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Settlement of Circular Footings on Layered Soils

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Introduction

Accurate estimation of settlement of footings due to external loads is an important requisite in foundation engineering. Since footings are quite often situated over soil layers, effect of layering must be taken into account in the settlement analysis, as it may have significant influence in the magnitude of settlement. Also, it is necessary to consider the effect of rigidity of the footing on settlement. It is known that with the exception of very thin footing slabs beneath frame structures, footings are usually almost rigid (Myslivec and Kysela, 1978). Under consideration in this paper is the problem of settlement analysis of rigid circular footings resting on a two-layer soil system. Elastic solutions to the above problem have been proposed by several investigators (e.g. Poulos, 1968; Chow, 1987; Conte and Dente, 1993). But, elastic solutions do not predict accurately the settlements at higher load levels, and a finite element analysis, together with appropriate constitutive modelling of the soils, can be used for this purpose. However, accurate modelling of soils is a difficult task even today and also finite element method requires expensive computer runs. For a foundation engineer who needs a reasonable estimate of settlement under various load levels, some alternate, simple technique is desirable. In this paper, it is intended to present a simple and effective method for prediction of settlement, at

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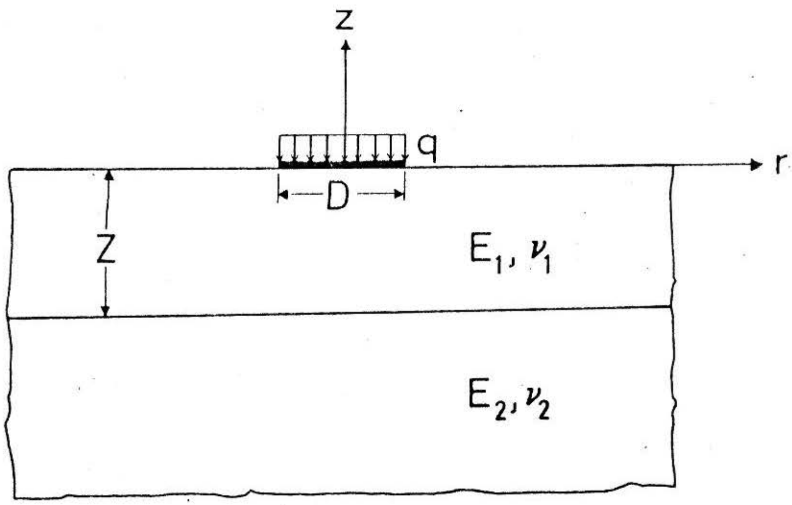

FIGURE 1 Definition Sketch

various load levels, of rigid circular footings resting on two-layered soils, considering the elastic-plastic nature of the soil beneath the footing.

Statement of the Problem

A strong soil layer of finite thickness Z with modulus of elasticity, E_1 and Poisson's ratio, ν_1 is underlain by a semi-infinite mass of a weak soil layer with elastic constants, E_2 and ν_2 (Fig. 1). On the surface, $z = 0$ of the layer rests a rigid circular footing of diameter, D. Under the action of a uniformly distributed load of intensity, q and the reactive normal pressure, p(r) the rigid footing undergoes uniform vertical displacement. It is required to determine the settlement of the footing for various values of q (i.e., at various load levels). Finite element method will be used for solving the above problem.

Theoretical Considerations

Schultze (1961) has shown that realistic contact pressure distribution beneath rigid footings on homogeneous soils will be elastic-plastic in nature. To obtain elastic-plastic solution, it is necessary to have expressions for contact pressure distribution under elastic and plastic conditions independently. Vinod et al. (1994) have proposed a method to obtain the elastic-plastic contact pressure distribution beneath rigid circular footings on

