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Bearing Capacity Analysis of Reinforced Two-Layered Soil System

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Introduction

Reinforced earth is widely in use as the construction material in formation of subgrade for roads, railway tracks and in air strips to reduce the settlement and to increase the bearing capacity. Binquet and Lee (1975) were the pioneers in carrying out an analytical study on the bearing capacity of reinforced soil beds. In their analysis Boussinesq's stress distribution, assuming semi-infinite medium was adopted to calculate the stress distribution on the plane of reinforcement and hence the maximum tensile stress in the reinforcement. They have considered three different failure mechanisms and given expressions for the tension in the reinforcement as a function of non-dimensional force and length parameters, and the BCR, (q/q_0). BCR is the ratio of the average contact pressures for the reinforced and unreinforced soils both measured at the same vertical displacement. However, it is necessary to consider reinforced soil system as a two layered medium with a top layer consisting of granular soil strengthened by horizontal layers of reinforcement. For the analysis of two layer soil system very few analytical techniques exist.

Semi-empirical methods to determine the bearing capacity of footings

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on a two layered soil system was developed by Yamaguchi (1963), Meyerhof (1974) and Hanna and Meyerhof (1980). These were primarily based on experimental investigations. Desai and Reese (1970) have used the finite element method to investigate the behaviour of circular footing on a two layered soil system using a nonlinear stress-strain behaviour. Numerous experimental studies on reinforced and unreinforced two layer soil systems, specifically for the design of unpaved roads ,have been carried out in recent years (Love, 1984; Poran, 1985; Fannin, 1986; Little, 1992; Brocklehurst, 1993). From above studies, it is observed that, with the inclusion of reinforcement there is a marked increase in load carrying capacity of the soil system, which improves with increase in either the clay substratum strength or the depth. Also it is extremely difficult and becomes expensive to do all different combinations of parameters in experimental model tests. Finite element modeling has the advantage over the conventional experimental modeling in that, parameters may be easily varied and details of stress distribution can be determined throughout the system.

In this paper, Binquet and Lee's approach has been extended for the design of reinforced soil bed as a two layered system. Stress distribution obtained from the finite element analysis has been used to calculate the nondimensional parameters for estimating the mobilization of tension in the reinforcement. Nondimensional parameters thus obtained are presented in the form of charts. These nondimensional parameters together with selected design variables such as footing width (B), footing depth (D), number of reinforcement layers (N), and reinforcement spacing (ΔH) can be used to calculate the thickness and length of tie reinforcements require to satisfy the performance criteria on settlement and bearing capacity.

Statement of the Problem

A footing of known width is resting on the surface of a two layered soil system, the strong upper layer overlaying a weak soil. The upper granular fill will have inclusion of a few horizontal layers of reinforcement at different depths as shown in the Fig. 1. For this non-homogeneous soil medium, a finite element analysis is used to obtain the stress distribution in the two layer soil system. The nondimensional force and length parameters, to estimate the tension in the reinforcing layers, are computed and presented.

Finite Element Analysis

In the analysis of two layered soil system the soil conditions are non-homogeneous, and the depth of stiffer layer is varied in order to arrive at an optimal thickness of upper granular layer. The analysis is made using the general purpose finite element program FEAP (Zienkiewicz, 1971). In the present investigation elastic theory has been used to estimate the stress

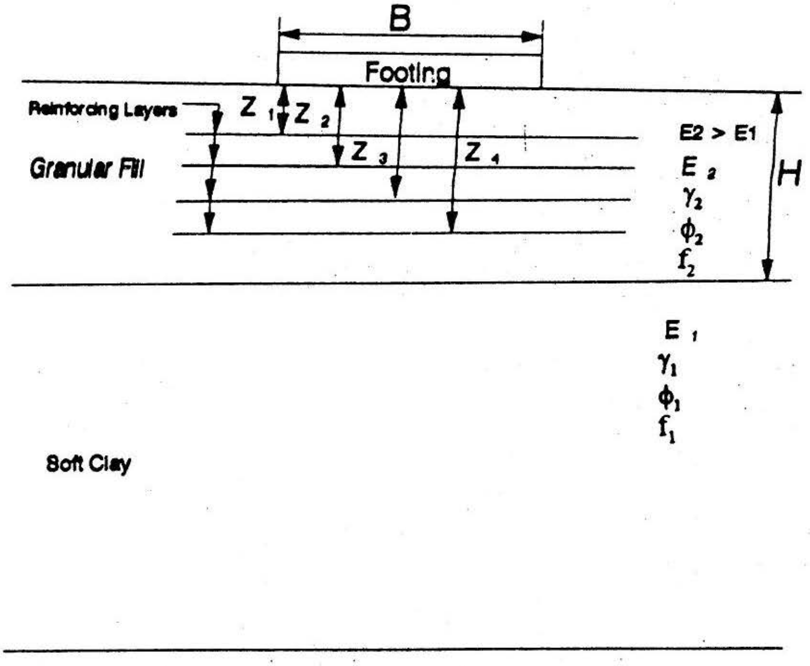


FIGURE 1 : Strip Footing on a Reinforced Two-layered Soil System

distribution in two layered soil system. The advantage of such an analysis is that stress distribution can be obtained at different sections of the whole soil system. In the present analysis, 4 noded quadrilateral elements have been used. The element number, dimensions along with mesh size have been arrived at by repeated mesh refinement. The final mesh size chosen has dimensions of 10B × 5B, where B being the width of the footing. Since it is a plane strain problem, in the analysis only half of the mesh width under the footing has been considered and the chosen mesh is presented in Fig. 2. The same mesh been used for all finite element runs in this paper. For the validation of the mesh, the results are compared with Boussinesq's solution for distribution of stresses in homogeneous semi-infinite soil mass as shown in Figs. 3 and 4. The vertical stress distributions obtained from finite element method compare very well with Boussinesq's solution along both vertical and horizontal planes at all sections.

