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New Method for Estimating Pile Group Efficiency

by

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Introduction

Piles are rarely used singly. Often a group of piles is provided to transfer structural loads to stronger and stiffer strata. Reliable static and dynamic methods of estimating the ultimate load capacity of single pile in clay are available (Poulos and Davis, 1980). Based on the predicted capacity of a single pile, the group capacity or its efficiency is estimated mostly by empirical methods notably by the method of Terzaghi and Peck (1948) and by the modification suggested by Poulos and Davis (1980). In this paper, a new rational method of estimating pile group capacity is presented incorporating the interaction effects between two piles.

Review of Literature

Poulos and Davis (1980) present a comprehensive review of the available methods, which are mostly empirical, of estimating efficiency of pile groups in clay. The simplest method, Feld's rule, suggests a reduction of load capacity of each pile in a group by 1/16th for each adjacent pile. An improvement over this rule is to replace 1/16 by 1/8. (s/d) and account for the effect of the spacing between the piles (s/d - is the ratio of spacing to diameter of the pile). The Converse-Labarre formula estimates the efficiency, η, of the pile group as

$$\eta = 1-(\xi/90)\,\{(n-1)\,m+(m-1)n\}/mn \qquad (1)$$

where m – is the number of rows, n– number of piles in a row, and ξ – arctan (d/s) in degrees. The formula accounts for the effects of the spacing between piles and the group size. Terzaghi and Peck (1948) had introduced a rational method of estimating pile group efficiency by identifying a block type of failure. The ultimate load, P_B, for the block type of failure is

$$P_B = B_G\, L_G\, C_b N_c + 2\,(B_G + L_G)\, L.C_\alpha \qquad (2)$$

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where B_G and L_G are the width and length of the block defined by the perimeter of the pile group, L – the length of piles, C_b and C_α – undrained cohesion at the base and the average adhesion along the shaft of the piles, respectively, N_C – the bearing capacity factor for the corresponding depth. Block type of failure implies mobilization of the strength by the soil on the surfaces defined by the block consisting of all the piles and the soil contained within the group. However, this method predicts an abrupt change in the mode of failure from block to individual pile failure as the spacing between the piles increases, and permits no gradual transition. Poulos and Davis (1980) have suggested an empirical rule to obviate the above shortcoming, as

$$\eta = 1/(1+1/\eta_B^2) \tag{3}$$

where $\eta_B = P_B/P_{u1}$, n – number of piles in a group, and P_{ul}—the ultimate load capacity of a single pile. Though lacking in a theoretical basis, the method of Poulos and Davis (1980) predicts the efficiency rather well.

Any rational method for the study of the interaction between piles and for the estimation of the efficiency of the pile group should include the effects of (*a*) the spacing, (*b*) the length of the piles, (*c*) the size of the group, i.e. the number and the arrangement of piles in the group, and (*d*) the variation of the strength of the soil with depth. The method developed in this paper includes all these effects and is based on an interaction analysis.

The Concept

Terzaghi and Peck (1948) introduced the concept that all the piles and the soil enclosed within the periphery ABCDE (Fig. 1) of the group fail as one single block, as one of the failure modes for the group. This concept suggests that while for a single pile, the load it can carry is limited by the shear resistance around the shaft surface and the bearing stresses at the tip, the load carrying capacity of a group of piles is governed by the resistance mobilized all along the outer periphery and the plan area of the group of piles (Fig. 1). In other words, the strength of the soil is mobilized not only next to the piles (points A, B, C, D, E, etc., Fig. 1) but also at all points on the lines connecting adjacent piles (points *P*, *Q*, *R*, *S* etc., Fig. 1) on the outer periphery of the group.

