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Method for Establishing Critical Failure Surface in Passive Pressure Determinations

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Introduction

The determination of earth pressures, both active and passive, has been a classical problem in geotechnical engineering as its forms the basis of solving problems like retaining walls, foundations in either compression or tension, anchors, sheet piles, etc. Limit equilibrium method is very frequently employed in the evaluation of these earth pressures. In this method, a certain failure surface is always assumed and the required magnitude of earth pressure is obtained by satisfying the statical equilibrium of the soil mass containing with in the most dangerous failure surface, the position of which, however, remains to be searched. In the active case, the results obtained from planar rupture surfaces and that from curved rupture surfaces differ insignificantly. But, in the passive case, the curved rupture surfaces are necessarily to be used. While solving the slope stability problems, Rendulic (1935) has considered a log-spiral failure surface, which was later employed by Terzaghi (1943) in determining the passive earth pressure. The results obtained with logarithmic spiral failure surface are almost close to that of Sokolovoski's (1960) slip line method. Although, the earth pressure coefficients from various other methods like Caquot and Kerisel (1948), Brinch Hansen (1953), Sokolovoski (1960), Lee and Herington, (1972), etc., are easily available, the availability of the earth pressure coefficients by the logarithmic spiral method, is perhaps still scarce. By making use of this method, Shields and Tolunay (1972) have developed passive earth pressure coefficients for a vertical wall with positive wall friction angle (δ), i.e., for the case when the wall moves relatively in a downward direction as compared to the soil wedge. By considering the problem of passive earth pressure for positive wall friction over an inclined retaining wall, Terzaghi

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(1943) obtained the solution for the bearing capacity of foundation. However, for negative wall friction, the solution with this method is still not available.

Eventhough the shape of the failure surface can be assumed, e.g., log-spiral, the nature of the same, convex or concave, at its lower portion will not be apparent. In the classical problem of bearing capacity, which deals with +ve δ, the nature of all the failure surfaces has been convex, but the same convex nature will not hold good for all inclinations of the retaining walls. Similarly, even for negative δ, as for example in anchor pull-out problems, the nature of the failure surfaces will not always be concave. It, therefore, becomes necessary to establish the nature of the failure surface which will be consistent with the given problem. What is presented in this paper is a method by which the nature of the failure surface can be determined. Also proposed here is a method by which the position of the critical failure surface can very rapidly be obtained. By making use of the methodology as proposed herein, a comprehensive set of passive earth pressure coefficients has been developed for an inclined retaining wall supporting sand for both positive and negative wall friction.

The Method

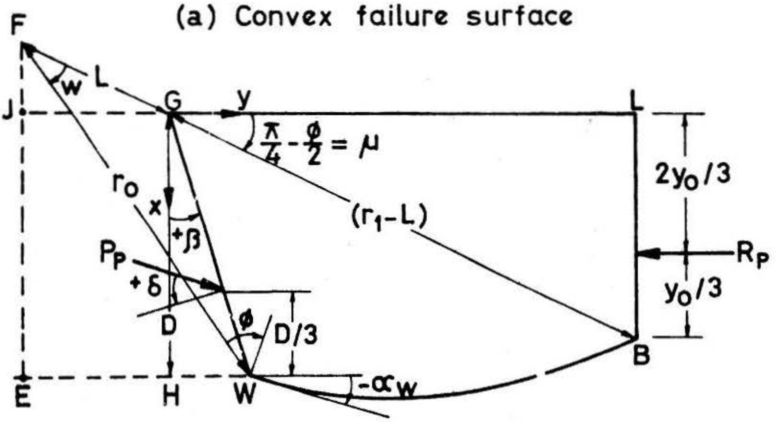
Determination of Nature of Failure Surface

Theories as well as experiments have indicated that depending upon the roughness, movement and the orientation of the wall, both for active and passive cases, two different natures of curvature, convex or concave, of the lower portion of the failure surface as shown in Figs. 1(a) and (b) generate in the soil mass. The nature of the failure surface will be referred to as convex/concave when the failure surface is curving inward/outward when viewed from with in the failure mass. The shape of the failure surface is taken as a combination of an arc of a log-spiral for the curved portion and a straight line in the Rankine passive zone as shown in the Fig. 1. By comparing the statically correct inclination of the failure surface at the base of the wall to the inclination of the same in the Rankine zone, the nature of the failure surface can be determined as follows :

Let, β = angle which the retaining wall makes with the vertical; it is considered as +ve when the wall rotates in a counter-clockwise direction with respect to the vertical as shown in Fig. 1.

