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One Dimensional Consolidation of Lightly OC Clays

by

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Introduction

Soft and highly compressible clays exist at many sites especially in coastal lowlands. Large settlements and slow rates of consolidation pose problems for the design of structures founded on them. Terzaghi's one dimensional theory of consolidation has withstood the test of time and is still widely used for predicting the degree of consolidation even though it is derived for a normally consolidated soil only. To monitor the in situ degree of consolidation, the pore pressures are usually measured and based on them, the degree of settlement is estimated. However, except in the case of simple linear theory of consolidation, the degree of pore pressure dissipation and settlement do not agree. Some of the nonlinear theories of consolidation (Davis and Raymond, 1965) explain and elaborate this difference for normally consolidated soils.

A soil in situ may often exist in an overconsolidated state for various reasons viz. cementation, isostatic uplift, glaciation, etc. (Brenner et al., 1980). Most soft clays exhibit pseudo-preconsolidation effect (Bjerrum, 1972) due to aging (creep or secondary compression), and behave as lightly over consolidated soils (Burland, 1971). The OCR due to aging ranges between 1.0 and 2.0 for plasticity index ranging from 0 to 100. The soft clays show overconsolidation effect due to desiccation also. Mesri and Rokhsar (1974) developed a very general theory of consolidation which accounts for finite strains, variable compressibility and permeability, secondary compression, and preconsolidation effects. A similar approach has been proposed by Tavenas *et al.* (1979), who in addition, consider unsaturated conditions and compressible pore fluid. Balasubramaniam and Brenner (1980) present a comprehensive review of theories of consolidation. No simple theory is available to predict the consolidation of lightly over consolidated soils. An attempt is made herein to extend the simple Terzaghi's theory for lightly over consolidated soils as a phase change process.

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Typical clay characteristics

Saga plain one of the lowlands, is underlain by Ariake clay whose thickness ranges between 15 m and 42 m. These clays have been deposited under marine and brackish environments (Ohtsubo et al., 1988). They are soft (undrained shear strength is as low as 7 MPa), and sensitive (sensitivity ranging between 20 to 200 with liquidity index varying from 1.3 to 3.0). The vertical and preconsolidation profiles with depth are depicted in Fig.1 (Miura et al., 1988), from which it may be noted that they are lightly over consolidated with OCR ranging between 1.0 and 4.0 except near the surface where it is very high due to desiccation. The void ratio - logorithm of effective stress relation for a typical sample shows C_c value of about 1.1 and a C_s value of 0.15. The coefficient of volume change, m_{vc}, for virgin compression, ranges (Fig. 2) between 3.0 to 0.3 m^2/MN while the coefficient for reloading, m_{vr}, lies in the range 0.5 to 0.05 m^2/MN. Thus, m_{vr} is a small fraction of m_{vc}.

Theory

A finite clay layer of thickness, H, consolidating under stress increment, $\Delta\sigma$, (Fig 3a), and governed by bi-linear void ratio-effective stress relation (Fig. 3b), is considered. The present theory incorporates all the assumptions (infinitesimal deformations, Darcy's law, one dimensional flow and deformation, linearity of void ratio - effective stress relation along the virgin compression curve, etc.) of Terzaghi theory except that the soil is lightly over consolidated. Point A (Fig. 3b) represents the initial state (e_0 and σ'_0) of the soil. If a uniform stress increment, $\Delta\sigma$, is applied to the soil represented by point A on the reloading part of the curve, the stress in the soil passes through point B to point C on the virgin compression line, provided the stress increment is large enough. The path A to B is in the $O.C.$ range while the path B to

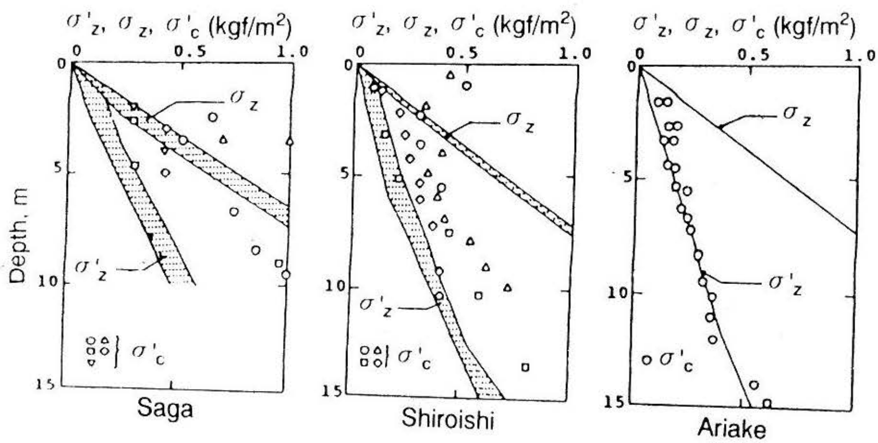

FIGURE 1 Typical Total, Effective and Preconsolidation Stresses in Soils of Saga Plain (Minura et al., 1988)

