Indian Geotechnical Journal, Vol. 23(4), 1993

Analysis of Reinforced Soil Beds

By

B.R. Srinivasa Murthy*

A. Sridharan**

and

Hans Raj Singh***

Introduction

Reinforced soil bed is a soil foundation containing horizontally embedded reinforcements. The reinforcements restrain the lateral deformation of the soil through frictional interaction by increasing lateral confinement. Several investigators have analysed the behaviour of reinforced soil beds. Of them, Binquet and Lee (1975) have proposed a method of analysis for the two dimensional plane—strain case of reinforced oil bed below strip footings. This elastic analysis, with several simplifying assumptions was based on superposition of components of load carried by soil and reinforcements. Perhaps due to limitations of some of the assumptions the mehcd does not predict the experimental results satisfactorily. It is attempted in this paper to present an improved method of analysis.

Theoretical Consideration

In any foundation problem the analysis should encompass evaluation of both the allowable soil pressure for a permissible settlement and ultimate bearing capacity based on shear failure. In this paper it has been attempted to evaluate both of them for the case of reinforced sand bed below strip footing. When a vertical load is applied on a footing, there will be downward movement of the footing associated with lateral flow of the soil. Fig. 1 schematically represents the condition. For the analysis of reinforced soil beds, the following assumptions, of which some of them are modified over those of Binquet and Lee (1975), are made.

1. The total load carried by the footing on a reinforced soil bed will have components in the form :

   (*a*) Load transferred through the soil directly, P_s and

*Associate Professor, **Professor, ***Former Research Student — Department of Civil Engineering, Indian Institute of Science, Bangalore 560 012, India.

(b) load transferred through reinforcements, P_R
i.e., $P = P_s + P_R$ where P is the total load.

2. The component of load directly transfered through the soil alone causes settlement of the footing.

3. The boundary between the downward and outward moving zones is the vertical plane passing through the edge of the footing.

4. With the application of the load, right angle kinks are formed in the reinforcement along the potential slip plane, which transmit the tension in the reinforcement as vertical force to resist the applied load.

5. Elastic theory is applied to estimate the stress distribution inside the soil mass.

6. Failure can occur in either of the modes of friction or tie failure, however it is assumed that the friction failure is critical for the evaluation of the mobilized tension in the reinforcement.

The improvement in bearing capacity/load carrying capacity has been defined by Binquet and Lee (1975) as :

$$\text{BCR} = q/q_o = P/P_s \tag{1}$$

where q and q_o are the average contact pressures below the footing for reinforced and unreinforced conditions repectively at the same settlement and P and P_s. are the respective total loads on the footing. For a given situation this could serve as an input parameter.

In this investigation the same definition has been adapted.

Analysis of Allowable Pressure :

Consider the strip footing as shown in Fig. 1. Now from definition, the total load on the footing on a reinforced soil bed at any settlement, S is

$$P = P_s + P_R \tag{2}$$

From the assumptions 1 and 2, P_s is the load on unreinforced condition at the same settlement, *i.e.*, obtained from the allowable soil pressure criterion at a settlement S.

With the formation of right angle kinks,

$$P_R = 2 \sum_{i=1}^{n} T_i \tag{3}$$

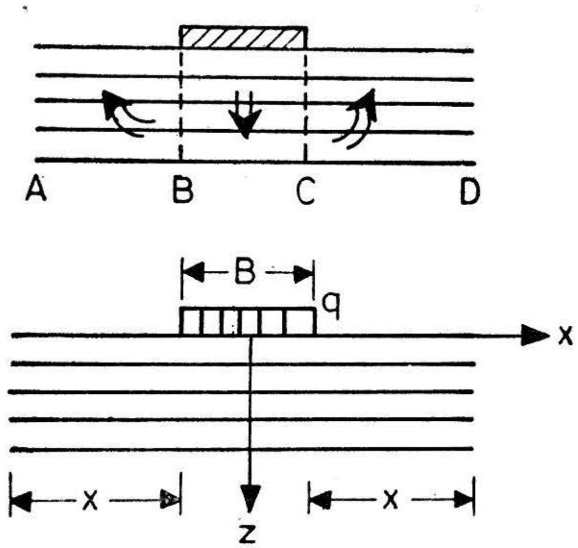

FIGURE 1 Assumed Failure Mechanism For Strip Footing on Reinforced Soil Bed

where T_i is the tension mobilized in the i^{th} layer of the reinforcement. Now the tension in the i^{th} layer of reinforcement per unit length of footing is due to the mobilized interfacial shearing resistance and is given by :

$$T = \int_{B/2}^{l/2} \frac{2\sigma_{vi}(x) w \tan \phi_{\mu i}}{S_x} dx \tag{4}$$

