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Determination of Stress Distribution under an Annular Footing

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Introduction

Circular foundations are generally provided for tall circular structures like overhead water tanks, cooling towers, smoke stacks and silos etc. The circular footing may either be solid circular or annular. In case of annular footings, the difference between maximum and minimum pressure is less as compared to solid circular footings. Therefore, a structure supported over a solid circular footing may tilt and undergo excessive settlement as compared to annular footing. It is due to these reasons that annular footing is preferred over solid circular footing. The problem of calculating the stresses due to annular footing has been treated by several investigators (Egorov, 1977; Kakroo, 1985) theoretically, assuming the soil as elastic material. To the knowledge of author, no analytical method for calculating the stress below annular footing exists besides only conservative approximation. In this study, tests have been conducted using circular and annular footings of external diameters of 400 mm having different annularities.

The stress analysis below annular footing has been carried out, the isobars plotted and compared with that of circular footing. The use of numerical Integration has also been suggested for determination of stresses under a uniformly loaded annular footing.

Stress Analysis

The bearing capacity of soil-foundation system is governed mostly by settlement criteria in case of sandy soils. The settlement of the foundation depends on the stress condition in the soil below the foundation. Boussinesq's classical equation assumes the material to be elastic, homogeneous and

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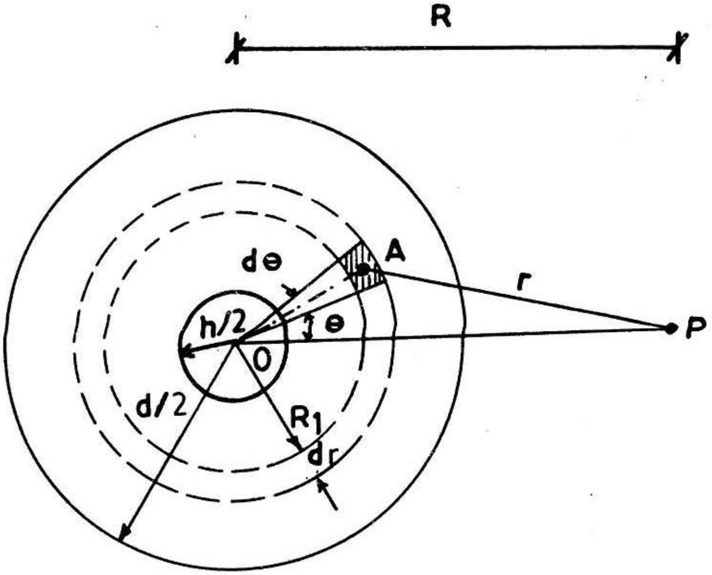

FIGURE 1 : Plan for Stress Below a Point Lying Outside Circular Area

isotropic. Though the soils are not truly elastic, yet the equation has profusely been used in Geotechnical Engineering. Numerical integration method has been used for the determination of stresses under a uniformly loaded annular footing.

Numerical Integration Method

Stresses in the soil under an annular footing carrying uniformly distributed load of intensity q are evaluated numerically using Boussinesq relationship. Annular footing is divided into concentric annular rings of thickness dr and inner radius being R_1. A small element in this annular ring subtending an angle dθ at the centre is considered as a point load for the evaluation of stresses at a general point P in the soil (Fig. 1). The point P is located at a depth z and is at a radial distance R. Total effect of the annular loaded area is obtained by integrating the stress due to elemental load over whole of the loaded area as given below.

$$\sigma_z = \left(\frac{3q}{2\pi}\right) \int_{R_1=h/2}^{d/2} \left[\int_0^2 \frac{R_1\, d_r}{\left\{1 + \left(r/z\right)^2\right\}^{5/2}}\, d\theta \right]$$

$$= \left(\frac{3q}{\pi}\right) \int_{R_1=h/2}^{d/2} \left[\int_0^{\pi} \frac{R_1\, d_r}{\left\{1+\left(r/z\right)^2\right\}^{5/2}}\, d\theta \right] \tag{1}$$

where $r = \sqrt{R^2 + R_1^2 - 2\,R\,R_1 \cos\theta}$

The above Eqn. (1) has been integrated numerically by converting it into the following form :

$$\sigma_{z/q} = \left(\frac{3}{2\pi}\right) \left[\sum_{R_1=h/2}^{d/2} \sum_{\theta=0}^{180} \frac{R_1}{\left\{1+\left(r/z\right)^2\right\}^{5/2}} \right] \frac{\delta r\, \delta\theta}{z^2}$$

$$= \left(\frac{3}{\pi}\right) \left[\sum_{i=0}^{n_1} \sum_{j=0}^{n_2} \frac{R_1 + i\,\delta r}{\left\{1+\left(r/z\right)^2\right\}^{5/2}} \right] \frac{\delta r\, \delta\theta}{z^2} \tag{2}$$

where n_1 and n_2 are the number of dimensions in radial and circumferential direction respectively. A computer programme has been developed, using the above algorithm.

