Technical Note

Dynamic Response of Soil Mass On Two Layered Non-cohesive Soil

by

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Introduction

When poor soil is replaced by compacted fill then optimum effective fill depth (the depth of fill beyond which the increase in resonant frequencies become insignificant) determination become the most important parameter from its design consideration.

A simple combination of analytical and empirical approach has been presented in this note to determine the effective depth within which the vibrating soil mass is resting below the footing under vertical vibration. Moreover an effort has been made to study the nature of variation of resonant frequency with the changes in moisture content of this soil.

Model Tests

A rectangular steel tank of dimension 122cm × 85cm × 115cm with 8mm thick rubber lining was used to perform the model tests. Moreover, 10mm thick thermocol sheets were placed at a distance of 5cm from the lining inside the tank to reduce the erratic reflections from the corners of the tank. To develop the constant amplitude force and to measure the frequency, amplitude, velocity of vibration etc. Vibration exciter SI–230, Power oscillator SI–29, vibration sensor SI–100 and vibration indicator SI–10 of SYSCON was used.

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Model footing tests over sand surface and over composite soil (cinder fill over sand bed) of square and circular footing were made at various (D/B) ratio with different moisture variation.

The variation of resonant frequencies w.r.t. various (D/B) ratio have been shown in Tables 1 and 2 and in Fig. 1 to Fig. 4. Variation of damping co-efficient w.r.t. (D/B) ratio has been shown in Table 3. Here D is the depth of cinder fill over the sand bed and B is the width of the footing for square base and diameter in case of circular base. The nature of variation of resonant frequencies of filling at different moisture content have also been shown in Fig. 1 to Fig. 3.

Determination of Effective Depth of Vibrating Soil Mass below the Footing

Assumption :

1. The soil masses are effectively vibrating below the foundation is equivalent to a spring having finite mass and length.

2. The vibrating soil mass lies below the footing in a truncated pyramidal zone for rectangular footing (as shown in Fig. 5) and in a truncated cone like zone for circular footing).

3. Mass of the spring in distributed property whereas stiffness is a discrete one.

Consider the system as shown in Fig. 6. For simplification, mass of the spring is considered as a distributed property and the stiffness of the spring as a discrete one. Considering the system as conservative, the equation of motion can be derived most conveniently by an energy approach. The displacement of mass m is denoted by x(t), length of the spring is L, then the displacement of an element of mass ρ dr at a distance r from the lower end of the spring can be assumed to be (r/L) x(t). Considering ρ be constant, the system kinetic energy can be written in the form.

$$T = \frac{1}{2} m \dot{x}^2(t) + \frac{1}{2} \int_0^L (r/L)^2 \dot{x}^2(t) \rho \, dr \tag{1}$$

And the potential energy

$$V = \frac{1}{2} x^2(t) \cdot K \tag{2}$$

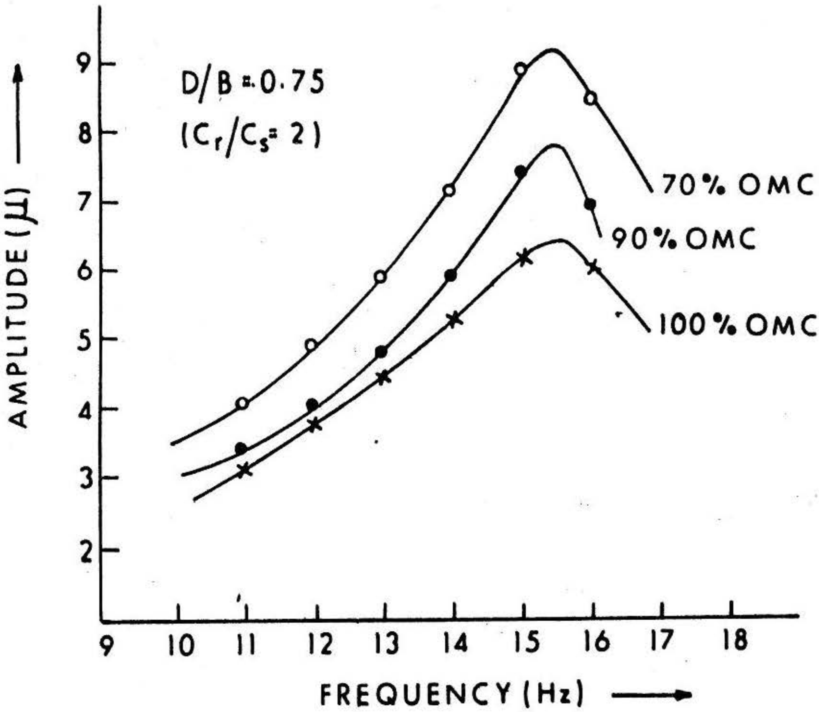

FIGURE 1 Response Curve for Vertical Vibration for D/B = 0.75 (Constant Amplitude Force)

TABLE 1
Magnitude of Resonant Frequency in Different Fill Depth and Moisture Content.

