No.	Test	Property	Value obtained
1.	Core cutter	Field density, γ_s	18.8 kN/m^2
2.	—	Natural water content	23.67 %
3.	Classification	Liquid limit	56.66 %
		Plastic limit	25.43 %
		Classification	CH
4.	Unconfined compression	Unconfined compressive strength, q_u	0.11 N/mm^2
5.	Triaxial (undrained)	Poisson's ratio, υ_s	0.40
6.	Wave propagation	Shear wave velocity, V_s	85.5 m/sec

TABLE 2
Machine and Foundation Data

Machine	Speed (rpm)	Pile group	Pile cap size		Machine weight (kN)	Total dead weight	Design Amplitude (peak-peak)
			Plan (m × m)	*Thickness* (m)		kN/pile	(micron)
I	9400	4 × 5*	4 × 7	1.20	340	160	8
II	11990	4 × 5	4 × 7	1.20	340	160	8
III	25000	3 × 3*	3 × 6	0.80	110	50	4
IV	20000	3 × 3	3 × 6	0.80	110	50	4

* Pile arrangement shown in Fig. 1.

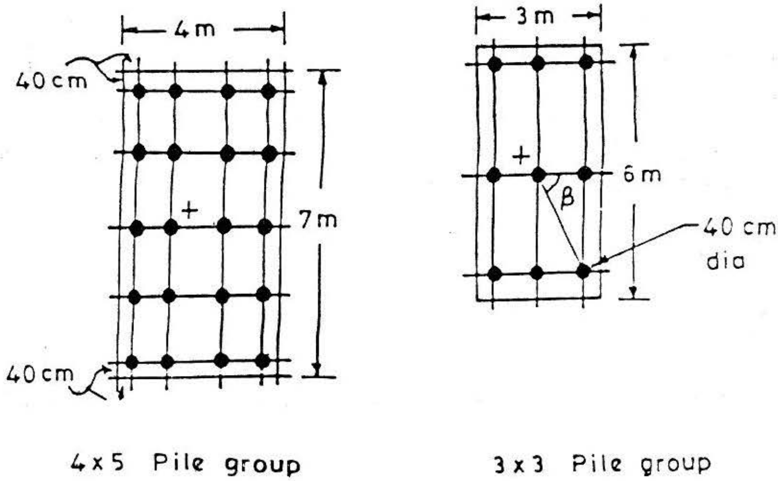


FIGURE 1 : Pile Arrangement for Compressor Foundations

Novak's Solution

Initially single pile stiffness and damping coefficients have been worked out for both vertical and horizontal motions by using expressions presented by Novak and El Sharnouby (1983). The stiffness and damping factors in these expressions depend upon the ratio of the Young's modulus of pile to the shear modulus of soil, the Poisson's ratio of soil and the pile slenderness ratio. The shear wave velocity through soil mass was measured in the field from wave propagation tests, from which the shear modulus of soil was computed. It was then corrected for the effects of confining pressure and strain level. Finally, the stiffness and damping coefficients of pile grous of different compressor units have been determined using the interaction factors given by Poulos (1968) for vertical motion and by Sharnouby and Novak (1985) for lateral motion.

The stiffness and damping contributed by the pile cap as given by Beredugo and Novak (1972) for horizontal motion and Novak and Beredugo (1972) for vertical motion, have also been considered.

Field Tests

Free and forced vibration tests were conducted on two prototype piles. Also, wave propagation tests were carried out on the soil strata which helped establish the shear modulus of the soil. The free vibration tests yielded the natural frequency of vibration of the piles in the horizontal direction. Finally steady state forced vibration tests were carried out to establish the resonant frequencies in both horizontal and vertical directions.