Considering the reinforced soil bed as a two layered system with the properties of layers as shown in the Fig. 1 The chosen stiffness ratio (E_2/E_1) and normalised thickness of the upper layer (H/B) are 10 and 1 respectively. Plots of distribution of vertical normal stresses at different vertical and horizontal sections for two layer and single layer system are given in Figs. 5 and 6. The vertical cross sections are taken at 0.05, 0.45,

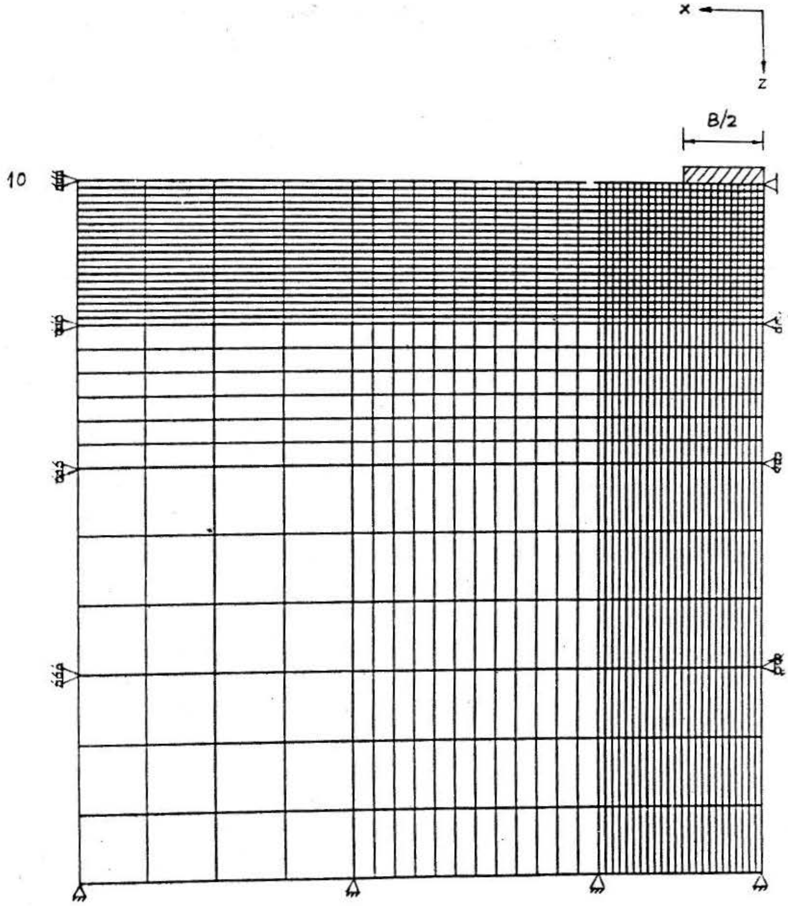


FIGURE 2 : Finite Element Mesh

1.45 and 2.55 cm from the centre of the footing . The horizontal cross sections are taken at depths 0.35, 0.65, 1.15 and 2.45 cm from the surface of the footing. The Fig. 5 shows the plots of vertical normal stress (σ_z) versus depth (z) for a two layered soil system and a single layered soil system at different horizontal distances from the centre of the footing. At *vertical cross section (e.g., x = 0.05 cm)* the stresses for two layered system are observed to be smaller than for a single layer system. This trend is not observed for the sections away from the center of the footing. It can be seen that, in two layer system the vertical normal stress distribution is distinctly different from that of single layer system. Figure 6 shows the plots of vertical normal stress distribution (σ_z) versus horizontal distance (x) at different depths (location of plane below the footing level z). At shallow depths there is not much of difference in the distribution of vertical normal stresses of two layer system. At greater depths, the difference in normal stress values is higher. Normal stress beneath the footing is higher for single layer, while

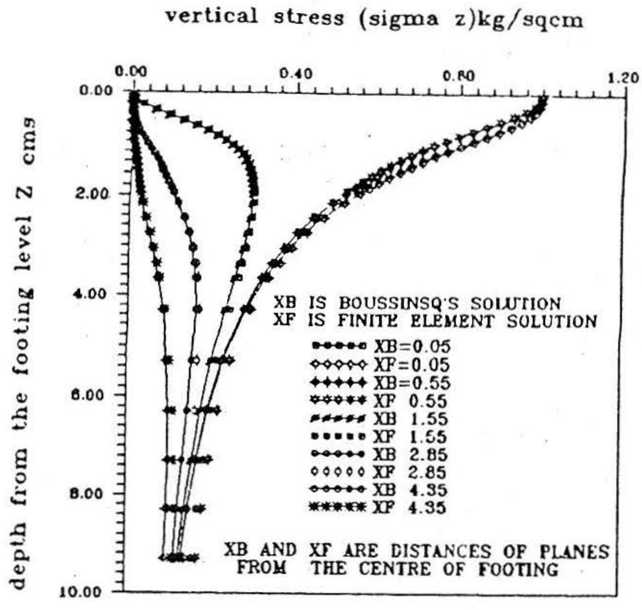


FIGURE 3 : Vertical Stress Distribution along Different Vertical Planes by Boussinesq's and Finite Element Solution