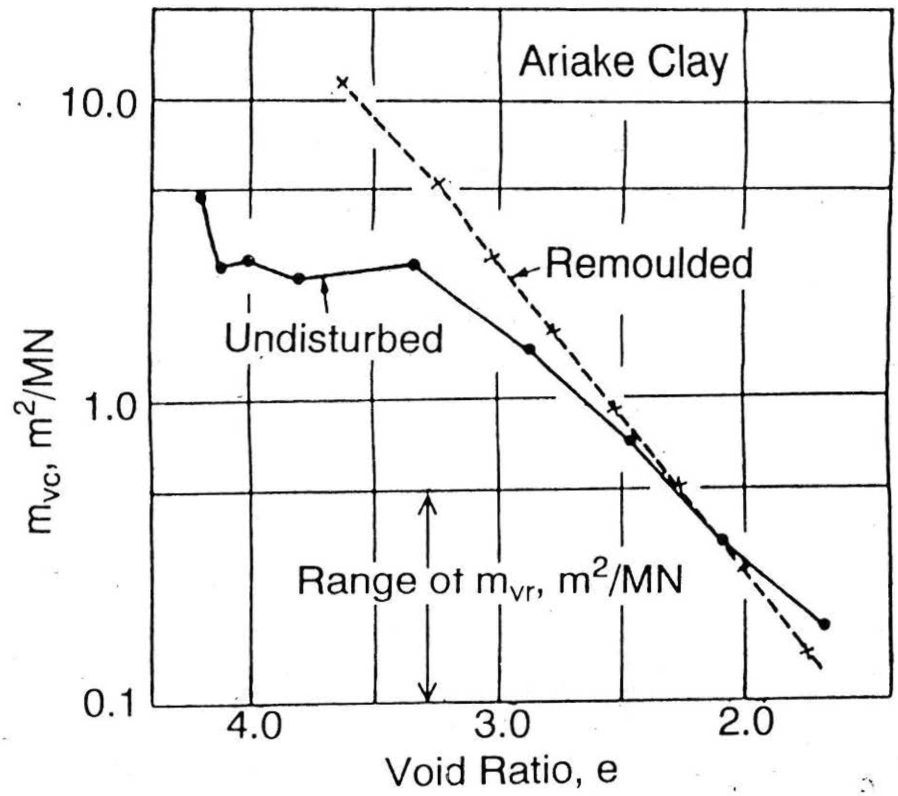

FIGURE 2 Typical m_{vc} - Void Ratio Relation for Undisturbed Clay (Onitsuka, 1988)

C is along the *N.C.* line. For simplicity AB and *BC* are considered to be straight lines. The slope of the $e - \sigma'$ curve for the recompression stage is defined as coefficient of volume change for recompression, m_{vr}, while the slope of *BC* is m_{vc}, the coefficient for virgin compression. The ratio, μ, defined as

$$\mu = m_{vc} / m_{vr} \tag{1}$$

signifies the relative magnitudes of the two compression coefficients. Even though during consolidation, every element of the soil experiences the same path ABC, the overall response of a clay layer of finite thickness will differ from it, since points at different depths pass through point B at different times. Initially *i.e.* at time, $t = 0$, all points will be at *A* (Fig. 3b). Once consolidation begins, points close to the drainage boundary traverse through the *OC* state (*A* to *B*) very rapidly and pass on to the *NC* stager (*B* – *C*). With increasing time, the depth over which the pore pressures have dissipated enough to make the effective stresses exceed the preconsolidation stress,

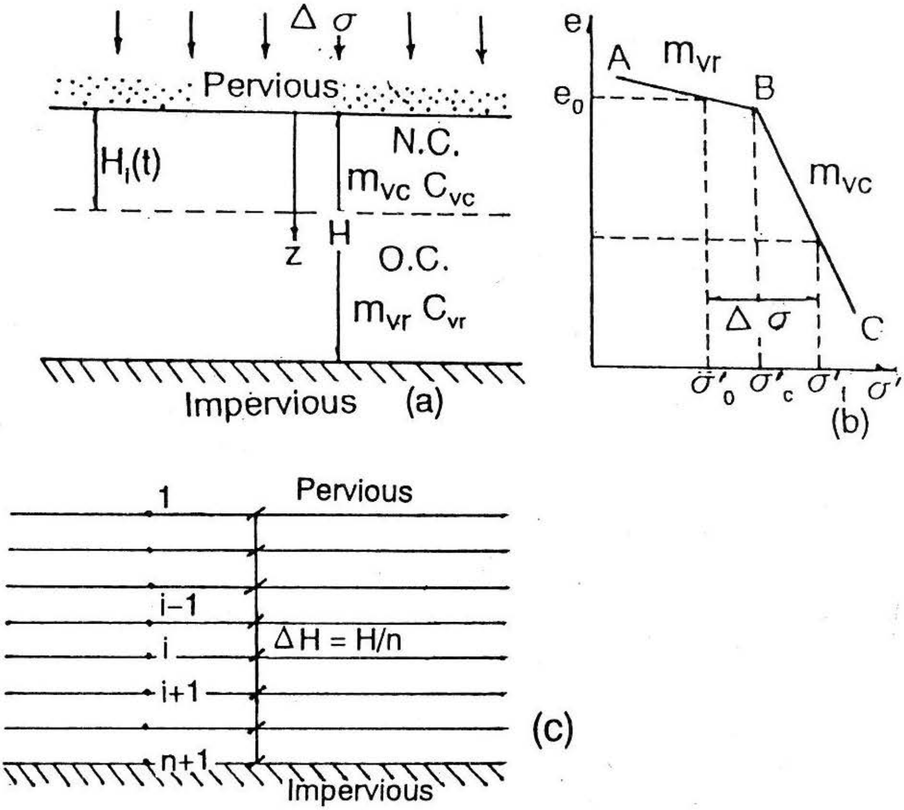

FIGURE 3 Definition Sketch (a) Consolidating Layer, (b) Bi-linear e − σ ' Relation and (c) Discretisation

σ'_c, increases. The boundary *M-M* (Fig. 3a) between the normally consolidated soil and the soil in the *OC* state moves down as function of time. The depth, H_i to the interface *M-M*, increases from 0 at time, $t = 0$ to H after a certain time, t_i', after which the whole consolidating layer follows the virgin compression line. The time, t_i , at which all the elements of the soil are in a N.C. state, depends on the stress increment, $\Delta\sigma$, and the *OCR* $(\sigma'_c - \sigma'_0)$. Hence, consolidation of a lightly *O.C.* soil is a moving (internal) boundary problem in which the interface between *NC* and *OC* states moves from the drainage boundary towards the impervious boundary as a function of time.

