

## **Comparison of Dynamic Response of Imperfectly Bonded Buried Thick and Thin Orthotropic Cylindrical Shells**

by

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### **Introduction**

In the past few years the study of the dynamic response of buried pipelines to seismic excitation has become important. Cast iron and steel pipes are now being slowly replaced by pipes made of the composite materials like reinforced plastic mortar (RPM). Unlike conventional isotropic materials, the strength and stiffness of a composite/orthotropic material in different directions can be controlled at the stage of fabrication of composite pipes. In the past four to five years Upadhyay et al., (1988) and Rao et al., (1989) have presented some research papers on the seismic response of buried orthotropic pipes/shells. These analyses have revealed that the introduction of orthotropy largely affects the dynamic response of buried pipelines. In all these works it has been assumed that the shell remains perfectly bonded to the surrounding medium, but in practice this idealization is never true. Chonan (1981) and Dutta et al., (1984) have studied the effect of bond imperfection between the shell and the surrounding medium on the dynamic response of buried shells. Both of these papers, however, have been concerned with shells of isotropic material only. Recently, Dwivedi et al., (1989, 1990) have studied the axisymmetric dynamic response of an imperfectly bonded buried orthotropic cylindrical shell subjected to p-wave and s-wave excitation, wherein effects of bond parameters on the radial and axial displacements have been discussed in detail. In both of these papers a thick shell theory formulation including the effects of shear deformation and rotary inertia was used.

For the dynamic case the author (1987) has compared the response of

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a buried orthotropic cylindrical shell as obtained from thick and thin shell theories under different surrounding soil conditions. However, in this work the shell is assumed to be perfectly bonded to the surrounding medium of infinite extent. In another paper, Chonan (1981) has compared the result of thick and thin shell theories for an imperfectly bonded cylindrical shell, but this paper is limited to an isotropic shell only.

For orthotropic pipes/shells, there appears to be no work available in which the response of imperfectly bonded buried thick and thin cylindrical shells are compared under different soil conditions. The main objective of comparing the results of thick and thin shell theories in this paper has been to study the nature of variation of difference between the two results as bond parameters are changed in different surrounding ground conditions. It is also of interest to see how the changes in results due to variation in bond parameters compare with differences in results of thick and thin shell theories. It is found that variations in bond parameter have a more significant effect on the shell response than the choice of shell model.

### Formulation and Governing Equations

The description and details of the formulation of the problem of dynamic response of a buried infinitely long thick orthotropic circular cylindrical shell including the effect of shear deformation and rotary inertia, imperfectly bounded to a linearly elastic, homogeneous and isotropic infinite medium, and subjected to a p-wave excitation, have already been presented by Dwivedi et al. (1989). Therefore, only those descriptions and equations are given, which are thought to be necessary and sufficient for the sake of completeness.

#### Thick Shell Equation

A thick cylindrical shell with mean radius  $R$  and thickness  $h$  imperfectly bonded the surrounding medium was considered. A cylindrical polar coordinate system  $(r, \theta, x)$ ,  $x$  coinciding with axis of the shell, was defined. The stress-strain relation of the material of the shell was taken in the form

$$\begin{aligned}\sigma_{xx} &= E_{x1}e_{xx} + E_{v1}e_{\theta\theta} \\ \sigma_{\theta\theta} &= E_{v1}e_{xx} + E_{\theta1}e_{\theta\theta} \\ \sigma_{xr} &= G_{x1}(2e_{xr})\end{aligned}\quad (1)$$

where  $E_{x1}$ ,  $E_{\theta1}$ ,  $E_{v1}$  and  $G_{x1}$  are the four independent elastic moduli of the shell material.

An imperfect bond between the shell and the soil was considered and it was assumed that this bond, which joined the shell and the continuum together, was thin, elastic and inertia-less. This implied that the stresses at the shell-soil interface were continuous. To take the elasticity of the bond into account, the normal and shear stresses in the bond were assumed to be proportional to the relative normal and tangential displacements between the shell and the continuum, i.e.

$$\sigma_{rr}^* \Big|_{r=R+h/2} = \left( S_r + Z_r \frac{\partial}{\partial t} \right) (dr^{(i)} + dr^{(s)} - w) \Big|_{r=R+h/2} \quad (2)$$

$$\sigma_{rx}^* \Big|_{r=R+h/2} = \left( S_x + Z_x \frac{\partial}{\partial t} \right) \{ d_x^{(i)} + d_x^{(s)} - u - (r-R)\psi_x \} \Big|_{r=R+h/2}$$

where  $\sigma_{rr}^*$  and  $\sigma_{rx}^*$  are, respectively, the normal and shear stresses at the outer surface of the shell due to the motion of the surrounding medium, and were calculated from the displacement field (incident + scattered) obtained for the medium, from Singh et al., (1987).  $S_r$  and  $Z_r$  are the stiffness and damping coefficients of the bond in the radial direction and  $S_x$  and  $Z_x$  are the stiffness and damping coefficient of the bond in the axial direction.  $U_r$  and  $U_x$  are the components of the displacement field in the medium with superscript i and s denoting the incident and the scattered field, respectively.

