Indian Geotechnical Journal, Vol. 24 (1), 1994

Dynamic Lateral Response of Piles

by

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Introduction

Pile foundations are nowadays used frequently to support a large number of structures *e.g.*, heavy machinery foundation, tall buildings offshore structures, nuclear power plants, etc. It is important to study the effect of dynamic loading due to operating machine, wind, waves and earthquake on the structures with compliant foundations particularly when such structures are constructed on sites having poor soil conditions. The performance of such dynamically loaded structures largely depends on the interaction between the pile and the surrounding soil. The complexity of soil-pile interaction phenomenon has created a lot of interest on the dynamic response of pile foundations.

During last two decades many analytical studies on the dynamic response of single pile subjected to horizontal surface loading are reported. Concept of equivalent cantilever by Hayashi et al., (1965), Prakash and Sharma (1968) and others relies on experiments and other evidence for energy dissipation or number of other factors. Yoshida and Yoshinaka (1972), Prakash and Chandrasekharan (1973) used discrete model and subgrade reaction theory in their analysis. Dynamic winkler foundation type approach presented by Kagawa and Kraft (1980), Dobry *et al.* (1982), Penzien et al. (1964), and Penzien (1970) used lump mass model to represent the soil-pile system. In more recent approaches Novak (1974), Kobori *et al.* (1977) and Novak and Nogami (1977), Novak and Aboul-Ella (1978), Velez *et al.* (1983) and others used more generalized continuum model to account for dynamic soil-pile interaction and make use of elastic and viscoelastic wave propagation. Novak and Sheta (1980), Novak and Mitwally (1988) and E1-Marsafawi *et al.* (1992) have introduced the concept of weak zone in their nonlinear analysis. Blaney *et al.* (1976), Kuhlemeyer (1979), Roesset and Angelides (1979) have utilized dynamic finite element technique and a consistent boundary matrix to simulate the effect of the far field.

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All these methods, *e.g.*, continuum model, finite element method, etc., interpret the dynamic soil-pile interaction as a frequency dependent complex stiffness or impedance functions established at the pile head. The real part of complex stiffness exhibits the stiffness and inertia characteristics of the soil-pile system and the imaginary part express the energy dissipation due to both geometric (radiation) damping and material (hysteretic) damping. The objective of this paper is to develop an approximate method which accounts for the soil-pile interaction for piles fully embedded in homogeneous soil medium, under coupled horizontal and rocking vibration in a relatively simple way. This method can be utilized by the practicing engier to predict the lateral dynamic response of pile foundations.

Theoretical Analysis

For the analysis of coupled horizontal and rocking vibration of pile supported footing, the proposed method assumes that (*i*) the pile is vertical, linearly elastic and of circular cross-section, fully embedded in homogeneous, isotropic and linearly hysteretic semi-infinite soil medium, (*ii*) perfect bonding exists between the pile and the surrounding soil while there is no contact between the footing and the soil, (*iii*) the soil reaction on pile tip is equal to that of a viscoelastic halfspace reaction.

Side Soil Reaction

A pile section at any depth Z (Fig.1) under steady-state horizontal motion $u(Z,t)$ due to harmonic horizontal force $P_0 e^{i\omega t}$ and/or moment $P(t) = P_0 e^{i\omega t}$ acting at its head in the vertical plane, where, P_0 and M_0 are the amplitudes of forcing functions, ω is the circular frequency, t = time and $i = \sqrt{(-1)}$, encounters a horizontal soil reaction (rotatory soil reaction is neglected). The complex horizontal soil reaction $S_u(Z,t)$ per unit length of pile may be expressed as:

$$S_u(Z,t) = [k_u(Z) + i\omega C_u(z)]\; u(Z,t) \tag{1}$$

where the stiffness coefficient $k_u(Z)$ per unit length of pile, varying with depth, computed by using Mindlin's (1936) solution (Saha and Gosh, 1986). The stiffness coefficient $k_u(Z)$ is expressed as;

$$\begin{aligned} k_u(Z) = {} & 16\pi G_s(1-\nu)/[(3-4\nu)/r_0 + 1/R_2 + 1/r_0 \\ & + (3-4\nu)\, r_0^2/R_2^3 + 2Z^2\{1 - 3r_0^2/R_2^2\}/R_2^3 \\ & + \{4(1-\nu)(1-2\nu)/[R_2 + 2z]\}\{1 - r_0^2/[R_2^2 + 2zR_2]\}] \end{aligned} \tag{2}$$