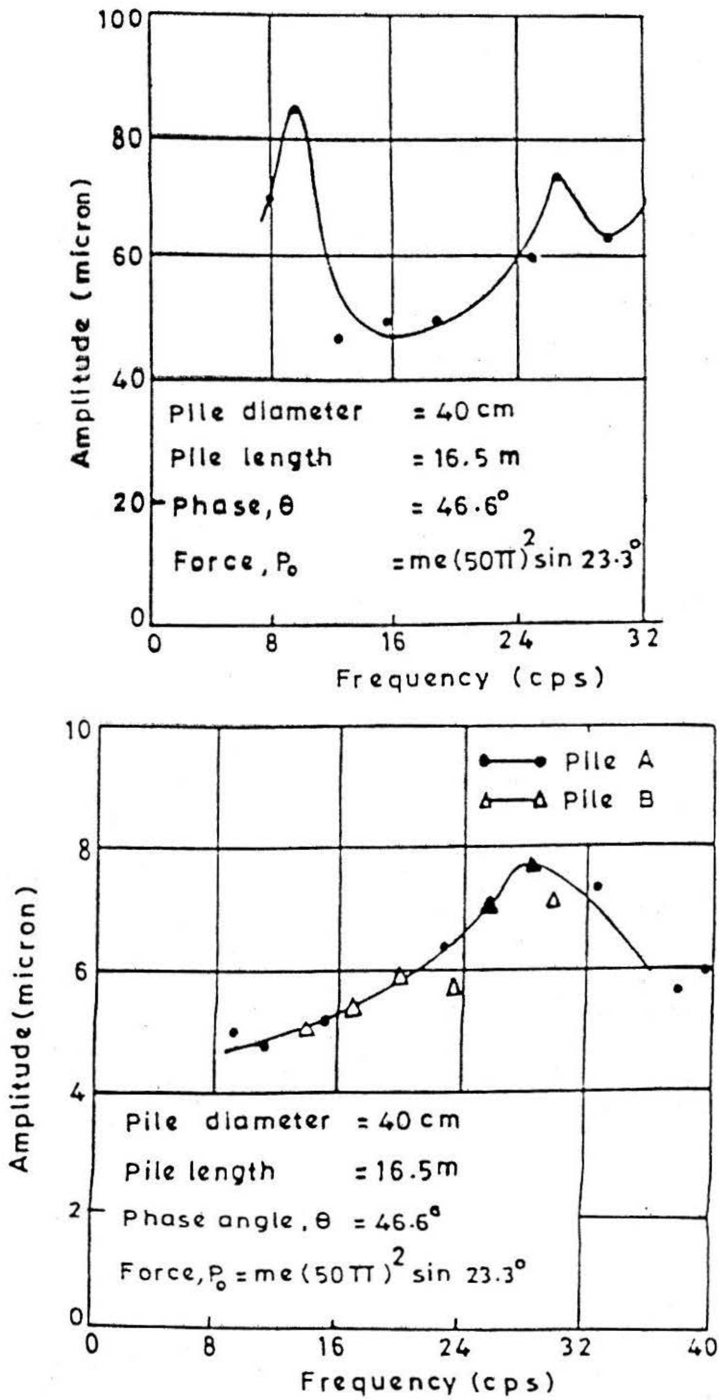


FIGURE 2 : Forced Vibration Tests on Single Pile
(a) Horizontal, (b) Vertical

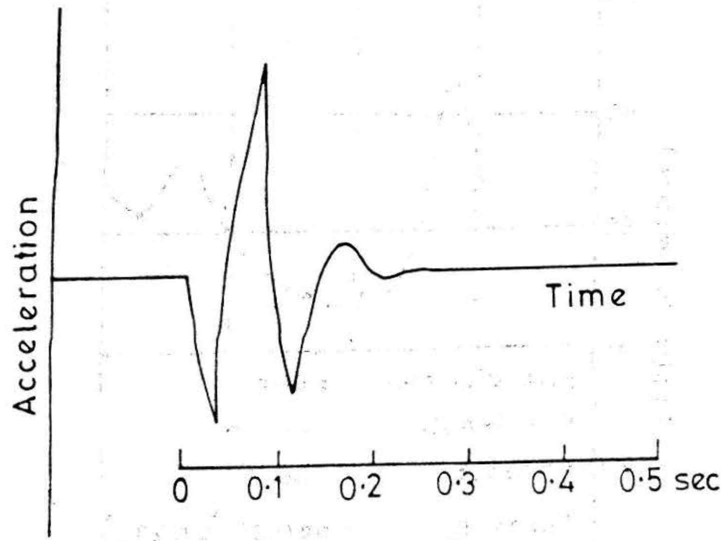


FIGURE 2(c) : Record of Free Vibration Test on Single Pile in Horizontal Mode

Wave Propagation Test (Hammer Test)

Using the standard hammer test data as per IS : 5249-1977, the velocity of shear wave propagation through the soil was obtained in the field at the pile cap bottom level as 85.5 m/s and has been included in Table 1.

Foreced Vibration Tests (Resonance Tests)

A pile cap was cast on top of each prototype pile constructed at site for the purpose of resonance tests. The pile cap was 0.7 m × 0.7 m × 0.5 m in size and is required for mounting the motor-oscillator assembly on the top of pile. Acceleration records were obtained on a direct ink writing pen recorder with linear response upto 70 cps. Both the vertical and horizontal vibration tests were conducted on the prototype cast-in-situ driven piles.

A typical resonance curve for the forced vibration test in horizontal direction is shown in Fig. 2(a), which shows a resonant peak at 10 cps. Also a second smaller peak is seen at a frequency of about 27 cps, which coincides with the vertical resonant frequency of 27.5 cps seen in Fig. 2b. However, the first natural frequency has only been considered in the analysis for pile stiffness since the higher modes of vibration are known to contribute very little to the vibration amplitudes.

It is also been from Figs. 2(a and b) that the vertical vibration amplitudes are much smaller than the horizontal ones, which is due to much higher vertical stiffness of pile, and limited power of the mechanical oscillator used during the tests. Failure of the mechanical system further limited the horizontal tests to one pile only. Therefore data of one pile alone is plotted in Fig. 2(a).

Free Vibration Tests

A single pile was excited in free vibrations in the horizontal direction by applying a lateral load through a 6 t capacity clutch capable of suddenly releasing the load. The reaction was taken from an adjacent pile through a 10 t chain-pulley block. The record of free vibration is shown in Fig. 2(c) from which the value of natural frequency of vibration os the pile has been obtained as 10.0 cps.

Vertical Stiffness and Damping Coefficients of Single Pile

Detailed computations of stiffness and damping coefficients for single piles as well as pile groups for the vertical and horizontal motions are presented below only for the foundations of machines I and II (4×5 pile group) only, whereas for the machines III and IV (3×3 pile group), the values are directly presented in various tables.

Novak's Solution

Stiffness Coefficient

Expression for the effective vertical stiffness for a single friction pile undergoing vertical vibrations is given by Eqn(1)(Novak and Sharnouby, 1983).

$$K_{v1} = \left(E_p \cdot A/r\right) \cdot f_{v1} \qquad (1)$$

where E_p = Young's modulus of pile material
$= 25 \times 10^6$ kN/m^2
A = Cross-sectional area of pile
r = radius of pile
f_{v1} = stiffness factor (Fig. 5)
= function of l/r and E_p/G_s

The shear modulus, G_s, which is initially computed from shear wave velocity obtained from wave propagation test, needs to be corrected for confining pressure as well as strain level. These corrections have been made and the computations are presented in Appendix-I. The corrected value of shear modulus is thus obtained as 23,900 kN/m^2. The ratios;

$$E_p/G_s = 25 \times 10^6 / 23900 = 1046$$

and pile slenderness,

$$l/r = 16.5 / 0.2 = 82.5$$

The corresponding value of stiffness factor (Fig. 5, Novak and Sharnouby, 1983),

$$f_{v1} = 0.0279$$

So that vertical stiffness of single pile (Eqn. 1),

$$K_{v1} = [25 \times 10^6 \times \pi \times (0.2)^2 / 0.2] \times 0.0279 = 438{,}250 \text{ kN/m}$$

Damping Coefficient

The expression for effective geometric damping coefficient for vertical motion in a single pile is given by Novak and Sharnouby (1983) as

$$C_{v1} = \left(\frac{E_p \cdot A}{v_s} \right) \cdot f_{v2} \tag{2}$$

where f_{v2} = damping factor

For $E_p/G_s = 1046$

and $l/r = 82.5$,

$$f_{v2} = 0.0515 \text{ (Fig.5)}$$

Therefore, $$C_{v1} = \left\{ \frac{25 \times 10^6 \times \pi \times (0.2)^2}{112} \right\} \times 0.0515 = 1{,}445 \text{ kN/m}$$

Corresponding value of damping ratio,

$$\zeta = \frac{C_{v1}}{2\sqrt{K_{v1} \cdot m_c'}} \tag{3}$$

where m'_c = mass of pile cap along with the mass of superstructure which would vibrate in phase with the cap (Table 2)

$= 160 / 9.8$

$= 16.3 \text{ kN-s}^2 / \text{m}$

Therefore, damping ratio,

$$\zeta = \frac{1445}{2\sqrt{438250 \times 16.3}}$$

$$= 0.270$$

Resonance Test

Vertical Pile Stiffness, as Obtained from Test

Weight of pile cap $= 0.7 \times 0.7 \times 0.5 \times 24$

$= 6.0$ kN

Weight of motor and oscillator $= 1.0$ kN

Assuming that weight of half the pile (which is considered to include the effective mass of the surrounding soil vibrating with the pile) would participate in vertical vibrations and lumpsed at the top of pile.