The new concept proposed herein is that for single piles, the strength of the soil is mobilized at points *A*, *B*, *C*, etc., adjacent to the pile and for piles in a group the point at which the soil mobilizes its full resistance extends from *A*, *B*, *C*, etc. Block failure results if this point reaches the midpoint, *P*, between two adjacent piles. The concept is illustrated as follows: consider the interaction between any two piles of diameter, *d*, and spaced at a distance, *s*. (Fig. 2*a*). Due to load *P* on pile *I*, the shear stresses, τ, are maximum close to the pile and decrease with distance, *x*, from the pile. Fig. 2*b*

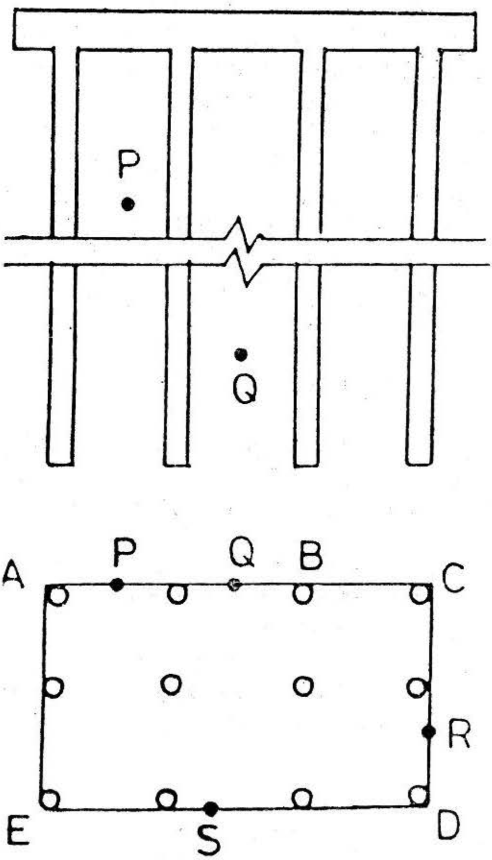


FIGURE 1 Block Failure of Pile Groups

depicts a conceptual variation of the normalized shear stresses with normalized distance from the pile. At large spacings (Curve 1 for $s > 12d$), there is no interference between the piles and the shear stresses reduce to negligible values at $x/s = 1.0$. As the piles are brought closer, the interference effect between the piles is significant because of which the shear stress variation could be as shown (Curves 2 and 3, Fig. 2*b*).

The total shear stress defined as the sum of the shear stresses due to loads transferred by piles I and II, varies with distance as shown in Fig. 2*c*. At large spacings the total shear stress is maximum only at points adjacent to the piles but is less than the maximum at points between the two piles. As the piles are brought closer, the interference effect is significant and large. The total shear stresses at points between the two piles, tend closer to the maximum value. At very close spacings (s/d 1.5 to 2.0) not only the total shear stresses at points adjacent to each pile but also the stresses at every point between the piles reach the maximum or the