Experimental Details

Size and Rigidity of Model Footings

Three sizes of mild steel model footings of 200 mm, 300 mm and 400 mm external diameter and annularity ratio i.e h/d = 0.0, 0.3, 0.4, 0.5, 0.6 and 0.7 were chosen. The thickness of model footing was decided on the basis of rigidity criteria laid down by Indian Standard Code IS : 2950 (Part I) 1971. The model footings were prepared using 20 mm thick mild steel plates so that the footings behave as rigid footings. The annular plate was mounted with a similar solid plate with the help of 100 mm × 40 mm × 20mm vertical legs for transferring the load to the footing plate. The steel plate at the top was grooved to accommodate a ball such that the model can be centered with the proving ring and load be applied concentrically. To simulate the roughness of the actual footings, the base was made rough according to IS : 1888 (1982).

Experimental Box

A rigid steel tank 2.0 m × 2.0 m × 1.0 m internal dimensions was designed and prepared for accommodating the bed of sand. The size of the tank was selected in order to keep the rupture zones and pressure bulb within boundaries. The tank was prepared with 4 mm thick steel plate and angle iron of 35 mm × 35 mm size. The steel tank was kept on a steel girder portal self straining loading frame which was designed for the purpose of loading arrangement. The tank was rested on four steel girders of loading frame 150 mm above the ground.

Preparation of Sand Bed

Vibration technique cannot be used for obtaining a uniform density of sand when earth pressure cells are to be embedded in it. Raining technique is quite suitable in such conditions (Walker and Whitkar, 1967). The density of sand in the raining technique depends upon the intensity of raining and the height of fall. A special type of raining equipment was designed and fabricated which consisted of container (2.50 m × 0.3 m × 0.15 m) made of mild steel plates having perforated plate at its base. The container was mounted on trolly which could move on a track laid at a suitable height above the tank.

The trolley was brought near one end of the tank and the container was filled with air- dry clean sand. The trolley was then manually moved slowly. As the trolley moved the sand was deposited in the tank by raining in layers of 100 mm. The intensity of raining was constant since the size and spacing of perforations were fixed. A large number of preliminary tests were conducted to obtain a relationship between the height of fall and the dry density of sand.

Pressure Cells

The strain gauge based pressure cells have been used for the measurement of stresses at various depths below the annular footing. The pressure cell was cut out of a stainless steel plate of 10 mm thick with over all finished thickness 9 mm with 25 mm external diameter. The pressure cells used here are of 400 kPa range. These cells have been used for measuring of stresses under static conditions. They are basically designed for application requiring flush diaphragm. Four conductor shielded cable terminates four arms of the wheat stone bridge formed by strain gauges bonded to the stainless steel diaphragm. In this study, eight pressure cells were used for measurement of pressure at various depths.

Calibration of Pressure Cells

Every pressure cell has been supplied with its calibration factor in terms of Kg/cm^2/micro strain per unit. This strain gauge based pressure cell was tested and calibrated on a precision Dead Weight Pressure Gauge Tester. The calibration data was available in terms of microstrain of output.

Switching Balancing Unit

Monitoring of data at many points one by one was served by versatile switching and balancing unit supplied by New Engg. Enterprises, Roorkee (India).

Universal Indicator

A carrier excited digital indicator was used to display outputs of pressure cells. It consists of a stable sine wave oscillator which provides excitation to the pressure cell and reference to the phase sensitive demodulator. Its highly sensitive carrier amplifier condition, the small amplitude signals provide a virtually drift free amplification. Its 3.5 digit display meter can be adjusted by front panel controls to give direct reading of measured physical or mechanical parameters.

Measurement of Pressure in the Soilmass

For determination of stresses in the sand at various depths below the footing, eight pressure cells were embedded at depths of 0.2 q and 0.5 q i.e. significant depth where q is the intensity of pressure below the surface of the footing depending upon the size and annularity of the footing. These pressure cells of 25 mm external diameter were placed on sand with their diaphragm facing towards bottom. As soon as the required level of sand was attained during the process of deposition of sand, four leads of the strain gauge of pressure cells were taken out horizontally at each level towards the side wall of the tank. The process of deposition of sand was continued after the pressure cells had been placed. These pressure cells were connected to a Switching Balancing Unit (SBU) and the SBU was connected to universal indicator digital display for displaying output of the pressure cells. It was fabricated in such a way that it can be used for placing the proving ring and jack at the centre of the tank so that the load application by hydraulic jack would always be on the centre of the tank.

Loading Arrangement

The steel girders were welded to suitably designed portal frame. A steel joist was bolted across the steel girders to support the reaction of