Effective weight of pile $= \pi \times (0.2)^2 \times (16.5 / 2) \times 24$

$= 24.9$ kN

Total effective lumped weight on pile, *W* $= 6.0 + 1.0 + 24.9$

$= 31.9$ kN

Measured natural frequency of vibration, fnv obtained from field tests (Fig.2b) $= 27.5$ cps

Therefore, vertical stiffness of single pile, Kv1 $= (2\pi)^2 \cdot (f_{nv})^2 \cdot W/g$

$= 4 \times \pi^2 \times (27.5)^2 \times 31.9/9.8$

$= 97.180$ kN/m

The level of strain reached during resonance test and that during the operating conditions of compressor are quite different. The vertical pile stiffness K_{v1}, therefore, also needs correction for strain level.

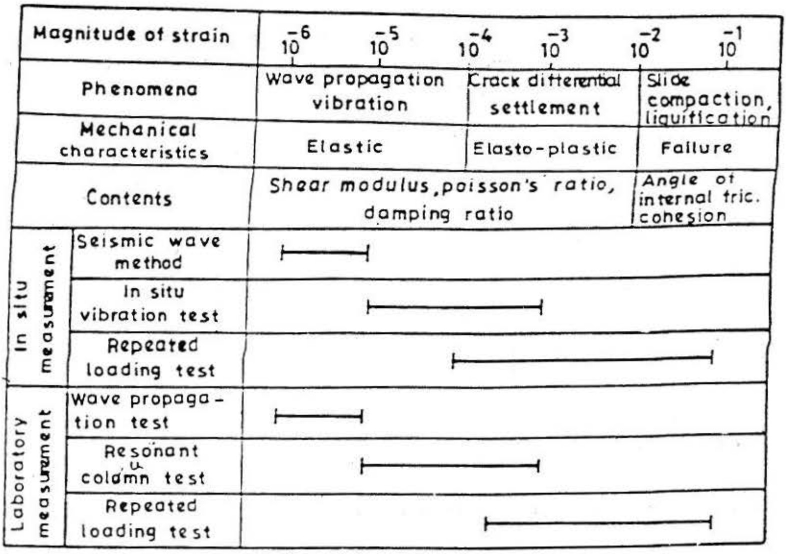


FIGURE 3 : Strain Levels Associated wih Different Tests (Ishihara, 1971)

Correction to Measured Stiffness of Pile

Strain level in resonance tests (Fig. 3) is reported to lie in the range of 10^{-5} to 10^{-3}. For the present case, it is expected that the strains will lie in the range 10^{-4} to 10^{-3} so that average strain level would be 5×10^{-4}.

Correction factor for G_s for an average strain level of 5×10^{-4} (Fig. 4)	$= 0.50$
Hence corrected value of shear modulus for resonance test	$= 27150 \times 0.50$
	$= 13{,}575$ kN/m^2
Ratio, E_p/G_s	$= 25 \times (10)^6/1357$
	$= 1{,}842$
Pile slenderness in resonance test	$= 82.5$
Corresponding value of stiffness factor (Fig. 5), f_{v1}	$= 0.0200$
Therefore, the stiffness ratio for the two strain levels	$= 0.0279/0.0200$
	$= 1.395.$

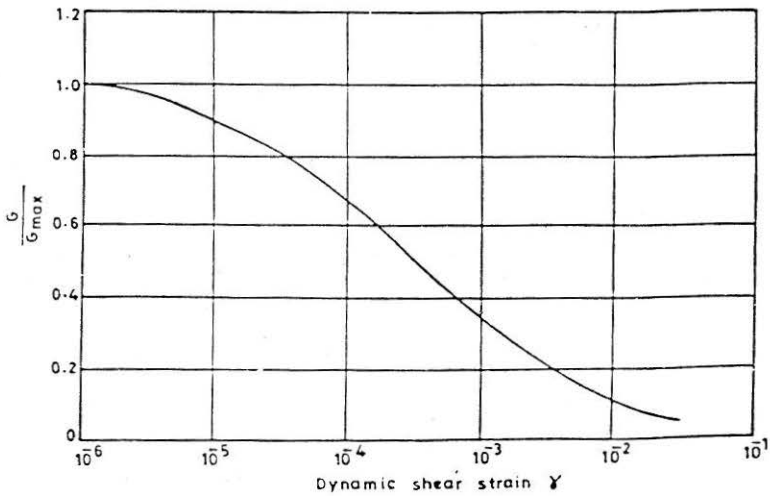


FIGURE 4 : Normalised Shear Modulus Vs. Shear Strain (Prakash, 1981)

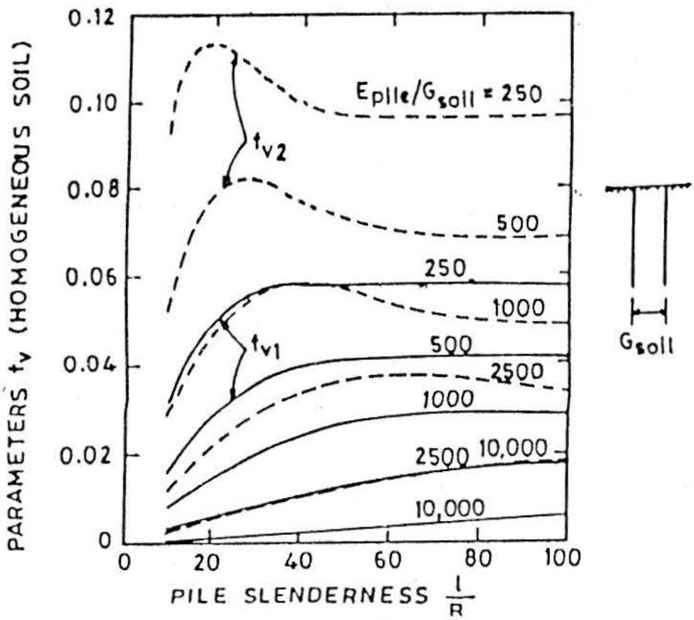


FIGURE 5 : Stiffness and Damping Parameters of Verticle Response for Relaxed-Tip Concrete Pile

Hence the vertical stiffness of single pile corresponding to the strain level during operating conditions from resonance test data,

$$K_{v1} = 1.395 \times 97180$$
$$= 135,570 \text{ kN/m}$$

Similar computations have also been made for the pile from the other pile group. The vertical stiffness and damping coefficients for single piles in two different pile groups (Fig. 1) are summerised in Table 3.