Following Terzaghi, the coefficient of permeability, k, and the two coefficients of volume change, m_{vr} and m_{vc},. are assumed to be constant during a particular load increment. Consequently, the two states, *OC* and *NC* of the soil are governed by the two coefficients of consolidation,

C_{vc}, and C_{vc}, inversely proportional to m_{vr}, and m_{vc}, respectively. The consolidation of a lightly OC clay layer of thickness, H, is akin to that of a two layered soil, but with a difference. The interface between the two layers moves as a function of time. At any time, t, the interface is at depth, H_i, from the top, and the consolidation phenomenon is governed by

$$\text{Layer I} \qquad \frac{\partial u_1}{\partial t} = C_{vc} \frac{\partial^2 u_1}{\partial z^2} \qquad \text{for } 0 < z < H_i \tag{2}$$

$$\text{Layer II} \qquad \frac{\partial U_2}{\partial t} = C_{vr} \frac{\partial^2 u_2}{\partial z^2} \qquad \text{for } H_i < z < H \tag{3}$$

where $u_1 = u_1(z,t)$ and $u_2 = u_2(z,t)$ are the pore pressures in the top (N C) and bottom (O C) layers respectively, z and t – the depth and the time variables. The boundary and the initial conditions are:

$$t > 0, \quad u = 0 \quad \text{at } z = 0 \tag{4}$$

$$t >= 0, \quad \partial u / \partial z = 0 \quad \text{at } Z = 0 \tag{5}$$

$$t > 0, \quad u_1 = u_2, \quad k\partial u_1 / \partial z = k\partial u_1 / \partial z, \text{ and } \sigma' = \sigma'_c \text{ at } z = H_i \tag{6}$$

$$t = 0, \quad u_1 = u_2 = \Delta\sigma = u_0 \quad \text{for } 0 < Z < H. \tag{7}$$

where u_0 is the intitial pore pressure. It is difficult to obtain an analytical solution to Eqs.(2) and (3) subject to be condition specified in Eqs.(4) through (7). The explicit finite difference approach is adopted to solve the above equations. Dividing the clay layer into n sublayers each of thickness $\Delta H = H/n$ (Fig.3c). Equations (2) and (3) are nondimensionalised as

$$W_{i,T+\Delta T} = W_{i,T} + \beta_c \left\{ W_{i-1,T} - 2W_{i,T} + W_{i+1,T} \right\} \qquad \text{for } i < I_t \tag{8}$$

and

$$W_{i,T+\Delta T} = W_{i,T} + \beta_r \left\{ W_{i-1,T} - 2W_{i,T} + W_{i+1,T} \right\} \qquad \text{for } i < I_t \tag{9}$$

where

$$W_{i,T} = u_{i,T}/u_0, \; \beta_c = C_{vc} \cdot \Delta t/(\Delta H)^2, \; \beta_r = C_{vr} \cdot \Delta t/(\Delta H)^2 = \mu . \beta_c, \; I_t = H_i/\Delta H$$

Since the permeabilities of the two zones are the same at point B on the interface, or $i = I_t$, it can be easily shown (Das, 1993), that

$$W_{i,T+\Delta T} = W_{i,T} + 2\beta_c/(1+\mu) \left\{ W_{i-1,T} - 2W_{i,T} + W_{i+1,T} \right\} \tag{10}$$

The impervious bottom boundary is accounted for by taking $W_{n+2,T} = W_{n,T}$ which implies Eq.(5). Initially, *i.e.*, at $T = 0$, the normalized pore pressures are

$$W_{i,0} = 1{\cdot}0 \quad \text{for all } i. \tag{11}$$

To initiate consolidation, the pore pressure at node 1, *i.e.* at the drainage boundary is equated to zero and a gradient for flow set up. In the numerical solution, at $T = 0$, the soil over its full depth is in lightly *OC* state and the interface between *N.C.* and *OC* states is at $Z = 0$ ($I_t = 1$). The pore pressures at the first time increment (ΔT) are evaluated from Eq. (9) and the effective stress, $\sigma_{i,T}$, calculated at each node as

$$\sigma_{i,T} = \sigma_0 + \Delta\sigma - W_{i,T}\,\mu_0 \tag{12}$$

and checked with the maximum past pressure, σ'_c: If

$$\sigma'_{i,T} > \sigma'_c \tag{13}$$

the soil goes into a *NC* state and the depth over which $\sigma'_{i,T} > \sigma'_c$ is noted. For the next time increment, Eq. (8) is used for the nodes wholly in the *NC* state, Eq. (9) for the nodes wholly in the *OC* state and Eq. (10) for the node on the interface, *i.e.* for $i = I_t$. These steps are repeated for each subsequent time increments and the progress of the *NC* /*OC* front and the respective consolidations are monitored. The time step, ΔT, and the sublayer depth, ΔH, are chosen such that it does not increase by more than one during any time step. That is, the transition from the *OC* state to the *NC* state takes place over only a small depth of the layer (ΔH). The accuracy of the solution can be further increased by choosing a sufficiently large value for n.

After the calculation of pore pressures at each time step, the normalized residual pore pressure, U_p, is calculated as

$$U_P = (1/H) \int_0^H u \, dz / u_0 \tag{14}$$

or numerically using the Simpson's rule.