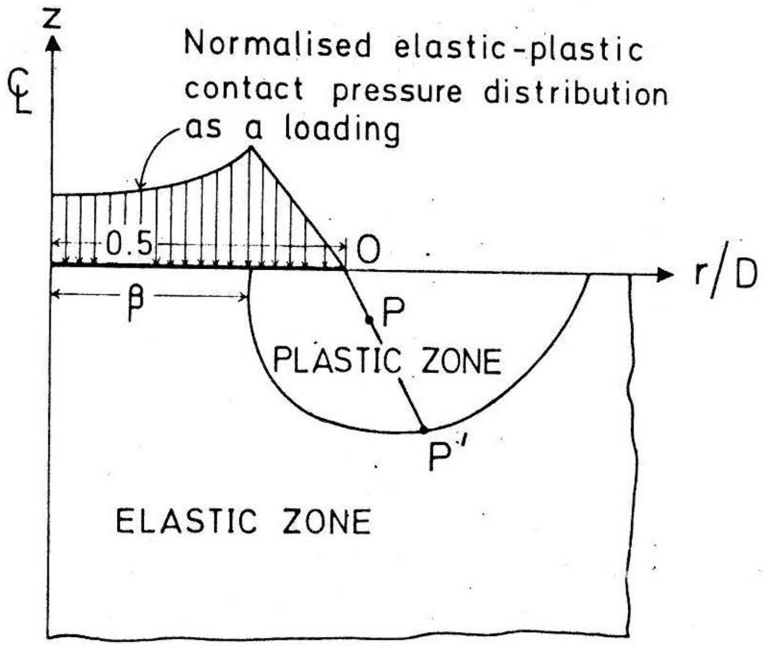

FIGURE 2 Elastic and Plastic Zones

a two-layer soil system. In this, expression for contact pressure under elastic conditions was obtained using the results of a finite element analysis, followed by a regression analysis. Contact pressure distribution under plastic conditions was taken to be conical in shape (Selvadurai, 1979). In a strict sense, this is valid only for surface footings on sand. For footings resting on cohesive soils and embedded footings, appropriate values of edge contact pressure should be used. However, as it will be shown later, adopting conical contact pressure distribution for surface loaded circular footings, the results of this study can be used for both cohesive and frictional soils. Using Schultze's approach, elastic and plastic solutions were then combined to obtain non-dimensional expressions for elastic-plastic contact pressure distribution beneath rigid circular footings on a two-layer soil system. Charts were also prepared to facilitate the determination of contact pressure distribution at any defined load ratio (k),

where $k = \dfrac{\text{average pressure acting over the footing}}{\text{ultimate bearing capacity of the footing}}$

In the present analysis, contact pressure distribution obtained as above will be used as loading on the circular footing. Fig. 2 shows a typical normalised elastic-plastic contact pressure $\left[p(r/D)/q\right]$ distribution beneath

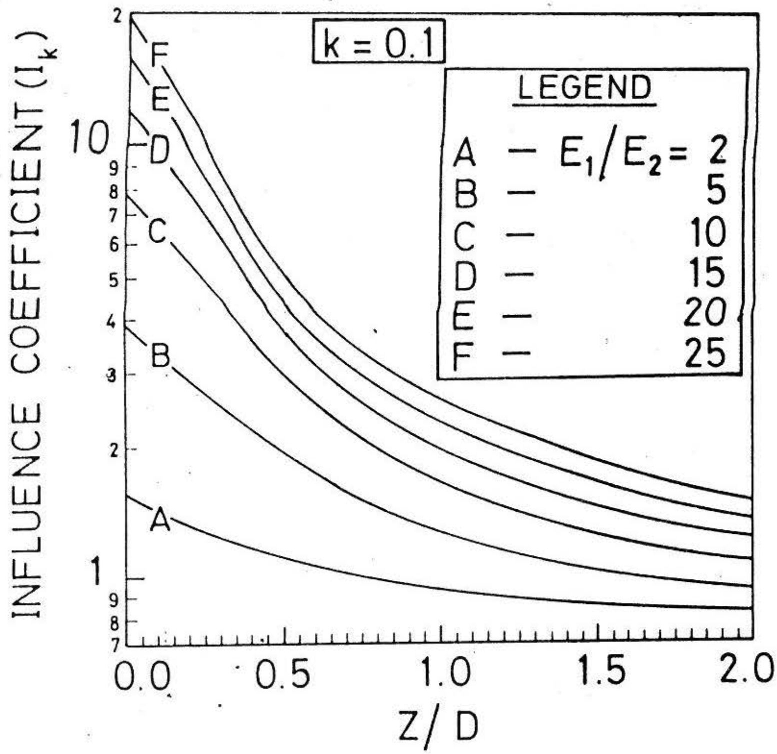

FIGURE 3 Influence Coefficients of Rigid Circular Footings on Layered Soils

the footing at some defined load ratio. In this figure, contact pressure is elastic in the region r/D = 0.0 to β and plastic in the region r/D = β to 0.5, where β is the normalised distance from the centre of the footing to the point of transition between elastic and plastic states.

It is a fact that for real foundation materials some yielding occurs near the edge of the footing for any load level and is accompanied by a redistribution of stress. A plastic zone develops around the edge of the footing, its size increasing with the load level, and the remaining foundation material is within elastic limits (Brown, 1968; Hooper, 1974; Chen, 1975). Accordingly, in the present study the foundation soil is divided into two zones – a plastic zone and an elastic zone (Fig. 2), the two zones being separated by the transition point at the footing level. Jumikis (1969) has indicated that in the case of a centrally and vertically loaded footing at failure, rupture surface is symmetrical and the footing settles uniformly. Further, the rupture surface which divides the plastic and elastic zones