Vertical Stiffness and Damping Coefficients for Group Piles

It is rare that the piles are installed singly. These are invariably in groups. Further, it is well established that stiffness and damping of pile groups are not, in general, a simple sum of stiffnesses or damping of individual piles.

Novak's Solution

Vertical Stiffness of Pile Group without Pile Cap

The vertical stiffness of the pile group excluding the pile cap stiffness is given by Poulos (1968) as

$$K_{vg} = \sum_1^N K_{v1} \Big/ \sum_1^N \alpha_A \tag{4}$$

where, N = no. of piles in group

α_A = vertical displacement interaction factor for a typical reference pile (Fig. 1) in the group relative to itself and to all other piles in the group

If S is spacing between the piles and r the pile radius, then

$$\alpha_A = f(S/r, \ l/2r \ \text{and} \ \nu_s)$$

TABLE 3
Vertical Stiffness and Damping Coefficients of Single Pile

Pile group	Vibration Mode	Single Pile			
		Resonance Test		Novak's Solution	
		K_{v1} (kN/m)	ζ_1 (%)	K_{v1} (kN/m)	ζ_1 (%)
4 × 5 (m/cs I & II)	Vertical	135570	-	438250	27.0
3 × 3 (m/cs III & IV)	Vertical	133800	-	447700	48.4

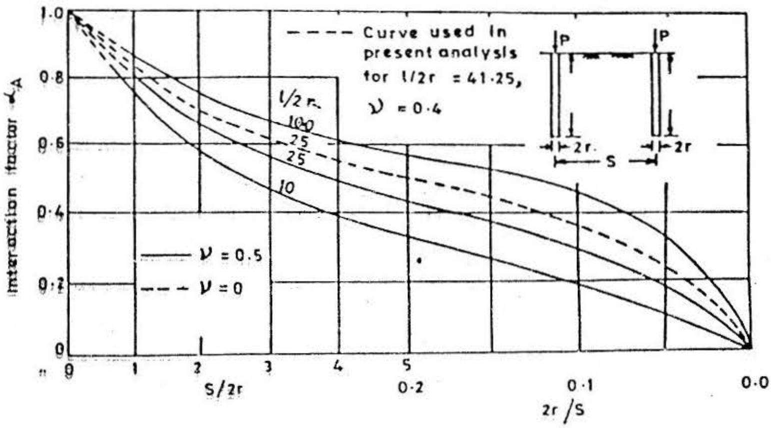


FIGURE 6 : Vertical Interation Factors for Group Action of Piles (Poulos, 1968)

Figure 6 (Poulos, 1968) gives variation of interaction factor, α_A with $S/2r$ for different values of $l/2r$. The value of α_A for the 4×5 pile group has been found to be 10.20. The details of computations of α_A are presented in Appendix-III.

Therefore, vertical stiffness of pile group without pile cap,

$$K_{vg} = \frac{20 \times 438250}{10.20}$$
$$= 859{,}300 \text{ kN/m}$$

Combined Geometric Damping of Pile Group without Pile Cap

The combined geometric damping of pile group without the contribution from pile cap is given by

$$C_{vg} = \sum_{1}^{N} C_{vi} \Big/ \sum \alpha_A \tag{5}$$

Therefore equivalent damping ratio of pile group without pile cap,

$$\zeta_{vg} = C_{vg} \Big/ 2\left(K_{vg} \times m'_{cg}\right)^{0.5} \tag{6}$$

where $\quad m'_{cg} = m'_c \times$ No. of piles in the group

$$\zeta_{vg} = 2833 \Big/ \left[2(859300 \times 16.3 \times 20)^{0.5}\right]$$
$$= 0.085$$

Pile Cap Stiffness (Novak and Beredugo, 1972)

The pile cap stiffness,

$$K_{vf} = G_s \times h \times \overline{S}_l \tag{7}$$

where, $\overline{S}_l$ = a constant for embedded pile cap (Table 4), and
h = pile cap thickness
= 1.2 m

The shear modulus, G_s of soil mass is to be corrected for the confining pressure and also for strain level. These computations are presented in Appendix-IV. The corrected value of G_s is thus 10320 kN/m.

Therefore,

pile cap stiffness, K_{vf} (Eqn. 7) = 10320 × 1.2 × 2.7
= 33,440 kN/m

Pile Cap Damping (Novak and Beredugo, 1972)

The contribution to damping due to the pile cap is given by

$$C_{vf} = h \cdot r \left(\frac{G_s \cdot \gamma_s}{g}\right)^{0.5} \cdot \overline{S}_2 \tag{8}$$

TABLE 4
Frequency Independent Constants for Embedded Pile Cap with Side Resistance

υ_s	$\overline{S}_1$*	$\overline{S}_2$*	$\overline{S}_{u1}$†	$\overline{S}_{u2}$†
0.0	2.7	6.7	3.6	8.2
0.25	2.7	6.7	4.0	9.1
0.4	2.7	6.7	4.1	10.6

* Novak and Beredugo (1972)
† Beredugo and Novak (1972)

where, $\bar{S}_2$ = 6.7 for υ_s equal to 0.4 (Table 4), and

r = equivalent radius of pile cap

$= (4 \times 7/\pi)^{0.5}$

$= 2.985$ m

Therefore, $$C_{vf} = 1.2 \times 2.985 \times \left(\frac{10320 \times 18.8}{9.8}\right)^{0.5} \times 6.7$$

$$= 3{,}380 \text{ kN-s/m}$$

Combined Stiffness of Pile Group

Combined stiffness of pile group including stiffness of pile cap,

$$\bar{K}_{vg} = K_{vg} + K_{vf}$$
$$= 859{,}300 + 33{,}440$$
$$= 892{,}740 \text{ kN/m}$$

Combined Damping of Pile Group

Combined geometric damping of pile group including that of pile cap,

$$\bar{C}_{vg} = 2{,}833 + 3{,}380$$
$$= 6{,}213 \text{ kN-s/m}$$

Damping Ratio

Damping ratio of the pile group including damping of pile cap,

$$\zeta_{vg} = \frac{\bar{C}_{vg}}{2\left(\bar{K}_{vg} \cdot m'_{cg}\right)^{0.5}} \tag{9}$$

$$= \frac{6213}{2(892740 \times 16.3 \times 20)^{0.5}}$$

$$= 0.182$$

Resonance Test

When in group, the vertical stiffness of single pile in a pile group is obtained by using a reduction factor suggested by Major (1980) and presented in Table 5.