In reference (Dwivedi et al., 1989) the shell was excited by a longitudinal wave of wavelength  $\Lambda (=2\pi/\xi)$  and speed  $c$  moving along the axis of the shell in the medium, and the final response equations were obtained in the matrix form

$$[A] \{M\} = \{F\} \quad (3)$$

where

$$\{M\} = \left[ \bar{U}, \bar{V}, \bar{W}, \frac{C_3}{Ru_0}, \frac{C_4}{Ru_0} \right]^T$$

$$\bar{U} = \bar{u}/u_0$$

$$\bar{V} = h\bar{\varphi}_x/2u_0$$

$$\bar{W} = \bar{w}/u_0$$

and  $C_3$  and  $C_4$  are arbitrary constants.  $u_0$  is a constant depending on the intensity of incident p-wave, and having the dimension on length.

The terms of matrices [A] and {F} are already given in reference (Dwivedi et al., 1989) but for readability these are reproduced here again

$$A_{11} = -i\beta \bar{h} \eta_2 / \eta_3$$

$$A_{12} = 2ik^2\beta$$

$$A_{13} = -\bar{h} \left[ \left[ 1 + \frac{(\bar{h})^2}{12} \right] \eta_2 / \eta_3 + \beta^2 k^2 \right] + \Omega^2 + \left( 1 - \frac{\bar{h}}{2} \right) \frac{\tau_r^2 \bar{\mu}_r (\bar{c})^2 \beta^2 I_0(\alpha_3)}{\delta_r I_1(\alpha_3)}$$

$$A_{14} = \left( 1 + \frac{\bar{h}}{2} \right) \bar{\mu} \left[ \begin{array}{l} \tau^2 \gamma^2 \left\{ K_0(d_1) + \frac{K_1(d_1)}{\alpha_1} \right\} - \frac{(\tau^2 - 2)\gamma^2}{\alpha_1} K_1(\alpha_1) \\ - \beta^2 (\tau^2 - 2) K_0(\alpha_1) \end{array} \right]$$

$$A_{15} = \left( 1 + \frac{\bar{h}}{2} \right) \bar{\mu} \left[ \begin{array}{l} i\tau^2 \beta \delta \left\{ k_0(\alpha_2) + \frac{\gamma K_1(\alpha_2)}{\delta \alpha_1} \right\} - \frac{i\beta \gamma (\tau^2 - 2)}{\alpha_1} K_1(\alpha_2) \\ - i\beta \delta (\tau^2 - 2) K_0(\alpha_2) \end{array} \right]$$

$$A_{21} = \frac{(\bar{h})^2}{12} \left[ \Omega^2 - \frac{\bar{h}\beta^2}{\eta_3} \right], \quad A_{22} = \left[ -\frac{(\bar{h})^2 \beta^2}{6\eta_3} - 2k^2 + \frac{(\bar{h})^2}{6} \Omega^2 \right],$$

$$A_{23} = -i\bar{h}\beta k^2, \quad A_{24} = \bar{h} \left( 1 + \frac{\bar{h}}{2} \right) \bar{\mu} \left[ -i\beta \gamma K_1(\alpha_1) \right]$$

$$A_{25} = \frac{\bar{h}}{2} \left( 1 + \frac{\bar{h}}{2} \right) \bar{\mu} \left[ (\beta^2 + \delta^2) K_1(\alpha_2) \right], \quad A_{31} = -\frac{\bar{h}\beta^2}{\eta_3} + \Omega^2 \bar{\epsilon}$$

$$A_{32} = \frac{\bar{h}}{6} A_{31}, \quad A_{33} = -A_{11},$$

$$A_{34} = \left( 1 + \frac{\bar{h}}{2} \right) \bar{\mu} \left[ -2i\beta \gamma K_1(\alpha_1) \right],$$

$$A_{35} = \frac{2}{\bar{h}} A_{25}, \quad A_{41} = A_{42} = 1, \quad A_{43} = 0,$$

$$A_{44} = \frac{\zeta_x \Gamma_x}{\Gamma_x - i\beta \bar{c} \zeta_x} \left[ \frac{2A_{24}}{\bar{h}(1 + \bar{h}/2)\bar{\mu}} \right] - i\beta K_0(\alpha_1),$$

$$A_{45} = \frac{\zeta_x \Gamma_x}{\Gamma_x - i\beta \bar{c} \zeta_x} \left[ \frac{A_{25}}{\frac{\bar{h}}{2}(1 + \bar{h}/2)\bar{\mu}} \right] + \delta K_0(\alpha_2),$$

$$A_{51} = A_{52} = 0, \quad A_{53} = 1,$$

$$A_{54} = \frac{\zeta_r \Gamma_r}{\Gamma_r - i\beta \bar{c} \zeta_r} \left[ \frac{A_{14}}{\bar{h}(1 + \bar{h}/2)\bar{\mu}} \right] + \gamma K_1(\alpha_1),$$

$$A_{55} = \frac{\zeta_r \Gamma_r}{\Gamma_r - i\beta \bar{c} \zeta_r} \left[ \frac{A_{15}}{\bar{h}(1 + \bar{h}/2)\bar{\mu}} \right] + i\beta K_1(\alpha_2),$$

$$F_1 = \left( 1 + \frac{\bar{h}}{2} \right) \bar{\mu} \left[ \{ \tau^2 e^2 - 2\beta^2 \} I_0(\alpha_1) + \frac{2}{\alpha_1} \gamma^2 I_1(\alpha_1) \right],$$