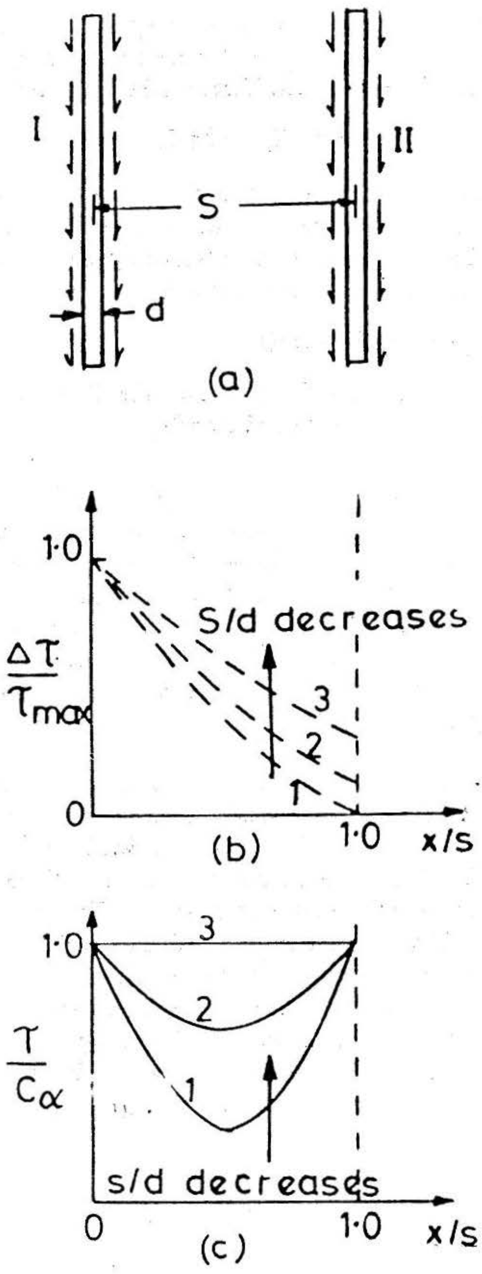


FIGURE 2 Variation of Stress Increment and Total Stresses with Distance

limiting value and the two piles and the enclosed soil, fail together as a single block. Hence the proposed extended Terzaghi-Peck concept can be stated as—if the total shear stress, i.e. the sum of the shear stresses due to loads transferred by two adjacent piles, equals the resistance offered by the soil at some point between the two piles, they fail as a group. At close spacings this criterion gives the block failure mode originally proposed by Terzaghi and Peck. The extended Terzaghi-Peck concept is expressed as

$$\tau_B = \tau\ (II) + \Delta\tau(I) \tag{4}$$

where τ_B is the total shear stress at any point B close to the pile shaft, τ(II)—the shear stress at B due to load on pile II itself, and $\Delta\tau$(I)—the shear stress at B due to the stresses transferred by pile I. Similar expression for the total normal stress, q_D, at point D on the base, is

$$q_D = q(II) + \Delta q(I) \tag{5}$$

where q(II) is the normal stress from load on pile II and Δq(I)—the normal stress at D due to stresses transferred by pile I.

The Method

In the present analysis, it is assumed that even though the stresses at the pile-soil interface may have reached near failure level, the influence of these stresses or loads at any point away from that pile-soil interface, can be estimated using linear elasticity theory. A similar assumption that nonlinear response is confined only to a narrow zone of soil adjacent to the pile shaft, whereas the bulk of the soil between the piles remains essentially elastic, has been used by Chow (1986).

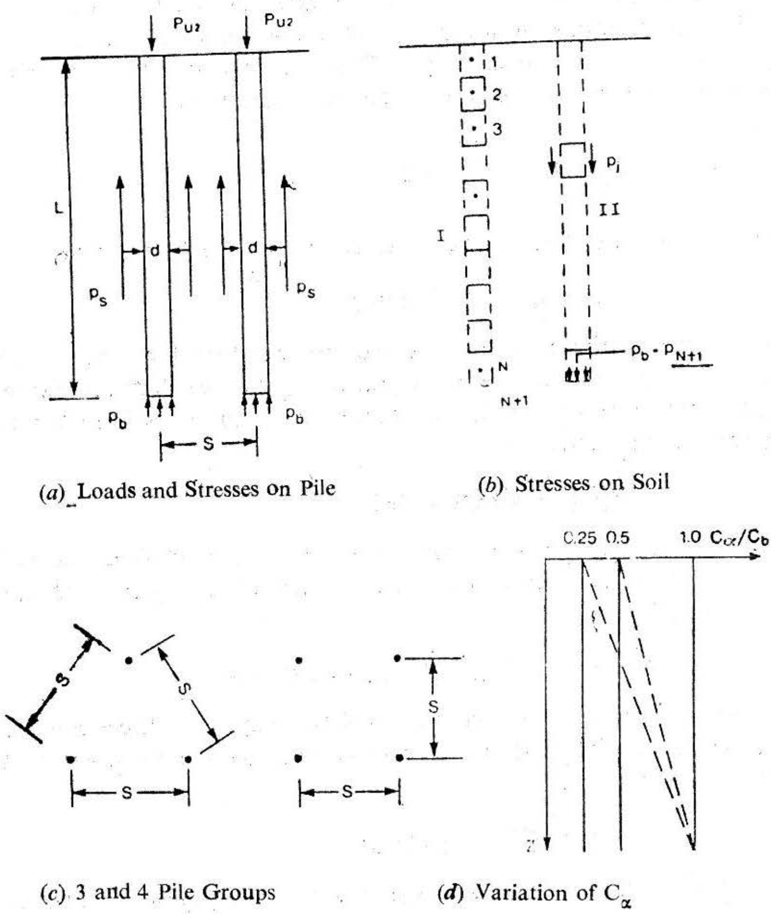
Two piles each of length, L, and diameter, d, and loaded by a load, P_{u2}, are considered (Figure 3*a*). The load on each pile is resisted by the shaft and the base loads (Fig. 3*b*). The pile is divided into n elements each of length L/n. Nodes 1 to n represent the centers of these elements, and node $(n+1)$ the center of the base of the pile. The stress, P_j, is the shaft resistance mobilized by the jth element and $P_{n+1}=P_b$—the base resistance. It can be surmised that the effects of stresses acting on pile I on the elements of pile II, are the same as those due to pile II on to pile I. The interaction between the two piles is studied by considering Δp_{ij}, the additional shear or vertical stress increments at nodes i on pile I, due to the stresses acting on shaft elements, j, or base of pile II. Using Mindlin's (1936) solution, ΔP_{ij} is written as