For any failure surface to be statically correct, it should satisfy : (i) the available boundary conditions; and (ii) the failure condition (i.e., the Mohr-Coulomb failure criterion). For the portion of a failure surface in the Rankine passive zone, these conditions automatically get satisfied. Whereas these conditions at the bottom of the wall can only be satisfied for a certain

(a) Convex failure surface



(b) Concave failure surface

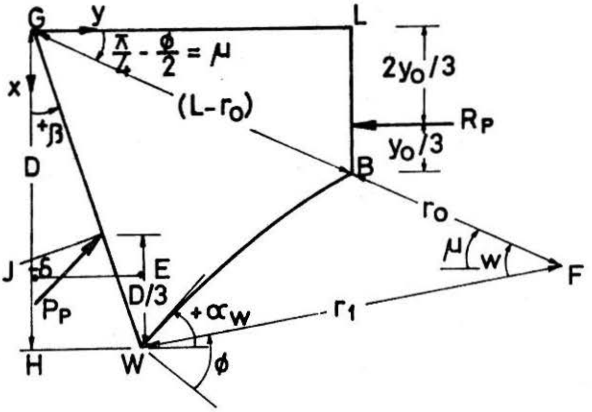


FIGURE 1 : Convex and Concave Failure Surfaces in Passive Case

inclination, α_w, the magnitude of which will become

$$\alpha_w = \beta - \frac{\Delta}{2} - \frac{\delta}{2} + \mu \tag{1}$$

in which, $\mu = \Pi / 4 - \phi / 2$;

Δ = principal value of $\sin^{-1}(\sin\delta / \sin\phi)$; and

α_w = statistically correct inclination of the failure surface with the horizontal at the bottom of the wall.

The above equation has been derived using Sokolovski's method.

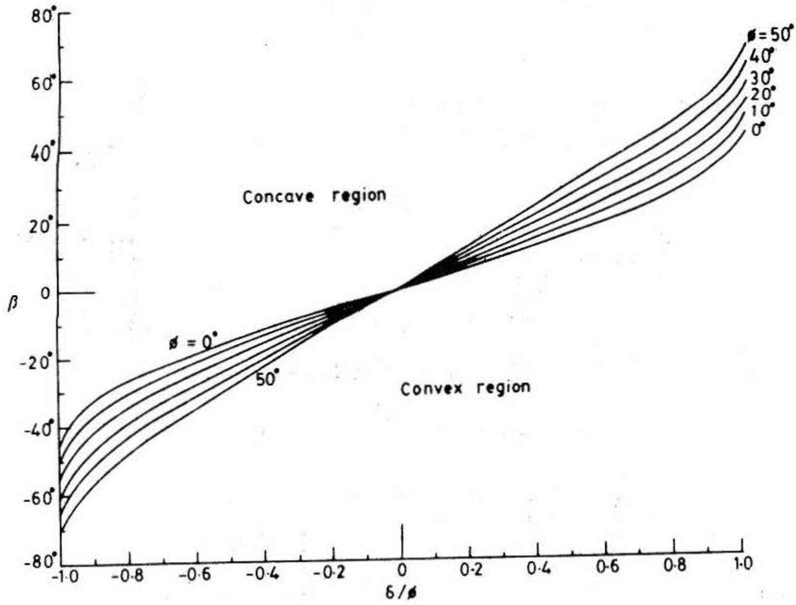


FIGURE 2 : Determination of Nature of the Failure Surface

In the Rankine passive zone, the failure surface becomes a straight line making an angle μ with the horizontal. To satisfy the geometry and the given stress conditions of the present problem, the path of the failure surface should be a continuous smooth curve. As a result, on making a comparison of the statical correct inclination of the failure surface at the wall with that of the inclination of the same in the Rankine passive zone, the following criterion can be established :

(i) $\alpha_w > \mu$: concave failure surface;
(ii) $\alpha_w < \mu$: convex failure surface; and
(iii) $\alpha_w = \mu$: straight line failure surface.

Based upon the above criteria, Fig. 2 has been developed. For given values of β and ϕ, it gives the required minimum δ / ϕ values for the development of convex failure surface. Similarly for given δ and ϕ values, it provides the maximum wall inclination angle β, for the generation of the convex failure surface. For a given problem, to know whether the convex of concave failure surface will develop, one need to plot a point on the Fig. 2 for the given co-ordinates β and δ / ϕ. If this point lies above the corresponding $\phi -$ curve, then only concave surface develops. If on the other hand, the point lies below the corresponding $\phi -$ curve, the failure surface will be convex. If the point lies on the corresponding $\phi -$ curve itself, the failure surface will be planar, being neither convex nor concave. In other words, a number of combinations of β, ϕ and δ/ϕ are possible for which the failure surfaces will only be planar.