For unequal pile spacing in two perpendicular directions in a (4 × 5) pile group (Fig. 1) :

Pile spacing in shorter direction, t = 1.0 m
Pile spacing in other direction, t = 1.55 m
Ratio, t/d in shorter direction = 2.50
Ratio, t/d in other direction = 3.875

An average value of the modified reduction factor, μ = 0.35 corresponding to an average value of t/d ratio of 3.0 (Table 5), is adopted in the subsequent analysis.

Hence, corrected single pile stiffness = $0.35 \times K_{vl}$, and
pile group stiffness, K_{vg} = $0.35 \times 135570 \times 20$
= 948,990 kN/m.

The vertical stiffness and damping coefficients for the two pile groups (Fig. 1) are summarised in Table 6.

TABLE 5
Pile Group Reduction Factors

Ratio Spacing, t / diameter, d	Modified reduction factor μ (Major, 1980)
—	1.00
6.0	0.63
4.5	0.58
3.0	0.35

TABLE 6
Vertical Stiffness and Damping Coefficients of Group Piles

Pile group	Vibration Mode	Single Pile			
		Resonance Test		Novak's Solution	
		K_{vg} (kN/m)	ζ_{vg} (%)	K_{vg} (kN/m)	ζ_{vg} (%)
4 × 5	Vertical	—	—	892740	18.2
		948990	—	859300*	8.5*
3 × 3	Vertical	—	—	840820	43.5
		602100	—	805860*	21.7*

* Excluding pile cap contributions

Horizontal Stiffness and Damping Coefficients of Single Pile

As in the case of vertical motion, the stiffness and damping coefficients for single piles and the various pile groups have been computed using Novak's solution and also utilising resonance test data.

Novak's Solution

Horizontal Stiffness of Single Pile

The stiffness of a single pile during horizontal vibrations is given by Novak and Sharnouby (1983) as

$$K_{ul} = \left(E_p \cdot I_p / r^3\right) f_{ul} \quad \text{for } l/r \geq 25 \tag{10}$$

where, I_p = moment of inertia of pile cross-section
$= \left(\pi r^4/4\right)$
$= 0.00125 \text{ m}^4$

and f_{ul} = stiffness factor in horizontal direction
$= f\left(l/r,\ \upsilon_s,\ E_p/G_s\right)$

Also, $\upsilon_s = 0.4$

and $l/r = 82.5$

The values of f_{ul} (Novak and Sharnouby, 1983) are presented in Table 7 for different values of the ratio, E_p/G_s and for both the free and fixed head piles.

TABLE 7
Values of f_{ul}, f_{u2} for $l/r > 25$ and $\upsilon_s = 0.40$

E_p/E_s	Fixed-head piles		Free-head piles	
	f_{ul}	f_{u2}	f_{ul}	f_{u2}
10,000	0.0047	0.0119	0.0024	0.0060
2,500	0.0132	0.0329	0.0068	0.0171
1,000	0.0261	0.0641	0.0136	0.0339
500	0.0463	0.1054	0.0231	0.0570

Note : Values reproduced in El Sharnouby and Novak (1985)

Corrections to be applied to the shear modulus for confining pressure and strain level are presented in Appendix-V, where the corrected value of G_s has been obtained as 17,420 kN/m. Thus, ratio $E_p / G_s = 1435$ for which the stiffness factor f_{ul} (Table 7) = 0.020.

Therefore, horizontal stiffness of single pile (Eqn. 10)

$$K_{u1} = \frac{25 \times 10^6 \times 0.00125}{(0.20)^3} \times 0.0200$$

$$= 78,125 \text{ kN/m}$$

Damping in Horizontal Mode for a Single Pile (fixed-head)

The damping in the horizontal mode for a single pile is given by (Eqn. 10),

$$C_{u1} = \left(\frac{E_p \cdot I_p}{r^2 \cdot v_s}\right) \cdot f_{u2} \tag{11}$$

where f_{u2} = damping factor presented in Table 7.

$f_{u2} = 0.0490$ for $l / r \geq 25$,

$\upsilon_s = 0.4$, and

$E_p / G_s = 1435$

Therefore, $$C_{u1} = \frac{25 \times 10^6 \times 0.00125}{(0.2)^2 \times 95.0} \times 0.0490$$

$$= 403 \text{ kN-s/m}$$

Damping ratio, ζ_{u1} $$= \frac{C_{u1}}{2\sqrt{K_{u1} \cdot m_c'}}$$

$$= \frac{403}{2\sqrt{78125 \times 16.3}}$$

$$= 0.179$$

Stiffness ratio, K_r $$= \frac{E_p \cdot I_p}{E_s \cdot L^4}$$

Therefore, $$= \frac{25 \times 10^6 \times 0.00125}{2 \times \{17420(1+0.40)\}(16.5)^4}$$

$$= 8.6 \times 10^{-6} \sim 10^{-5}$$

Resonance Test

Average lumped weight of pile may be computed assuming that length of pile equal to ten pile diameters participates in horizontal vibration, since the pile deflected shape and hence the pile-to-pile interaction facators remain constant beyond this limiting pile length (Sharnouby and Novak, 1985).

Weight of pile lumped at pile top, $= (1/2)\times 10\times 0.4\times \pi\times (0.2)^2\times 24$

$= 6.0$ kN

Weight of pile cap $= 6.0$ kN

Weight of motor and oscillator $= 1.0$ kN

Total lumped weight $= 6.0 + 6.0 + 1.0$

$= 13.0$ kN

Measured resonant frequency (Fig. 2a) $= 10.0$ cps

The horizontal stiffness of single free-head pile, K_{u1} $= 4\pi^2 \cdot \frac{W}{g} \cdot f_{nu}^2$

$$= 4\times \pi^2 \times \frac{13.0}{9.8}\times (10)^2$$

$= 5{,}240$ kN/m

The pile stiffness needs to be corrected for strain level and pile head fixity condition. This correction is shown in Appendix-VI and the corrected value of K_{u1} is obtained as 16,280 kN/m.