$$F_2 = -\frac{\bar{h}}{2} \left( 1 + \frac{\bar{h}}{2} \right) \bar{\mu} \left[ 2i\gamma\beta I_1(\alpha_1) \right], \quad F_3 = \frac{2}{\bar{h}} F_2,$$

$$F_4 = \frac{\zeta_x \Gamma_x}{\Gamma_x - i\beta \bar{c} \zeta_x} \left[ \frac{F_2}{(1 + \bar{h}/2)\bar{\mu}} \right] + i\beta I_0(\alpha_1),$$

$$F_5 = \frac{\zeta_r \Gamma_r}{\Gamma_r - i\beta \bar{c} \zeta_r} \left[ \frac{F_1}{(1 + \bar{h}/2)\bar{\mu}} \right] + \gamma I_1(\alpha_1),$$

where  $\Omega^2 = (\rho_0 h R / G_{xl}) \omega^2 = \beta^2 \bar{h} (\bar{c})^2 \tau^2 \bar{\rho} \bar{\mu}$  is the non-dimensionalized frequency of the shell with

$$\bar{h} = h/R,$$

$$\bar{\rho} = \rho_0/\rho_m,$$

$$\bar{\mu} = \mu/G_{xl},$$

$$\bar{c} = c/c_1,$$

$$\beta = \zeta R = 2\pi R/\Lambda,$$

$$\tau^2 = c_1^2/c_2^2 = 2(1 - \nu_m)/(1 - 2\nu_m)$$

( $\nu_m$  being the Poisson's ratio of the surrounding medium) and  $c_1 = \{(\lambda + 2\mu)/\rho_m\}^{1/2}$  and  $c_2 = \{\mu/\rho_m\}^{1/2}$  as longitudinal and shear wave speeds, respectively, in the medium depending upon the Lamé's constants ( $\lambda$  and  $\mu$ ) and the density,  $\rho_m$ , of the medium (soil).

The other constants appearing in equation (4) were defined as

$$\alpha_1 = (1 + \bar{h}/2)\gamma,$$

$$\gamma = \beta \sqrt{1 - (c/c_1)^2},$$

$$\alpha_2 = (1 + \bar{h}/2)\delta,$$

$$\delta = \beta \sqrt{1 - (c/c_2)^2},$$

$$\epsilon = \bar{c}\beta,$$

$$k = \pi/\sqrt{12} \quad (\text{shear correction factor}).$$

$\eta_1 = E_{\theta 1}/E_{xl}$ ,  $\eta_2 = E_{\nu 1}/E_{xl}$  and  $\eta_3 = G_{xl}/E_{xl}$  were the non-dimensionalized orthotropy parameters of the shell.  $\zeta_x = \mu/S_x R$  and  $\Gamma_x = \mu/Z_x c_1$  were the non-dimensionalized stiffness and damping coefficients of the bond in the axial direction and  $\zeta_r = \mu/S_r R$  and  $\Gamma_r = \mu/Z_r c_1$  those in the radial direction.

The displacement vector  $\{M\}$ , contains the non-dimensional axial and radial displacement components ( $\bar{U}$  and  $\bar{W}$ ) of the shell which are expressed in terms of intensity of p-wave.  $I_n$  and  $K_n$  ( $n=0, 1$ ) are the modified Bessel functions of first and second kinds. When the arguments of  $I_n$  and  $K_n$  are imaginary (for  $c/c_1 > 1.0$ ), these functions become  $J_n$  and  $Y_n$  respectively;  $K_n$  can, of course, be alternatively expressed in terms of the Hankel function.

## Thin Shell Equations

The geometrical coordinates and notations for the description of various quantities remain the same for the thick shell model. By following the procedure detailed in the paper by Dwivedi et al. (1989), the final response equation for the thin shell can be expressed as

$$[A']\{M'\} = \{F'\} \quad (5)$$

which is identical to equation (3).

The elements of the matrices  $[A']$ ,  $\{M'\}$  and  $\{F'\}$  are found as

$$A'_{11} = \frac{i\bar{h}\beta}{\eta_3} \left[ \eta_2 + \frac{(\bar{h})^2 \beta^2}{12} \right], \quad A'_{12} = \frac{\bar{h}\eta_1}{12\eta_3} \left[ 12 + (\bar{h})^2 + (\bar{h})^2 \beta^4 / \eta_1 \right] - \Omega^2,$$

$$A'_{13} = -A_{14}, \quad A'_{14} = -A_{15}, \quad A'_{21} = A_{31}, \quad A'_{22} = A_{11},$$

$$A'_{23} = A_{34}, \quad A'_{24} = \frac{2}{h} A_{25}, \quad A'_{31} = A_{41}, \quad A'_{32} = A_{43}, \quad A'_{33} = A_{44},$$

$$A'_{34} = A_{45}, \quad A'_{41} = A_{51}, \quad A'_{42} = A_{53}, \quad A'_{43} = A_{54}, \quad A'_{44} = A_{55},$$

$$F'_1 = -F_1, \quad F'_2 = \frac{2}{h} F_2, \quad F'_3 = F_4, \quad F'_4 = F_5$$