Determining the Position of Critical Failure Surface

After identifying the nature of the failure surface, any number of failure surfaces the focus of which should lie on the line GB making an angle μ with the horizontal as shown in Fig. 1(a) for convex nature and Fig. 1(b) for concave nature, can be tried. The required magnitude of passive force, P_p, the position of which is assumed to act at one-third height of the wall, can be calculated by taking the moment of all the forces acting on soil wedge GLBW about the focus F. Among all these failure surfaces, the one which corresponds to a minimum value of P_p will be the critical.

The approximate position of the critical, can be determined by satisfying the statically correct orientation of the failure surface at the bottom of the wall. The geometric details of such failure surface can be determined by knowing the value of α_w from Eqn. (1). With reference to Figs. 1(a) and (b), it can be seen that,

(i) For convex failure surface :

$$r_0 = \frac{D}{\cos\beta} \times \frac{\cos(\mu+\beta)}{\sin w} \tag{2}$$

$$L = \frac{D}{\cos\beta} \times \frac{\cos(w+\mu+\beta)}{\sin w} \tag{3}$$

$$r_1 = r_0\, e^{w \tan\phi} \tag{4}$$

(ii) For convave failure surface :

$$r_1 = \frac{D}{\cos\beta} \times \frac{\cos(\mu+\beta)}{\sin w} \tag{5}$$

$$L = \frac{D}{\cos\beta} \times \frac{\cos(w-\mu-\beta)}{\sin w} \tag{6}$$

$$r_0 = r_1 / e^{w \tan\phi} \tag{7}$$

wherein,
r_o = length of initial radius vector,
r_1 = length of final radius vector,
L = distance between the focus and the top point of the retaining wall, and
D = vertical height of the retaining wall.

Extensive computations by varying ϕ from 10° to 50°, β from $-45°$ to $+45°$ and δ from $-\phi$ to $+\phi$, have shown (Tables 1 to 11) that the magnitudes of passive earth pressure as calculated on the basis of a failure surface which satisfies statically correct orientation at the base of the wall, are almost the same as determined with the critical failure surface. In the Tables 1 to 11,

$$k_{p\gamma} = \text{passive earth pressure coefficient}$$

$$= \frac{P_{p\gamma} \cos\delta}{\left(\gamma D^2/2\right)} \tag{8}$$

$$\text{Error (\%)} = \frac{\left[k_{p\gamma}\right]_{\text{approxi.}} - \left[k_{p\gamma}\right]_{\text{critical}}}{\left[k_{p\gamma}\right]\text{critical}} \times 100 \tag{9}$$

wherein, $[k_{p\gamma}]_{\text{approxi.}}$ = the value of $k_{p\gamma}$ as calculated from the failure surface which satisfies statically correct orientation with the wall; and

$[k_{p\gamma}]_{\text{critical}}$ = the value of $k_{p\gamma}$ as calculated from critical failure surface

TABLE 1

Passive Earth Pressure Coefficients ($k_{p\gamma}$) and Error for $\delta/\phi = +1.0$

ϕ	β						
	- 45°	- 30°	- 15°	0°	15°	30°	45°
	3.627*	2.469	1.916	1.637	1.520	1.535	1.712
10°	4.424†	2.765	2.035	1.684	1.536	1.539	1.712
	21.989‡	12.000	6.210	2.868	1.053	0.221	0.000
	35.540	16.513	9.144	5.783	4.098	3.237	2.888
30°	48.779	20.832	10.725	6.384	4.316	3.302	2.889
	37.250	26.157	17.292	10.390	5.324	2.014	0.008
	1528.556	422.453	140.447	54.200	24.022	12.272	7.568
50°	2152.826	563.017	177.565	64.993	27.293	13.225	7.568
	40.840	33.273	26.428	19.913	13.615	7.768	0.000

* earth pressure coefficients by exact critical failure surface.

† earth pressure coefficients by the approximate critical failure surface.

‡ error (%)

TABLE 2
Passive Earth Pressure Coefficients ($k_{p\gamma}$) and Error for $\delta / \phi = +0.8$

ϕ	β						
	- 45°	- 30°	- 15°	0°	15°	30°	45°
	3.499*	2.398	1.871	1.607	1.499	1.520	1.702
10°	3.618†	2.433	1.881	1.609	1.499	1.520	1.702
	3.394‡	1.457	0.497	0.104	0.000	0.000	0.012
	31.069	14.653	8.249	5.307	3.825	3.071	2.774
30°	31.707	14.867	8.314	5.322	3.827	3.071	2.774
	2.053	1.462	0.783	0.279	0.041	0.000	0.000
	1122.887	312.764	105.599	41.711	19.081	10.140	6.352
50°	1132.717	313.189	105.603	41.713	19.081	10.141	6.352
	0.875	0.136	0.004	0.003	0.002	0.001	0.001