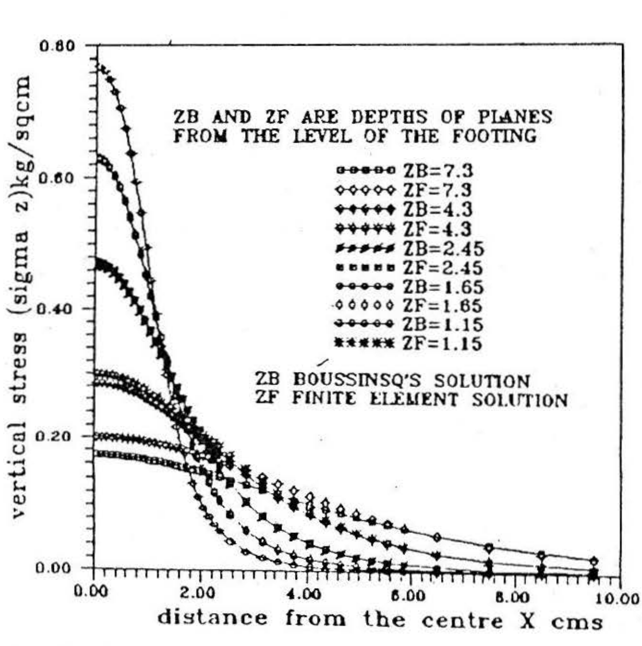


FIGURE 4 : Vertical Stress Distribution along Different Horizontal Planes by Boussinesq's and Finite Element Solution

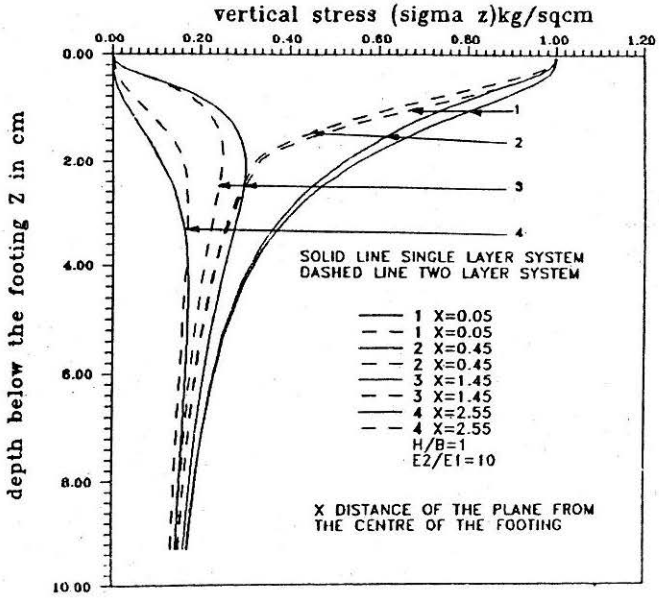


FIGURE 5 : Comparison of Vertical Stress Distribution versus Depth (z) for Single Layer and Two Layer Soil System

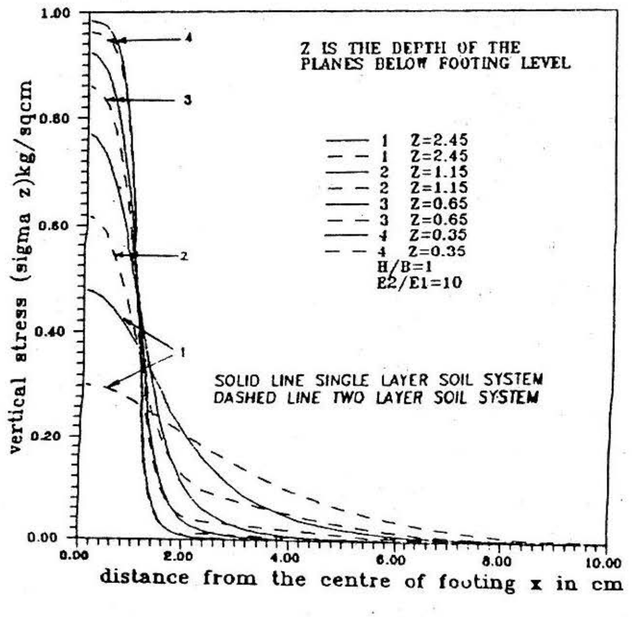


FIGURE 6 : Comparison of Vertical Stress Distribution versus Horizontal Distance (x) for Single Layer and Two Layer Soil System

for sections away from the center line, it is higher for two layer system. As stress distribution for two layer system is different when compared to single layer semi-infinite system, the stress distribution for two layered system has to be considered in calculating nondimensional parameters to estimate BCR.