The degree of settlement, U_s, is calculated as follows: The final settlement of the clay layer, S_f , is

$$S_f = \left\{ m_{vr}(\sigma'_c - \sigma'_0) + m_{vc}(\sigma'_0 + \Delta\sigma - \sigma'_c) \right\} H \quad \text{or} \tag{15}$$

$$= m_{vc}\,\Delta\sigma\,H \left\{ (OCR - 1)/\mu + SIR - OCR \right\} / SIR \tag{16}$$

where

$$OCR = \sigma'_c/\sigma'_c, \; SIR = \Delta\sigma/\sigma'_0 \text{ and } \mu = m_{vc}/m_{vr.}$$

The settlement, $\Delta S_{i,t}$, of the ith layar at any time, t, is

$$\text{if } \sigma' < \sigma'_c \;\; \Delta S_{i,t} = m_{vr}\,(\sigma' - \sigma'_0)\,\Delta H \tag{17}$$

$$= m_{vc} \,.\, \Delta\sigma \,.\, \Delta H \left\{ 1/SIR + 1 - W_{i,T} \right\} \tag{18}$$

and

$$\text{if } \sigma' > \sigma'_c \;\; \Delta S_{t,T} = \left\{ m_{vr} \cdot (\sigma'_c - \sigma'_0) + m_{vc} \cdot (\sigma' - \sigma'_c) \right\} \Delta H \tag{19}$$

$$= m_{vc} \,.\, \Delta\sigma \,.\, \Delta H \left\{ (OCR - 1)/\mu \,.\, SIR + 1/SIR + 1 - OCR/SIR - W_{i,T} \right\} \tag{20}$$

The total settlement, S_t , of the layer at time, t, is

$$S_i = \sum_{1}^{n} \Delta S_{i,t} \tag{21}$$

The settlement of the I_t^{th} layer is calculated as the average of Eqs.(18) and (20). The degree of settlement, U_s , is

$$U_s = S_t / S_f \tag{22}$$

Results

Initially, the accuracy of the numerical solution is checked by analyzing a normally consolidated soil, *i.e.* with $OCR = 1.0$ but varying the number of elements, n, into which the consolidating soil, is divided. The value of β_r is taken as equal to 1/6 to keep the numerical errors in the finite difference equations to a negligible level. A value of n equal to 20 has been chosen as any further increase in its value did not improve the accuracy. The results agree closely with Terzaghi's values. A parametric study is carried out with the following ranges for the parameters considered:

$$\mu : 2 - 50; \quad OCR : 1.0 - 2.5; \quad SIR : 1.0 - 5.0$$

Transition from O.C. to N.C. Phase

Firstly the progress of consolidation with time is studied with respect to the movement of the normally consolidated/over consolidated (*NV/OC*) interface with time (Fig. 4). H_i is the normalized depth to the *NC/OC* interface or front. At $T = 0$, the front is at the top and H_i is equal to zero. With increasing time, the front moves down and reaches the bottom *i.e.* $H_i / H = 1.0$, after a certain time, beyond which the whole soil behaves as *NC* soil. Therefore at any time, the soil above H_i is in a *NC* State while the soil below H_i is in the *OC* state. The rate of movement of the front or interface is a function of the parameters μ , The movement of the front is faster for soils with higher values of the compressibility ratio, μ, since the coefficient of consolidation for the *OC* phase is higher if μ, is larger. Comparing curves for *OCR* equal to 1.1 and 2.0 (Fig. 4), soil with a lower *OCR* transits sooner into the *NC* state since even a small stress increment of $0.1\,\sigma'_0$ is adequate to change the state of the soil from *OC* to *NC* state. And if *SIR* is small,

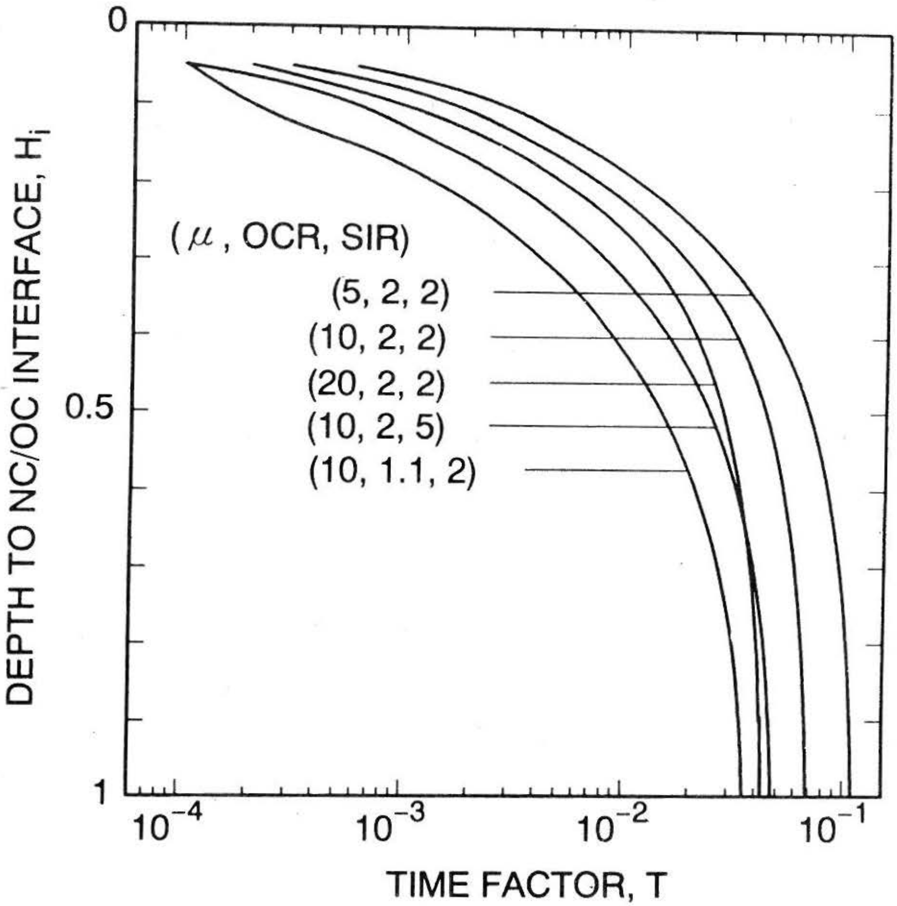

FIGURE 4 Movement of N.C./O.C. Interface with Time

e.g. 1.1, for a soil with an *OCR* of 2.0, the soil is almost wholly in the reloading stage and hence the *NC/OC* front moves much faster than the curve for $SIR = 2.0$, with μ and *OCR* remaining the same.