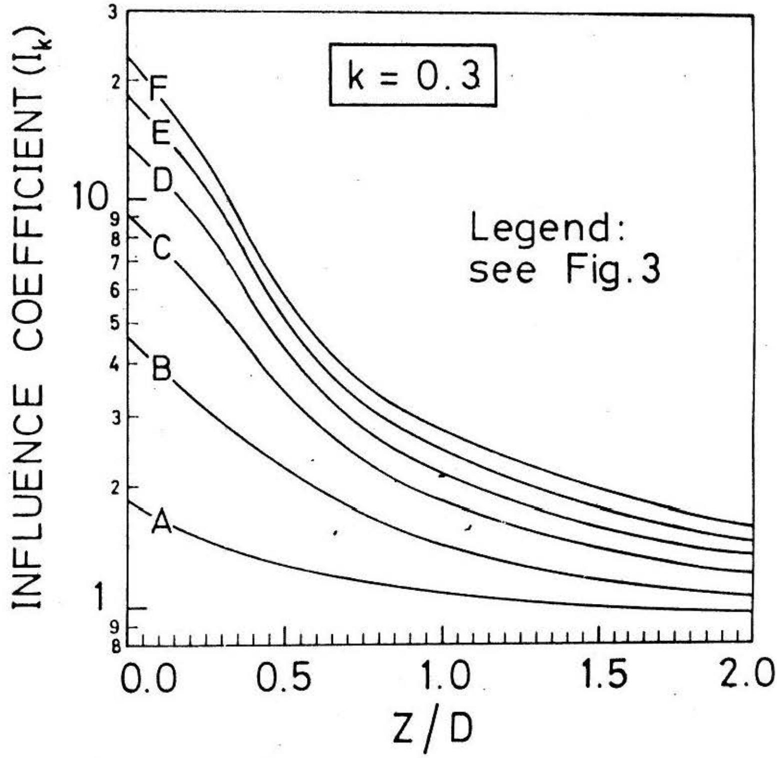

FIGURE 4 Influence Coefficients of Rigid Circular Footings on Layered Soils

coincides remarkably well with the mathematical curve of a logarithmic spiral of the form:

$$r_i = r_o e^{a\theta} \tag{1}$$

where,
r_i = radius vector from pole O to the spiral curve
r_o = *initial radius vector on the polar axis*
θ = angle of amplitude between r_o and r_i
a = constant

In the present study, it is assumed that at all load levels, elastic and plastic zones are separated by a logarithmic spiral with $r_o = (0.5 - \beta)$ taking care of the size of the plastic zone.

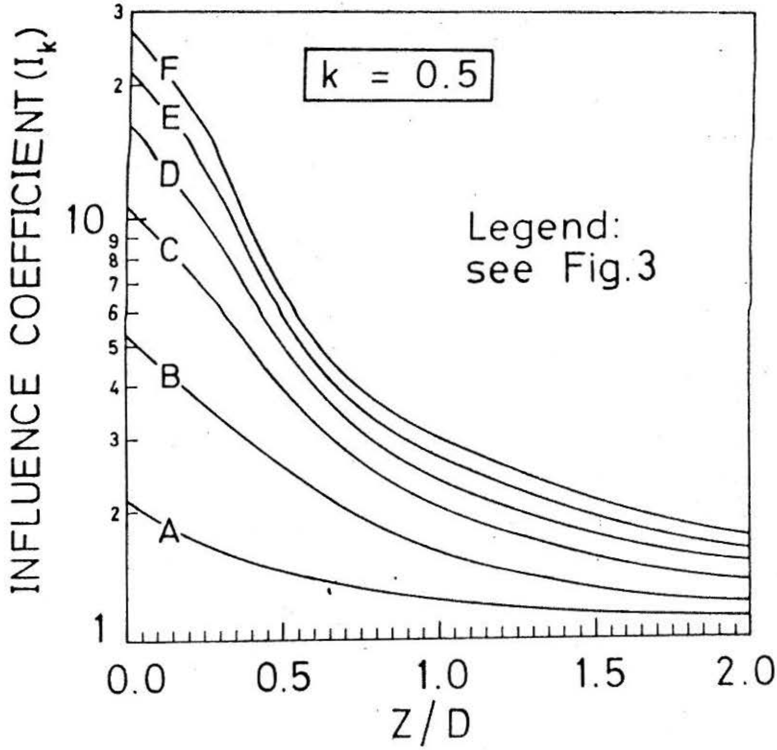

FIGURE 5 Influence Coefficients of Rigid Circular Footings on Layered Soils

It is difficult to assign a value for the equivalent elastic modulus in the plastic zone. It is to be noted here that the particulate nature of soil brings in a stable state even for plastic softening conditions. Therefore, an equivalent modulus can be assigned for the soil within the plastic zone. Though the boundary between elastic and plastic zones is defined by Eqn. 1, it is likely that complete flow condition exists only in the vicinity of pole O, where E can be assumed to be zero. At any point, P within the plastic zone, the equivalent elastic modulus is assumed to vary as a function of the radial distance of the point, P from the pole as follows:

$$E_P = E_k (OP/OP')^f \tag{2}$$

where, k = 1 or 2 depending upon whether the point P is in layer 1 or 2

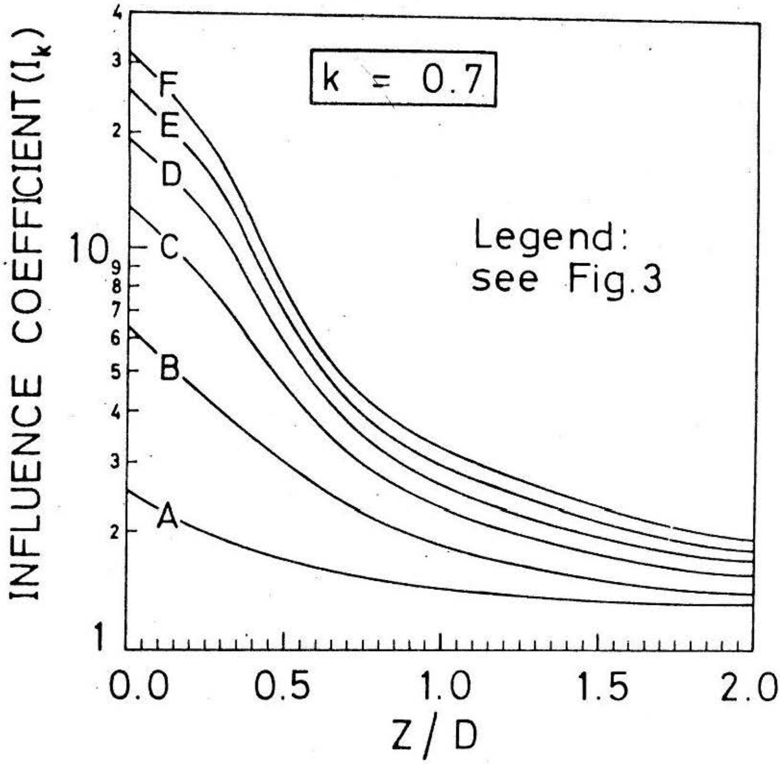

FIGURE 6 Influence Coefficients of Rigid Circular Footings on Layered Soils

f = constant

OP and OP' are as shown in Fig. 2.

In the present approach, the two boundary conditions namely, the elastic-plastic contact pressure distribution and uniform settlement of the footing are simultaneously satisfied. In contrast, in the conventional elasto-plastic analysis, only one of the above boundary conditions will be specified since the constitutive law for the soil is already defined. In the present study, the equivalent elastic modulus within the plastic zone and the size of the plastic zone (defined by parameters 'f' and 'a') are suitably modified to satisfy the above two boundary conditions. In a way, this can be regarded as an approximate method of defining the constitutive law for the soil with easily measurable properties. The relationships thus established rigorously satisfy the field boundary conditions.

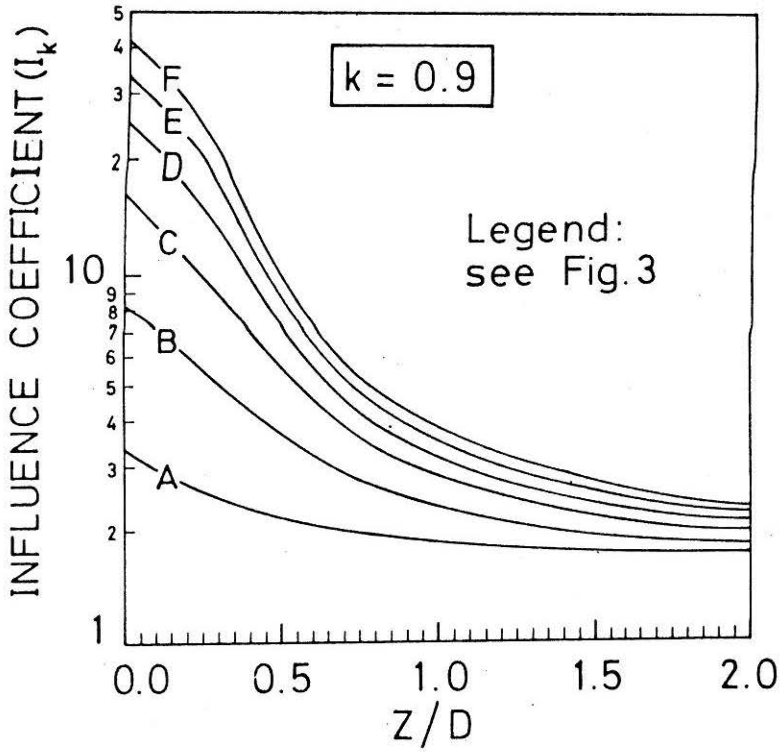

FIGURE 7 Influence Coefficients of Rigid Circular Footings on Layered Soils

Finite Element Analysis

Analysis is made using the general purpose finite element program FEAP (Zienkiewicz, 1979). Necessary changes are made in the program so that elastic modulus as given by Eq. 2 is used at all quadrature points in the evaluation of the stiffness matrix. Elastic-plastic contact pressure distribution obtained with $\nu_1 = \nu_2 = 0.35$ is used in the present analysis as it has been found that variations in the values of Poisson's ratio of soils does not appreciably alter the contact pressure pattern (Vinod et al. 1994). Influence of variation in Poisson's ratio on settlement will be discussed later. Consistent nodal loads acting on nodes at the soil-footing interface are evaluated using the known elastic-plastic contact pressure distribution and they depend on E_1/E_2, Z/D and k. Apart from these nodal loads, the values of 'a' and 'f' are also input parameters in the finite element analysis. Different combinations of 'a' and 'f' are tried and the combination which gives nearly equal settlement for all the nodes at soil-footing interface is