where

G_S = shear modulus of soil

ν = Poisson's ratio of soil

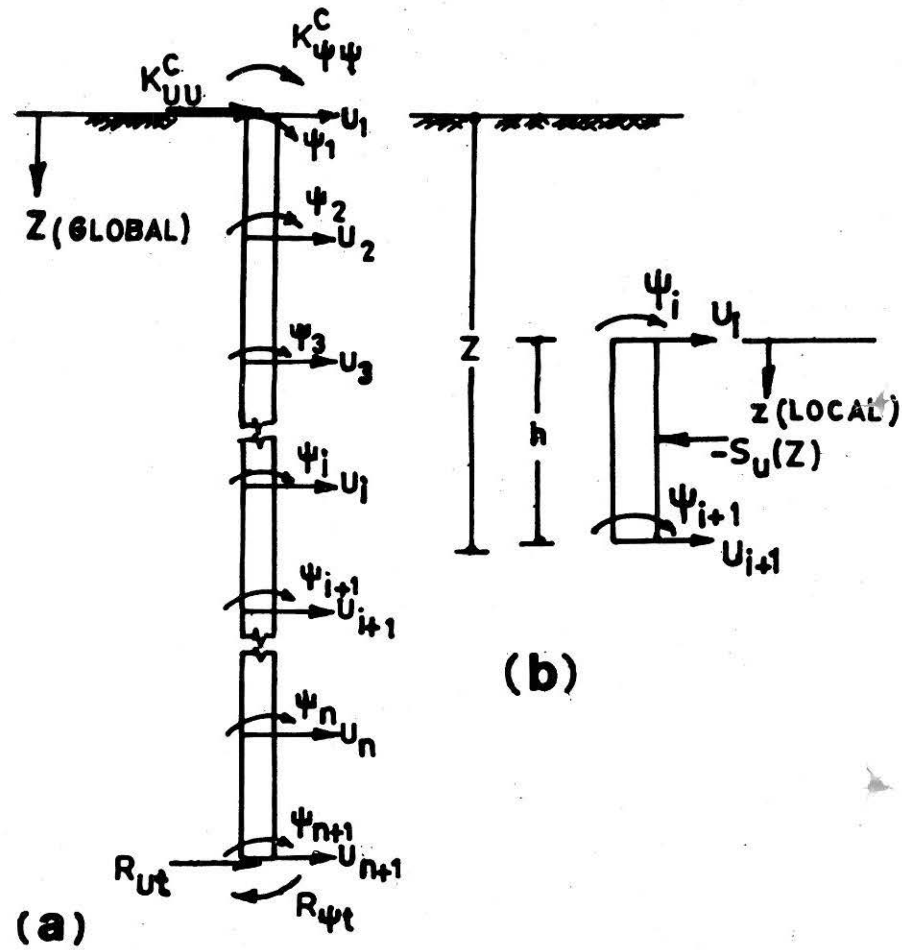

FIGURE 1 (a) Discretisation of Pile Embedded in Homogeneous Medium, and (b) i^{th} element.

r_0 = radius of the pile

$R_2 = \sqrt{(r_0^2 + 4Z^2)}$

Equivalent viscous damping coefficient $C_u(Z)$, per unit length of pile comprises of radiation damping, C_{ur} and material damping, $C_{um}(Z)$ and expressed as;

$$C_u(Z) = C_{ur} + C_{um}(Z) \tag{3}$$

Viscous damping component due to energy radiation (radiation damping), per unit length of pile projected on a vertical plane is assumed to be equal to the equivalent viscous damping per unit length of a rigid square plate of side = $2a$ *which is equal to the diameter of the pile* ($2r_0$), *resting on an elastic halfspace and vibrating in vertical direction. It can be expressed as*

$$C_{ur} = \frac{1}{2\omega} G_s C_{v2} \tag{4}$$

in which G_s is the shear modulus of the soil medium C_{V2} is the halfspace damping parameter in vertical mode of vibration which is function of Poisson's ratio (ν) of the soil and dimensionless frequency $\alpha_p = \omega\alpha / V_s$, *where* V_s is the shear wave velocity of the soil.

Wong and Luco (1976) have given compliance function for massless rigid square plate, vibrating vertically on an elastic halfspace for $\nu = 1/3$. These compliance function are normalised for the effect of Poisson's ratio as suggested by Rücker (1982) to obtain normalized halfspace parameters. The expression for normalized damping parameter C^n_{v2} can be written as

$$C^n_{v2} = 1\cdot7629\,\alpha_p + 16\cdot125\,\alpha^2_p - 53\cdot5227\,\alpha^3_p + 95\cdot8953\,\alpha^4_p$$
$$- 97\cdot5389\,\alpha^5_p + 56\cdot4098\,\alpha^6_p - 17\cdot2907\,\alpha^7_p + 2\cdot1801\,\alpha^8_p \tag{5}$$

Once the normalized damping parameter is known from Eq. 5, the damping parameter C_{v2} for any Poisson's ratio is determined as follows:

$$C_{v2} = C^n_{v2} / (1 - \nu) \tag{6}$$

Material damping due to hysteretic action in the soil material is generally specified as a frequency independent damping into β. For a given soil β is mainly a function of the amplitude of induced shear strain (Hardin and Drnevich, (1972). It is well known that the exact distribution of material damping has no substantial effect on pile response as long as the average value of β over the pile length remains the same. Hence in the present analysis the damping ratio, β, is taken to be constant. The material damping can approximately be related to β (Gazetas and Dobry, 1984) using

$$C_{um}(Z) = 2\beta \frac{k_u(Z)}{\omega} \tag{7}$$

Tip Reactions

For relaxed pile tip, motion of the pile tip generates soil reactions. The soil reactions action on the pile tip for horizontal translation and rocking motions are assumed to be equal to the viscoelastic halfspace reactions. These reactions can approximately be obtained from elastic halfspace reactions on rigid circular disc in horizontal and rocking modes of vibrations respectively. The soil reactions for unit amplitude of motion of the pile tip can be expressed as

Horizontal Reaction $$R_{ut} = G_b r_0 [\, C^v_{u1} + i\, C^v_{u2} \,] \tag{8a}$$

Rocking Reaction $R_{\psi t} = G_b r_0^3 [C^v_{\psi 1} + iC^v_{\psi 2}]$ (8b)

where

$$C^v_{j1} = C_{j1} - \beta C_{j2} \quad (8c)$$

$$C^v_{j2} = C_{j2} + 2\beta C_{j1} \quad (8d)$$

G_b = shear modulus of the soil below pile tip

$Cj1, Cj2$ = elastic halfspace stiffness and damping parameters respectively, in horizontal($j = u$) and rocking ($j = \psi$) mode, are functions of ν and dimensionless fequency $\alpha_{0b} = \omega r_0 / V_b$, V_b being the shear wave velocity in the soil below tip.