Where $\sigma_{vi}(x)$ is the vertical normal stress at a point (x, z_i) due to both applied load P and overburden.

w is the width of the strip reinforcement

S_X is the horizontal spacing of the reinforcement

$\phi_{\mu i}$ is the mobilized angle of interfacial frictional the i^{th} layer level
l is the length of reinforcement

B is the width of the footing

In the above equation, the vertical normal stress at any point (x, z_i) has two components in the form :

$$\sigma_{vi}(x) = \sigma_{vi1}(x) + \sigma_{vi2}. \tag{5}$$

where $\sigma_{vi1}(x)$ is due to the applied load P

σ_{vi2} is due to ov.rburden pressure $= \gamma z_i$

Using Boussinesq's theory, the vertical normal stress $\sigma_{vi1}(x)$ can be expressed in the form:

$$\sigma_{vi1}(x) = (P/B)\, I_{xi} \tag{6}$$

Where P is the load on the footing per unit length

B is the width of the footing and

$$I_{xi} = \frac{1}{\pi}\left[\tan^{-1}\left(\frac{(z_i/B)}{(x/B)-0.5}\right) - \tan^{-1}\left(\frac{(z_i/B)}{(x/B)+0.5}\right) - \frac{(z_i/B)\ \{(x/B)^2-(z_i/B)^2-0.25\}}{\{(x/B)^2-(z_i/B)^2-0.25\}^2+(z_i/B)^2}\right] \tag{7}$$

Now for the first layer

$$\sigma_{v11}(x) = P/B\, (I_{x1})$$

The vertical normal stress $\sigma_{v21}(x)$ on the second layer will be influenced by the load carried by the first layer of reinforcement. Similarly $\sigma_{v31}(x)$ etc are influenced by P_{R1}, P_{R2} etc. Since the component of the load carried by the reinforcements will be distributed beyond the loaded area, it is assumed that the load inducing vertical stress on the second layer is $(P - P_{R1})$ and similarly for the i^{th} layer $(P - \sum_{j=1}^{(i-1)} P_{Rj})$ is the load on the footing inducing vertical stress. From the above and equations (4), (5), (6) and (7), the equation for tension in various reinforcement layers can be written as.

$$T_1 = \frac{2w \tan \phi_{\mu 1}}{S_x}\left[PI_1+\gamma z_1\left(l/2-B/2\right)\right]$$

$$T_2 = \frac{2w \tan \phi_{\mu 2}}{S_x}\left[\left(P-2T_1\right)I_2+\gamma z_2\left(l/2-B/2\right)\right]$$

$$T_i = \frac{2w \tan \phi_{\mu i}}{S_x}\left[\left(P-2\sum_{j=1}^{i-1} T_j\right)I_i+\gamma z_i\left(l/2-B/2\right)\right] \tag{8}$$

where $I_1, I_2, \ldots, I_i$ are the non-dimensional stress influence factors obtained by integrating the nondimensional stress influence factor at any point, (x, z_i) given by equation (7) over the length of reinforcement. Fig. (2) shows the plots of these factors as function of (x/B) and (z/B)

In the above equation, T_i depends on the mobilized $\phi_{\mu i}$ which essentially depends on the relative movement between soil and reinforcement. In the present case this relative movement will vary with depth, being maximum at the first layer. It is very difficult to evaluate the magnitude of the relative movement at different layer levels.