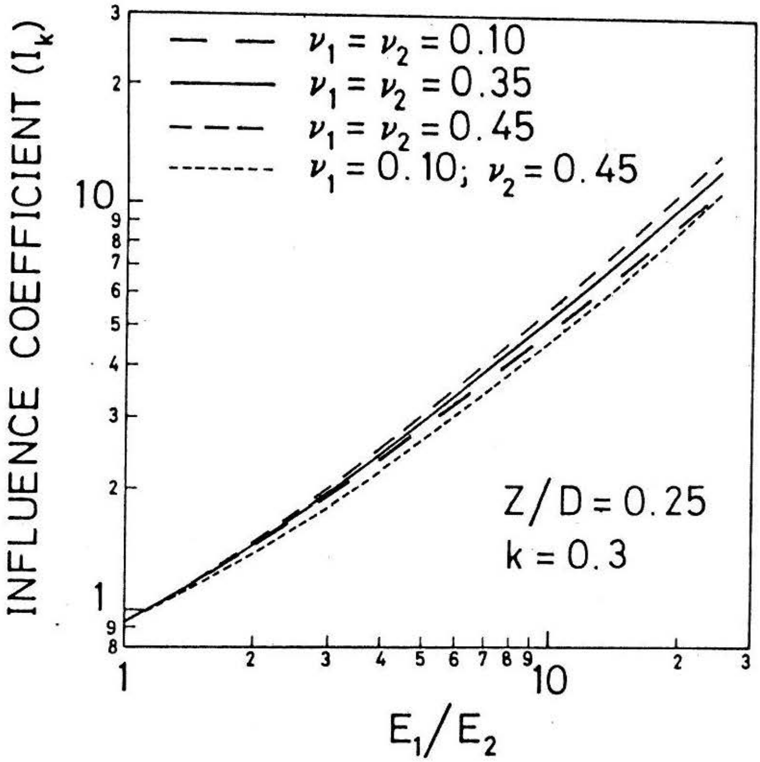

FIGURE 8 Influence of Poisson's Ratio of Strong and Weak Soils on I_k

obtained. The average of the settlement of all nodes at the interface is calculated (δ_{av}). Since the elastic modulii at all points in the system have *been modified suitably and since the realistic contact* pressure distribution has been used in the analysis, the conventional equation for settlement may be used for layered soils also for any defined load ratio in an equivalent form as follows :

$$\delta_k = \frac{q_k \, D\left(1-\nu_1^2\right) I_k}{E_1} \qquad (3)$$

where, δ_k = settlement of the circular footing on two-layer soil system at load ratio = k

q_k = average pressure acting over the circular footing on two-layer soil system at load ratio = k

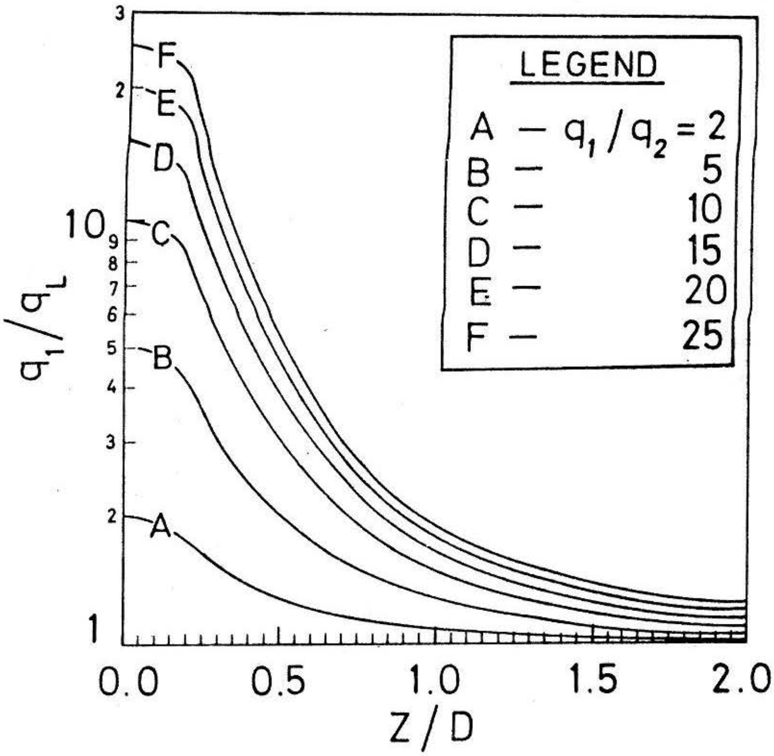

FIGURE 9 Ultimate Bearing Capacity of Rigid Circular Footings on Layered Soils

I_k = influence coefficient for circular footing on two-layer soil system at load ratio = k.

As mentioned earlier, δ_k is obtained from finite element analysis (= δ_{av}) for a given two-layer system and at a given load ratio. Now, using the known values of q_k, D, ν_1 and E_1, I_k can be calculated using Eqn. 3. The magnitude of I_k will depend on E_1, E_1/E_2, Z/D and k.