## Results and Discussions

Results are presented here mainly to study the effect of the bond parameters  $\zeta_x$ ,  $\zeta_r$ ,  $\Gamma_x$  and  $\Gamma_r$  on the nature of variation of difference between the results of thick and thin shell theories under different soil conditions and different angles of wave incidence. For this purpose only one bond parameter has been varied at a time, keeping the other parameter fixed. The effects of soil condition and angle of wave incidence have been shown by changing  $\bar{\mu}$  and  $\theta$  respectively. While studying the effect of imperfect bonding the orthotropy parameters have been kept constant as  $\eta_1=0.5$ ,  $\eta_2=0.05$  and  $\eta_3=0.02$ . Some other parameters which are taken to be constant throughout are  $\bar{h} = 0.10$ ,  $\bar{\rho} = 0.30$  and  $v_m = 0.25$ . The bond parameters have been varied between zero and are hundred.  $\zeta_x = \zeta_r = \Gamma_x = \Gamma_r = 0$  represent the condition of a perfect bond between the shell and the surrounding soil. When one of these parameters is varied,

the other three are kept at zero. A nearly grazing angle of incidence is represented by  $\theta = 5^\circ$ ; and  $60^\circ$  represents a general incidence angle of the wave. Corresponding to soft, medium hard and very hard ground conditions,  $\bar{\mu}$  has been taken to be 0.01, 0.1 and 1.0, respectively.

Figure 1 shows the comparison of axial displacement ( $\bar{U}$ ) for two shell models under a very hard soil condition, for a nearly grazing angle of incidence ( $\phi = 5^\circ$ ) as the axial stiffness parameter  $\zeta_x$  is varied. Results are not shown for other soil conditions (i.e.  $\bar{\mu} = 0.01$  and 0.1) as both the shell models give the same value of axial displacement, irrespective of variation in the parameter  $\zeta_x$ . The Fig. 1 shows that there is some difference in axial displacement of two shell models when the value of  $\zeta_x$  is very low or when there is a perfect bond between the shell and the surrounding soil. At higher values of  $\zeta_x$  this difference vanishes completely. However, the axial displacement is largely affected by changing the axial stiffness parameter  $\zeta_x$ . As the angle of incidence is increased to  $60^\circ$  it is observed from Figs. 2 & 3 that in medium hard ( $\bar{\mu} = 0.1$ ) or very hard ( $\bar{\mu} = 1.0$ ) surrounding soil the axial displacement as obtained from the two shell theories differs only when  $\zeta_x$  is very low. The value of  $\bar{U}$  is greater with the thin shell theory approximation. This difference in  $\bar{U}$  as the choice of shell model is changed from thick to thin is visible only for smaller value of  $\zeta_x$ , and when the soil is medium hard or very hard.

Figures 4 & 5 show the comparison of axial displacement for the two shell models as  $\Gamma_x$  is varied under different soil conditions. Results are shown for one value of  $\phi$  ( $= 60^\circ$ ) only as no difference in  $\bar{U}$  is observed for the grazing angle of incidence. For a medium hard soil ( $\bar{\mu} = 0.1$ ) there is little difference in  $\bar{U}$  and for a low value of  $\Gamma_x$  ( $= 0.1$ ). However, this difference  $\bar{U}$  for the two shell theories is comparable with the perfect bond condition, i.e.  $\Gamma_x = 0$  (Fig. 4). From Fig. 5 exactly similar behaviour is observed when the surrounding medium is very hard, but the difference in  $\bar{U}$  is visible even for  $\Gamma_x = 1$ . Once  $\Gamma_x$  is more than 1.0 it becomes immaterial whether a thick or thin shell theory formulation is incorporated. The difference in  $\bar{U}$  for the two shell models increases with increasing value of  $\beta$ , but this is true only when  $\Gamma_x$  is very low ( $= 0.10$ ). It is also observed that the difference between the axial displacement of the two shell models are comparable to differences observed due to change in nature of conduct between the shell and the soil, i.e., from perfect bond ( $\Gamma_x = 0$ ) to imperfect bond ( $\Gamma_x = 0.1$ ). However, displacement  $\bar{U}$  is significantly affected by variation in the axial damping parameter  $\Gamma_x$ .

Figures 6 & 7 show the comparison of radial displacement  $\bar{W}$  for a



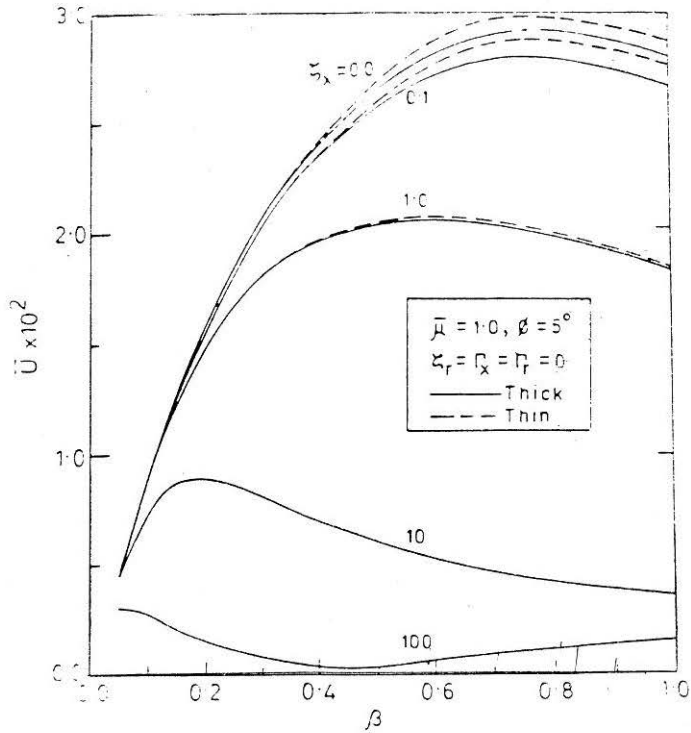


FIGURE 1. Axial Displacement ( $\bar{U}$ ) versus Wavelength Parameter ( $\beta$ ) for  $\bar{\mu} = 1.0$  and  $\phi = 5^\circ$  with  $\zeta_x$  as Parameter.