Halfspace parameters in horizontal mode (C_{u1} & C_{u2}) and rocking mode ($C_{\psi 1}$ & $C_{\psi 2}$)are derived after Arnold et al. (1955) and Moore (1975) respectively. Expression for these parameters are as follows;

$$C_{ji} = \sum_{l=0}^{8} \alpha_l a_{0b}^l \quad (9)$$

The coefficients α_l of the polynomials for $j = u$ and $j = \psi$ and $i = 1, 2$ are given in Tables 1 and 2 respectively, for different values of ν.

When

(i) $R_{ut} \to \infty$, then $u(1) \to 0$ (10)

i.e. the horizontal motion of the pile tip vanishes which corresponds to hinge tip condition, where 1 is the length of the pile and

(ii) R_{ut} and $R_{\psi t} \to \infty$, then $u(1)$ and $\psi(1) \to 0$ (11)

i.e. at the tip both horizontal and rocking motion of the pile vanishes, corresponding to fixed tip condition.

Equation of Motion

The embedded pile is discretized into a number of elements as shown in Fig.1*a*. Governing differential equation of horizontal motion of i^{th} element (Fig. 1*b*) is

$$m\frac{\partial^2 u(Z,t)}{\partial t^2} + c\,\frac{\partial u(Z,t)}{\partial t} + EI\,\frac{\partial^4 u(Z,t)}{\partial Z^4}$$

$$[K_2(Z) + i\omega C_u(Z)]\,u(z,t) = 0 \quad (12)$$

where, m = mass per unit length of pile, c = coefficient of pile internal damping, E = Young's modulus of pile material and I = moment of inertia

TABLE 1
Coefficient (α_l) for Halfspace Stiffness and Damping Parameters in Horizontal Mode

α_l	C_{u1} Poisson's Ratio (v)			C_{u2} Poisson's Ratio (v)		
	0.25	0.33	0.40	0.25	0.33	0.40
α_0	4.8117	4.9919	5.1363	0.0000	0.0000	0.0000
α_1	-0.0237	-0.2666	-0.3229	2.6517	2.7221	2.7410
α_2	0.3293	2.4657	1.6252	0.4129	0.3962	0.7360
α_3	-0.4414	-8.6713	-1.4472	-0.4002	-0.411	-0.5369
α_4	-1.2354	17.5108	-2.4356	0.0212	-0.5861	-0.2748
α_5	1.8856	-23.6073	3.5514	0.4211	0.7806	0.6885
α_6	-0.4976	19.3192	-0.7002	-0.1357	-0.2161	-0.1869
α_7	-0.3841	-8.4427	-0.7474	-0.2286	-0.1775	-0.1651
α_8	0.1724	1.5000	0.2875	0.1187	0.0924	0.0800

TABLE 2
Coefficients α_l) for Halfspace Stiffness and Damping Parameters in Rocking Mode

α_l	$C_{\psi 1}$ Poisson's Ratio (ν)			$C_{\psi 2}$ Poisson's Ratio (ν)		
	0.25	0.33	0.40	0.25	0.33	0.40
α_0	3.5903	4.1604	4.6592	0.0000	0.0000	0.0000
α_1	0.0442	0.1540	0.3253	0.1402	0.1862	0.2257
α_2	-3.1681	1.2512	5.0929	-1.3206	-2.1306	-2.8239
α_3	13.9574	-13.9884	-38.3061	10.3798	15.7100	20.2794
α_4	-39.0952	37.4635	104.0810	-31.5741	-48.5119	-63.0589
α_5	58.0007	-53.3897	-150.2913	52.9503	82.2342	107.4348
α_6	-46.0276	43.8411	121.9951	-48.7489	-76.3972	-100.2298
α_7	18.6417	-19.3834	-52.4408	23.0217	36.4197	47.9837
α_8	-3.0381	3.524	9.2800	-4.3517	-6.9583	-9.2104

of pile cross section. The harmonic motion

$$u(z,) = u(z)\, e^{i\omega t} \tag{13}$$

where u(z) is the complex amplitude of horizontal motion of depth z of i^{th} element.

Neglecting the coefficient of pile internal damping and combining the Eqs. 12 and 13 the complex amplitude of horizontal motion can be expressed as

$$u(z) = C_1 \cosh\lambda \frac{z}{h} + C_2 \sinh\lambda \frac{z}{h} + C_3 \cos\lambda \frac{z}{h} + c_4 \sin\lambda \frac{z}{h} \tag{14}$$

in which $C_1, C_2, C_3, C_4 =$ integration constants, h = height/length of the element and the complex frequency parameter,

$$\lambda = h \left[\frac{1}{EI} - \left\{ m\omega^2 - k_u(Z) - i\omega C_u(Z) \right\} \right]^{0.25} \tag{15}$$

The moment M and horizontal transverse force H are

$$M = -EI \frac{d^2 u(z)}{dz^2} \tag{16}$$

$$H = -EI \frac{d^3 u(z)}{dz^3} \tag{17}$$

Numerical Solution

The impedance function of the embedded pile can be obtained from the expression for complex amplitude of horizontal motion by employing matrix stiffness method. This method is based on the formulation of element dynamic stiffness matrix (Novak and Aboul-Ella, 1978).