Results and Discussion

The procedure described in the previous section has been used to obtain the influence coefficients for a rigid circular footing on a strong soil of finite thickness underlain by a semi-infinite mass of a weak soil with

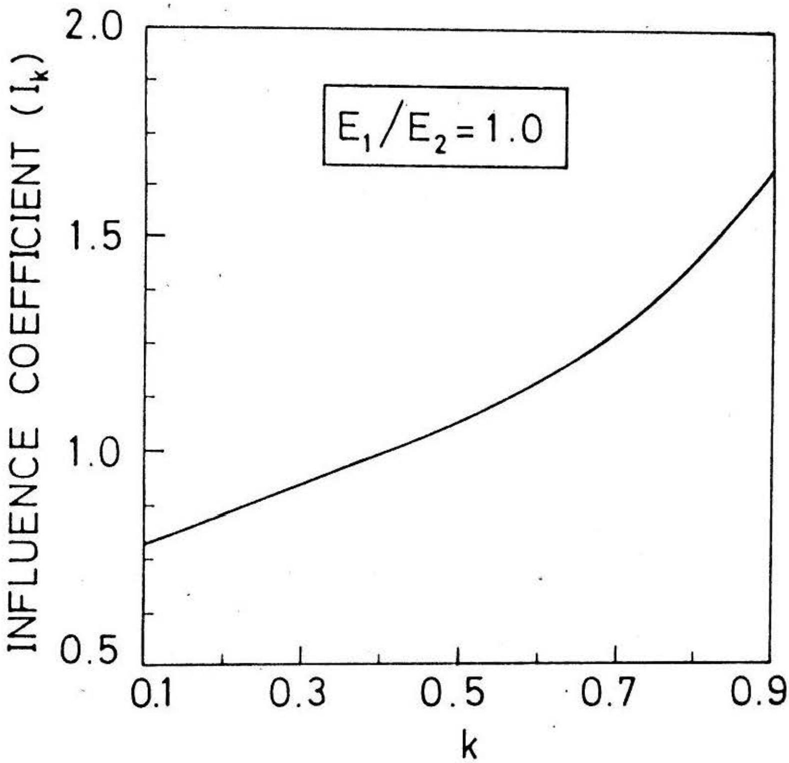

FIGURE 10 Influence Coefficients of Rigid Circular Footings on Homogeneous Soils

$E_1/E_2 = 2$, 5, 10, 15, 20 and 25; $Z/D = 0.25$, 0.50, 0.75, 1.00 and 2.00; and $k = 0.1$, 0.3, 0.5, 0.7 and 0.9. The results are presented in Figs. 3 to 7. It is seen from these figures that I_k and consequently, the settlement is significantly different for various values of E_1/E_2 particularly for smaller thickness of the upper soil. Also, most of the variation in I_k takes place within $Z/D = 1.00$. Comparison of the figures reveals that for a given E_1/E_2 and Z/D, there is a significant increase in the magnitude of the influence coefficient with increase in load ratio. This clearly shows the necessity of using appropriate value of influence coefficient, commensurate with the load ratio, in the equation for settlement prediction. Use of elastic solutions will obviously result in underestimation of settlements at higher load ratios.

All the above analyses were carried out with $\nu_1 = \nu_2 = 0.35$. Effect of variation in ν_1 and ν_2 on influence coefficients was also studied and the results are presented in Fig. 8. The study was conducted for a two-layered

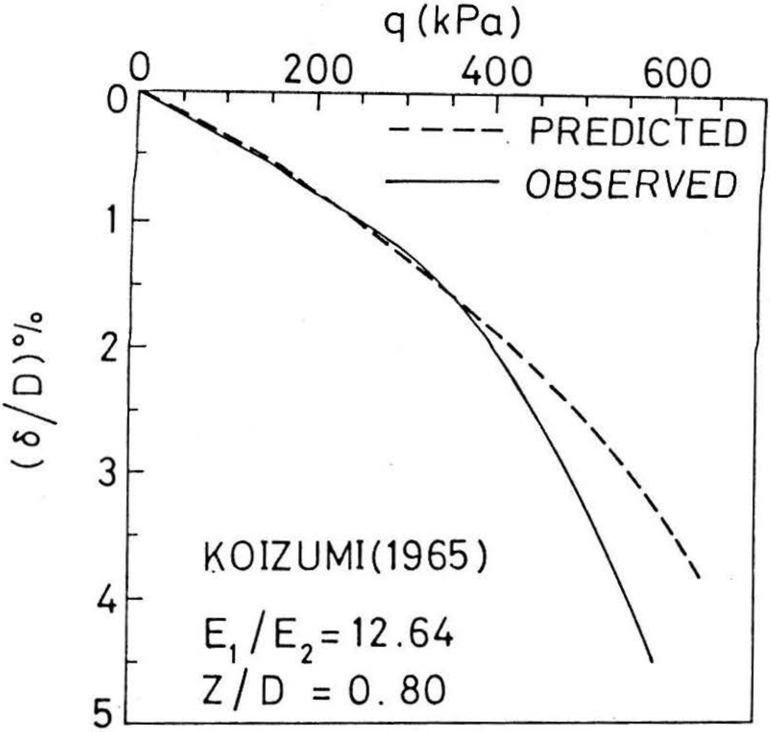

FIGURE 11 Comparison of Predicted and Observed Settlements for Layered Soils

soil with $Z/D = 0.25$ (presence of strong layering effect) and at a load ratio, $k = 0.3$ (common working load level). From Fig. 8, it is seen that variations in the Poisson's ratio of strong and weak soils do have a slight influence on the magnitude of influence coefficient. However, the difference compared to $\nu_1 = \nu_2 = 0.35$ case is not appreciable. Hence, for all practical purposes, the results presented in Figs. 3 to 7 may be used.