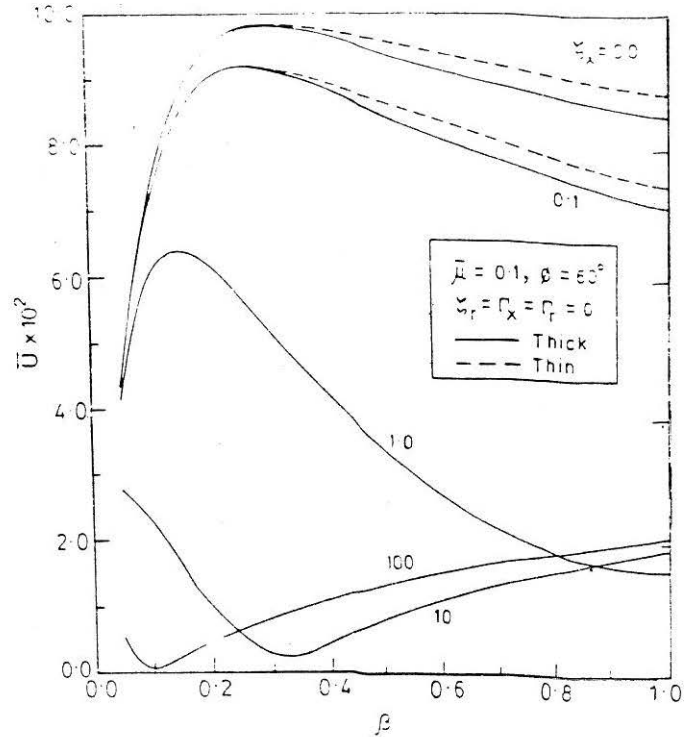


FIGURE 2. Axial Displacement ( $\bar{U}$ ) versus Wavelength Parameter ( $\beta$ ) for  $\bar{\mu} = 0.1$  and  $\phi = 60^\circ$  with  $\zeta_x$  as Parameter.

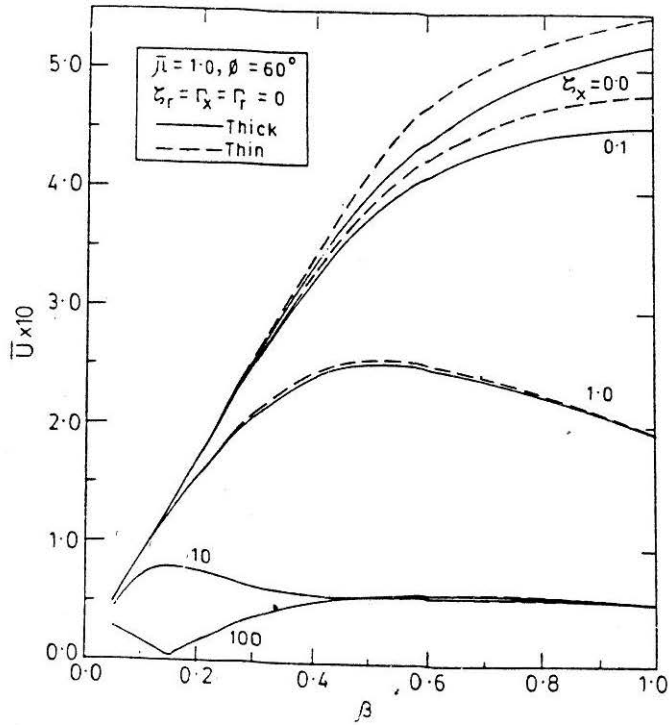


FIGURE 3. Axial Displacement ( $\bar{U}$ ) versus Wavelength Parameter ( $\beta$ ) for  $\bar{\mu} = 1.0$  and  $\phi = 60^\circ$  with  $\zeta_x$  as Parameter.