The horizontal stiffness and damping coefficients for single piles in the two pile groups (Table 2) are summarised in Table 8.

TABLE 8
Horizontal Stiffness and Damping Coefficients of Single Piles

Pile group	Vibration Mode and Head-Fixity	Single Pile			
		Resonance Test		Novak's Solution	
		K_{u1} (kN/m)	ζ_{u1} (%)	K_{u1} (kN/m)	ζ_{u1} (%)
4 × 5	Free Head	8140	—	40430	13.1
	Fixed Head	16280	—	78125	17.9
3 × 3	Free Head	8320	—	41820	23.2
	Fixed Head	16640	—	80590	31.9

Horizontal Stiffness and Damping Coefficients for Group Piles

Novak's Solution

Pile group stiffness excluding pile cap stiffness

The horizontal stiffness of pile group excluding the pile cap stiffness is given by Novak and Sharnouby (1985) as,

$$K_{ug} = \sum_{1}^{N} K_{u1} \Big/ \sum_{1}^{N} \alpha_L \tag{15}$$

where, N = number of piles in the group,

α_L = pile interaction factor in horizontal motion
= $f(\upsilon_s,\ l/r,\ S/2r,\ \beta)$ (where β is the angle made by line joining any pile and the reference pile with the direction of load)

Figure 7 (Novak and Sharnouby, 1985) gives variation of α_L with S/2r for two extreme values of β (0° and 90°). The computations for interaction factor, α_L for the group are presented in Appendix-VII wherein its value has been found to be 5.04 . Therefore, horizontal stiffness of pile group (without pile cap),

$$K_{ug} = 20 \times 78125 / 5.04$$
$$= 310{,}020 \text{ kN/m}$$

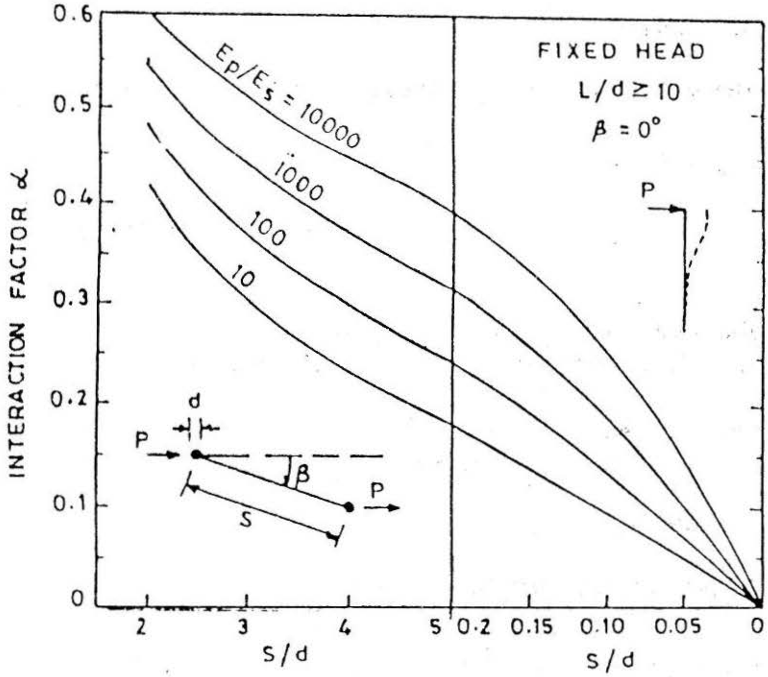
Pile Group Damping Without Pile Cap

Combined pile group damping without pile cap is given by :

$$\text{Cug} = \sum_{1}^{N} C_{u1} \Big/ \sum_{1}^{N} \alpha_L \tag{16}$$
$$= 20 \times 403 / 5.04$$
$$= 1{,}600 \text{ kN/m}$$

$$\text{Damping factor, } \zeta_{ug} = \frac{C_{ug}}{2\sqrt{K_{ug} \cdot m'_{cg}}} \tag{17}$$

$$= 1600 \Big/ \left[2\sqrt{310020. \times 16.3 \times 20}\right]$$
$$= 0.080$$



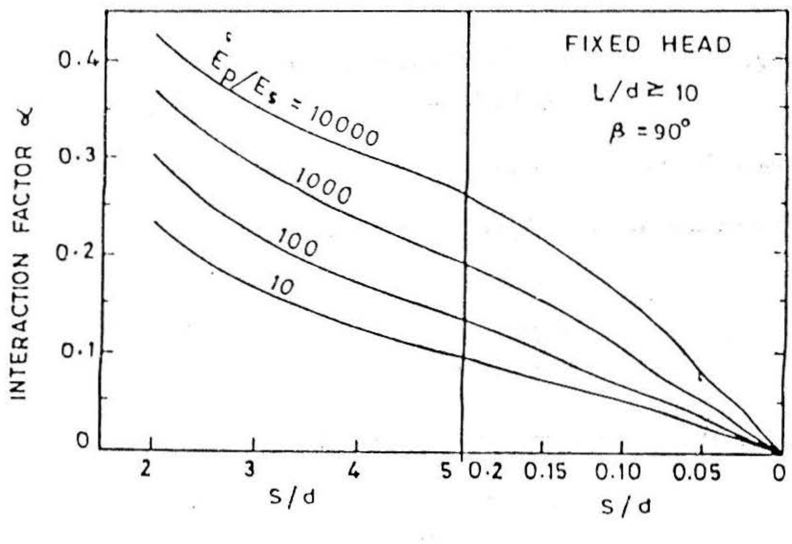


FIGURE 7 : Horizontal Interation Factors for Fixed-Head Piles (Novak and Sharnouby, 1985)