In order to make use of the proposed method, it is required to know the ultimate bearing capacity of the circular footing on the two-layer soil system (q_L). For this, the method suggested by Srinivasa Murthy et al. (1994) may be used. The method is essentially same as that presented in theoretical considerations of this paper which resulted in a global constitutive model for the two-layer soil system and the results are reproduced in Fig. 9. Knowing the bearing capacities of the strong and weak soils under their homogeneous conditions (q_1 and q_2 respectively) and the ratio of the upper

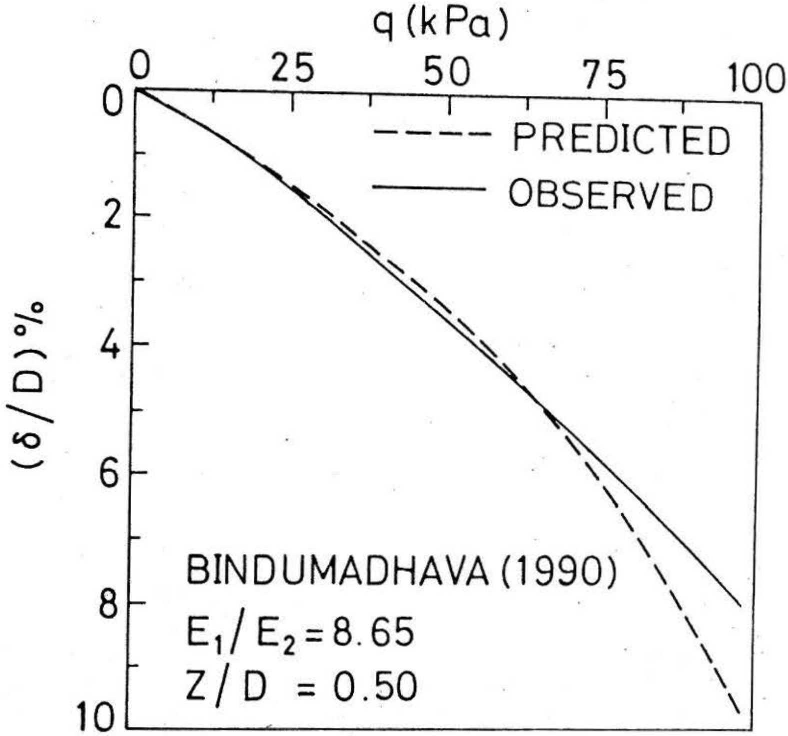

FIGURE 12 Comparison of Predicted and Observed Settlements for Layered Soils

layer thickness to the footing diameter (Z/D), q_1/q_L and hence, q_L can be easily computed using the above figure. Since the computation of q_1 and q_2 takes care of the nature of soils i.e., cohesive or frictional, it is appropriate to examine the applicability of the method to all types of soils.

Various steps in the determination of settlement of a rigid circular footing resting on a given two-layer soil system may, now, be summarised as follows :

1. With the known values of c and ϕ, obtain the ultimate bearing capacities of strong and weak soils under homogeneous conditions (q_1 and q_2 respectively).

2. With the known values of q_1, q_2, Z and D, obtain the ultimate bearing capacity of the circular footing on two-layer system (q_L) using Fig. 9.