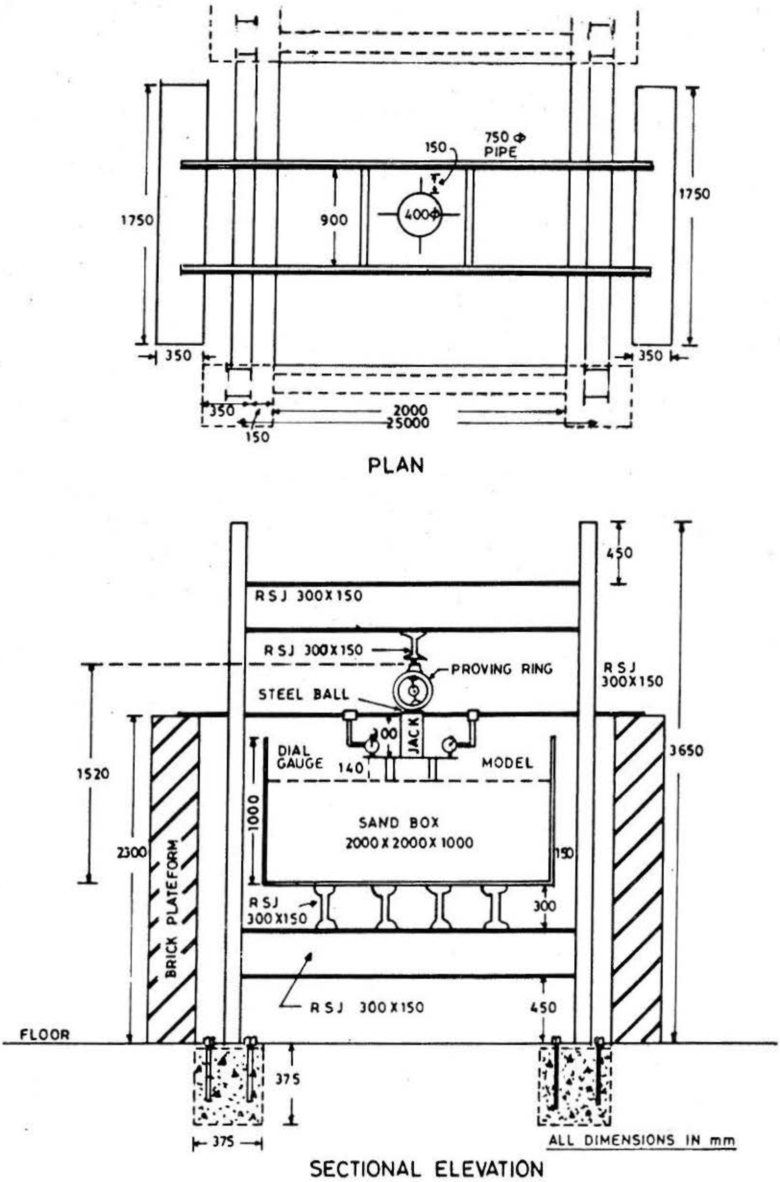

FIGURE 2 : Details of Experimental Set-up

hydraulic jack. The jack was located just above the centre of the footing. A proving ring of 50 t capacity was used to measure the load applied. Loads were applied to the footing through a remote control hydraulic jack (Fig. 2).

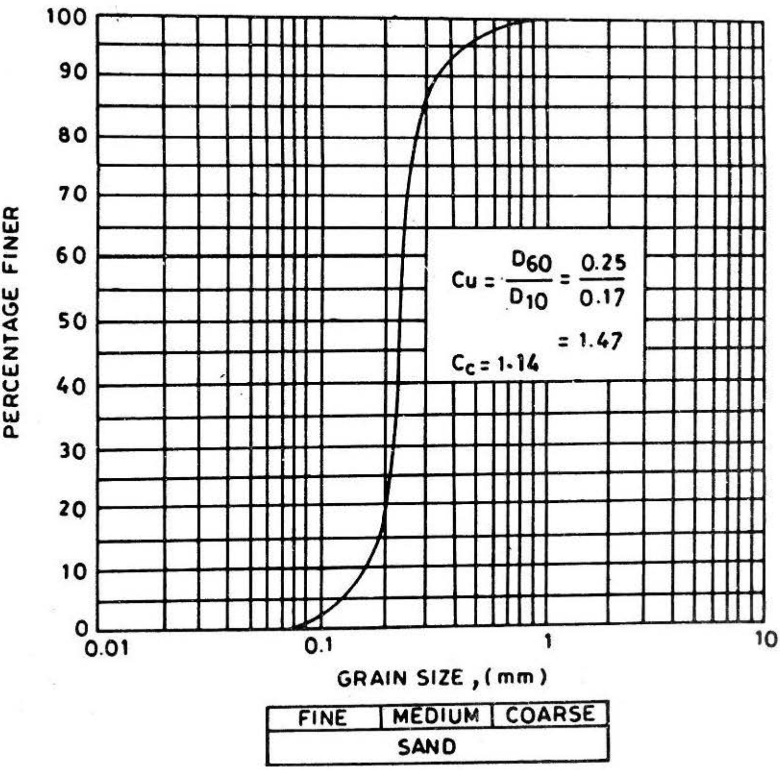

FIGURE 3 : Particle Size Distribution for Sand

Soil Used

In this study, medium, uniform river sand was used. The grain size distribution curve is shown in Fig. 3. The properties of the sand used are as follows :

Uniformity coefficient = 1.47
Effective size = 0.19 mm
Coefficient of curvature = 1.14
Angle of internal friction, ϕ = 42°
Average bulk unit weight, γ = 16.08 kN/M^3

According to Indian standard code IS : 1498 - 1970, the soil is poorly graded sand (SP). The angle of shearing resistance was obtained from triaxial shear test for confining pressure from 50 kPa to 100 kPa.

The leads in 4 numbers from each pressure cells (2 for excitation and

2 for output) were taken outside. The SBU consists of 10 channels. Eight channels were used for eight pressure cells. The SBU was connected to the digital universal indicator. After balancing the universal indicator and SBU, the stress was increased by recording the reading which appeared on digital display and multiplied by the respective calibration factor of pressure cell.

After the maximum load had been applied, the jack pressure was released gradually. The footing was then removed. After every test, the tank was emptied. The sand removed was dried, sieved and re-deposited.