By assuming that the prismoidal wedge with vertical face through the edge of the footing moves vertically down and the soil in the adjoining

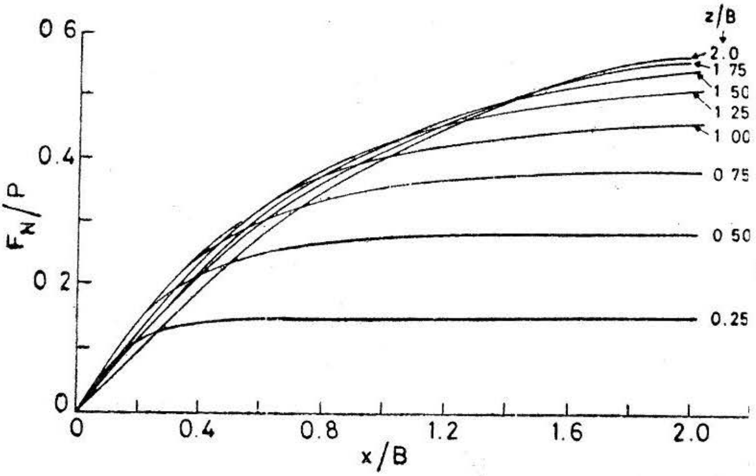

FIGURE 2 Non-dimensional Normal Force Factor for Strip Reinforcements Beneath Strip Footing

zone moves horizontally, relative movement of the soil against an inextensible reinforcement can be estimated. The surface settlement, S of the footing results in a variable settlement at different layer levels. Further, at large depths the settlement due to surface loading is negligible. It is assumed that the vertical settlement at any level is proportional to the vertical stress at that point. By adapting Boussinesq's theory this results in about 30% of surface settlement at a depth of B and neglegible settlement at $2B$ (Fig. 3).

For stiff reinforcements the relative desplacement, is equal to the settlement of the soil at that level. Now S_i is the settlement at i^{th} layer level for a surface settlement of S which is given by :

for $z_i \leqslant B$.

$$S_i = \left(\frac{B-z_i}{B}\right) 0.7S + 0.3S \tag{9a}$$

for $z_i > B$,

$$S_i = \left(\frac{2B-z_i}{B}\right) 0.3S \tag{9b}$$

The mobilized $\phi_{\mu i}$ at any relative displacement can be evaluated from the results of sliding tests with reinforcement in the bottom half and soil in the top half of the shear box.

Thus all the terms in equation (8) are known for a given configuration of reinforcement from which the load carrying capacity at each layer level can be computed in terms of applied load P from which P and BCR

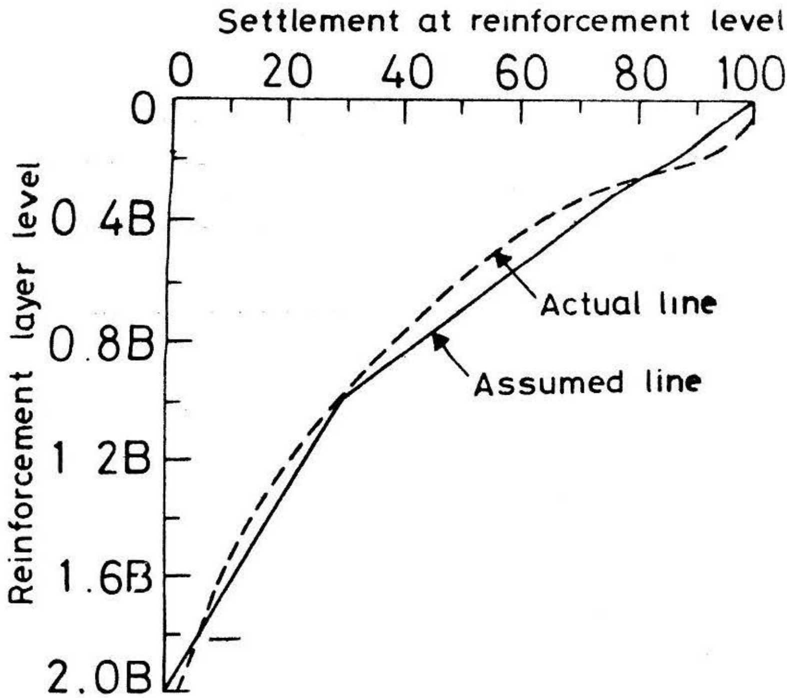

FIGURE 3 Effective Settlement at Different Reinforcement Layer Level

can be computed. The final designing is complete when the desired bearing capacity ratio is reached at a particular layer level. The cross section of the reinforcement is designed for the layer where maximum tension is mobilised.