Element Dynamic Stiffness Matrix: Coefficient of the element dynamic stiffness matrix can be evaluated from Eqs. 14, 16 and 17. Applying horizontal displacements $u = 1$ and rotations $\psi = 1$ at the ends of an element one at a time, the element dynamic stiffness matrix for horizontal translation and rotation $[k_d]$, is

$$[k_d] = \frac{EI}{h} \begin{bmatrix} F_6(\Lambda)/h^2 & F_4(\Lambda)/h & F_5(\Lambda)/h^2 & F_3(\Lambda)/h \\ F_4(\Lambda)/h & F_2(\Lambda) & F_3(\Lambda)/h & F_1(\Lambda) \\ F_5(\Lambda)/h^2 & F_3(\Lambda)/h & F_6(\Lambda)/h^2 & F_4(\Lambda)/h \\ -F_3(\Lambda)/h & F_1(\Lambda) & -F_4(\Lambda)/h & F_2(\Lambda) \end{bmatrix} \tag{18}$$

where the dimensionless functions,

$$F_1(\Lambda) = -\frac{1}{\phi} \lambda (\sinh\lambda - \sin\lambda)$$

$$F_2(\Lambda) = -\frac{1}{\phi}\lambda\,(\cosh\lambda.\,\sin\lambda - \sinh\lambda.\,\cos\lambda)$$

$$F_3(\Lambda) = -\frac{1}{\phi}\lambda^2(\cosh\lambda - \cos\lambda)$$

$$F_4(\Lambda) = \frac{1}{\phi}\lambda^2(\sinh\lambda.\,\sin\lambda)$$

$$F_5(\Lambda) = \frac{1}{\phi}\lambda^3(\sinh\lambda + \sin\lambda)$$

$$F_6(\Lambda) = -\frac{1}{\phi}\lambda^3(\cosh\lambda.\,\sin\lambda + \sinh\lambda.\,\cos\lambda)$$

$$\phi = \cosh\lambda.\,\cos\lambda - 1$$

The structural (overall) dynamic stiffness matrix $[K]$, which relates the nodal force $\{X\}$ and displacements $\{\delta\}$ of the embedded pile, is obtained by assembling the element dynamic stiffness matrices along with appropriate tip condition. Pile tip condition has a significant effect on the dynamic behaviour particularly on short pile. Hence nodal force and displacements can be related as,

$$\{X\} = [K]\,\{\delta\} \tag{19}$$

Complex Stiffness of Pile Head: Complex stiffness of the pile at its head is the external force required to produces unit horizontal displacement or rotation at its head. Hence,

$$\begin{bmatrix} K^c_{uu} & K^c_{u\psi} \\ K^c_{\psi u} & K^c_{\psi\psi} \\ 0 & 0 \\ 0 & 0 \\ \cdot & \cdot \\ \cdot & \cdot \\ 0 & 0 \\ 0 & 0 \end{bmatrix} = [K] \begin{bmatrix} u_1=1 & u_1=0 \\ \psi_1=0 & \psi_1=1 \\ u_2 & u_2 \\ \psi_2 & \psi_2 \\ \cdot & \cdot \\ \cdot & \cdot \\ u_{n+1} & u_{n+1} \\ \psi_{\nu+1} & \psi_{\nu+1} \end{bmatrix} \tag{20}$$

where, K^c_{uu}, $K^c_{\psi\psi}$ are the external complex horizontal force and moment required to produce unit horizontal displacement and rotation at the pile head, termed as complex horizontal and rotational stiffness respectively. $K^c_{u\psi} = K^c_{\psi u}$ is termed as complex cross stiffness $u_1, u_2 \ldots\ldots\ldots u_{n+1}$ and $\psi_1, \psi_2 \ldots\ldots\ldots\ldots \psi_{\nu+1}$ are the nodal horizontal and rotational displacements, n being the number of elements.

Complex pile head stiffnesses may be written as:

Horizontal stiffness $K^c_{uu} = K_{uu} + i\,\omega\, C_{uu}$

where true stiffness $K_{uu} = \text{real}\, K^c_{uu} = \frac{E\ I}{r_o^3} f_{u1}$ (21)

and equivalent horizontal damping

$$C_{uu} = \frac{1}{\omega}\, \text{imag}\, K^c_{uu} = \frac{E\ I}{r_0^2 V_s} f_{u2} \quad (22)$$

f_{u1}, f_u2 = dimensionless horizontal stiffness and damping parameters.

Rotatory stiffness $K^c_{\psi\psi} = K_{\psi\psi} + i\ \omega\ C_{\psi\psi}$

where true rotatory stiffness $K_{\psi\psi} = \text{real}\ K^c_{\psi\psi} = \frac{E\ I}{r_0} f_{\psi 1}$ (23)

and equivalent damping, $c_{\psi\psi} = \frac{1}{\omega}\, \text{imag}\, K^c_{\psi\psi} = \frac{E\,I}{V_s} f_{\psi 2}$ (24)

$f_{\psi 1}, f_{\psi 2}$ = dimensionless cross stiffness and damping parameters.

Cross-stiffness $K^c_{u\psi} = K_{u\psi} + i\ \omega\ C_{u\psi}$

where true cross stiffness $K_{u\psi} = \text{real}\ K^c_{u\psi} = \frac{E\,I}{r_0^3} f_{c1}$ (25)

and equivalent cross damping

$$C_{u\psi} = \frac{1}{\omega}\, \text{imag}\ K^c_{u\psi} = \frac{E\,I}{r_o V_s} f_{c2} \quad (26)$$

f_{c1}, f_{c2} = dimensionless cross stiffness and damping parameters.