Pore Pressure Dissipation

Figure 5 presents the variation of normalized residual pore pressure, U_P, with the time factor, T, for different values of μ, which represents the ratio of the coefficients of volume change for virgin compression and reloading. For low values of μ ($=2$), the variation U_P with T is nearly the same as in Terzaghi's theory for normally consolidated soil. U_P decreases uniformly with time factor. With increasing clauses of μ, implying higher coefficients of consolidation for the reloading stage, since m_{vr} is smaller compared to m_{vc},

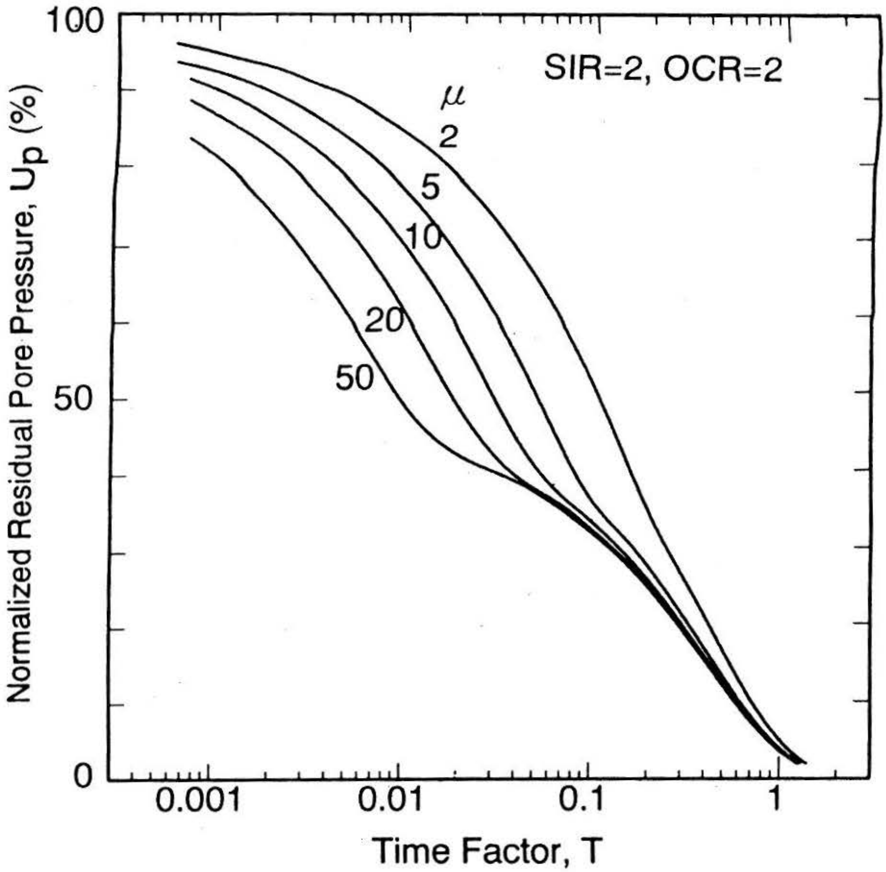

FIGURE 5 U_p - T Relations - Effect of μ

the pore pressure dissipation is faster in the early time periods but becomes slower at later times. For μ equal to 5 and 10, the transition from higher to lower rates of dissipation takes place at T equal to about 0.1 and 0.01 respectively. The initial rate of pore pressure dissipation is very high for μ = 50 since m_{vr} is very small. The overall rate of pore pressure dissipation slows down considerably when the upper part of the consolidating layer transits into the normal consolidation state and controls the dissipation of the pore pressure in the lower part which may still be in the reloading state. Mesri and Choi(1979) report field observations where pore pressures show rapid decrease initially and remain stationary for some time thereafter while settlement continue. At large times, *i.e.* $T \geq 0.5$, all the curves for μ = 2