$$\Delta p_{ij} = \sum_{C_1}^{C_2} \sum_{0}^{2\pi} (p_j\, d.r)/16(1-\nu)\ [(1-2\nu)\ \{1/R_1^3 - 1/R_2^3\} + 3(z-c)^2/R_1^5$$

$$\{3(3-4\nu)z(z+c)^2 - 3c(z+c)\ (3z+c)\}/R_2^5 + 30\ cz\ (z+c)^2\}/R_2^7]dz.d\theta \tag{6}$$

or $$\Delta p_{ij} = a_{ij}\,.\,p_j \tag{7}$$

where a_{ij}—the interaction coefficient for shear stress at node i on pile I



(*a*) Loads and Stresses on Pile (*b*) Stresses on Soil

(*c*) 3 and 4 Pile Groups (*d*) Variation of C_α

FIGURE 3 Definition Sketch

due to the stresses on element j of pile II, $R_{1/2}^2 = \{r^2 + (z +/- c)^2\}$, $r^2 = (x^2 + y^2)$, x, y and z coordinates of node i relative to element j at depth c, $c_1 = (j - 1/2)\ L/n$, $c_2 = (j + 1/2)\ L/n$ (Poulos and Davis, 1974), and ν the Poisson's ratio of the soil. A similar expression is written for Δp_{bj}—the increment in stress at the base of the pile. Eq. (6) does not contain the modulus of deformation. Hence it is assumed to be valid even when the soil is non-homogeneous with respect to strength. The shear stresses can not be superposed directly. Their components on to a given vertical plane have been computed and added. It is noted that

$$\alpha_{ij} = f\,(L/d,\ s/d,\ \nu) \tag{8}$$

where L/d and s/d are the length to diameter and spacing to diameter ratios respectively. α_{ij} can be evaluated by integrating over the surface of each element of the pile or by using the resultant force $\Delta P_{ij} = p_j \cdot \pi dL/n$ ($p_b \cdot$

$\pi d^2/4$ for the base). For $|i-j| >= 3$, the latter approach which is simpler, proved to be accurate to within 1% of the exact value, and as such, is retained for the rest of the analysis. The interaction due to all the elements on pile *II*, on element *i* of pile *I*, can be summed up, as

$$\Delta p_i = \sum_{j=1}^{n+1} \Delta p_{ij} = \sum_{j=1}^{n+1} a_{ij}.p_j \tag{9}$$