Results and Discussion

The stresses were computed at various depths and distances for different annularity ratios 0.3, 0.4, 0.5, 0.6 and 0.7 for 400 mm outer diameter annular footings (Shah, 1994). The stresses at different depths for h/d = 0.3 have been given in Table 1 and the same have been plotted in the form of isobars in Fig. 4. Isobars have also been drawn for circular footing (h/d = 0.0) of 400 mm diameter, (Fig. 4) for comparing with (h/d = 0.3) annular footing. Similarly other isobars have been drawn for h/d ratios of annular footings (Figs. 5, 6, 7 and 8).

In order to verify the results of the computation of stresses using the principle of numerical integration, the stresses have been measured in the soilmass experimentally by pressure cells under annular footings of diameter 400 mm having annularity ratio 0.3, 0.4, 0.5, 0.6 and 0.7. The results were computed at q = 50 kPa and 100 kPa and verification was also done. The location of pressure cells have been shown in Fig. 9.

The observed vertical stresses measured by pressure cells have been given in Table 2. The experimental values of σ_Z/q have also been compared with the theoretical values of stresses computed by computer program.

There is about 20% difference between theoretical and experimental values but the theoretical values are more than the experimental values. Therefore, the stresses under the annular footing can be predicted safely by this technique.

The observed values of σ_Z/q have also been compared with the theoretical values by plotting graph between σ_Z/q versus z/B (where $B = (d-h)/2$ and z is depth). The comparison of theoretical and measured stresses for 400 mm diameter footing for annularity ratio 0.3 has been shown in Fig. 10. The results show similarity in the value of observed and theoretical stresses.

TABLE 1 : Vertical Stress Under Footings (Theoretically Computed)

Annular Footing

Annularity Ratio = 0.3 mm
External Diameter = 400 mm
Internal Diameter = 120 mm
Radial Distance = 150 mm

Circular Footing

External Diameter = 400 mm

S.No.	Depth (mm)	$\sigma_{z/q}$	
		Annular Footing	Circulation Footing
1.	20.0	0.4679	0.4681
2.	40.0	0.4686	0.4598
3.	60.0	0.4424	0.4460
4.	80.0	0.4234	0.4305
5.	100.0	0.4031	0.4142
6.	120.0	0.3825	0.3976
7.	140.0	0.3622	0.3807
8.	160.0	0.3426	0.3638
9.	180.0	0.3239	0.3471
10.	200.0	0.3061	0.3305
11.	220.0	0.2894	0.3144
12.	240.0	0.2736	0.2987
13.	260.0	0.2587	0.2836
14.	280.0	0.2447	0.2690
15.	300.0	0.2315	0.2551
16.	320.0	0.2191	0.2291
17.	340.0	0.2073	0.2291
18.	360.0	0.1963	0.2171
19.	380.0	0.1860	0.2058
20.	400.0	0.1762	0.1951
21.	420.0	0.1671	0.1850
22.	440.0	0.1585	0.1755
23.	460.0	0.1505	0.1666
24.	480.0	0.1429	0.1582
25.	500.0	0.1358	0.1503