Ultimate Bearing Capacity

In spite of designing the reinforced soil bed for allowable bearing pressure, often the system has to be checked for the factor of safety under ultimate conditions. Two modes of failure viz. reinforcement pull out or reinforcement breakage conditions could be visualised under ultimate conditions.

Pull out failure

In the case of reinforcement pullout failure, there will be simultaneous frictional pullout of the reinforcement and shear failure of the soil. The ultimate bearing capacity of the soil can be obtained by appropriate classical equations, say Terzhaghi's equation for a surface footing on sand in the form :

$$q_u = 0.5 \gamma B N_\gamma \qquad (10)$$

where γ is the unit weight of the soil

B is the width of the footing and

$N\gamma$ is the bearing capacity factor

While evaluating P_{Ri} under ultimate condition for each of the layer, the value of $\phi_{\mu i}$, can be taken as residual. By combining the two components,

$$P_u = 0.5\,\gamma B^2 N\gamma + \sum_{i=1}^{n} P_{Ri} \tag{11}$$

where

$$P_{Ri} = \frac{4w \tan \phi_{\mu r}}{S_x}\left[\left(P_u - \sum_{j=1}^{i-1} P_{Rj}\right) I_i + \gamma z_i \left(l/2 - B/2\right)\right] \tag{12}$$

where $\phi_{\mu r}$ is the residual angle of interfacial friction. Using equations (11) and (12), for defined number of layers of reinforcement P_u can be calculated.

Tie failure

From the known tie cross section, the ultimate load carrying capacity of the reinforced soil bed can be evaluated for a condition of tie failure. Since, at the time of reinforcement breakage, the soil may not be stressed to its ultimate strength, the principle of superposition with ultimate strength of soil, is not valid. In this case the ultimate load on the footing will be such that the maximum tension mobilized in a layer will be equal to its ultimate tensile strength. Since, the reinforcement has not failed by friction, the ϕ_μ mobilised will be less than the peak value. Since assuming peak value of ϕ_μ, results in a lower bound value of the P_{ults} it is assumed that during tie failure, $\phi_\mu = \phi_{\mu peak}$. For a given reinforcement configuation the maximum tension occurs in k^{th} layer on which $\left(P - \sum_{j=1}^{k-1} P_{Rj}\right) I_k$ is maximum. From the table prepared for evaluation of allowable soil pressure, this can be identified.

Then $T_k = \sigma_y wt$

$$= \frac{2w \tan \phi_{\mu peak}}{S_x}\left[P_{ut} - \sum_{j=1}^{k-1} P_{Rj} I_k + \gamma z_k \left(l/2 - B/2\right)\right] \tag{13}$$

$$P_{ut} = \frac{\sigma_y t S_x}{2 \tan \phi_{\mu peak}} + \sum_{j=1}^{k-1} P_{Rj} I_k + \gamma z_k \left(l/2 - B/2\right)$$

where σ_y yield strength of the reinforcement

t is the thickness of the reinforcement

Experimental Verification

To examine the validity of the proposed method of analysis model plate load tests on both unreinforced and reinforced sand were conducted. In the reinforced case tests were carried out with two different thicknesses of reinforcements to achieve specifically both frictional and tie failure conditions. Table 1 gives the properties of the sand under test conditions.

TABLE 1

Properties of Sand

Dry density	1.73 gm/cm^3
Relative Density	85%
Angle of Internal Friction	45°
Uniformity Coefficient	3.5
Coefficient of Curvature	0.97

The tests were conducted in a square tank of 3.9m size and depth 2.1m. The size of the footing was 152 × 915mm. The reinforcements were of aluminium strips of thicknesses 0.54mm, and 0.3mm, of width 25.4 mm and of length 457mm. Three layers of reinforcements at a vertical spacing of 38.1mm were used, with the first layer being at a depth of 38.1 mm. The tests were carriedout till the ultimate failure was reached. It is to be noted that the frictional strengths of the two types of reinforcements were same while the tie strengths were different.