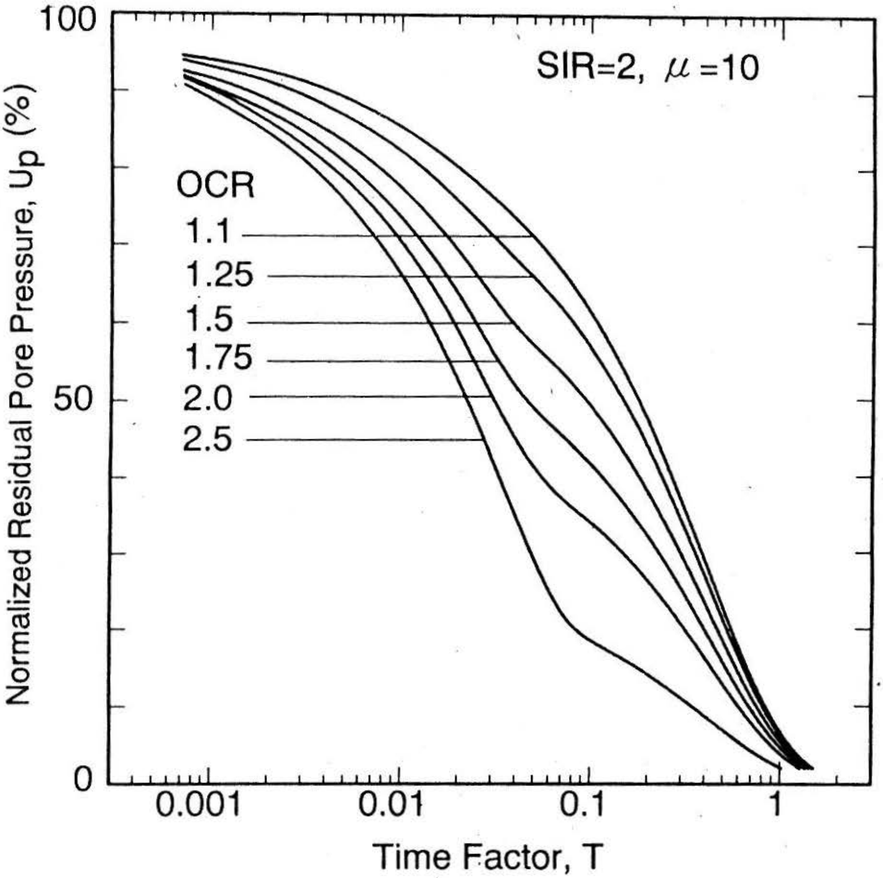

FIGURE 6 U_P - **T Relations - Effect of OCR**

to 50 tend to get closer and merge at about $T = 1.2$. The ratio μ effects the rate of pore pressure dissipation for U_P values up to about 10%.

A similar pattern of behaviour is observed for U_P versus T variation for soils with different *OCR* (Fig.6). For soils with low ($OCR \leq 1.1$), the U_P *vs* T relation is identical to the Terzaghi curve. The rate of pore pressure dissipation increases in the early stages of consolidation, with increasing values of *OCR*. For soils with *OCR* values of 1.1, 1.75, and 2.5, the normalized residual pore pressure values are about 73%, 54%, and 35% respectively at $T = 0.05$. The transition from faster to slower rates of pore pressure dissipation takes place at increasingly later times for increasing values of *OCR*. For soils

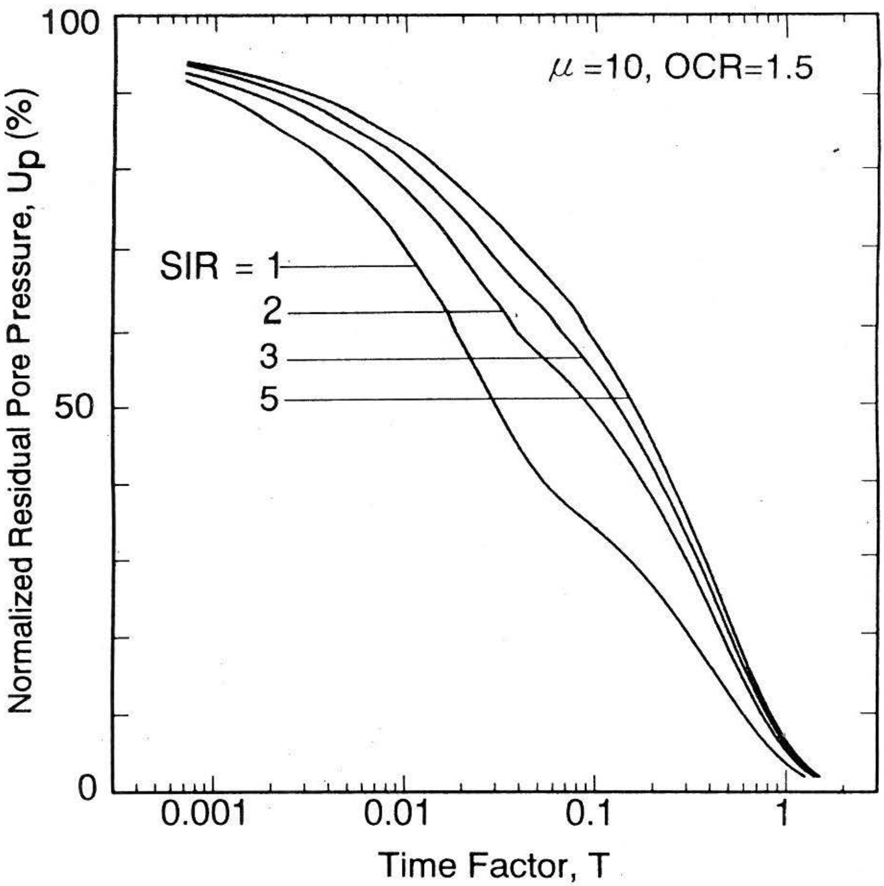

FIGURE 7 U_p - T Relations - Effect of SIR

with high *OCR* values, a major part of the stress increment is in the reloading stage and soil passes into the *N.C.* state at later times wherein the dissipation rates are slower.

The stress increment ratio, SIR, also has a significant effect on U_P vs T relation (Fig. 7). For a *N.C.* soil obeying linear void ratio - effective stress relation, SIR is not a parameter of significance. However, if the soil exhibits non-linear stress - strain behaviour or secondary compression, *SIR* is shown (Davis and Raymond, 1965, Wahls, 1962, *etc.*) to influence the consolidation phenomenon. A similar effect due to *SIR* can be noted in Fig. 7. A higher value of *SIR* implies a smaller range of over consolidation and closer is the $U_P - T$ variation to the linear Terzaghi theory. For smaller values of *SIR*,