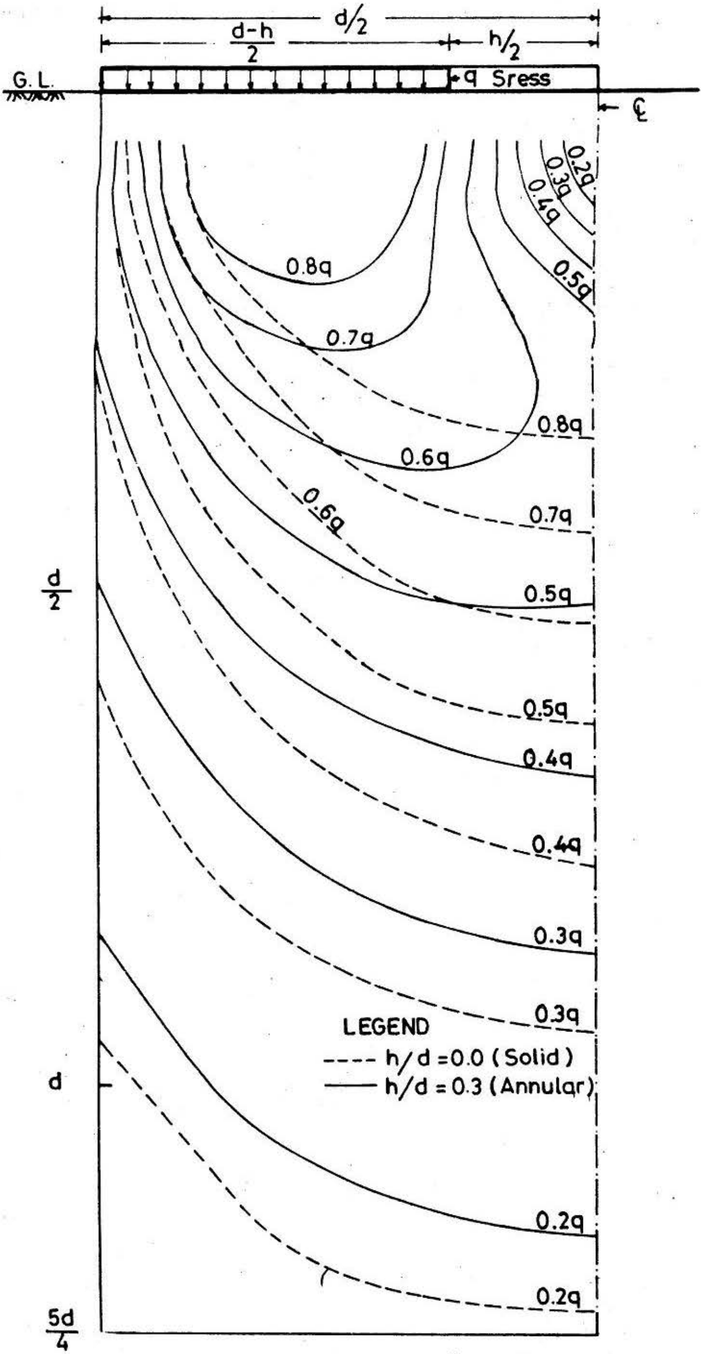

FIGURE 4 : Comparision of Isobars for Solid Circular and Annular Footing of 400 mm Diameter (h/d = 0.3)

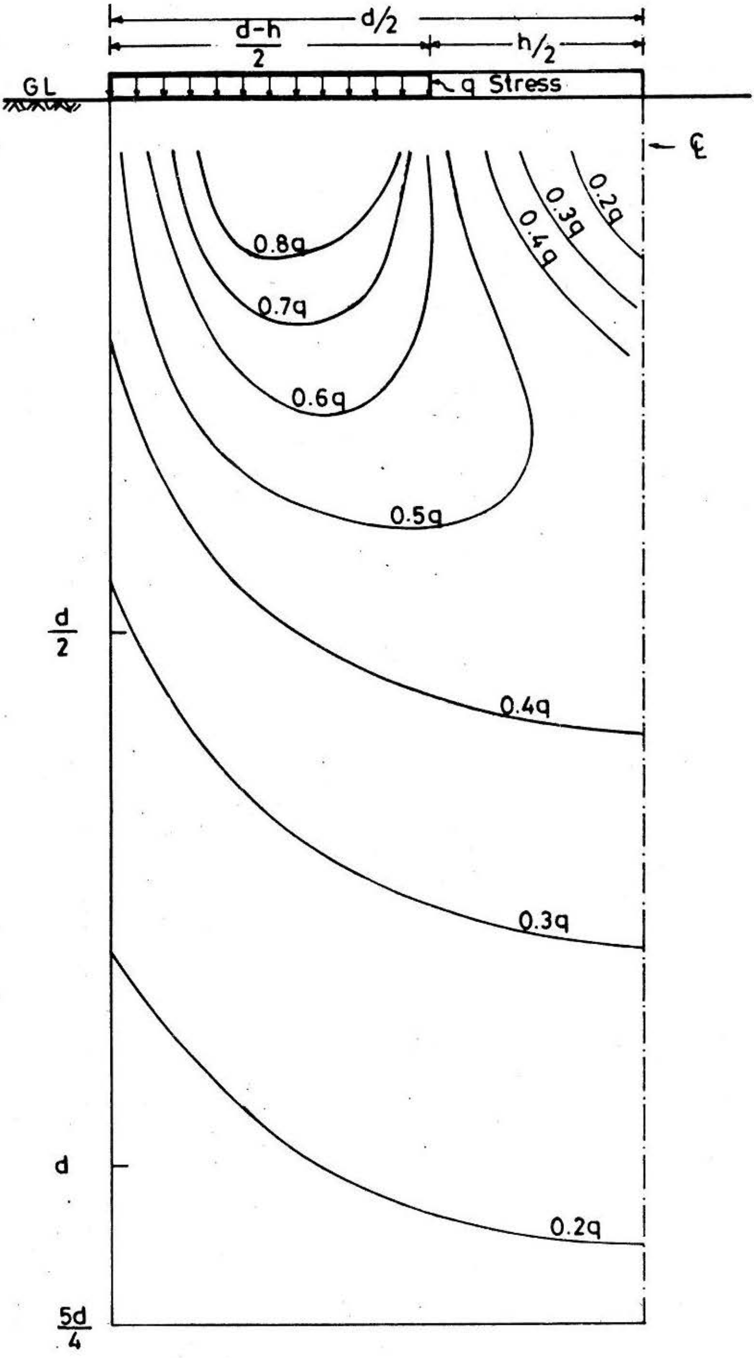

FIGURE 5 : Isobars for Annular Footing of 400 mm Diameter (h/d = 0.4)