Fill Depth (cm)	D/B	Resonant Frequency at		
		70% OMC (CPS)	80% OMC (CPS)	100% OMC (CPS)
3.75	0.25	13.78	13.86	14.30
7.5	0.5	14.40	14.55	14.65
11.25	0.75	15.40	15.50	15.55
13.5	0.90	15.45	15.55	15.65
15	1.0	15.65	15.65	15.65
18.75	1.25	15.65	15.65	15.65

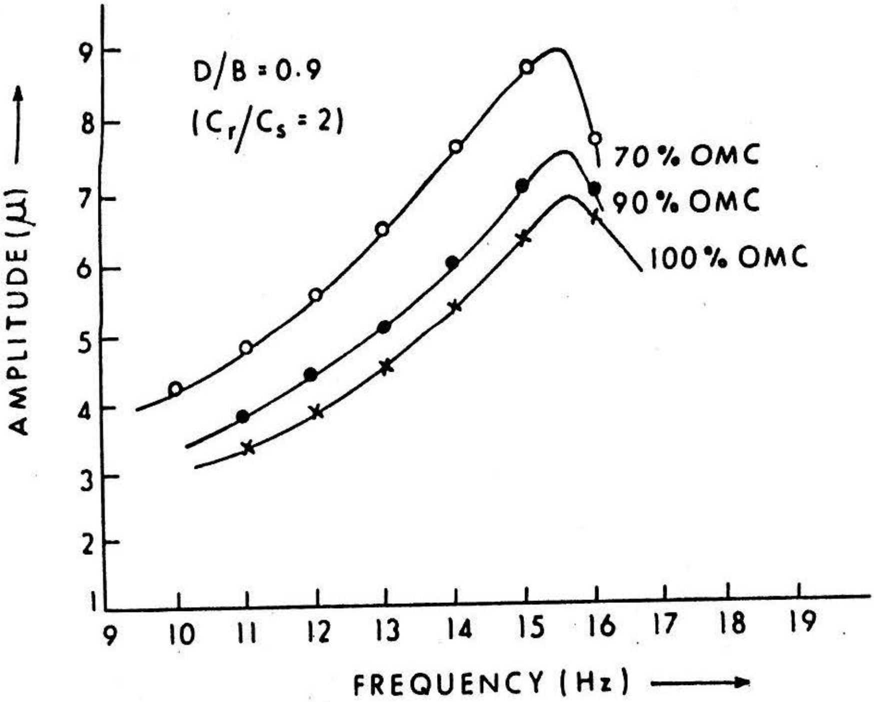

FIGURE 2 Response Curve for Vertical Vibration for D/B = 0.90

TABLE 2

Magnitude of Resonant Frequency with Various (D/B) Ratio for Surface Footings with Circular Base.

D/B	0.0	0.25	0.50	0.75	1.0	1.25
Resonant Frequency (CPS)	13.75	15.10	15.75	16.25	17.25	17.25

TABLE 3

Values of Damping Co-efficient at Different Fill Depth (For Square Footing).

D/B	0.25	0.50	0.75	0.90	1.0	1.25
Damping Co-efficient	0.0592	0.0870	0.1016	0.1096	0.1102	0.1102