The total stress transferred by each element *i* on pile *I*, would be

$$p_i + \Delta p_i = p_i + \Sigma\, a_{ij}\, p_j \tag{10}$$

However, the maximum stress that can be transferred to the soil should equal its strength at that depth : equal to $C_{\alpha j}$ for the shaft resistance, and $N_c C_b$ for the base or the tip resistance. Eq. (10) is written for all the elements (or the nodes) as

$$\{[U] + [a]\}\, \{p\} = \{A\} \tag{11}$$

where $\{U\}$ – the unit matrix, $\{a\}$ – the influence coefficient matrix, $\{p\}$— the vector of soil resistance mobilized on each pile along the shaft and the base of the pile, and

$$\{A\}^T = \{C_{\alpha_1}, C_{\alpha_2}, \ldots, C_{\alpha n}, C_b\, N_c\} \tag{12}$$

It is common practice to take $N_c = 9.0$ for clay in undrained condition. Variation of C_α with depth for any known profile can be postulated. Eq. (11) is solved to obtain

$$\{p\} = \{[U] + [a]\}^{-1}\, \{A\} \tag{13}$$

The ultimate load, P_{u2}, on each pile in a two pile grpup, is obtained as

$$P_{u2} = \Delta A_s.\ \Sigma_j\, p_j + A_b \cdot p_b \tag{14}$$

where $\Delta A_s = \pi dL/n$ and $A_b = \pi d^2/4$. For a single floating pile, the ultimate load capacity, P_{u1}, is

$$P_{u1} = \Delta A_s.\ \Sigma_j\, C_{\alpha j} + A_b\, C_b\, N_c \tag{15}$$

The efficiency η_2, of the two pile group is defined as

$$\eta_2 = P_{u2}/P_{u1} \tag{16}$$

A reduction factor, R_α, which gives the reduction in the load capacity in each pile due to the presence of the adjacent pile, is defined as

$$R_\alpha = P_{u1}/P_{u2} - 1.0 \tag{17}$$

η_2 and R_a are interrelated as

$$\eta_2 = 1/(1 + R_a) \tag{18}$$

To generalize this approach, a three pile group (an equilateral triangular arrangement) and a four pile group (2^2 group or square grid), are considered (Fig. 3*c*). Eq. (11) is modified for the three pile group as

$$\{[U] + 2[\alpha(s)]\}.\{p\} = \{A\} \tag{19}$$

and for the four pile group

$$\{[U] + 2\,[\alpha(s)] + [\alpha(\sqrt{2}.s)]\}.\{p\} = \{A\} \tag{20}$$

where $[\alpha(s)]$ is the influence coefficient matrix for two piles spaced at a distance, s, and $[\alpha(\sqrt{2}.s)]$ is the corresponding matrix for a spacing of $\sqrt{2}.s$. The efficiency of the three pile group, $\eta_3 = P_{u3}/P_{u1}$, and that of the four pile group, $\eta_4 = P_{u4}/P_{u1}$, are calculated directly in a manner similar to the one described above (Eqs. (5) through (16)). η_3 and η_4 are also obtained by the method of superposition, as

$$P_{u3}\,\{1 + 2R_\alpha\,(s)\} = P_{u1}$$

or

$$\eta_3 = 1/\{1+2R_\alpha\,(s)\}$$

and

$$P_{u4}\,\{1 + 2R_\alpha\,(s) + R_\alpha\,(\sqrt{2}.s)\} = P_{u1}$$

or

$$\eta_4 = 1/\{1+2R_\alpha\,(s) + R_\alpha\,(\sqrt{2}.s)\} \tag{22}$$

where $R_\alpha\,(s)$ and $R_\alpha\,(\sqrt{2}.s)$ are the reduction factors for piles spaced at s and $\sqrt{2}.s$ respectively. As can be intuitively expected the two approaches agree well (Table 1), indicating that the method of superposition can be adopted for the analysis of pile groups.