Analysis of Reinforced Soil Beds as Two Layer Soil System

The method chosen to analyse the behaviour of two layered soil system is based on theoretical analysis as suggested by Binquet and Lee (1975). Figures 7(a to 7(c) show the assumed mechanism of failure, components of force and nondimensional force and length parameters respectively for single layer system as proposed by Binquet and Lee. Similarly for the analysis of two layered soil system bearing capacity calculation requires an evaluation of three key forces for the selected design parameters such as,tensile force developed in the reinforcement (T_D), breaking resistance of reinforcement (R_y) and pullout resistance of the reinforcement (T_f) for each of the reinforcing layers. For detailed explaination one can refer Binquet and Lee

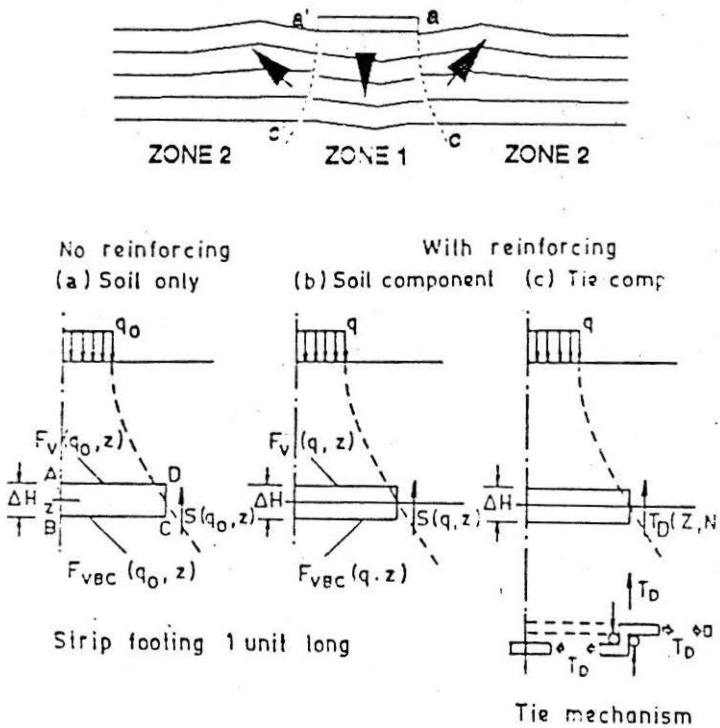
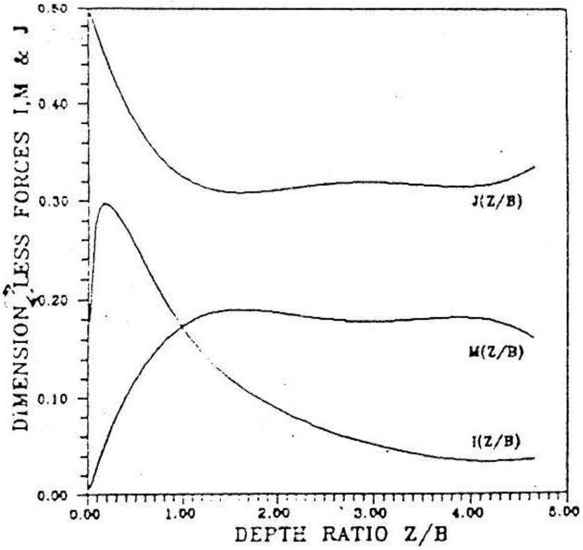


FIGURE 7 : Reinforced Soil Bed in a Single Layer System
(a) Assumed Failure Mechanism
(b) Components of Force in Bearing Capacity Calculations



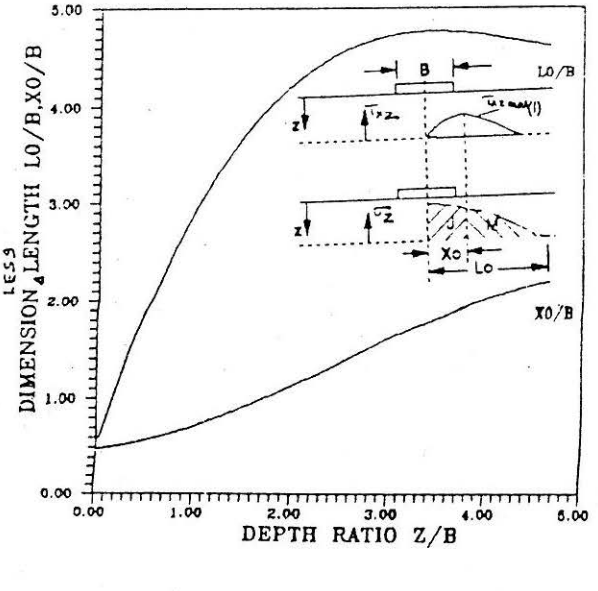


FIGURE 7 : Reinforced Soil Bed in a Single Layer System
(c) Non-Dimensional Force and Length Parameters
(After Binquet and Lee, 1975)

(1975), however the relevant equations for T_D, R_y and T_f are given as below.

The tensile force developed in the reinforcement is :

$$T_D(z, N) = \frac{1}{N}\left[J\left(\frac{z}{B}\right) B - I\left(\frac{z}{B}\right) \Delta H\right] q_0 \left(\frac{q}{q_0} - 1\right) \tag{1}$$

in which, N = number of reinforcement layers,
ΔH = spacing between the reinforcement,
z = depth of the reinforcement layer from the footing, and
q/q_0 = required bearing capacity ratio.

Nondimensional parameters J and I are given as follows :

$$J\left(\frac{z}{B}\right) = \frac{\int_0^{X_0} \sigma_z\left(\frac{z}{B}\right) dx}{q\,B} \tag{2}$$

$$I\left(\frac{z}{B}\right) = \frac{\tau_{xz\,\max\left(\frac{z}{B}\right)}}{q} \tag{3}$$

The breaking resistance of reinforcement (Ry) is given by :

$$R_y = \frac{w\ N_R\ t\ f_y}{F\,S_y} \tag{4}$$

in which, N_R = number of ties per unit length,
$F\,S_y$ = factor safety,
w = width of a single tie,
t = thickness of a single tie, and
f_y = yield or breaking strength of the tie material.