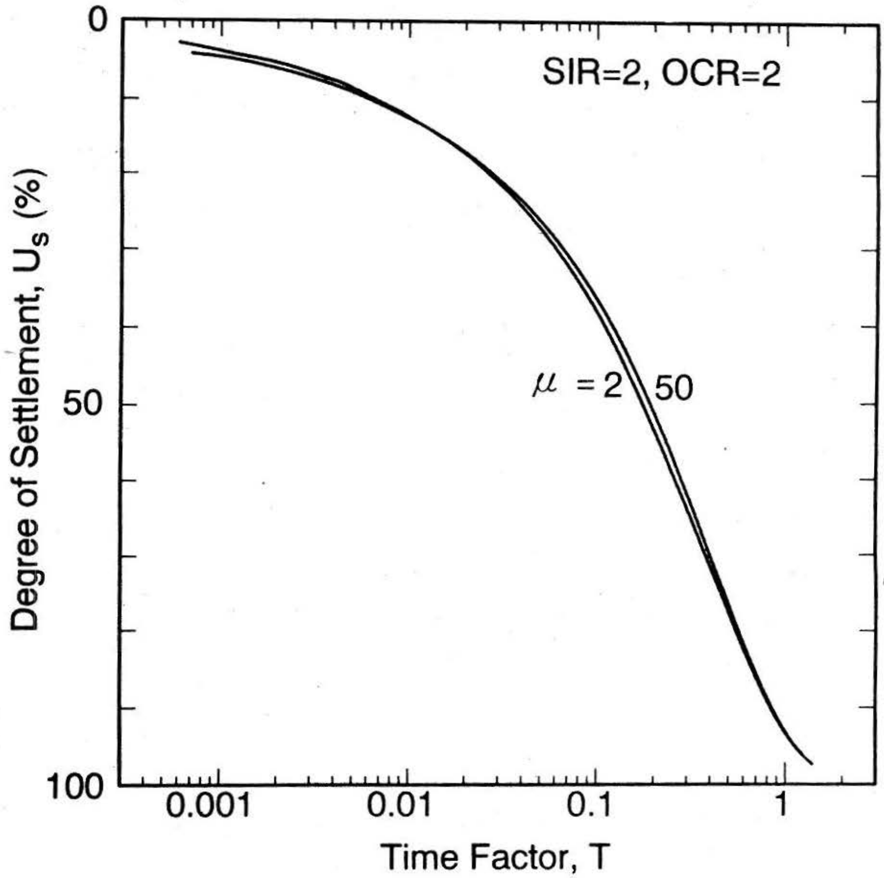

FIGURE 8 U_s-T Relations — Effect of μ

the curves move downward indicating faster rates of pore pressure dissipation because of larger contributions of the *OC* stage.

Degree of Settlement

The degree of settlement, U_s, versus time factor, T, relations as effected by the ratios μ, *OCR*, and *SIR*, are presented in Figs. 8, 9 and 10, respectively. Interestingly compared to the effects of these parameters on $U_p - T$ relationships, their influence $U_s - T$ on relations is relatively small. Results of Mesri and Rokhsar (1974) and Tavenas *et al.* (1979) also show similar feature. In particular, for μ increasing from 2 to 50, the difference in the two $U_s - T$ curves (Fig. 8) is very small (< 3 %) and Terzaghi's relation is close enough for all practical purposes.

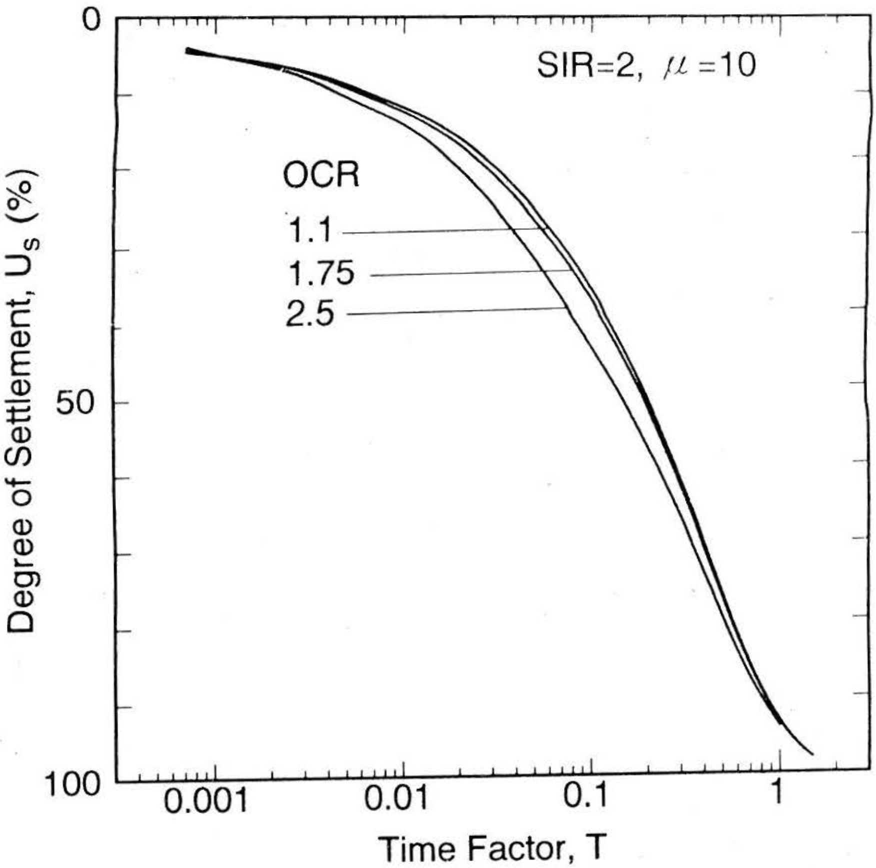

FIGURE 9 U_s-T Relations — Effect of OCR

Deviations from Terzaghi's theory become apparent (Fig. 9) in the $U_s - T$ relationships with increasing values of *OCR*. Compared to a *N.C.* consolidated soil, a lightly *O.C.* soil exhibits faster rates of consolidation. For an *OCR* of 2.5, the degree of settlement is about 5 to 7 % more than that for *N.C.* soil for *T* values in the range 0.01 to 0.4. In the early and the final stages of consolidation, the $U_s - T$ relations are independent of the *OCR* of the soil. The stress increment ratio also has a relatively small effect on the $U_s - T$ variation. For a soil with μ = 10 and *OCR* of 1.5, the difference in curves (Fig.10) with *SIR* of 1.0 and 5.0 is less than 2 to 3% in U_s values at any time factor. A large SIR, as mentioned earlier, implies that consolidation progresses predominantly in the *N.C.* range and the $U_s - T$ relations are