Response of Pile Supported Footing

Once the stiffness and damping constants at the head of a pile are established, the stiffness and damping constants at the center of gravity (C.G) of the footing can be expressed as;

$$\left\{\begin{array}{ll} K^f_{uu} & = K_{uu} \\ c^f_{uu} & = C_{uu} \\ K^f_{\psi\psi} & = K_{\psi\psi} + K_{uu}.Z_c^2 - 2K_{u\psi}.Z_c \\ C_{\psi\psi} & = C_{\psi\psi} + C_{uu}.Z_c^2 - 2K_{u\psi}.Z_c \\ K^f_{u\psi} & = K_{u\psi} - K_{uu}.Z_c \\ C^f_{u\psi} & = C_{u\psi} - C_{uu}.Z_c \end{array}\right\} \quad (27)$$

where the superscript f indicates the respective magnitudes of stiffness and damping constants of the footing and Z_c is the distance between the pile head and the C.G. of the footing. With the stiffness and damping constants, obtained from Eq. 27, the response of the footing under coupled excitation may be determined as that for a shallow footing. Generally such response are expressed in terms of dimensionless sliding amplitude A_u and rotational amplitude, A_ψ [Saha and Ghosh, 1982].

Comparison with Experiments

In order to check the validity and accuracy of the proposed approximate theory, the theoretical results are compared with the reported experimental results of Novak and Grigg (1976) for response of footing (mass = 453.1 kg, mass moment of inertia $I_\psi = 17.97$ kg m^2), supported on single black steel pile (outer diameter = 0.06 m., flexural rigidity = 56880.5 $N\ m^2$, slenderness ratio = 77.9) embedded in homogeneous soil medium (Poisson's ratio = 0.25, unit weight = 1792.0 kg/m^3 and shear wave velocity = 175.0 m/sec), under horizontal excitation. details of the test description are available in literature (Novak and Grigg, 1976). Figures 2 and 3 show the experimental

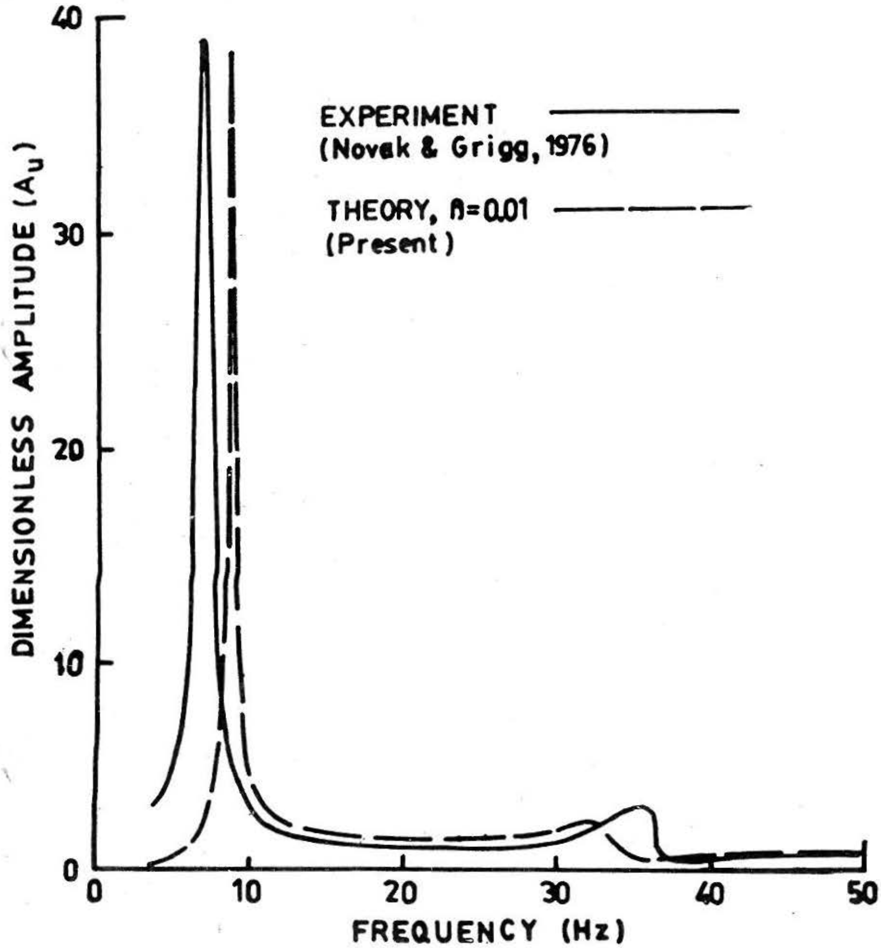

FIGURE 2. Comparison of Experimental and Theoretical Response of Single Pile Foundation Under Lateral Excitation (Sliding Component).