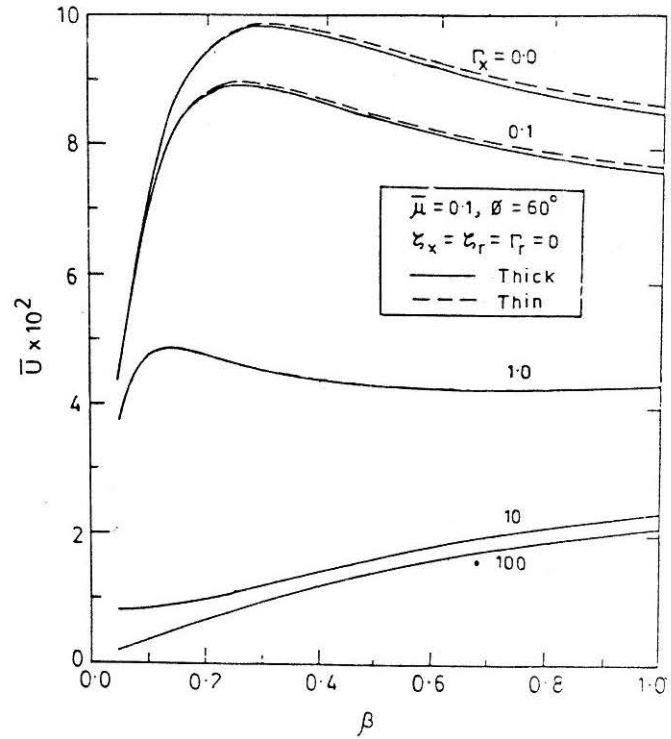


FIGURE 4. Axial Displacement ( $\bar{U}$ ) versus Wavelength Parameter ( $\beta$ ) for  $\bar{\mu} = 0.1$  and  $\phi = 60^\circ$  with  $\Gamma_x$  as Parameter.

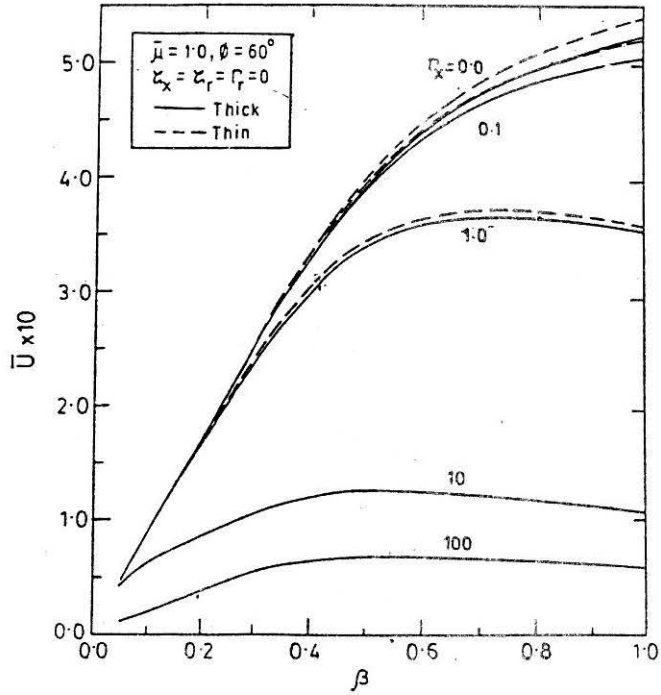


FIGURE 5. Axial Displacement ( $\bar{U}$ ) versus Wavelength Parameter ( $\beta$ ) for  $\bar{\mu} = 1.0$  and  $\phi = 60^\circ$  with  $\Gamma_x$  as Parameter.

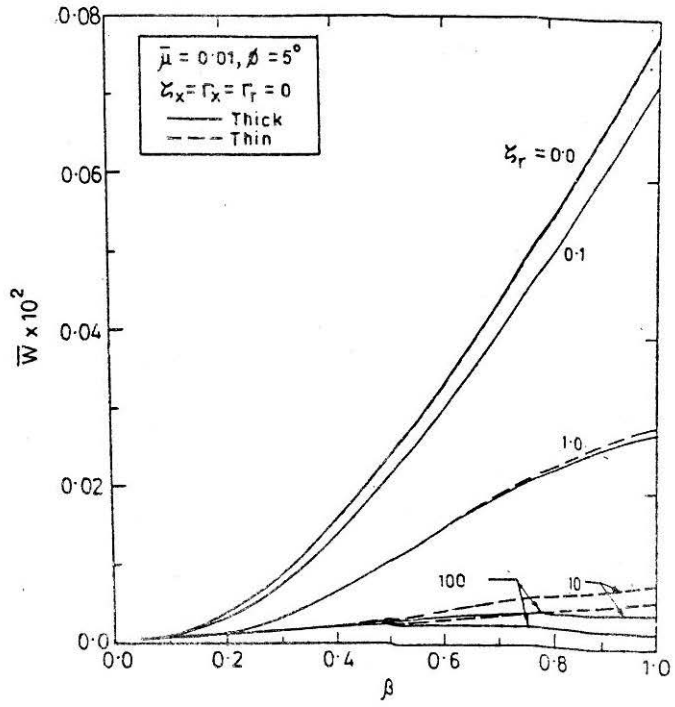


FIGURE 6. Radial Displacement ( $\bar{W}$ ) versus Wavelength Parameter ( $\beta$ ) for  $\bar{\mu} = 0.01$  and  $\phi = 5^\circ$  with  $\zeta_r$  as Parameter.