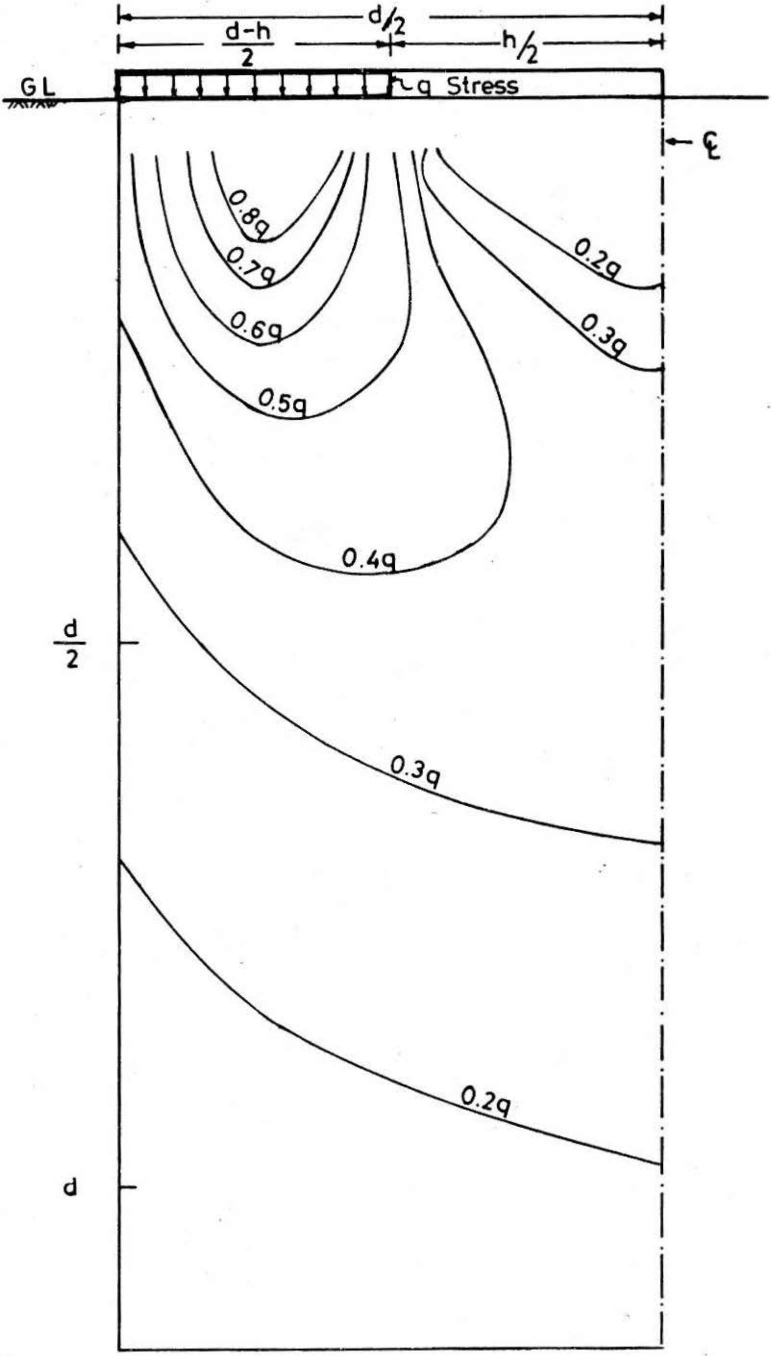

FIGURE 6 : Isobars for Annular Footing of 400 mm Diameter (h/d = 0.5)