TABLE 3
Passive Earth Pressure Coefficients ($k_{p\gamma}$) and Error for $\delta / \phi = +0.6$

ϕ	β						
	- 45°	- 30°	- 15°	0°	15°	30°	45°
	3.361*	2.319	1.820	1.570	1.471	1.496	1.689
10°	3.391†	2.326	1.821	1.570	1.471	1.496	1.690
	0.886‡	0.296	0.059	0.001	0.000	0.000	0.031
	26.490	12.683	7.261	4.752	3.483	2.841	2.617
30°	26.493	12.683	7.261	4.752	3.483	2.841	2.619
	0.011	0.002	0.001	0.001	0.000	0.000	0.083
	763.552	212.481	72.601	29.388	13.946	7.771	5.158
50°	920.548	226.665	74.219	29.587	13.968	7.771	5.160
	20.561	6.675	2.228	0.677	0.164	0.000	0.039

* earth pressure coefficients by exact critical failure surface.

† earth pressure coefficients by the approximate critical failure surface.

‡ error (%)

TABLE 4
Passive Earth Pressure Coefficients ($k_{p\gamma}$) and Error for $\delta / \phi = +0.4$

ϕ	β						
	- 45°	- 30°	- 15°	0°	15°	30°	45°
	3.214*	2.232	1.761	1.526	1.435	1.467	1.676
10°	3.218†	2.233	1.761	1.526	1.435	1.467	1.677
	0.125‡	0.021	0.000	0.000	0.000	0.031	0.047
	21.987	10.702	6.239	4.159	3.103	2.581	2.449
30°	22.225	10.737	6.246	4.160	3.103	2.582	2.455
	1.083	0.321	0.116	0.040	0.000	0.032	0.229
	476.453	132.238	46.009	19.325	9.673	5.750	4.130
50°	738.202	148.888	47.514	19.457	9.673	5.750	4.141
	54.937	12.591	3.272	0.682	0.000	0.001	0.257

TABLE 5
Passive Earth Pressure Coefficients ($k_{p\gamma}$) and Error for $\delta / \phi = +0.2$

ϕ	β						
	- 45°	- 30°	- 15°	0°	15°	30°	45°
	3.056*	2.138	1.696	1.476	1.394	1.434	1.662
10°	3.056†	2.138	1.696	1.476	1.394	1.435	1.663
	0.004‡	0.011	0.009	0.000	0.000	0.081	0.059
	17.731	8.801	5.239	3.566	2.717	2.315	2.284
30°	18.166	8.863	5.249	3.566	2.717	2.319	2.293
	2.458	0.703	0.192	0.000	0.000	0.173	0.387
	269.507	75.501	27.300	12.180	6.554	4.225	3.329
50°	450.094	83.805	27.855	12.200	6.554	4.234	3.346
	67.006	10.998	2.031	0.161	0.000	0.210	0.508

* earth pressure coefficients by exact critical failure surface.

† earth pressure coefficients by the approximate critical failure surface.

‡ error (%)

TABLE 6
Passive Earth Pressure Coefficients ($k_{p\gamma}$) and Error for $\delta / \phi = +0.0$

ϕ	β						
	- 45°	- 30°	- 15°	0°	15°	30°	45°
10°	2.888*	2.035	1.624	1.420	1.348	1.398	1.648
	2.892†	2.036	1.624	1.420	1.349	1.400	1.649
	0.135‡	0.064	0.001	0.000	0.024	0.149	0.068
30°	13.866	7.052	4.303	3.000	2.344	2.058	2.125
	14.298	7.105	4.308	3.000	2.346	2.067	2.137
	3.109	0.752	0.128	0.000	0.102	0.391	0.533
50°	137.373	40.100	15.559	7.549	4.434	3.122	2.706
	195.527	42.368	15.653	7.549	4.442	3.137	2.726
	42.333	5.657	0.605	0.000	0.175	0.482	0.738

TABLE 7
Passive Earth Pressure Coefficients ($k_{p\gamma}$) and Error for $\delta / \phi = -0.2$

ϕ	β						
	- 45°	- 30°	- 15°	0°	15°	30°	45°
10°	2.709*	1.923	1.544	1.358	1.298	1.359	1.634
	2.718†	*1.924*	*1.544*	*1.358*	*1.299*	1.363	1.635
	0.331‡	0.100	0.000	0.000	0.099	0.225	0.073
30°	10.498	5.505	3.460	2.482	1.996	1.816	1.974
	10.799	5.534	3.460	2.482	2.002	1.829	1.987
	2.873	0.521	0.000	0.000	0.308	0.664	0.654
50°	63.838	20.355	8.741	4.680	3.012	2.314	2.205
	75.003	20.705	8.741	4.686	3.026	2.333	2.226
	17.490	1.720	0.000	0.124	0.465	0.797	0.970

* earth pressure coefficients by exact critical failure surface.