Pile Cap Stiffness

The stiffness of pile cap (Beredugo and Novak, 1972) is given by

$$K_{ug} = G_s \cdot h \cdot S_{u1} \tag{18}$$

where S_{u1} = a constant for embedded pile cap (Table 4) = 4.1

Soil shear modulus after strain level correction = 10,320 kN/m^2

Therefore, $K_{uf} = 10320 \times 1.2 \times 4.1 = 50{,}770$ kN/m

Pile Cap Damping

As per Beredugo and Novak (1972)

$$C_{uf} = \left(h \cdot r\sqrt{G_s \cdot \gamma_s/g}\right) \cdot \bar{S}_{u2} \tag{19}$$

where $\bar{S}_{u2}$ = a constant for embedded pile cap (Table 4) = 10.6

Therefore, $= 1.2 \times \left\{2.985\sqrt{(10320 \times 18.8/9.8)}\right\} \times 10.6 = 5{,}342$ kN-s/m

Combined Pile Group Stiffness

Combined stiffness of pile group and pile cap,

$$\bar{K}_{ug} = K_{ug} + K_{uf} = 310{,}020 + 50{,}770 = 360{,}790 \text{ kN/m}$$

Combined Pile Group Damping

$$\bar{C}_{ug} = C_{ug} + C_{uf} = 1600 + 5342 = 6{,}942 \text{ kN-s/m}$$

Damping factor including damping due to pile cap,

$$\zeta_{ug} = \frac{\overline{C}_{ug}}{2\sqrt{(K_{ug} \cdot m'_{cg})}}$$

$$= \frac{6942}{2\sqrt{(360790 \times 16.3 \times 20)}}$$

$$= 0.320$$

Resonance Test

The modified reduction factor, μ equals 0.35 (Table 5) corresponding to an average spacing to diameter (t/d) ratio of 3.0 (actural t/d values being 2.5 and 3.8 / 5).

Hence corrected single pile stiffness in the group $= 0.35 \times K_{ul}$
$= 0.35 \times 16280$ kN/m

or pile group stiffness, K_{ug} $= 0.35 \times 16280 \times 20$
$= 113{,}960$ kN/m.

The horizontal stiffness and damping coefficients for the two pile groups (Table 2) are summarised in Table 9.

TABLE 9
Horizontal Stiffness and Damping Coefficients of Group Piles

Pile group	Vibration Mode and Head-Fixity	Single Pile			
		Resonance Test		Novak's Solution	
		K_{ug} (kN/m)	ζ_{ug} (%)	K_{ug} (kN/m)	ζ_{ug} (%)
4 × 5	Horizontal	—	—	360790	32.0
	Fixed Head	113960*	—	310020*	8.0*
3 × 3	Horizontal	—	—	293260	76.3
	Fixed Head	74790*	—	240170*	18.4*

* Excluding pile cap contributions

Discussion

Single Piles

For the case of both the vertical and horizontal pile stiffness, the stiffness coefficients obtained by the Novak's solution are much greater than those obtained by the resonance test method (Tables 3 and 8). The ratios being roughly 3 and 5 respectively. Large discrepancies between theory and experiments have also been reported by other investigators (Novak and Grigg, 1976; Novak and Sheta, 1982). The assumption of soil homogeneity can result in considerable over estimation of pile stiffness reaching hundreds of percent and similar errors can occur in the prediction of damping (Novak and Sheta, 1982). The main reasons for such large differences is the reduction of shear modulus of soil towards the ground surface and loss of contact between the pile and soil resulting in pile separation. Other factors contributing to errors are the assumed strain levels and the pile and soil masses participating in vibrations.

Pile Groups

The discrepancy between the theoretical and experimental stiffness apparently reduces in the case of group piles compared to the single pile case. This is on account of the higher interaction factors used with the experimental values (as given by Major, 1980) than with the theoretical values (Poulos, 1968; Novak and Sharnouby, 1985) as given in Table 10. The former, unlike the latter, are independent of the number of piles in the group, depending only on the pile spacing expressed as a ratio of the pile diameter (Table 5).

TABLE 10
Comparison of Interaction Factors

Pile group	No. of Piles	Vertical Vibration		Horizontal Vibration	
		Resonance Test	Novak's Solution	Resonance Test	Novak's Solution
4 × 5	20	0.35	0.098*	0.35†	0.198*
3 × 3	9	0.50	0.200*	0.35†	0.331*

* Values inversed for comparison

† Assumed same as for vertical vibration

Damping Coefficients

These have been worked out in the Novak's solution from the expressions for geometric damping in the elastic half space (Tables 6 and 9). The damping coefficients expressed as percentage of critical damping without pile cap are generally much smaller (by a factor of 2 to 4) than those with the pile cap contribution. However, a real possibility of loss of contact between the pile cap and the soil exists (Novak and Sheta, 1982) with consequent loss of the large amount of damping contributed by the pile cap – soil interaction. A much smaller value of damping should therefore be used in the design.

Conclusions

1. The stiffness and damping coefficients of single piles predicted by theory (Novak and others), assuming homogeneous soil condition, may result in considerable overestimation reaching hundreds of percent.

2. The pile group reduction factors given by Major (1980) can be relatively much greater compared to the interaction factors given by Poulos (1968) and by Novak and Sharnouby (1985). Major's factors have a further limitation of being strictly applicable only to the vertical motion of the pile group.

3. Where field tests are not feasible, the theoretical values of pile stiffnesses need to be computed using more realistic assumptions like non-uniform soil conditions with the shear modulus reducing towards the ground surface and possible loss of contact between the foundation and soil. This has been recently confirmed through laboratory tests by Srinivasulu et al. (1996)

4. Where field tests are carried out, the effects of strain level as specified for a particular application, confining pressure and pile length need to be considered in arriving at realistic values of the pile stiffnesses.

5. Damping in pile foundations may be much below the predicted damping values. In particular, the damping due to pile cap may not be realised in practice due to separation from the soil. It is therefore suggested that a damping value of not more than 10 percent of the critical may be used for design which is on the conservative side.

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Appendix-I

Correction to Shear Modulus,* G_s *for Confining Pressure

Shear wave velocity at 2.0 m depth as measured from hammer test (Table 1) = 85.5 m/sec.

Shear modulus at bottom of pile cap, G'_s

$$= V_s^2 \cdot \gamma_s / g$$
$$= (85.5)^2 \cdot 18.8/9.8$$
$$= 14{,}000 \text{ kN/m}^2 \qquad (1.1)$$

Confining pressure at bottom of pile cap, σ'_o

$$= \gamma H (2 K_o + 1)/3 \qquad (1.2)$$

where, H = pile cap bottom level = 2.0 m

$$\sigma'_o = (18.8 \times 2.0)(2 \times 0.5 \times 1)/3$$
$$= 25 \text{ kN/m}^2$$

Considering that depth of soil upto 2/3rd of pile length below is effective, the average depth for computing the confining pressures

= (1/2) × (2/3 of pile length)

= 1/3 of pile length.