Pile Group Efficiency

Rigorous Method : Consider a general pile group of size $M \times N$ (Fig. 4). Eq. (21) or Eq. (22) can be generalized for the ultimate load capacity P_{ij} of any given pile, A, defined by its indices i and j, as

$$\sum_{k=1}^{M}\sum_{l=1}^{N} R_\alpha\,(S_{ijkl})\,P_{kl} = P_{u1} \tag{23}$$

where $R_\alpha\,(s_{ijkl})$ is the reduction factor due to the influence of pile $B(k,l)$ on pile $A(i,j)$, s_{ijkl}—the distance or spacing between piles A and B, P_{kl}—the ultimate load on pile $B(k,l)$ in the group. It should be noted that if $i = k$ and $j = l$,

$$R_\alpha\,(S_{ijkl}) = 1.0 \tag{24}$$

that is, the influence of a pile on itself equals one. Eq. (23) is written for all piles in the group, i.e. $i = 1$ to M and $j = 1$ to N, and combined to get

$$[R_\alpha]\,\{P\} = \{1\}\,P_{u1} \tag{25}$$

TABLE 1

Comparison of Group Efficiency by the Exact and Superposition Method : Three Pile Group

L/d		2	3	4	7	10
		C_α/C_b = 1.0 and Constant				
10	E	.8581	.9085	.9528	.9726	.9866
	MS	.8651	.9154	.9585	.9772	.9899
100	E	.8333	.8834	.9277	.9483	.9644
	MS	.8340	.8842	.9289	.9496	.9655
		C_α/C_b = 0.25 and Constant				
10	E	.8755	.9196	.9570	.9741	.9868
	MS	.8832	.9251	.9618	.9781	.9989
100	E	.8380	.8874	.9296	.9897	.9654
	MS	.8408	.8873	.9302	.9506	.9664

E—Exact Method MS—Method of Superposition

where $[R_\alpha]$ is a square matrix of size $(M \times N)$, consisting of reduction factors $R_\alpha\,(S_{ijkl})$, $\{P\}$—a vector of ultimate pile loads, and $\{1\}$—a unit vector, i.e.

$$\{1\}^T = \{1, 1, ..., 1\} \tag{26}$$

both vectors being of size $(M \times N)$. However, because of symmetry, the actual number of unknown pile loads will be smaller. For square groups, i.e. $M = N$, the number equations to be solved is $M(M+2)/8$ if M is even, and $(M+1)\ (M+3)/8$ if M is odd.

Approximate Method : A simplification is possible if as a first approximation, Eq. (25) is written as

$$P_{ij} \sum_{k=1}^{M} \sum_{l=1}^{N} R_\alpha\,(S_{ijkl}) = P_{u1} \tag{27}$$

With this approximation, the load on each pile in the group is obtained directly without the need to solve a set of simultaneous equations, as

$$P_{ij} = P_{u1}/[\sum_{k=1}^{M} \sum_{1=1}^{N} R_\alpha\,(S_{ijkl})] \tag{28}$$