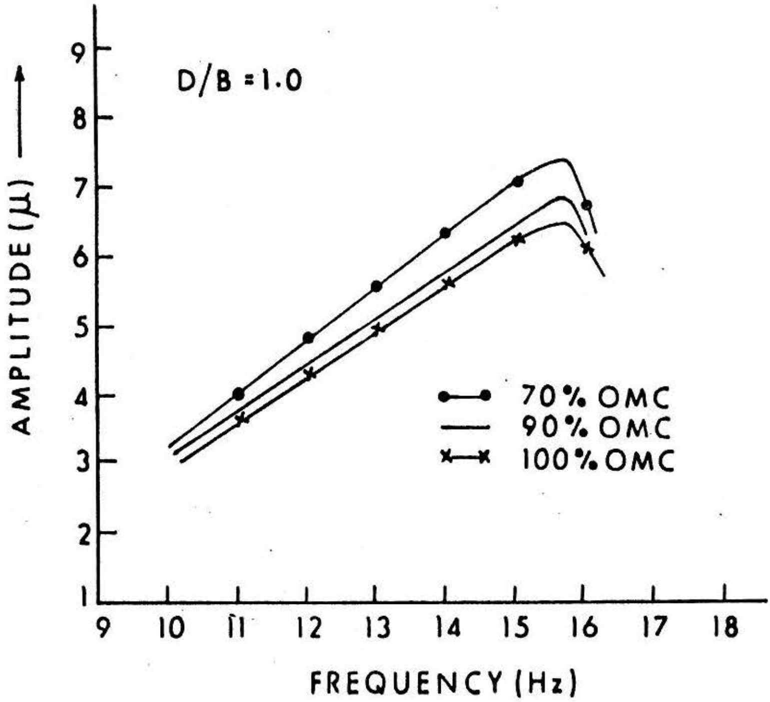

FIGURE 3 Response Curve for Vertical Vibration for D/B = 1.00

∴ Total energy = $E = T + V$

Again $\frac{dE}{dt} = 0$ for conservative system, one can get

$$\left(m+\frac{\rho L}{3}\right)x(t)+K\,x(t)=0 \qquad (3)$$

Where $\left(m+\frac{\rho L}{3}\right)$ is considered as the effective mass of the system. This implies that the spring contributes one-third of its mass to the inertia of the system i.e. the mass of the soil which is vibrating with the foundation can be expressed as $\frac{\rho L}{3}$

According to Pauw (1953) the angle 0 can be assumed as $\tan^{-1}\frac{1}{2}$.

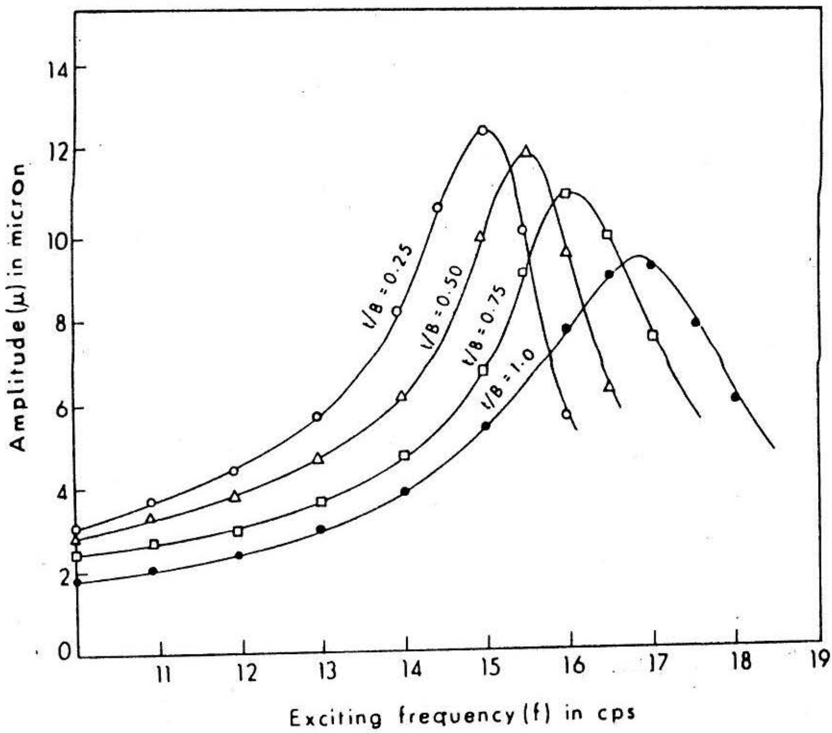

FIGURE 4 Frequency Response Curve for Surface Footings Circular Base on Compacted Fill with D/B Ratios of 0.25, 0.50, 0.75 and 1.00