The pullout resistance of the reinforcement T_f is given by :

$$T_f(z) = 2\ f\ LDR\left[M\left(\frac{z}{B}\right) B\ q_0 \left(\frac{q}{q_0}\right) + \gamma\left(L_0 - X_0\right)(z + D)\right] \tag{5}$$

in which,

f = soil tie coefficient ($\tan\phi_f / FS_f$; where ϕ_f is the angle of internal friction, F S_f is the factor of safety against friction),

D = surcharge load of soil of depth above the base of the footing,

y = density of the overburden soil, and

LDR = Linear Density Ratio, is the total width per unit length of tie.

Nondimensional parameter M(z/B) in Eqn. 5 is given by :

$$M\left(\frac{z}{B}\right) = \frac{\int_{X_0}^{L_0} \sigma_z\left(\frac{z}{B}\right) dx}{q\,B} \quad (6)$$

Comparison of the two allowable tie resistance forces R_y and T_f (Eqns. 4 and 5) with the driving force T_d (Eqn. 1) will define whether the tie pullout or tie breaking is the most critical for that depth. Assuming a stronger reinforcement, tie pull out will become the criteria for design by equation $T_D(z, N)$ to $T_f(z)$ for each layer. BCR for each layer can be written as :

$$BCR = \frac{q}{q_0} = \frac{J\left(\frac{z}{B}\right) B - I\left(\frac{z}{B}\right) \Delta H + \frac{2\ f\ N\ LDR\ \gamma}{q_0} (L_0 - X_0)(z + D)}{J\left(\frac{z}{B}\right) B - I\left(\frac{z}{B}\right) \Delta H + 2\ f\ N\ LDR\ B\ M\left(\frac{z}{B}\right)} \quad (7)$$

For any set of design parameters, the two tie forces T_D and T_f are functions of BCR. Along with design parameters, nondimensional parameters I(z/B), J(z/B), M(z/B), X_0/B and L_0/B completely define the bearing capacity ratio of reinforced soil beds. In the present analysis the result of finite element runs on two layer soil system have been made use of in determining these nondimensional parameters for various combinations of stiffness of the layers (E_2/E_1), reinforcing depth (z/B) and depth of upper layer (H/B). The results are presented in the following section.

Results and Discussions

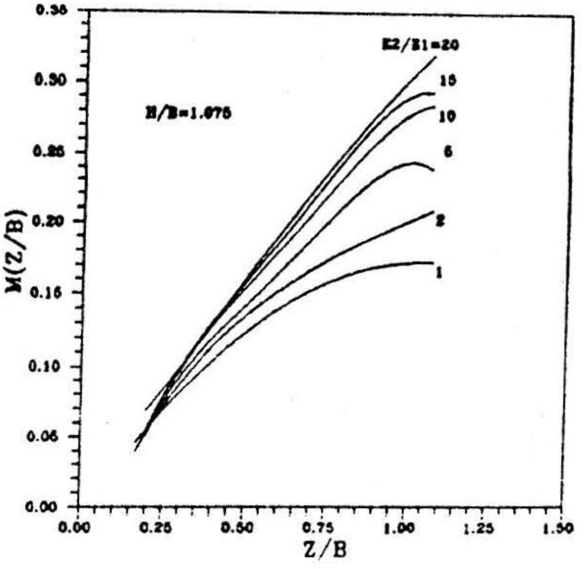
For evaluation of the nondimensional force parameters (M(z/B), J(z/B) and I(z/B) for two layer soil systems, the reinforcement layers are assumed to be always within the upper layer. The depth of upper layer is varied form 0.275 B to 1.075 B. For each depth of the upper layer (H/B) different depths of the reinforcing layer (z/B) have been considered for evaluation of nondimensional parameters. The ratio of stiffness (E_2/E_1) or the upper to lower soil layer is varied from 1 to 20.

In the present analysis, the finite element results are utilized to estimate the stresses. Nondimensional parameters M(z/B), J(z/B) and I(z/B) are calculated using Eqns. 2, 3 and 6. The nondimensional parameters M(z/B) and I(z/B) along with nondimensional length parameter L_0/B have been presented in Figs. 8, 9 and 10 respectively. In the present analysis the non-dimensional parameter J(z/B) is not presented, however one can calculate J(z/B) as [q – M(z/B)] where q is the uniform contact pressure on the footing. The values of nondimensional parameters evaluated for E_2/E_1 = 1, compare well with that of *Binquet and Lee (1975) which is given in the* Fig. 7c.