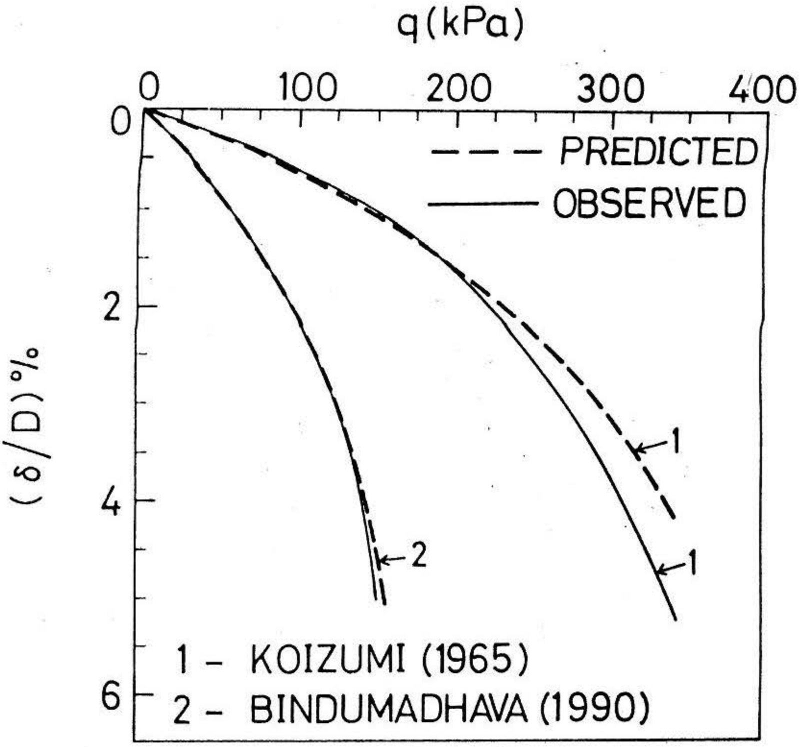

FIGURE 13 Comparison of Predicted and Observed Settlements for Homogeneous Soils

Now, for any defined load ratio (k),

$$q_k = k \cdot q_L$$

3. With known values of E_1/E_2 and Z/D, obtain the influence coefficients at different load ratios using Figs. 3 to 7.

4. Substitute the known values of D, E_1, ν_1, q_k and I_k in Eqn. 3 and estimate the settlements at various load ratios.

As far as the authors are aware of, elastic-plastic solutions for settlement prediction are not available even for homogeneous soils. Hence, the analysis described in the previous section was carried out for homogeneous soils ($E_1 = E_2$) and the results are presented in Fig. 10. The figure presents the variation in the magnitude of influence coefficient with

TABLE 1

Comparison of Predicted and Observed Settlements for Layered Soils

Source of data	Soil type	E_1/E_2	Z / D	(δ / D)%			
				k = 0.3		k = 0.5	
				predicted	observed	predicted	observed
Koizumi (1965)	Stiff clay overlying Soft clay	2.50	0.58	1.03	1.00	1.92	1.83
		12.64	0.80	0.85	0.83	1.60	1.58
Desai and Reese (1970)	Stiff clay overlying Soft clay	2.29	0.48	0.73	0.77	1.44	1.58
Bindumadhava (1990)	Sand overlying Saw dust	3.22	0.50	1.44	1.43	2.72	2.48
		8.65	0.50	2.00	2.17	3.75	3.94
			1.50	1.01	0.96	1.83	2.00

load ratio. Using this figure, one can compute the settlement of the rigid circular footing resting on a homogeneous soil for any defined load ratio. It is again evident that use of elastic solutions (where I = Π/4) will result in underestimation of settlements.

Comparison Study

In order to verify the accuracy of the proposed method, comparison with experimental data available in literature has been made. Predicted and observed pressure vs. settlement curves for two typical cases of layered soils are presented in Figs. 11 and 12. Table 1 shows the comparison of predicted and observed settlements at k = 0.3 and k = 0.5 for 6 cases of circular footing resting on a two-layer system. Comparison study for homogeneous soil conditions is presented in Fig. 13 and Table 2. It can be seen that the proposed method gives fairly accurate predictions of the observed experimental results, for both layered and homogeneous soil conditions beneath the footing.

TABLE 2

Comparison of Predicted and Observed Settlements for Homogeneous Soils

Source of data	Soil type	(δ / D)%			
		k = 0.3		k = 0.5	
		predicted	observed	predicted	observed
Koizumi (1965)	Clay	1.08	1.04	2.08	1.88
		0.78	0.72	1.50	1.52
		0.46	0.48	0.88	0.80
		1.26	1.15	2.42	1.91
Desai and Reese (1970)	Clay	0.67	0.73	1.28	1.30
Hans Raj Singh (1990)	Sand	1.34	1.41	2.59	2.63
Bindumadhava (1990)	Sand	0:88	0.90	1.71	1.68
		0.67	0.70	1.30	1.40
		2.08	1.96	4.01	3.30

Concluding Remarks

A simple method to estimate the settlement, at various load levels, of a rigid circular footing resting on a two- layer soil system is presented. The method can handle soil layers with both cohesion and friction. The results of the analysis are presented in the form of a chart. Knowing the elastic modulii of the strong and weak soils and the upper layer thickness, one can easily compute the settlements for any defined load ratio. Comparison of the results predicted by the present method with those observed experimentally has been made. The good agreement of the results indicates the validity of the method. It is believed that the solutions presented will be adequate for practical design purposes.

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Notation

- a – constant in the expression of logarithmic spiral
- D – diameter of circular footing
- E_1, E_2 – elastic modulii of upper and lower soils respectively
- f – constant in the expression of elastic modulus within the plastic zone
- I_k – influence coefficient for circular footing on two-layer soil system at defined load ratio, k
- k – load ratio
- q_k – average pressure acting over the circular footing on two-layer soil system at defined load ratio, k
- q_L – ultimate bearing capacity of circular footing on two-layer soil system
- Z – thickness of upper soil layer
- δ_k – settlement of circular footing on two-layer soil system at defined load ratio, k
- ν_1, ν_2 – values of Poisson's ratio of upper and lower soils respectively