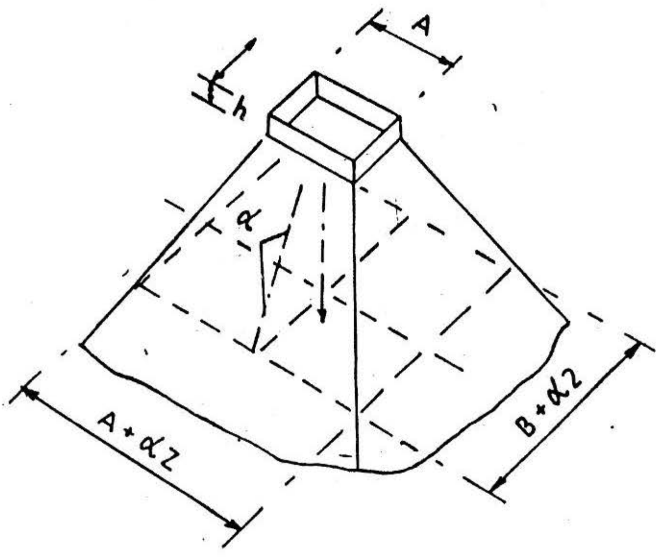

FIGURE 5 Effective Zone of Vibrating Soil Mass Below the Footing

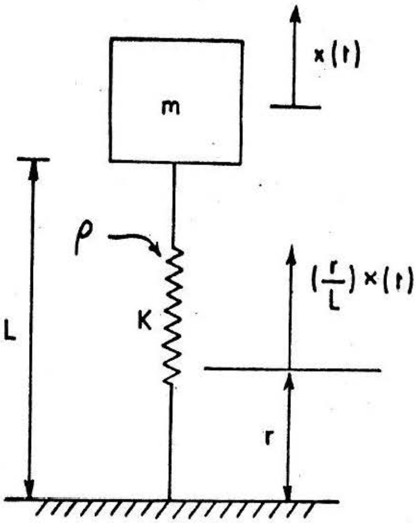

FIGURE 6 Equivalent Mass Spring System

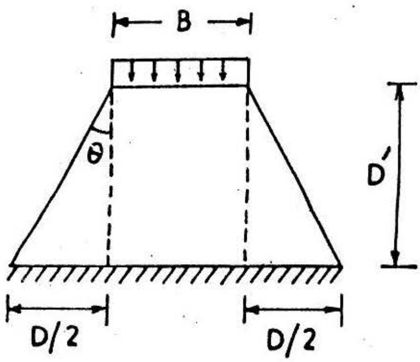

FIGURE 7 Sectional View of the Effective Zone of Vibration for a Square Footing

D′ = effective depth of the vibrating soil mass as shown in Fig. 7. A and B are the length and width of the foundation respectively. The area of the truncated pyramid now can be expressed as

$$A = (B + D')^2 \text{ considering square footing.}$$

So, effective mass of the vibrating soil can be written as

$$m_s = (B + D')^2 \rho' (D'/3) \tag{4}$$

From Pauw's (1953) approach, for non-cohesive soil the effective mass of the vibrating soil can be expressed as

$$m_s = \rho' B^3 C_m \tag{5}$$

where C_m is a constant, depends on the shape and equivalent soil surcharge of foundation. Now equating the Eqns. 4 and 5

$$(D')^3 + 2(D')^2 B + D' B^2 = 3B^3 C_m \tag{6A}$$

From this cubic equation the value of D can be determined. While considering circular footing Eqn. 6A would be in the following form

$$(D')^3 + 2(D')^2 B + D' B^2 = 3.819\, B^3 C_m \tag{6B}$$

Table 4
Values of (D/B) Ratio w.r.t. (A/B) Ratio.

$\left(\frac{A}{B}\right)$ Ratio	$\left(\frac{h}{B}\right)$	$C_m\left(\frac{A}{B}\right)$	C_m	$3B^3 C_m$	$D(C_m)$	(D/B)
1	1.5625	1.233	1.233	12484	14.4	0.96
2	1.5625	1.428	0.714	7220	10.8	0.72
3	1.5625	1.623	0.541	5478	9.3	0.62
4	1.5625	1.752	0.438	4434	8.2	0.54
Strip p4	1.5625	-	-	-	-	0.50