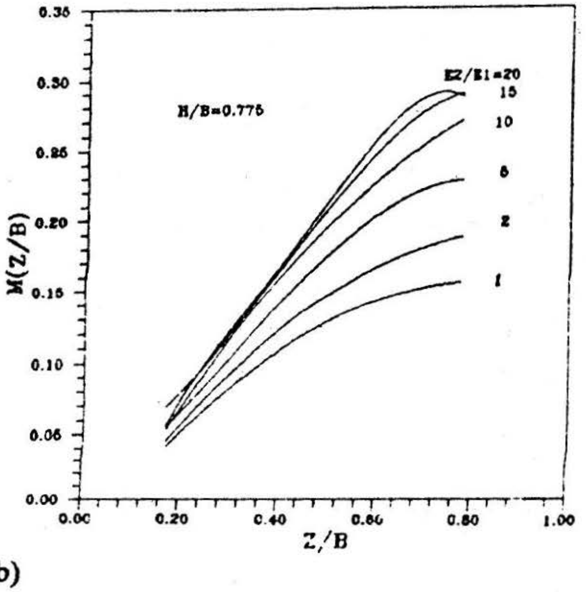
The variation of nondimensional parameter M(z/B) with depth of reinforcing layer z/B for a particular depth of upper layer (H/B) and stiffness ratio (E_2/E_1) are presented in Figs. 8(a) to 8(d). The variation of M(z/B) with depth of reinforcing layer (z/B) unlike in single layer system depends much on the stiffness ratio, (E_2/E_1). It can also be concluded that the depth of upper granular layer (H/B) has marked influence on the nondimensional parameter M(z/B). It is evident from Figs. 8(a) to 8(d) that there is an optimum value of depth of granular layer (z/B) for which improvement in parameter M(z/B) is maximum. Increase in M(z/B) will increase pullout resistance (T_f) of reinforcement and in turn the bearing capacity ratio of two layer system. It can be seen from the figures that if the value of z/B is quite small (reinforcing depth is shallow), the high stiffness of the upper layer has a limited effect. However, as the reinforcing depth (z/B) increases then the value of M(z/B) increases with the increase in stiffness ratio (E_2/E_1).

The variation of the nondimensional parameter I(z/B) with reinforcing depth (z/B) for the two layered soil systems are presented in Figs. 9(a) to 9(d). The value of I(z/B) for two layered soil system is higher than that of the single layer soil system up to the thickness of the upper layer (z/H = 1), beyond which it decreases in comparison to single layer system. Figure 10 shows the variation of L_0/B with depth z/B. The value of L_0/B increases with the increase in stiffness ratio E_2/E_1. for two layer soil system the value of the nondimensional length parameter (x_0/B) does not vary much compared to that of single layer system except for very high stiffness of upper layer. Thus, we can use Fig. 7c for finding x_0/B values for two layer soil system.

The design parameters such as B, ΔH, N and t etc., used for the calculation of bearing capacity and the obtained results for both single layer and two layer soil system are presented in Table 1, for comparison. Nondimensional force parameters estimated from charts in arriving at the solution is also presented in Table 1. From Table 1, it is observed that, the

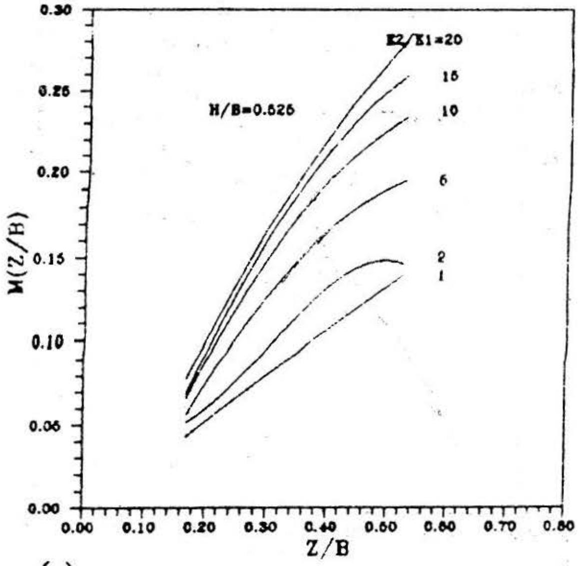


(a)

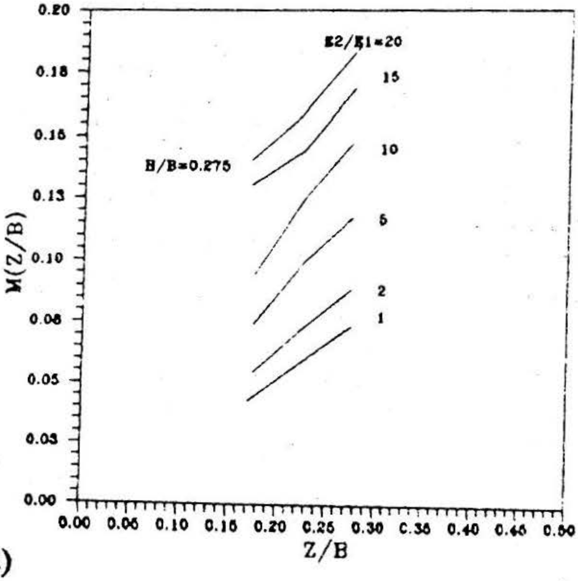


(b)

FIGURE 8 : Plot of M(z/B) versus z/B for Two Layer System (a) H/B = 1.075, (b) H/B = 0.775

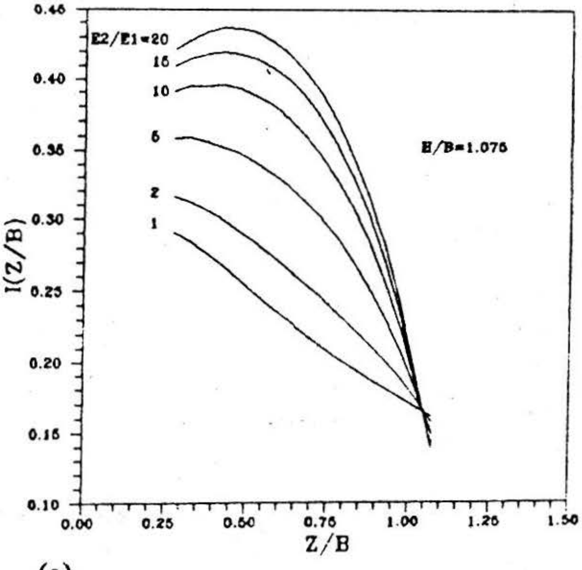


(c)