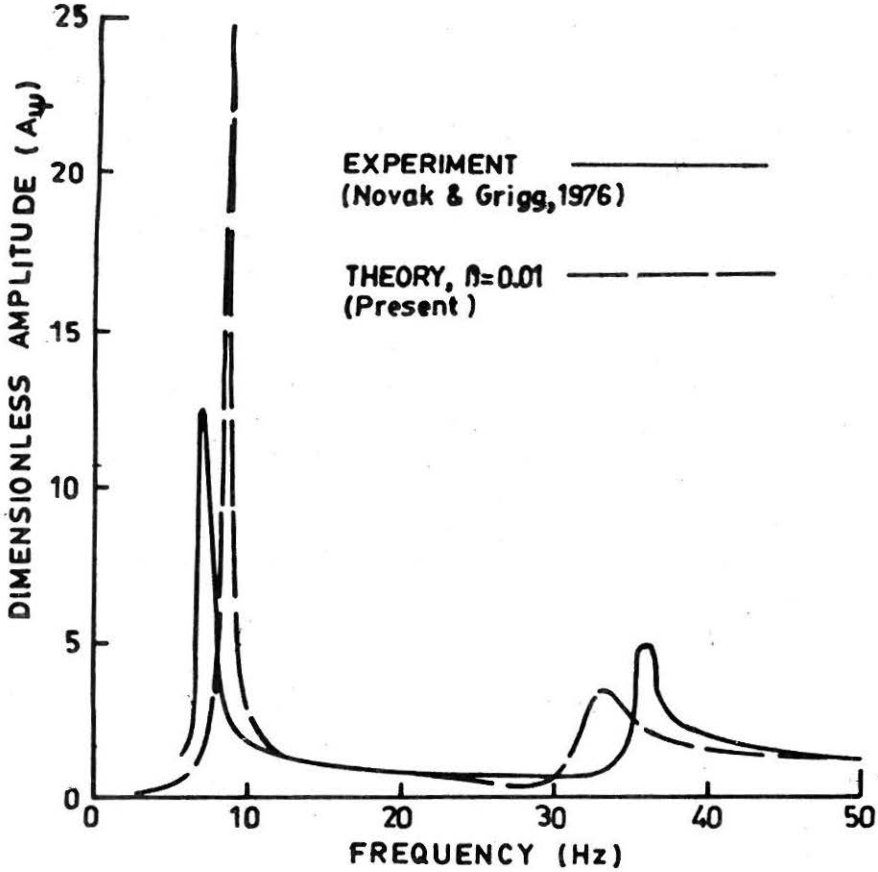

FIGURE 3 Comparison of Experimental and Theoretical Response of Single Pile Foundation Under Lateral Excitation (Sliding and Rocking Component).

and theoretical response for sliding and rocking components of the pile supported footing. The predicted curves are given for $\beta = 0.01$. It is observed that the experimental first and second resonant frequencies are 7.0 Hz and 36.0 Hz while the predicted values are 8.6 and 32.5 Hz. The theoretical sliding component has a good match with the experimental resonant amplitude. For rotational component, it is seen that the first resonant amplitude while the second resonant is in good agreement. Since in any dynamic analysis the determination of resonant frequency rather than amplitude is more relevant, this simple method may be used to predict the dynamic response of pile supported footing.

Theoretical Results and Discussion

The effect of wave velocity ratio V_s / V_c, V_c being the compressional wave velocity in the pile, and material damping on the response of footing (mass

= 5400.0 kg, $Z_c = 0.50$ m and height of excitation above C.G. of footing = 0.75 m), supported on fixed tip single concrete pile (r_0 = 0.15 m) are presented in Fig. 4. It is observed that the resonant frequencies and amplitude increase with the increase of wave velocity ratio while material damping reduces the resonant amplitude and its effect on resonant frequency is insignificant.

The variation of dimensionless stiffness and damping parameters of fixed tip concrete pile with dimensionless frequency α_0 are shown in Fig. 5. The horizontal stiffness parameter (f_{u1}) decreases with the frequency and becomes negative at $\alpha_0 \approx 0.85$. This trend is observed irrespective of slenderness ratio (l/r_0), wave velocity ratio and Poisson's ratio of soils. The rotational stiffness parameter ($f_{\psi 1}$), initially decreases gently with frequency but the trend is

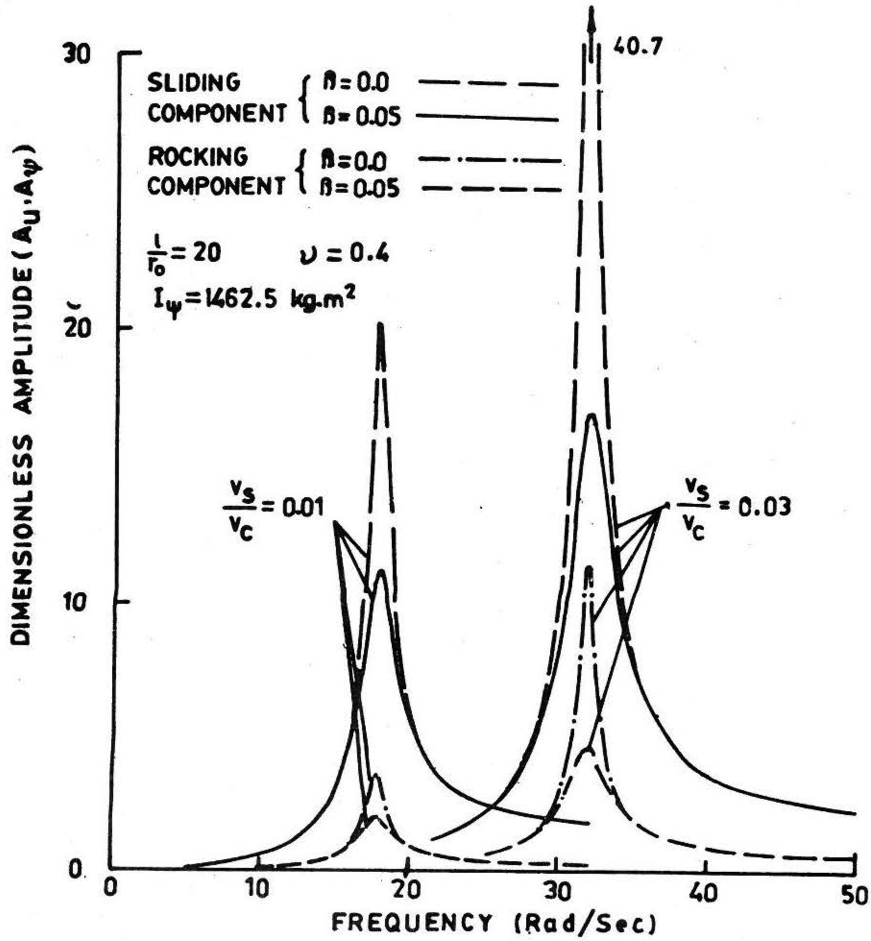

FIGURE 4 Effect of Wave Velocity Ratio and Material Damping (β) on Dynamic Response of Fixed Tip Pile.