† earth pressure coefficients by the approximate critical failure surface.

‡ error (%)

TABLE 8
Passive Earth Pressure Coefficients ($k_{p\gamma}$) and Error for $\delta / \phi = -0.4$

ϕ	β						
	-45°	-30°	-15°	0°	15°	30°	45°
10°	2.515*	1.802	1.456	1.291	1.244	1.320	1.619
	2.527†	1.802	1.456	1.291	1.246	1.324	1.621
	0.486‡	0.005	0.000	0.002	0.196	0.296	0.073
30°	7.671	4.189	2.270	2.016	1.678	1.591	1.830
	7.827	4.189	2.720	2.020	1.688	1.606	1.843
	2.035	0.002	0.000	0.236	0.623	0.961	0.741
50°	27.892	10.160	4.904	2.902	2.040	1.706	1.786
	29.293	10.160	4.904	2.916	2.058	1.726	1.808
	5.025	0.000	0.001	0.479	0.839	1.183	1.242

TABLE 9
Passive Earth Pressure Coefficients ($k_{p\gamma}$) and Error for $\delta / \phi = -0.6$

ϕ	β						
	-45°	-30°	-15°	0°	15°	30°	45°
10°	2.303*	1.666	1.359	1.216	1.851	1.279	1.605
	2.315†	1.666	1.359	1.218	1.189	1.283	1.606
	0.528‡	0.000	0.000	0.148	0.315	0.342	0.065
30°	5.372	3.070	2.085	1.603	1.389	1.381	1.689
	5.426	3.070	2.086	1.613	1.403	1.398	1.702
	0.996	0.000	0.039	0.594	1.020	1.220	0.766
50°	11.864	5.002	2.720	1.774	1.356	1.231	1.420
	11.867	5.002	2.735	1.791	1.374	1.252	1.442
	0.023	0.000	0.542	0.955	1.348	1.693	1.573

* earth pressure coefficients by exact critical failure surface.

† earth pressure coefficients by the approximate critical failure surface.

‡ error (%)

TABLE 10
Passive Earth Pressure Coefficients ($k_{p\gamma}$) and Error for $\delta / \phi = -0.8$

ϕ	β						
	- 45°	- 30°	- 15°	0°	15°	30°	45°
10°	2.064*	1.507	1.249	1.133	1.123	1.236	1.591
	2.064†	1.507	1.249	1.136	1.128	1.240	1.591
	0.001‡	0.000	0.021	0.304	0.398	0.311	0.046
30°	3.528	2.147	1.532	1.238	1.126	1.184	1.552
	3.528	2.147	1.542	1.252	1.141	1.199	1.562
	0.000	0.000	0.657	1.085	1.343	1.262	0.649
50°	4.762	2.367	1.447	1.035	0.858	0.849	1.089
	4.762	2.384	1.465	1.052	0.876	0.868	1.109
	0.000	0.740	1.250	1.639	2.014	2.236	1.813

TABLE 11
Passive Earth Pressure Coefficients ($k_{p\gamma}$) and Error for $\delta / \phi = -1.0$

ϕ	β						
	- 45°	- 30°	- 15°	0°	15°	30°	45°
10°	1.635*	1.279	1.104	1.035	1.056	1.193	1.576
	1.635†	1.279	1.104	1.035	1.056	1.193	1.576
	0.000‡	0.000	0.000	0.000	0.000	0.000	0.000
30°	1.829	1.301	1.032	0.905	0.884	0.999	1.415
	1.829	1.301	1.032	0.905	0.884	0.999	1.415
	0.002	0.000	0.000	0.000	0.000	0.000	0.000
50°	1.499	0.930	0.659	0.529	0.485	0.535	0.782
	1.499	0.930	0.659	0.529	0.485	0.535	0.782
	0.001	0.000	0.000	0.000	0.000	0.002	0.000

* earth pressure coefficients by exact critical failure surface.

† earth pressure coefficients by the approximate critical failure surface.

‡ error (%)

It can be seen that the magnitude of error for negative δ in almost all the cases is quite less, and the failure surface as obtained by satisfying the statically correct orientation of the failure surface with the wall, becomes itself the critical. Even for the positive δ with the value of β greater than zero, the magnitude of error still does not exceed about 20%. However, for negative values of β, the error becomes as high as 67%. In all the cases in general, the magnitude of error increases with decreasing values of β (more negative) and with increasing magnitudes of δ (more positive) both for positive and negative δ.