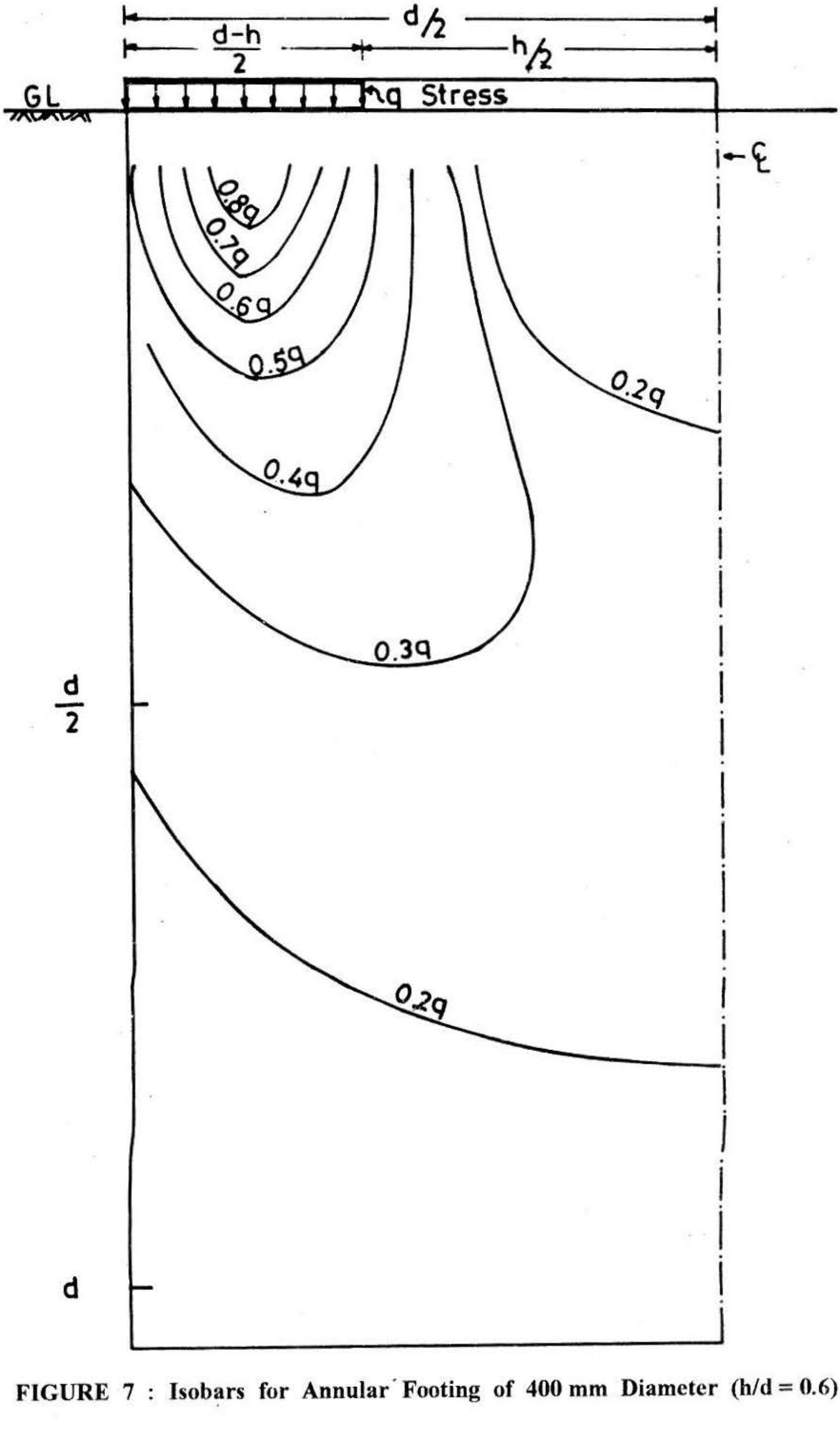

FIGURE 7 : Isobars for Annular Footing of 400 mm Diameter (h/d = 0.6)

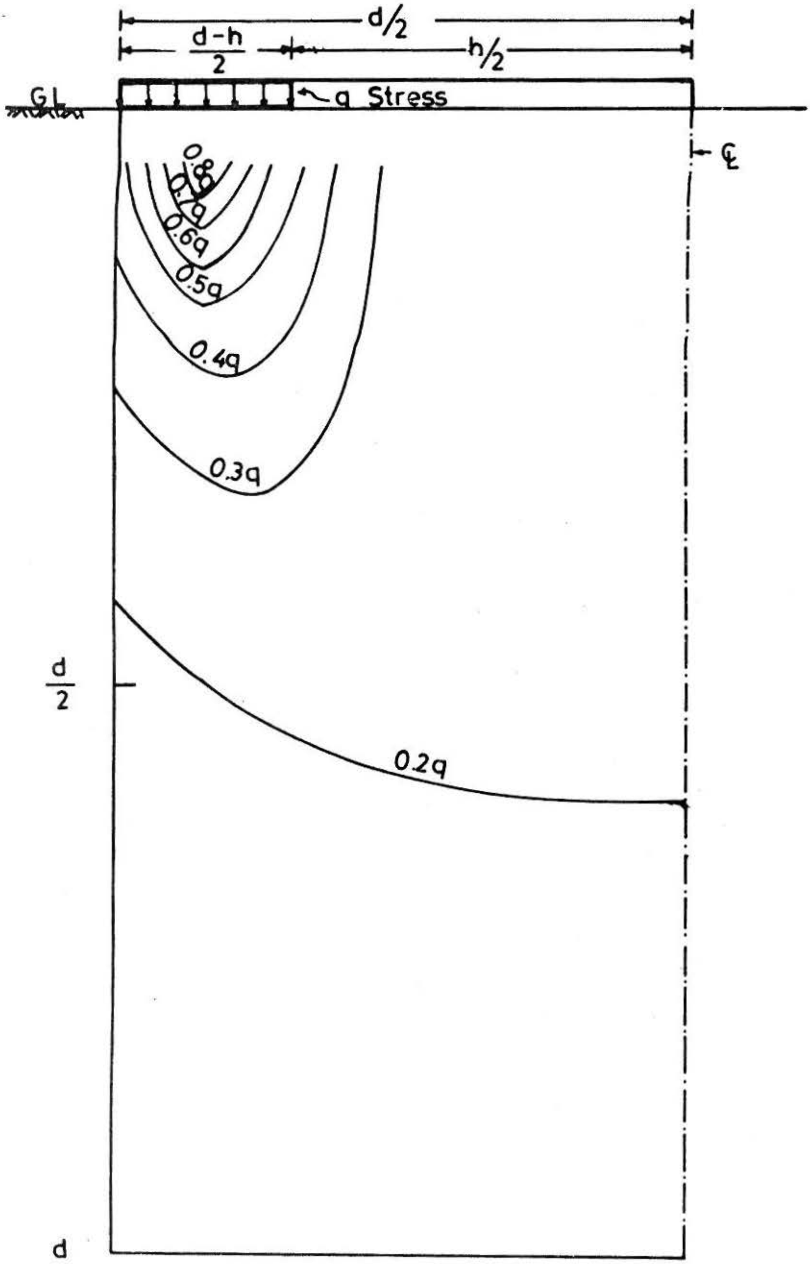

FIGURE 8 : Isobars for Annular Footing of 400 mm Diameter (h/d = 0.7)