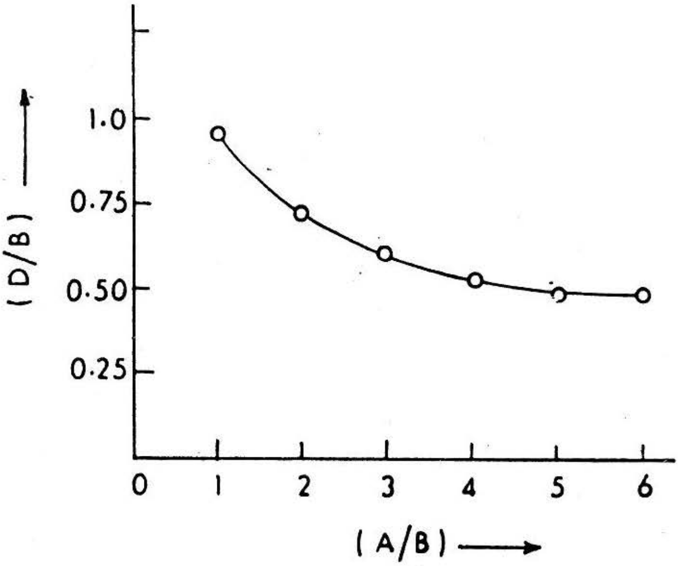

FIGURE 8 Effect of Shape on D/B Ratio

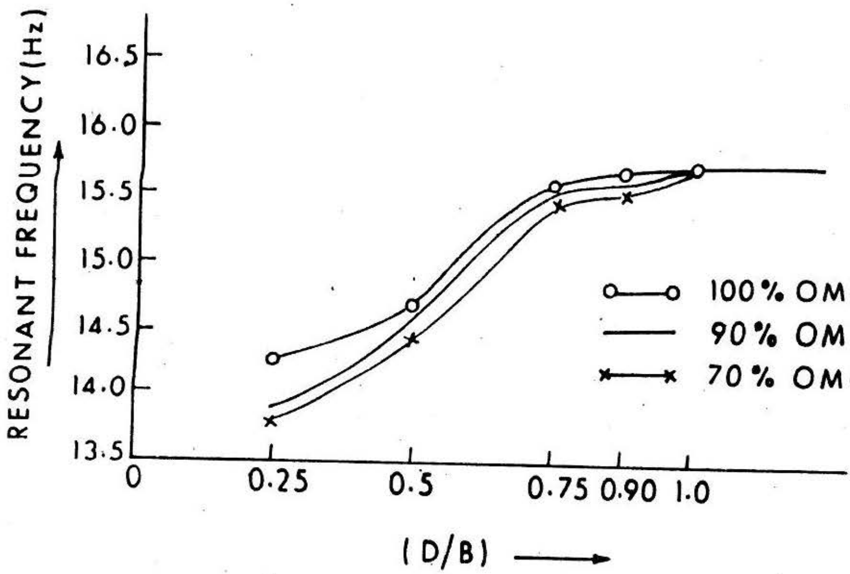

FIGURE 9 Effect of Fill Depth on Resonant Frequency

Discussion on Different Approaches

From those data one can conclude that when the ratio between width and height of equivalent soil surcharge remains constant, then with the increase of A/B ratio, the value of effective depth of vibrating soil mass gradually decreases. Observing (Table 4, *Fig. 8) the results derived from* model tests can be compared satisfactorily with the analytical values derived from Eqn. 6A or 6B. The acceptability of this model test results in field condition was verified by the hints given by Gazetas and Roesset (1979). Fig. 10 and Fig. 11 was originally curves drawn by Gazetas corresponding to $C_r/C_s = 1$ and $C_r/C_s = 2$ *have been considered here as these are relevant* for present work. Corresponding to the mass ratio for footings once over sand and once over composite soil strata, dimensionless frequency at resonance was measured from curve. Here C_r and C_s are the shear wave velocity of underlying and overlying soil layer respectively. Shear-wave velocity in cinder and sand was found as 119.23 m/sec. and 249.2 m/sec respectively. Once from the curve of Gazetas and once from model tests, the increase in the dimensionless frequency at resonance was determined when the footing was transferred from homogenous half space to composite medium. Table 5 indicates the comparison between the results obtained from model tests with the values obtained from Gazetas's curve for strip footing.

Here, the most important fact is that among several shapes of shallow footing the maximum effective depth of vibrating soil mass was 100% of footing width and the minimum was determined as 50% of footing width, beyond this limit, one can easily assume that the underlying soil masses do not take part effectively in vibration. So the underlying soil in that case may be analogous to rigid rock base as inferred by Gazetas.