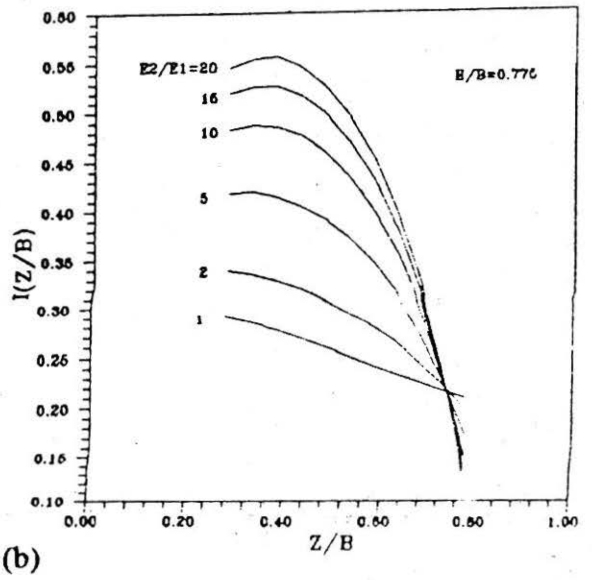


(d)

FIGURE 8 : Plot of M(z/B) versus z/B for Two Layer System (c) H/B = 0.525, (d) H/B = 0.275

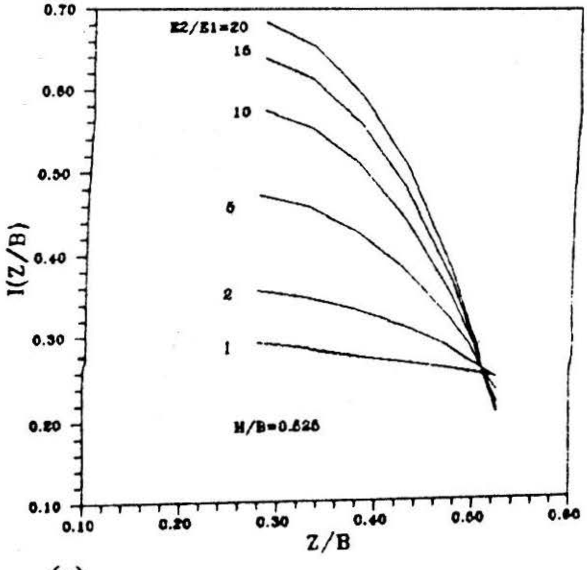


(a)

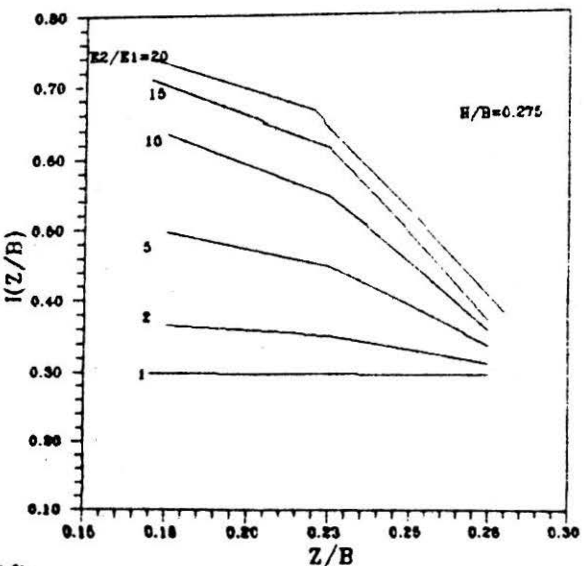


(b)

FIGURE 9 : Plot of I(z/B) versus z/B for Two Layer System (a) H/B = 1.075, (b) H/B = 0.775

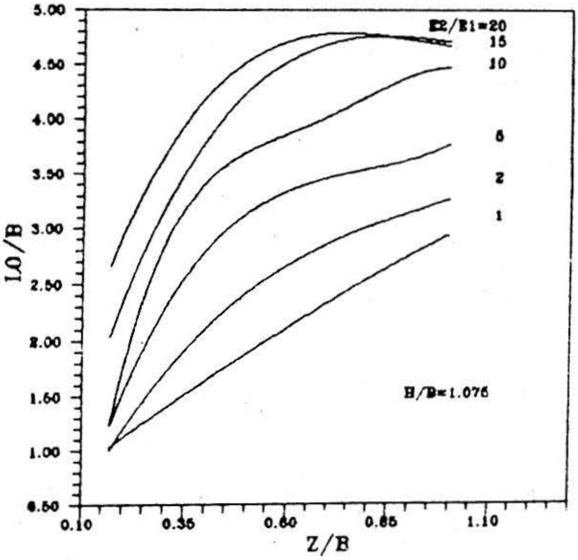


(c)