Confining pressure at a depth of one-third the pile length below bottom of pile cap, σ_o

$$= 18.8 \times \left(\frac{16.5}{3} + 2.0\right)(2 \times 0.5 \times 1)/3$$
$$= 94 \text{ kN/m}^2$$

Shear modulus value (after confining pressure correction), G_s	$= \left(\sigma_o/\sigma_o'\right)^{0.5} \cdot G_s'$	(1.3)

Correction to Shear Modulus, G_s ***for Strain Level***

Strain level in hammer test (Fig. 3; Ishihara, 1971)	$= 10^{-6}$
Correction factor for shear modulus (Fig. 4; Prakash, 1981)	$= 1.0$
Design strain for 4 micron amplitude	$= 0.004/400$ $= 10^{-5}$
Correction factor for shear modulus (Fig.4)	$= 0.88$
Therefore, correction factor for strain level	$= 0.88 / 1.0$ $= 0.88$
Shear modulus after strain level correction, Gs	$= 27150 \times 0.88$ $= 23{,}900$ kN/m^2

APPENDIX-II

Computation of Interaction Factors, α_A ***for Vertical Stiffness of Piles***

S.No.	No. of piles	Distance in plan from reference pile S(m)	$\frac{S}{2r}$	Pile interaction factor α_A
1	1*	0.0	0.0	1.0
2	2	1.0	2.5	0.65
3	2	1.55	3.88	0.56
4	4	1.844	4.61	0.52
5	1	2.0	5.0	0.50
6	2	3.1	7.75	0.43
7	4	3.257	8.14	0.41
8	2	2.53	6.33	0.46
9	2	3.69	9.23	0.39

* Reference pile

For the 4 × 5 pile group,

$$
\begin{aligned}
l/2r &= 16.5/0.4 \\
&= 41.25 \\
N &= 20
\end{aligned}
$$

thus,

$$
\begin{aligned}
\alpha_A &= [1 + 2\times 0.65 + 2\times 0.56 + 4\times 0.52 + 1\times 0.50 \\
&\quad + 2\times 0.43 + 4\times 0.41 + 2\times 0.46 + 2\times 0.39) \\
&= 10.20
\end{aligned}
$$

APPENDIX-III

Computation of Corrected Value of G_s ***for Pile Cap Stiffness***

Overburden above mid-height of pile cap $= 2.0 - 0.6$

$= 1.4$ m

Confining pressure at a depth of one-half the pile cap thickness, σ_o $= 18.8 \times 1.4(2\times 0.5 \times 1)/3$

$= 17.5$ kN/m^2

Confining pressure at top of pile, σ'_o $= 25.0$ kN/m^2

Modified shear modulus for confining pressure effect, G_s $= G'_s\,(\sigma_o/\sigma'_o)^{0.5}$

$= 14000 \times (17.5/25)^{0.5}$

$= 11.730$ kN/m^2

where G_s is taken from *Eqn.* 2.

Correction factor for strain level (Fig. 4) $= 0.88$

Corrected shear modulus, G_s $= 0.88 \times 11.730$

$= 10{,}320$ kN/m

APPENDIX-IV

Correction to Shear Modulus

Shear modulus G_s at the bottom of pile-cap (Eqn. 2)	$= 14000 \text{ kN/m}^2$
Confining pressure at top of pile, σ'_o (Eqn. 3)	$= 25 \text{ kN/m}^2$

Considering the length of pile participating in lateral vibrations as 10 pile diameters (Sharnouby and Novak, 1985),

Mean depth for computing the dynamic shear modulus	$= 2.0 + (1/2) \times 10 \times 0.4$ m
	$= 4.0$ m
Therefore, confining pressure at a depth of 4.0 m, σ_o	$= 18.8 \times 4.0(2 \times 0.5 \times 1)/3$
	$= 50 \text{ kN/m}^2$
Shear modulus of soil as modified for confining pressure, G_s (Eqn. 4)	$= 14000 \times (50/25)^{0.5}$
	$= 19{,}800 \text{ kN/m}^2$

The value of shear modulus thus obtained is to be further corrected for strain level.

Corrected G_s (Eqn. 5)	$= 19800 \times 0.88$
	$= 17{,}420 \text{ kN/m}^2$

APPENDIX-V

Correction to Pile Stiffness in Horizontal Mode (Resonance Test)

Design amplitude (Table 1)	= 4 micron
Design strain level	$= 0.004 / 400$
	$= 10^{-5}$
Corresponding correction factor (Fig. 4)	= 0.88
Average strain level during resonance test (Fig. 3)	$= 5 \times 10^{-4}$
Corresponding correction factor (Fig. 4)	= 0.50

Therefore, correction factor for shear modulus $= 0.88 / 0.50$
$= 1.8$

For clayey soils,
(Prakash and Chandrasekaran, 1977), $K_{ul} \propto (G_s)^{0.75}$
so that, $K_{ul} = 5240 \times (1.8)^{0.75}$
$= 8{,}140$ kN/m

The stiffness of actual pile will remain the same as that for test pile for l/r > 25.

For fixed-head pile,
(Prakash and Chandrasekaran, 1977), $K_{ul} = 8140 \times (1.20/0.85)^2$
$= 16{,}280$ kN/m

APPENDIX-VI

Computation of Interaction Factors, α_L for Horizontal Stiffness of Group Piles

S.No.	No. of piles	Distance S, from Ref. Pile No. 1 (m)	$\frac{S}{2r}$	Departure angle β (deg)	Pile interaction factor α_L
1	1	0.0	0.0	0	1.0
2	2	1.0	2.5	0	0.47
3	2	1.55	3.875	90	0.23
4	4	1.844	4.61	57.17	0.24
5	1	2.0	5.0	0	0.30
6	2	3.1	7.75	90	0.12
7	4	3.257	8.14	72.12	0.12
8	2	2.53	6.33	37.77	0.21
9	2	3.69	9.22	57.17	0.12

For the pile group of 20 (= N) piles, Group interaction factor,

$$\sum \alpha = [1 + 2 \times 0.47 + 2 \times 0.23 + 4 \times 0.24 + 1 \times 0.30 + 2 \times 0.12 + 4 \times 0.12 + 2 \times 0.21 + 2 \times 0.12)$$
$$= 5.04$$