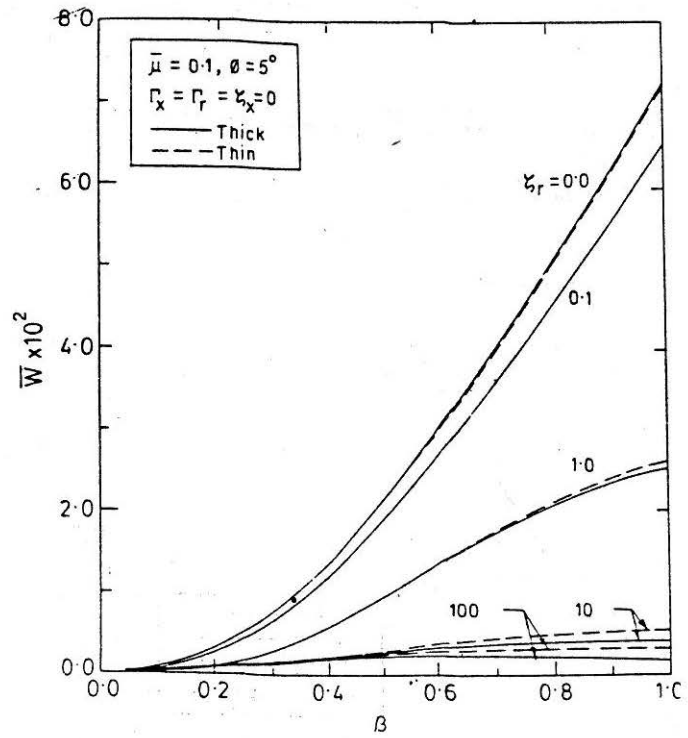


FIGURE 7. Radial Displacement ( $\bar{W}$ ) versus Wavelength Parameter ( $\beta$ ) for  $\bar{\mu} = 0.1$  and  $\phi = 5^\circ$  with  $\zeta_r$  as Parameter.

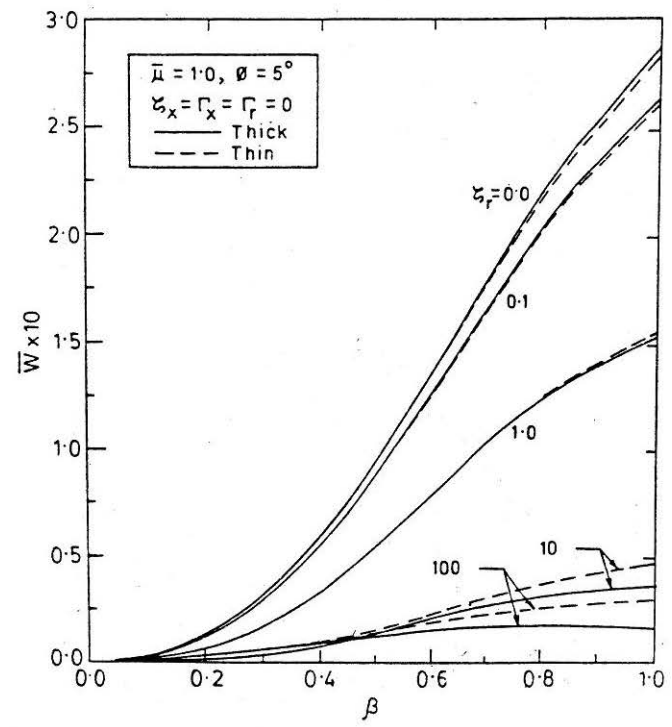


FIGURE 8. Radial Displacement ( $\bar{W}$ ) versus Wavelength Parameter ( $\beta$ ) for  $\bar{\mu} = 1.0$  and  $\phi = 5^\circ$  with  $\zeta_r$  as Parameter.

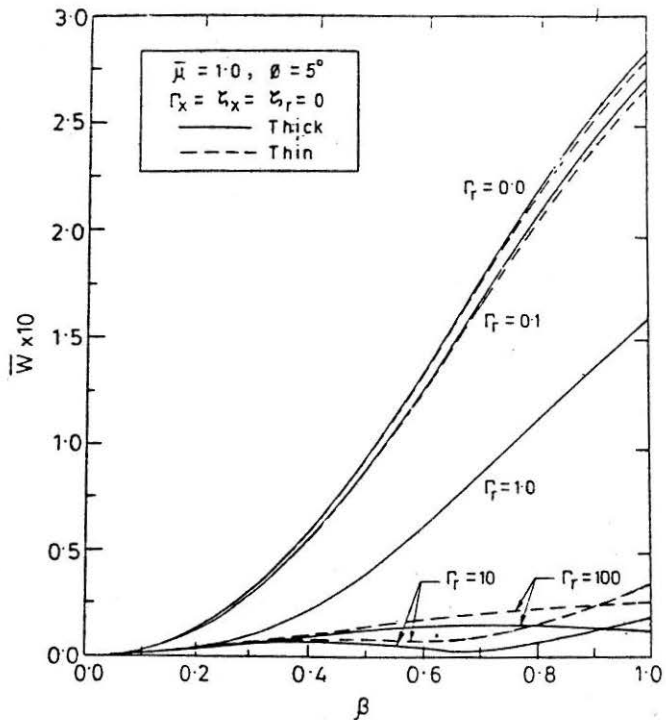


FIGURE 9. Radial Displacement ( $\bar{W}$ ) versus Wavelength Parameter ( $\beta$ ) for  $\bar{\mu} = 1.0$  and  $\phi = 5^\circ$  with  $\Gamma_r$  as Parameter.