Conclusions

1. The effective mass of the soil participating in vibration can be expressed as ρ L/3.

2. If the width and equivalent soil surcharge of the foundation remains constant, then the effective depth of vibrating soil mass is maximum for footing having (A/B) = 1.0 and minimum for strip foundation.

Moreover, for composite soil layers having $C_r/C_s = 2$, the effective depth of vibrating soil mass would be in the domain of $0.5 \leq D/B \leq 1.0$ for a shallow foundation under vertical vibration under constant amplitude force.

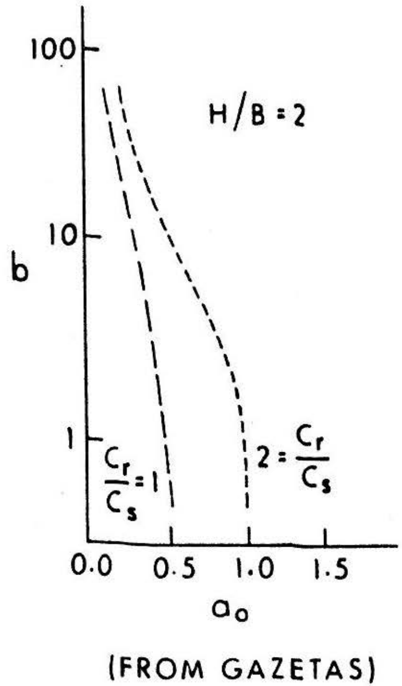

FIGURE 10 Dimensionless Frequency at Resonance vs. Mass Ratio from Gazetas

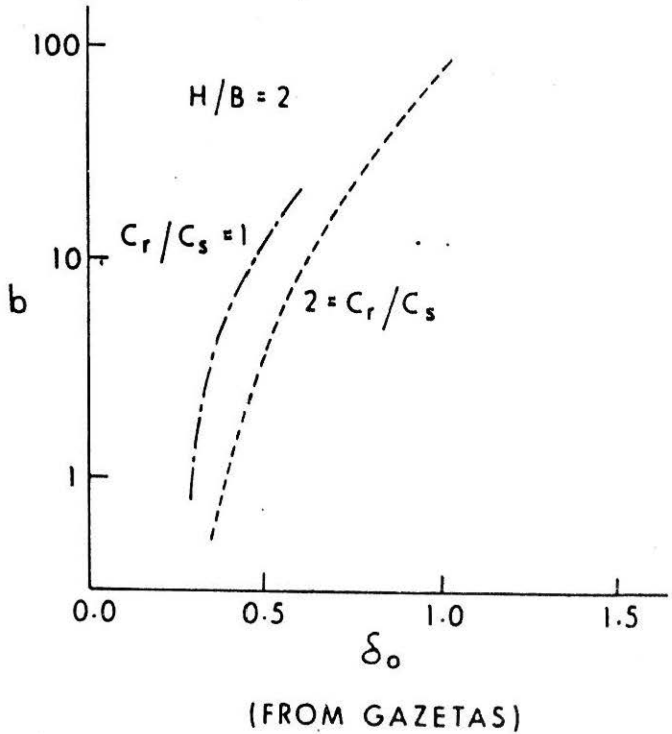

FIGURE 11 Dimensionless Peak Amplitude vs. Mass Ratio from Gazetas

Table 5

Comparision of Frequency Factor Ratio and Peak Amplitude Ratio between Gazetas and Present Approach.

Shape of Footing	Mass Ratio	C_r/C_s	a_{02}/a_{01} from		δ_{02}/δ_{01} from	
			Gazetas	Model Test	Gazetas	Model Test
Circular	1.86	1.0	1.974	2.467	1.35	1.34
	1.876	2.0				
Square	0.826	1.0	1.909	2.37	1.44	1.45
	0.831	2.0				

Notes :

a_{01}, d_{01} = Dimensionless frequency factor and dimenstionless peak amplitude respectively which was derived considering the soil as homogenous medium

a_{02}, d_{02} = Dimensionless frequency factor and dimensionless peak amplitude respectively which was derived considering the soil as composite medium.

Cinder Data

Optimum moisture content = 34%
Maximum dry density = 1.22 t/m^3
Bulk density = 1.64 t/m^3
ϕ = 36°
C = 0.05 kg/cm^2
Mass of square footing = 8.265×10^{-4} $t\,sec^2\,m^{-1}$
Mass of circular footing = 1.34×10^{-3} $t\,sec^2\,m^{-1}$
Constant amplitude force = 0.0028 L

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