Fig. 4 indicates the load settlement curves (each being the average of 3 tests) for both unreinforced and reinforced conditions. The predicted curve for reinforced condition is also superposed. The computed and experimental loads and bearing capacity ratio values at different settlement levels have been compared in Table 2. The required ϕ_μ has been obtained from the sliding test results (Fig. 5). In this test aluminium strip was fastened to a wooden block, and was placed in the bottom half of direct shear box. The sand at the same condition of plate load testing was placed in the top box. The ultimate load from both friction and tie failure considerations have been calculated and are presented in Table 3. The results clearly indicate that friction failure occured when tie strength is large and vice versa, which could be predicted reasonably. It may be noted that the angle of interfacial friction being same for both the reinforcements, the present approach, based on the criticality of friction failure, predicts identical behaviour.

TABLE 2

Computation of Load at Different Settlements and its Comparison with Experimental Values

No. of Layers = 3, l=457mm, B =152mm w=25.4mm, S_x=50.8mm

S mm	z_i mm	S_i mm	$\phi_{\mu i}$ deg	$2I_l$	P_{Ri} (in Tonnes)	ΣP_{Ri} (in Tonnes)	P_s tonnes	P_{expt} tonnes (BCR)	P_{pred} tonnes (BCR)
2	38.1	1.65	36.5	0.1536	0.114P+0.015				
	76.2	1.30	35.0	0.2784	0.173P+0.024	0.455P+0.071	0.875	1.80	1.74
	114.3	0.95	33.0	0.3628	0.168P+0.032			(2.06)	(1.99)
4	38.1	3.30	39.0	0.1536	0.124P+0.016				
	76.2	2.60	38.8	0.2784	0.196P+0.030	0.505P+0.080	1.500	3.20	3.20
	114.3	1.70	36.8	0.3628	0.185P+0.034			(2.13)	(2.13)
8	38.1	6.60	36.0	0.1536	0.112P+0.015				
	76.2	5.20	37.0	0.2784	0.186P+0.038	0.500P+0.810	3.1000	5.40	6.36
	114.3	3.80	38.4	0.3628	0.202P +0.038			(1.74)	(2.06)
12	38.1	9.90	35.6	0.1536	0.110P+0.015				
	76.2	7.80	35.6	0.2784	0.177P+0.025	0.476P+0.074	4.4000	7.40	8.16
	114.3	5.70	36.2	0.3628	0.189P+0.034			(1.76)	(1.95)
16	38.1	13.20	35.6	0.1536	0.110P+0.015				
	76.2	10.40	35.6	0.2784	0.177P+0.025	0.472P+0.075	4.400	9.00	8.48
	114.3	7.60	35.6	0.3628	0.185P+0.035			(2.05)	(1.93)

Note : P is in tonnes.

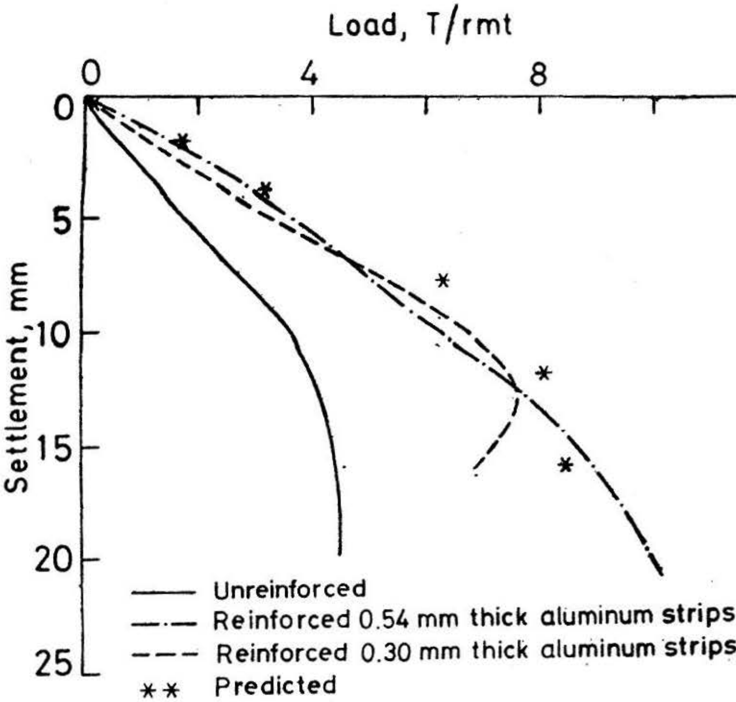

FIGURE 4 Load Verses Settlement Curves for Reinforced and Unreinforced Conditions