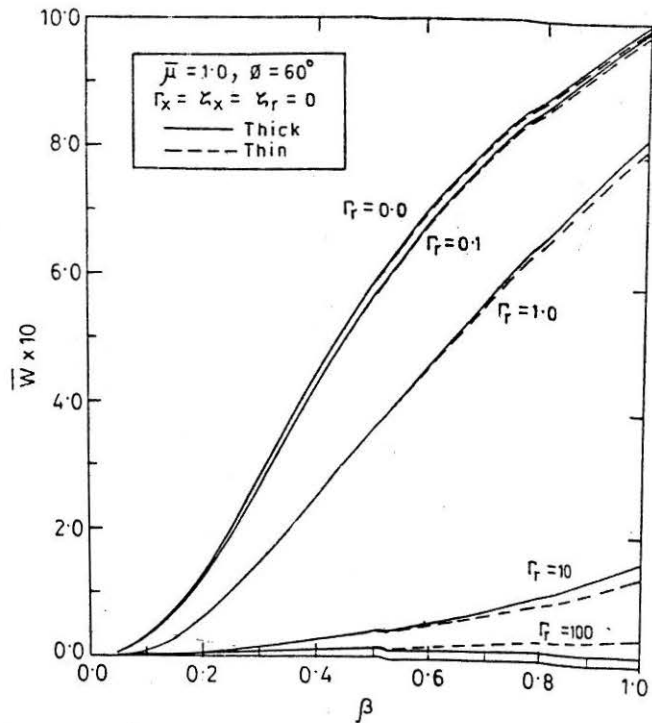


FIGURE 10. Radial Displacement ( $\bar{W}$ ) versus Wavelength Parameter ( $\beta$ ) for  $\bar{\mu} = 1.0$  and  $\phi = 60^\circ$  with  $\Gamma_r$  as Parameter.

grazing angle of incidence ( $= 5^\circ$ ) under different ground conditions and radial stiffness parameter  $\zeta_r$ . Fig. 6 shows that soft and sandy soil conditions, differences in radial displacement for two shell models exist at higher values of  $\zeta_r$ . This difference in  $\bar{W}$  goes on increasing with increasing value of  $\zeta_r$ . For a perfect bond or when  $\zeta_r$  is very low ( $= 0.1$ ), both the shell theories give the same value of radial displacement. The same conclusion stands true for all other soil conditions, however, result has been shown only for a case when the rigidity of the ground is comparable to that of the shell (Fig. 7). Fig. 8 predict that at higher angle of incidence ( $\phi = 60^\circ$ ) the nature of changes in differences of radial displacement for two shell models remains the same as discussed for the grazing angle of incidence.

It can be observed from these Fig. (6-8) that the differences in  $\bar{W}$  due to two shell models are significantly visible only when radial stiffness parameter  $\zeta_r$  is high ( $= 10$  or  $100$ ) and this difference is augmented the as soil becomes harder and harder. This difference in  $\bar{W}$  trends to increase with increasing  $\beta$ . Irrespective of soil conditions the difference in  $\bar{W}$  decreases with increasing angle of incidence. The change in the value of  $\bar{W}$  is very much more prominent when compared to the changes in the difference of  $\bar{W}$  obtained from two shell theories. However, the changes in  $\bar{W}$  are comparable to the difference in  $\bar{W}$  when value of  $\zeta_r$  is  $10$  or  $100$ . This trend is visible irrespective of  $\phi$  or  $\bar{\mu}$ .

Figures 9 & 10 show the results for varying the radial damping parameter  $\Gamma_r$ , when the angle of incidence is  $\phi = 5^\circ$  and  $60^\circ$ , respectively. The results are shown for a fixed soil condition ( $\bar{\mu} = 1.0$ ), as for other values of  $\bar{\mu}$  the difference in  $\bar{W}$  is not very prominent. As appears from Fig. 9, the magnitude of the difference in  $\bar{W}$  obtained from two shell theories is varying with variation in value of radial damping parameters  $\Gamma_r$ . The amplitude of radial displacement is more with the thick shell theories assumption when there is a perfect or nearly a perfect bond between the shell and surrounding medium. However, this difference is too small to be neglected. For  $\Gamma_r = 10$  or  $100$  the amplitude of  $\bar{W}$  is higher with thin shell theory approximation and the difference in  $\bar{W}$  seems to be increasing rapidly with increasing value of  $\beta$ . From Fig. 10 it is evident that at a higher angle of incidence ( $\phi = 60^\circ$ ) the difference in  $\bar{W}$ , for  $\Gamma_r = 10$  or  $100$  increasing value of  $\beta$ . Fig. 9 & 10 reveal that the difference in radial displacement decreases with increase in angle of incidence and the significant changes in difference of radial displacement occur only at higher value of  $\Gamma$ .

## Conclusions

Based on the results discussed through Figs. 1-10, the main conclusions of the paper are as follows.

1. For all the angles of incidence and under soft surrounding soil, the variation in  $\zeta_x$  or  $\Gamma_x$  does not produce any difference in axial displacement,  $\bar{U}$ , as obtained from two shell theories, whereas a difference in radial displacement,  $\bar{W}$ , is obtained as  $\zeta_r$  or  $\Gamma_r$  is varied.

2. The differences in axial displacements for the two shell models are visible only for smaller values of  $\zeta_x$  and  $\Gamma_x$ , but the difference in radial displacement is observed when  $\zeta_r$  or  $\Gamma_r$  is high or very high.

3. Variation in bond parameters bring out prominent changes in the values of displacement ( $\bar{W}$  and  $\bar{U}$ ) compared to the differences in  $\bar{W}$  or  $\bar{U}$ , obtained by thick and thin shell theories.

4. The difference in displacements ( $\bar{W}$  and  $\bar{U}$ ) for the two shell models increases, at the smaller wavelength, as the surrounding soil becomes harder.

5. For the wavelength in the range of  $\beta < 0.50$  displacements are not very different, whether a thick or thin shell model is used.

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