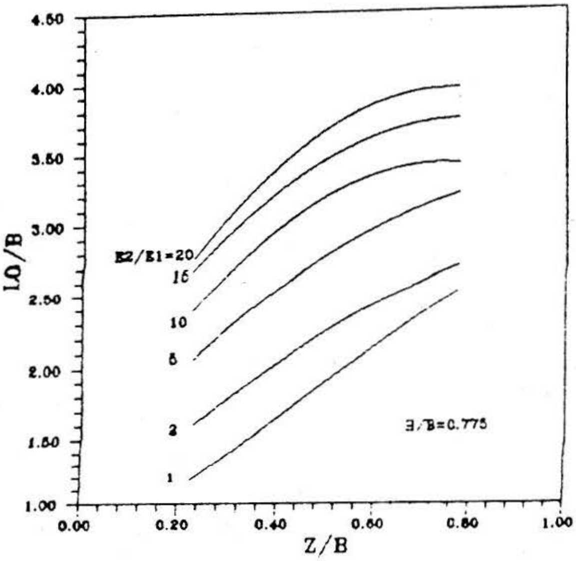


(d)

FIGURE 9 : Plot of I(z/B) versus z/B for Two Layer System (c) H/B = 0.525, (d) H/B = 0.275

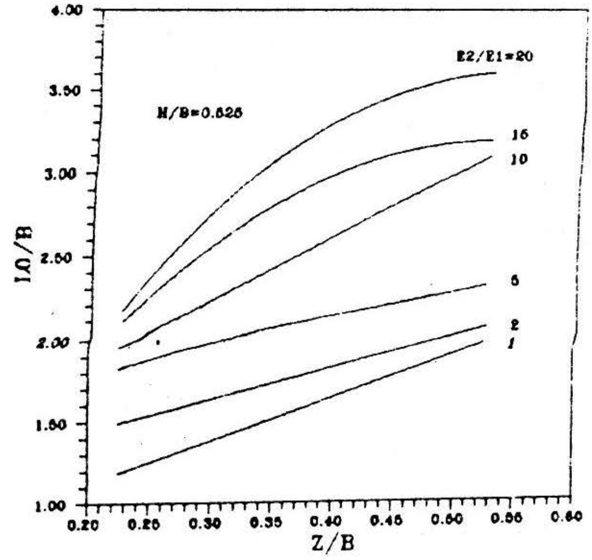


(a)

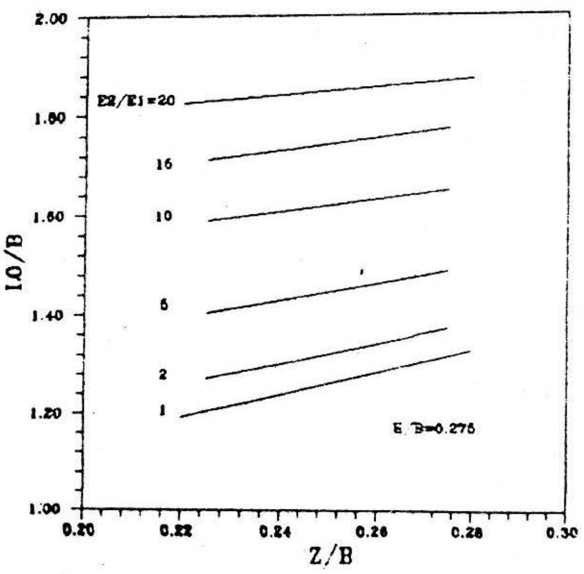


(b)

FIGURE 10 : Plot of L_0/B versus z/B for Two Layer System (a) H/B = 1.075, (b) H/B = 0.775



(c)



(d)

FIGURE 10 : Plot of L_0/B versus z/B for Two Layer System (c) H/B = 0.525, (d) H/B = 0.275

values of L_0/B for two layer soil system are always higher than that of the single layer soil system. Nondimensional parameter L_0/B is the one, which contributes to the increase in bearing capacity. The obtained L_0/B values as presented in Table 1 are always lower than the width of the unpaved road. It is general practice to provide the reinforcement for the whole width of the road. thus the presented L_0/B values in the Table 1 are always conservative. This concept will be very useful in the analysis of reinforced unpaved roads.

Conclusions

From finite element method of analysis of two layered soil system below a strip footing, it has been shown that the vertical normal stress distribution in the upper layer is totally different from that of semi- infinite soil system. In the analysis of two layered soil system, this variation in the stress distribution has been considered. A simple analytical method as proposed by Binquet and Lee for analyzing a reinforced single layer soil system beneath a strip footing has been extended for two layered system. It is observed that nondimensional parameters for two layer soil system are distinctly different from that of a single layer. this change in nondimensional parameters will yield higher values of bearing capacity of the two layer soil system. It is also observed that, the thickness of the upper layer plays a vital role in the increase in bearing capacity of the two layer soil system. The analysis throws light on the fact that, using stiffer layer of soil over a soft clay improves the bearing capacity of the system.

Table 1. Summary of Bearing Capacity Calculations.

	One layer system $f_l = 0.155$ $\gamma = 1.6\ t/m^3$ $E_2/E_1 = 1$			Two layer system $f_2 = 0.20$ $\gamma_2 = 2.0\ t/m^3$ $\gamma_1 = 1.6\ t/m^3$ $E_2/E_1 = 5$		
Reinforcement layer	1	2	3	1	2	3
z/B	0.275	0.525	0.775	0.275	0.525	0.775
M(z/B)	0.059	0.139	0.145	0.082	0.155	0.189
J(z/B)	0.441	0.361	0.355	0.418	0.345	0.31
I(z/B)	0.297	0.249	0.213	0.358	0.342	0.3
L_0/B	1.1	1.6	2.25	2.4	3.1	3.5
X_0/B	0.525	0.525	0.7	0.525	0.6	0.7
q/q_0	1.12	1.69	4	1.705	4.55	7.31

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