From Fig. 2 also, one can get an idea of the accuracy of the results. If the point with co-ordinates β and δ / ϕ is close to the corresponding ϕ curve the error will be quite small.

In all the cases, the computational time in searching the critical failure surface can be drastically reduced, if a start is being made with a failure surface which satisfies the statically correct orientation with the wall.

Variation of Earth Pressure Coefficients

Computed values of passive earth pressure coefficients $k_{p\gamma}$ as defined earlier, have been plotted as a function of δ / ϕ on a semi-log graph, with

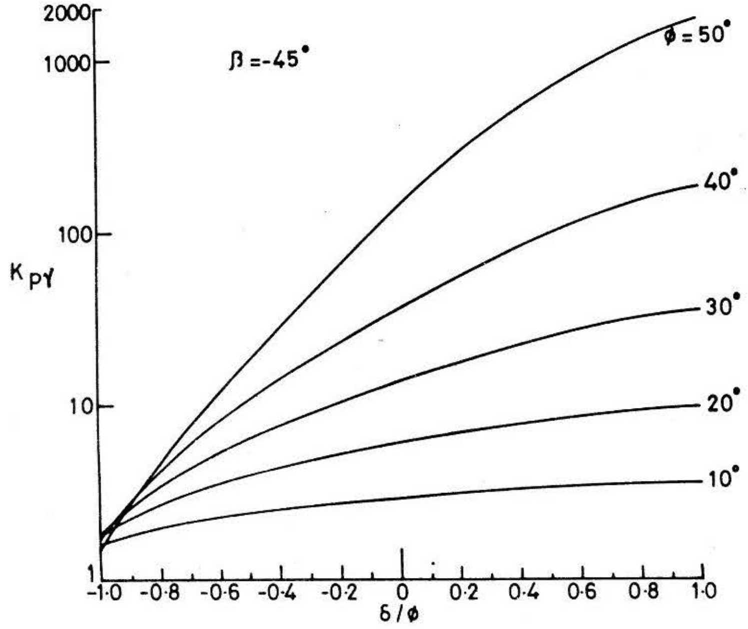


FIGURE 3 : Passive Earth Pressure Coefficients $k_{p\gamma}$ for $\beta = -45°$

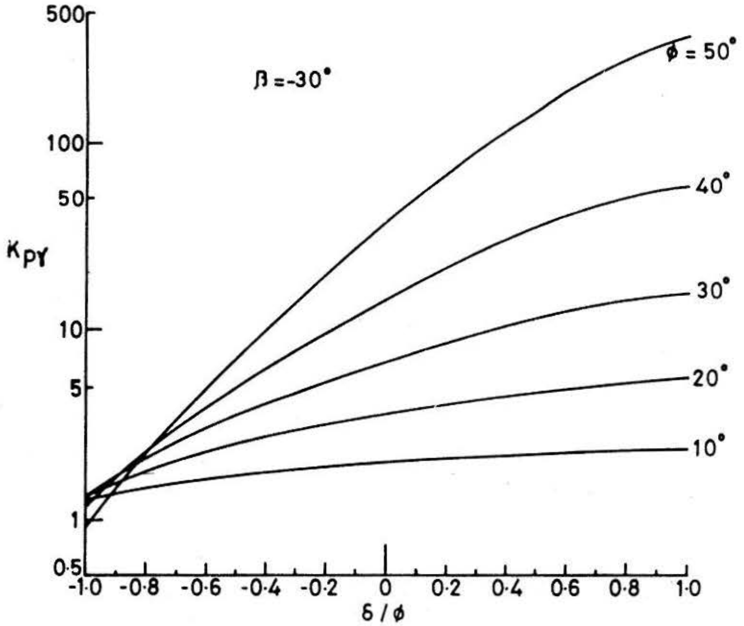


FIGURE 4 : Passive Earth Pressure Coefficients $k_{p\gamma}$ for $\beta = -30°$

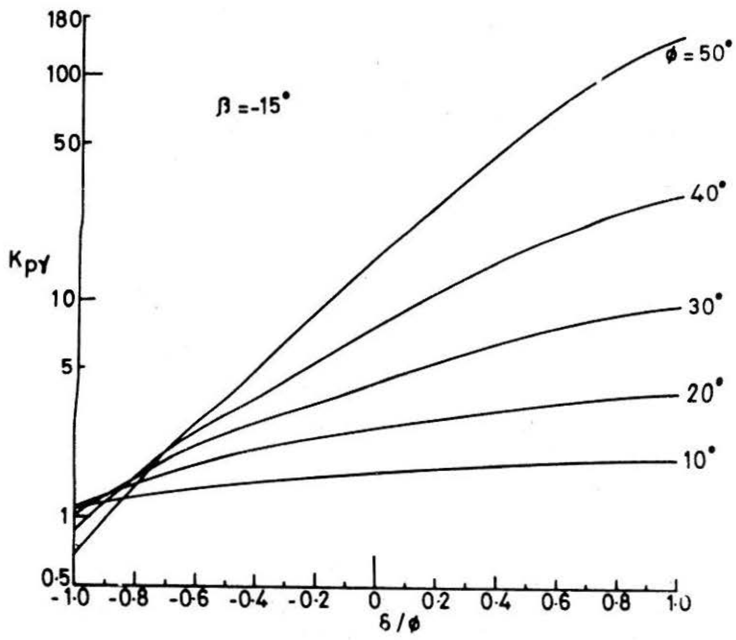