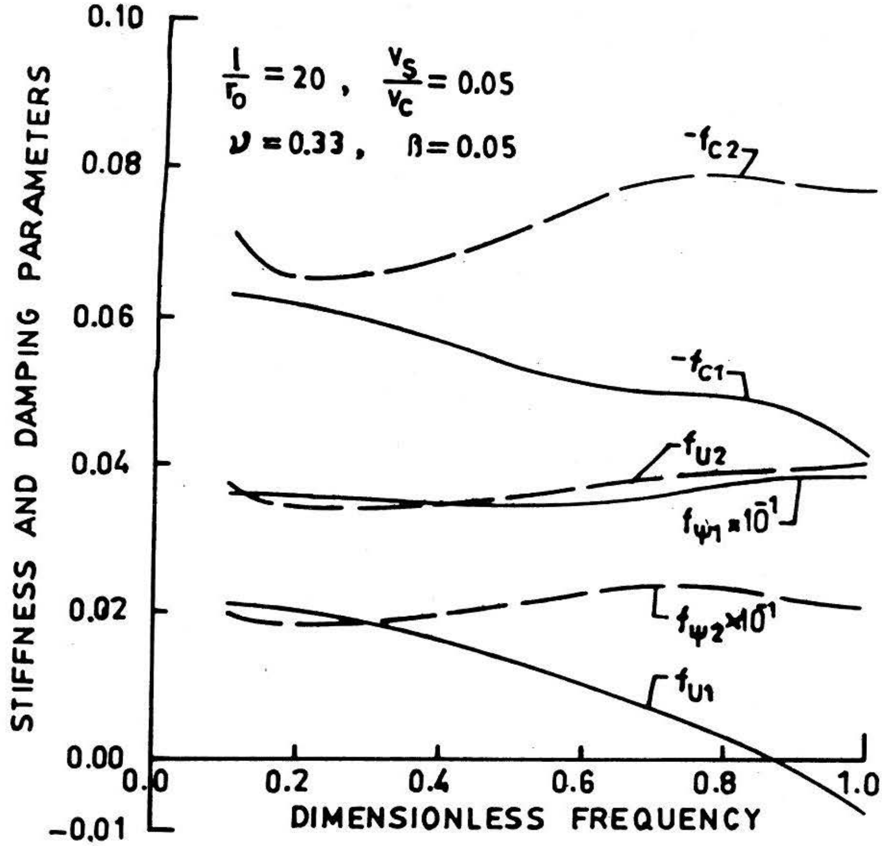

FIGURE 5 Variations of Stiffness and Damping Parameters with Dimensionless Frequency ($a_0 = \omega r_0/V_s$).

reversed at higher frequency ($\alpha_0 > 0.6$) where as the cross stiffness parameter (f_{c1}) sharply decreases upto $\alpha_0 \approx 0.6$ and becomes more or less constant within the frequency range $0.6 < \alpha_0 < 0.85$, followed by decreasing trend ($a_0 > 0.9$). The damping parameters ($f_{u2}, f_{\psi2}, f_{c2}$) decreases at lower frequency range followed by an increase at $\alpha_0 \approx 0.2-0.3$ and a decreasing trend for $f_{\psi2}$ and f_{c2} beyond $\alpha_0 \approx 0.7$.

Figure 6 illustrates the variation of dimensionless stiffness and damping parameters with wave velocity ratio for fixed tip concrete pile at a given frequency. The magnitude of these parameters increases with the increasing wave velocity ratio. The variation of cross stiffness and damping parameters is very sharp and nearly proportional to wave velocity ratio compared to the