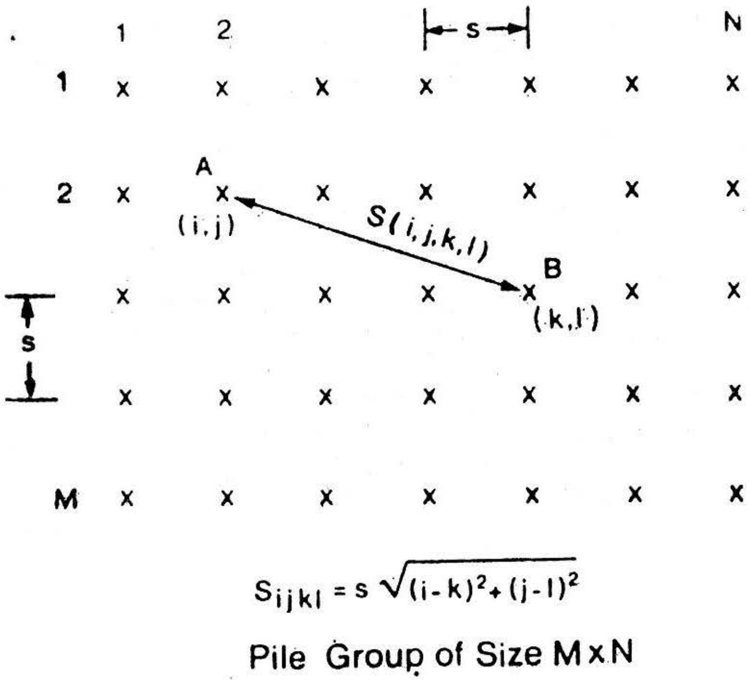


FIGURE 4 Interaction of Pile B (k, l) on Pile A (i, j)

or

$$\eta_{ij} = 1/[\sum_{k=1}^{M} \sum_{l=1}^{N} R_a(s_{ijkl})] \tag{29}$$

The total load, P_G, carried by the group, is

$$P_G = \sum_{i=1}^{M} \sum_{j=1}^{N} P_{ij} \tag{30}$$

and the efficiency, η_G of the group, is

$$\eta_G = P_G/(M\times N\times P_{u1}) = \Sigma\,\Sigma\, \eta_{ij}/(M\times N) \tag{31}$$

where η_{ij} is the efficiency of pile $A(i, j)$, given by Eq. (29). The percentage of the load carried by a pile in a group, $P_P(i, j)$, is

$$P_P(i, j) = \eta_{ij}/(\sum_{i=1}^{M} \sum_{j=1}^{N} \eta_{ij}) \times 100 \tag{32}$$

Results

The pile shaft is divided into ten equal parts, i.e. $n=10$, and the influence coefficients α_{ij} computed. To check the accuracy of the discretization, results with $n=10$, are compared with those with $n=20$. The difference between the two sets is less than 1%. For further work $n=10$ has been adopted. A parametric study has been carried out for different L/d ratios (10, 20, 50 and 100), s/d ratios (2, 3, 5, 7, 10, 20 and 50), and constant and linear increase with depth distributions of C_α with depth (Fig. 3d).

The influence of one pile on another is studied (Fig. 5) through the reduction factor, R_α, which estimates the reduction in load carrying capacity in a pile due to the presence of another pile. Three variations of adhesion with depth shown as inset in Fig. 5 are considered. R_α decreases with increasing spacing between the two piles. The decrease in R_α is rapid for short (L/d=10) piles, and for C_α/C_b=1.0, the values decrease from about 0.075 for s/d=2 to 0.005 for s/d=10. For long (L/d=100) piles, R_α values are more than those for L/d=10 and decrease less rapidly with s/d. Thus the zone of influence for each pile appears to be larger for long piles than that for short ones. The reduction factor, R_α, for piles in soils with C_α increasing with depth is bounded by the values for pile in soils with constant C_α with depth. A comparison of some of the