FIGURE 5 : Passive Earth Pressure Coefficients $k_{p\gamma}$ for $\beta = -15°$

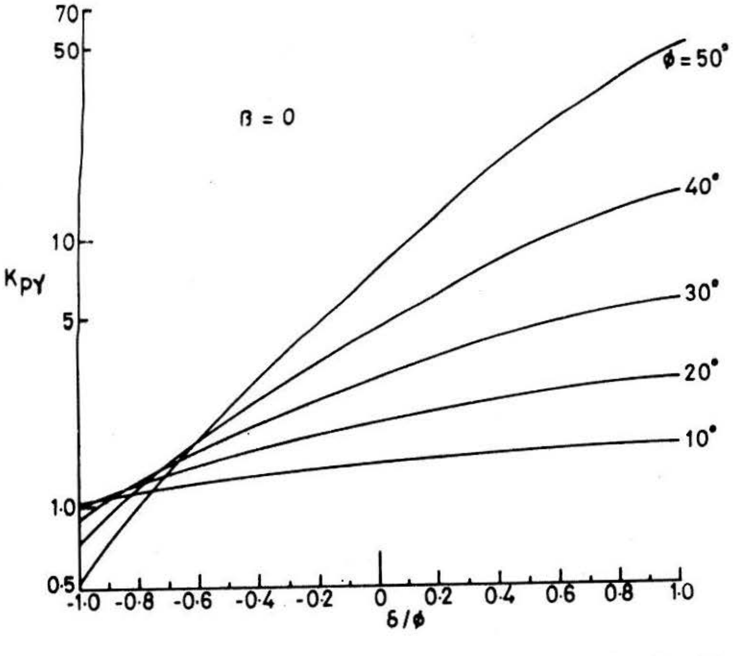


FIGURE 6 : Passive Earth Pressure Coefficients $k_{p\gamma}$ for $\beta = 0°$

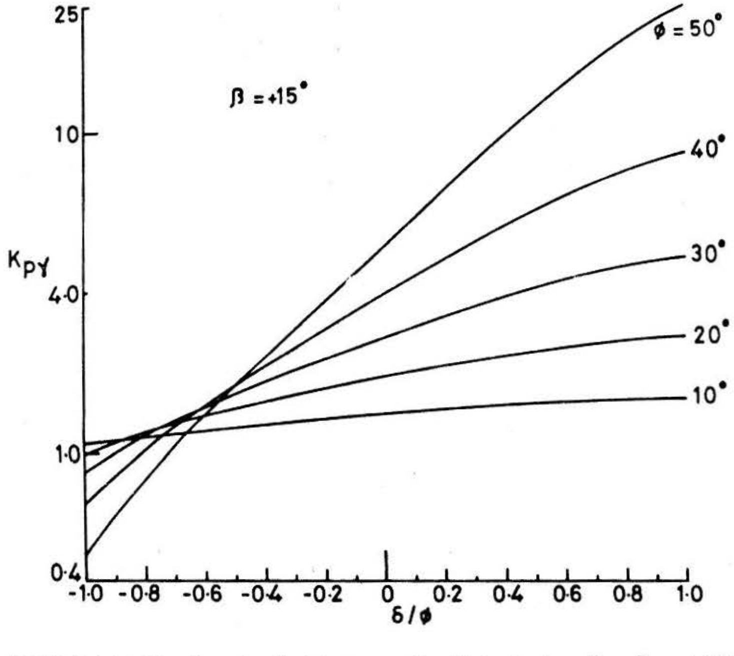


FIGURE 7 : Passive Earth Pressure Coefficients $k_{p\gamma}$ for $\beta = +15°$

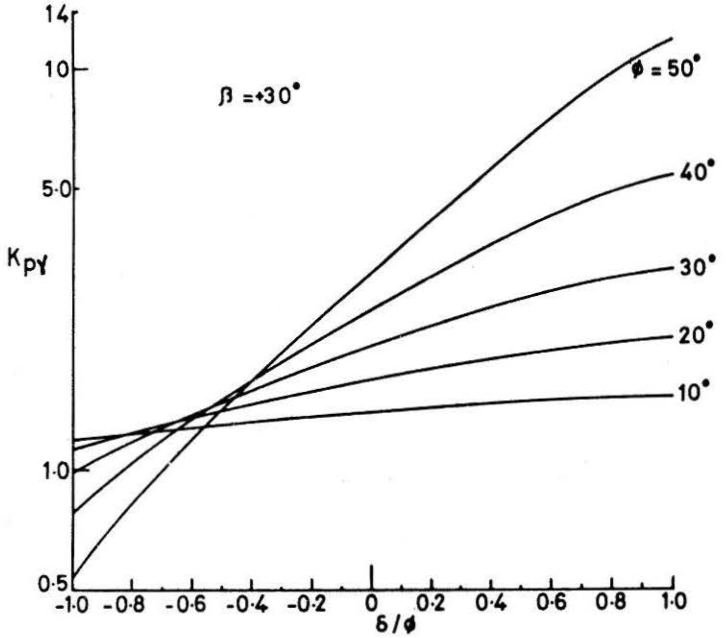


FIGURE 8 : Passive Earth Pressure Coefficients $k_{p\gamma}$ for $\beta = +30°$

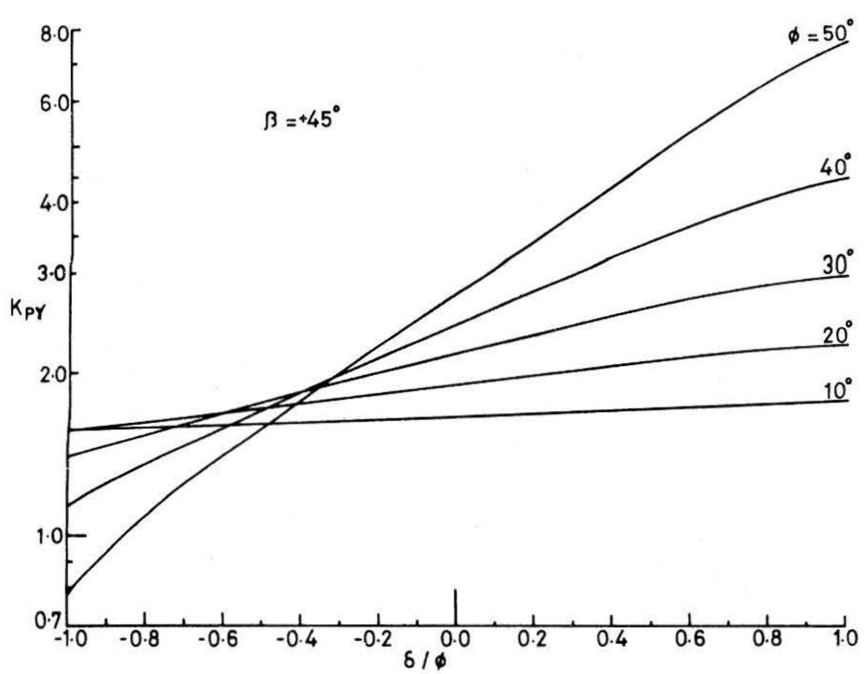


FIGURE 9 : Passive Earth Pressure Coefficients $k_{p\gamma}$ for $\beta = +45°$

δ / ϕ on the linear axis and kpg on the log-scale. The variation has been shown in Figs. 3 to 9. The values of $k_{p\gamma}$ increase continuously with increase in ϕ / δ from -1.0 to $+1.0$ for all values of ϕ and β. For higher values of ϕ and for more negative values of β, the values of $k_{p\gamma}$ are generally higher.

Conclusions

A method has been developed for establishing the nature of the failure surface for given retaining wall and soil conditions. A direct method for locating an approximate critical position of the failure surface has been suggested in the determination of passive earth pressure coefficients. In many of the cases, particularly for -ve δ, such established failure surface itself practically becomes the critical. In all the cases, however, the search for the critical failure surface can be done with the minimal computational effort, if a start is made with the proposed approximate critical failure surface. Further, a comprehensive set of passive earth pressure coefficients in sand has been developed by using the composite log-spiral failure surface for both positive and negative wall friction on an inclined retaining wall.

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