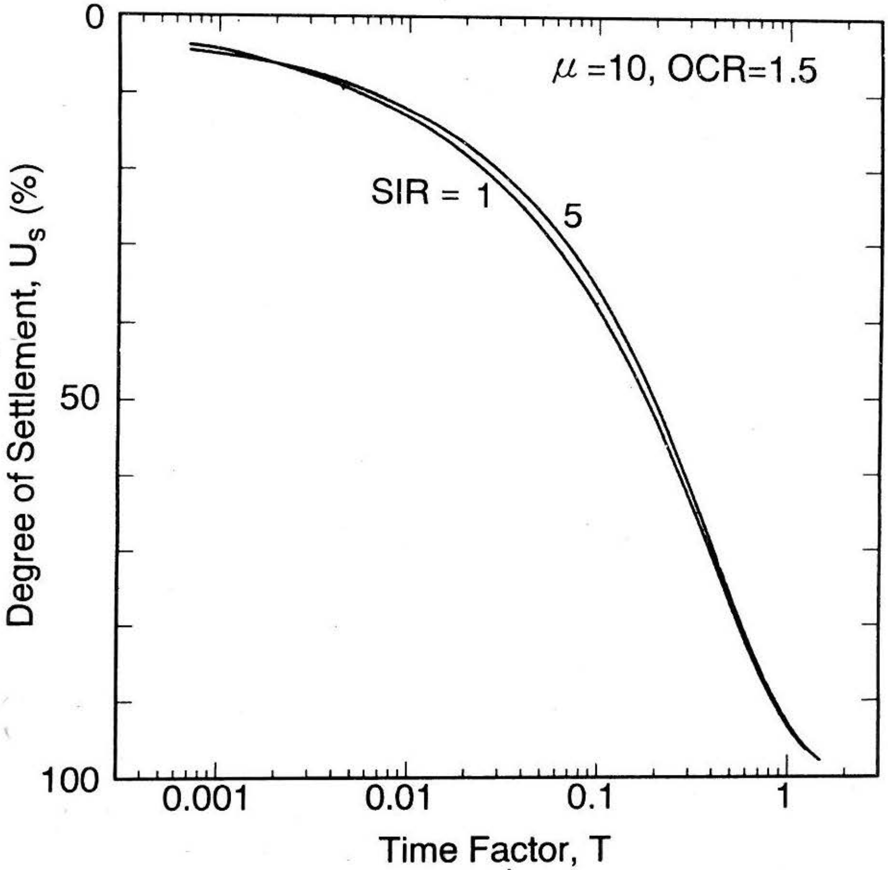

FIGURE 10 U_s-T Relations — Effect of SIR

nearly identical with Terzaghi's. For *SIR* equal to 1.0 the soil experiences over consolidation effects over half the stress increment (since $OCR = 1.5$), and virgin loading for the remaining stress increment. The $U_s - T$ relation for this case lies below the curve corresponding to $SIR = 5.0$, indicating a slightly faster rate of settlement because the soil is in the lightly *OC* range over a significant part of the stress increment.

Conclusions

Many natural clays are lightly over consolidated with *OCR* ranging between 1.0 and 4.0. Under a given stress increment, the soil initially goes through a reloading stage while the effective stresses are smaller than the precon-solidation stress, σ'_c, and then pass on to the normal or virgin compression state at later times. Since the soil near the drainage boundary gets consolidated first, the soil layers close to it reach the virgin compression phase soon and control the dissipation of pore pressure through the soil. The soil layer now comprises of a normally consolidated layer over an *OC* zone with the interface between the two travelling down as a function of time. An extension of Terzaghi's consolidation theory as a phase change process, is presented here as applicable to lightly *OC* soils. The governing equations are solved numerically. The normalized residual pore pressure, U_P, versus time factor, T, and degree of settlement, U_s, versus T relationships are obtained for the ranges of parameters studied. Because of the bilinear void ratio - effective stress relationship, the $U_P - T$ and $U_S - T$ relationships differ. A parametric study brings out that the $U_P - T$ relationships are *significantly affected by the ratios* μ $(= m_{vc} / m_{vr})$, *OCR* and the *SIR* $(+ \Delta\sigma / \sigma'_0)$ while the $U_s - T$ relationships are not. Lightly over consolidated soils in general exhibit faster rates of pore pressure dissipation and only slightly higher rates of settlement than normally consolidated soils.

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Notation

C_{vc}, C_{vr}	-	coefficients of consolidation for *NC* and *OC* phases;
e	-	void ratio;
e_0, e_f	-	initial and final void ratios;
e_c	-	void ratio at preconsolidation stress;
H	-	thickness of clay layer;
H_i	-	depth to *NC/OC* interface;
I_t	-	$H_i/\Delta H$;
k	-	coefficient of permeability ;
m_{vc}. m_{vr}	-	coefficient of volume compression for NC and OC phases;
n	-	number of sublayers;
OCR	-	over consolidation ratio;
SIR	-	stress increment ratio;
S_f, S_t	-	final settlement and settlement at time t;
T	-	$C_{vc}.t/H^2$ - time factor;
t	-	time
U_P	-	normalized residual pore pressure;
U_s	-	degree of settlement;
u	-	pore pressure;
u_0	-	initial pore pressure;
u_1, u_2	-	pore pressures in top (N.C.) and bottom (O.C.) layers;
W	=	u/u_0 - normalized pore pressure;
Z	=	z/H - normalized depth;
z	-	depth;
β_c, β_r	=	C_{vc} (or C_{vr})$\Delta T/(\Delta H)^2$
ΔH	=	H/n;
ΔT	-	increment in time factor;
$\Delta\sigma$	-	applied stress increment;
μ	=	m_{vc}/m_{vr};
σ'	-	effective stress;
σ_0	-	initial effective stress;
σ'_c	-	preconsolidation stress.