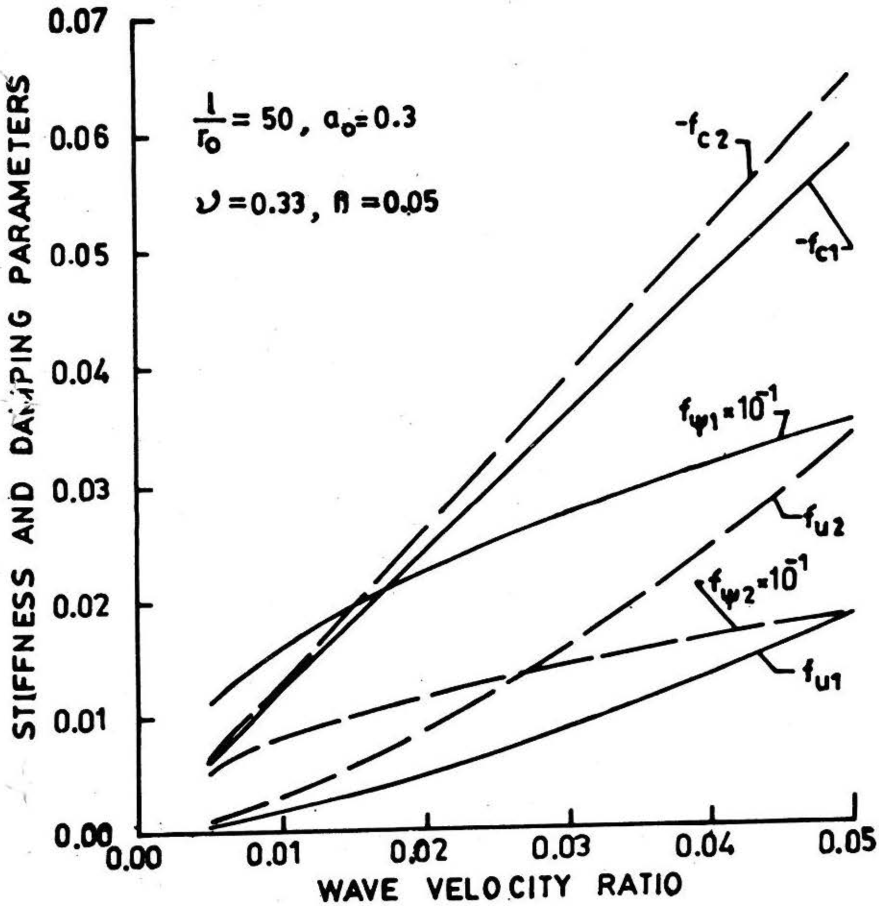

FIGURE 6 Variations of Stiffness of Damping Parameters with Wave Velocity Ratio V_s / V_c.

horizontal and rotational stiffness and damping parameters.

Typical variation of dimensionless stiffness and damping parameters with slenderness ratio at a given frequency and wave velocity ratio for a fixed tip concrete pile are shown in Fig.7. The stiffness parameters decrease and damping parameters increase sharply with increasing slenderness ratio upto 20.0 and becomes constant at $l / r_0 \approx 30.0$.

Conclusions

An approximate method of analysis is presented for determining the dynamic response of laterally loaded pile. The proposed method uses Mindlin's static solution and theory of plate vibration over elastic halfspace along with

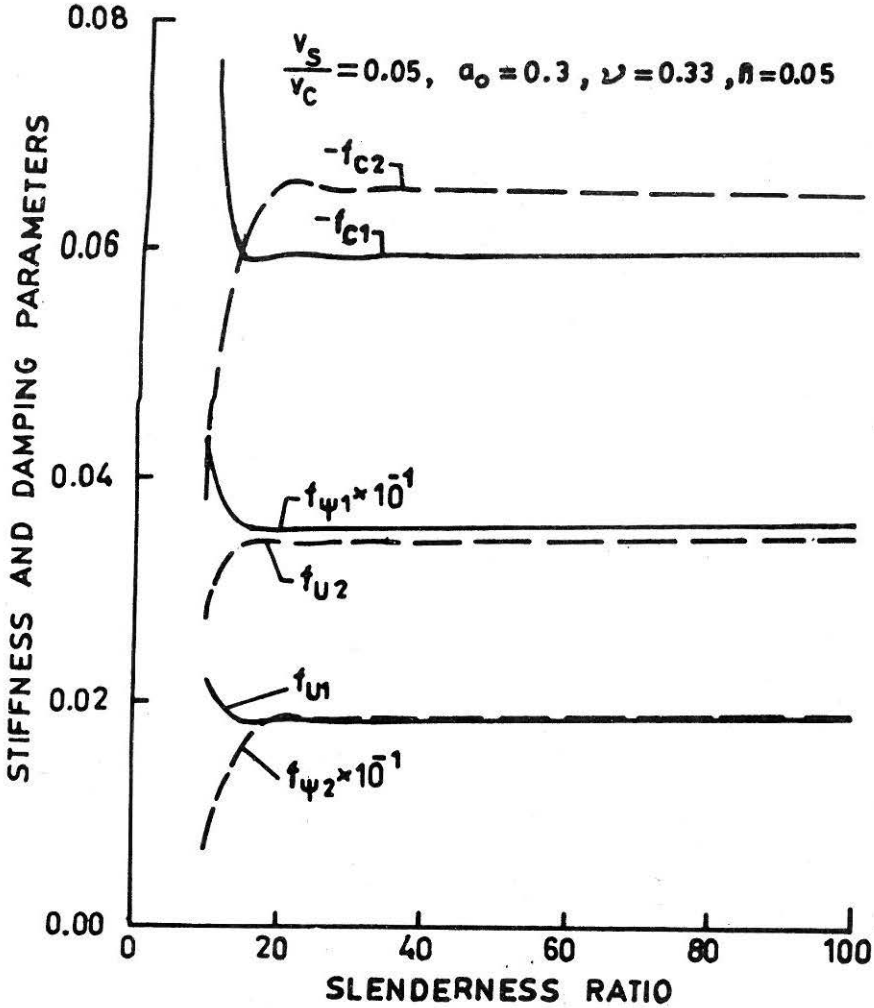

FIGURE 7 Variations of Stiffness and Damping Parameters with Slenderness Ratio (l/r_0).

correspondence principle of viscoelasticity for estimating impedance function. The theoretical results are compared with the reported experimental results. The effect of various factors on stiffness and damping parameters are studied. From the above discussion the following conclusions are tentatively drawn.

1. The proposed method of analysis can predict satisfactorily the response of footings resting on a single pile subject to horizontal excitation.

2. Material damping has a significant effect on resonant amplitude whereas negligible effect on resonant frequency.

3. The dimensionless stiffness and damping parameters are found to be

frequency dependent and vary with the wave velocity ratio and slenderness ratio ($l / r_0 \leq 30$).

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