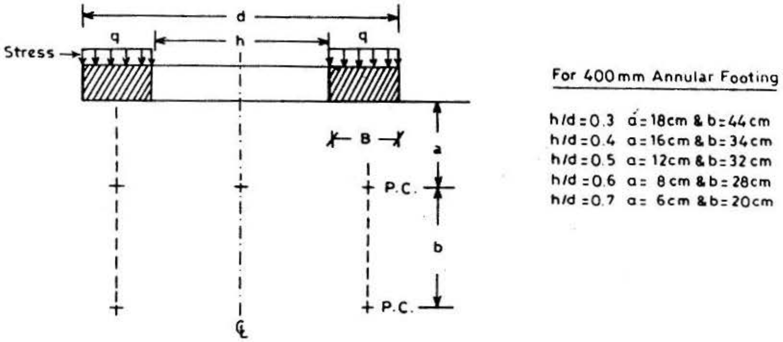

FIGURE 9 : Location of Pressure Cells

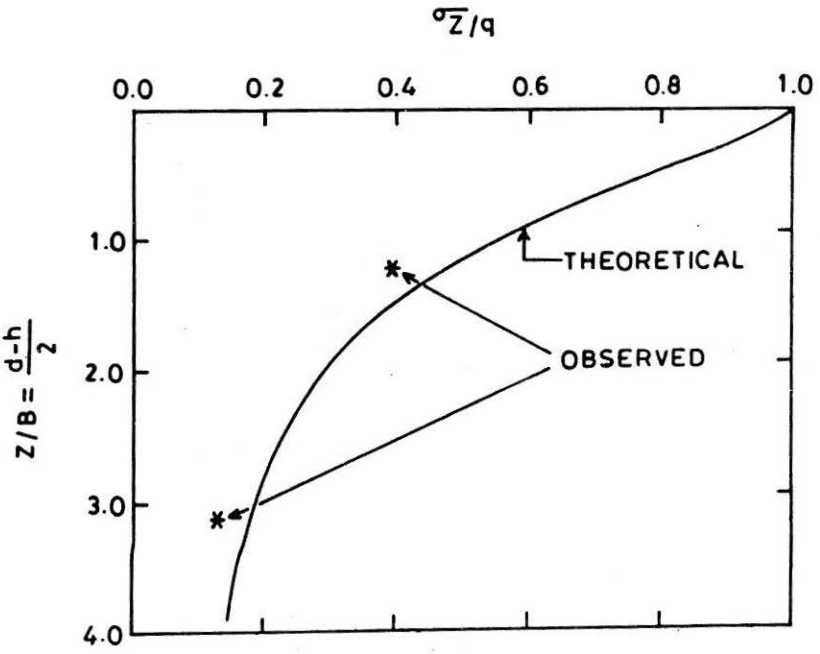

FIGURE 10 : Comparison of Theoratical and Observed Stresses for *400 mm diameter Plate having h/d = 0.3*

Conclusions

The stress analysis below the annular footing shows that the stress concentration occurs near the footings as compared to circular footing where the stresses are dispersed in deep layers. It was deduced from the computed results that the significant depth i.e. the isobars corresponding to 0.5 q and 0.2 q stress decreases due to annularity of the footing.

TABLE 2 : Comparison between Experimental and Theoretical Values of σ_z/q

Annularity Ratio (h/d) = 0.3
External Diameter, d = 400.0 mm
Internal Diameter, h = 120.0 mm

S.No	Location of Pressure Cell		Pressure Cell Used		Universal Indicator Reading	σ_z/q for q = 50kPa	Universal Indicator Reading	σ_z/q for q = 100 kPa
	Depth (mm)	Radial distance (mm)	Numbers	Value of K				
1.	180.0	150.0	1251	0.0222	9.036	0.2006	18.07	0.4012
2.	"	"	1252	0.0290	6.868	0.1992	13.75	0.3985
3.	"	"	1253	0.0225	8.360	0.1881	16.72	0.3763
4.	"	"	1254	0.0548	3.551	0.1946	7.102	0.3892
5.	440.0	"	1255	0.0465	1.447	0.0673	2.892	0.1347
6.	"	"	1257	0.0179	3.977	0.0712	7.908	0.1425
7.	"	"	1258	0.0188	3.452	0.0649	6.904	0.1298
8.	"	"	1259	0.0209	3.181	0.0665	6.368	0.1331

TABLE 3 : Comparison between Experimental and Theoretical Values of σ_z/q

Annularity Ratio (h/d) = 0.3
External Diameter, d = 400.0 mm
Internal Diameter, h = 120.0 mm

S.No	Location of Pressure Cell		Theoratical σ_z/q	Experimental σ_z/q	Average Experimental σ_z/q	Percentage Difference
	Depth (mm)	Radial distance (mm)				
1.	180.0	150.0	0.4480	0.4012	0.3912	14.51
2.	"	"	"	0.3984		
3.	"	"	"	0.3762		
4.	"	"	"	0.3892		
5.	440.0	"	0.1620	0.1346	0.1349	20.08
6.	"	"	"	0.1424		
7.	"	"	"	0.1298		
8.	"	"	"	0.1330		

A software programme has been developed to predict the stress below the annular footing of different annularity ratio at desired depth. The stresses experimentally observed and theoretically computed by software at same depth have been compared. There is about 20% difference between observed and theoretical values of vertical stresses. Theoretical values are on the higher side. Therefore it is safe to adopt the developed software programme for prediction of stresses under annular footing.

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