TABLE 3

Comparison of Predicted and Experimental Ultimate Loads (tonnes/rmt)

$P_{s\ ult}$ exp	P_{ult} exp	P_{ult} predicted	
		Friction failure	Tie failure
4.4	9.0*	8.48	13.94
4.4	7.8**	8.48	7.66

*friction failure (for 0.54 mm thick aluminium strip)

**tie failure (for 0.3 mm thick aluminium strip)

Note : For tie failure the maximum tension is in third layer and is given by 0.202 P + 0.038 (from Table 2) and is equal to σ_y wt, where σ_y = 1100 kg/cm^2 and w = 25.4 mm

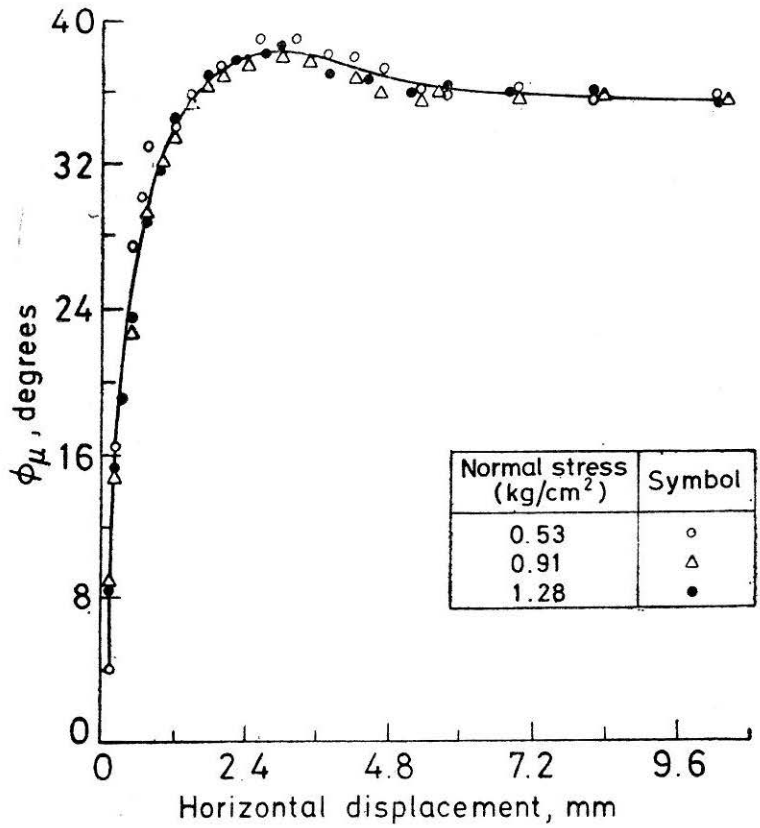

FIGURE 5 **Interfacial Friction Angle Versus Displacement Plot for Sliding Tests Between Sand and Aluminium**

Concluding Remarks

It has been attempted to develop an improved method of analysis for reinforced soil bed below strip footings by considering the mobilized frictional strength. From this, tension in each layer of reinforcement can be computed directly. Further while computing the frictional strength of lower layers, that component of the load carried by the upper layers has been assumed to be distributed uniformly beyond the loaded area and hence not available for mobilization of frictional strength. From this approach it is possible to design the reinforced soil bed for an allowable soil pressure corresponding to a permissible settlement. Also, the ultimate capacity can be checked for both pullout and tie failure conditions. The validity of the approach has been brought out in relation to experimental data.

REFERENCES

BINQUET, J. and LEE, K.L. (1975) : "Bearing capacity analysis of reinforce eart slabs". *Journal of Geotechnical Engineering Division*, ASCE, Vol. 101, No. GT12, pp. 1257 1275.

BINQUET, J. and LEE, K.L. (1975) : "Bearing capacity tests on reinforced earth slabs". *Journal of Geotechnical Engineering Division*, ASCE, Vol. 101, No. GT12, pp. 1276-1295.