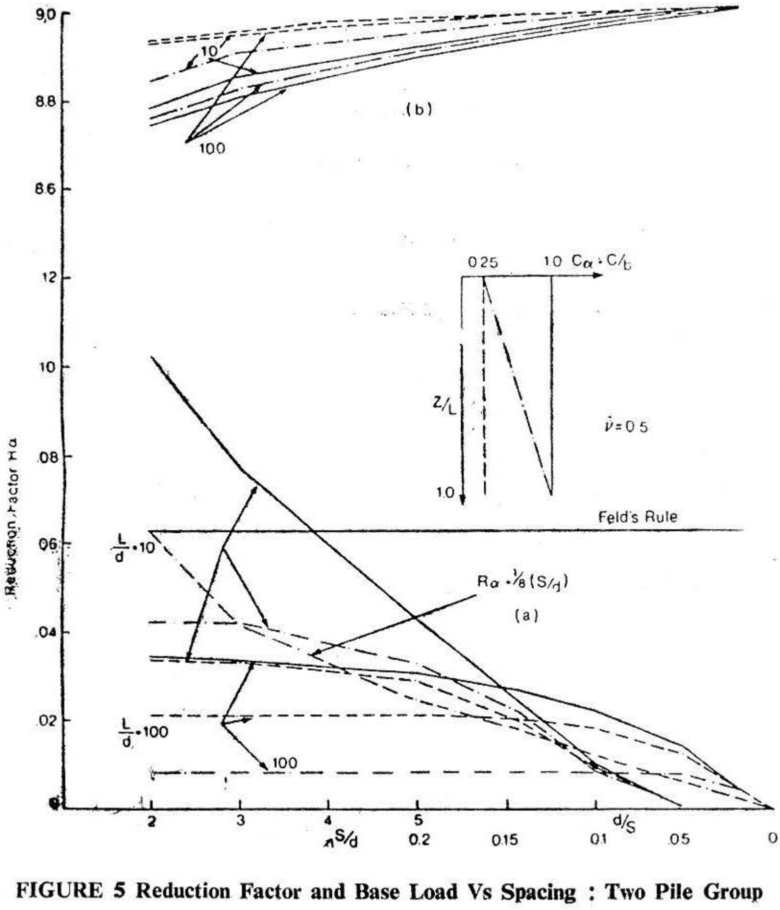


FIGURE 5 Reduction Factor and Base Load Vs Spacing : Two Pile Group

empirical rules with the reduction factor, brings out that Feld's rule is overly conservative while the relation $R_\alpha = 1/8(s/d)$ does not predict the near constancy of R_α with spacing for long piles nor the effect of the L/d ratio. The variation of base resistance with spacing is shown in Fig. 5*b*. With increasing spacing, the base load or the point resistance (p_b/C_b) of the pile tends to 9.0, the value for a single pile. The reduction in base load is more for long than short piles because the former transfers more shaft resistance than the latter. The reduction in the base load for $L/d=100$ and $C_\alpha/C_b=1.0$, is of the order of 3% and much less for the other cases. The effect of interaction between two piles at $s/d=2$ (close to block failure mode), on the mobilized maximum shaft resistance is significantly large and about 22% for a short ($L/d=10$) pile in a soil with constant value of adhesion. The reduction in shaft resistance near the top and tip of the pile is less, of the order of 10% for similar piles in soils with linearly increasing adhesion. In case of long piles ($L/d=100$), the reduction in mobilized shaft resistance is uniform over most of the shaft length, and is about 20% in both the soil types.

Extending the analysis to three pile group (Fig. 3*c*), the efficiency is evaluated directly and by the method of superposition (Eq. 21). From Table 1, it can be noted that the method of superposition is valid and the difference between the two methods is less than about 0.6%. A similar conclusion but with the errors of the order of 1% can be arrived at for the four pile group. As such, the method of superposition can be adopted for the study of large groups of piles noting that the errors due to the method of superposition vanish at spacings greater than (4 to 5) d.

The predictions from the proposed method are compared (Fig. 6) with the well documented classical results of Whitaker (1957). Predictions based on three possible variations (shown as inset in Fig. 5) of soil strength with depth are shown and appear to predict the group efficiency reasonably well. Whitaker (1957) suggests an almost linear variation of group efficiency with spacing while Eq. 3 (Poulos and Davis, 1980) indicates a hyperbolic increase of η_G with spacing. The predictions from the present theory are closer to the reported values of Whitaker (1957). The agreement between the present analysis and Whitaker's results is much better (Fig. 6*a*) for 3^2 pile groups. For 5^2 and 7^2 groups (Fig. 6*b*), the agreement is good but the present theory under estimates the efficiency of the group. The percentage of loads carried by corner, center of edge, and centre piles in a 3^2 group, for spacings of $2d$ and $4d$, and the percentages of loads carried by piles A, B, C, D, E and F in a 5^2 group, are compared (Table 2) with the predictions from the present analysis. Considering the limitations of the experimental results, the effects of installation of piles on the soil properties and of the assumption made in the present theory, the agreement between the experimental results and the predictions, appears reasonable.

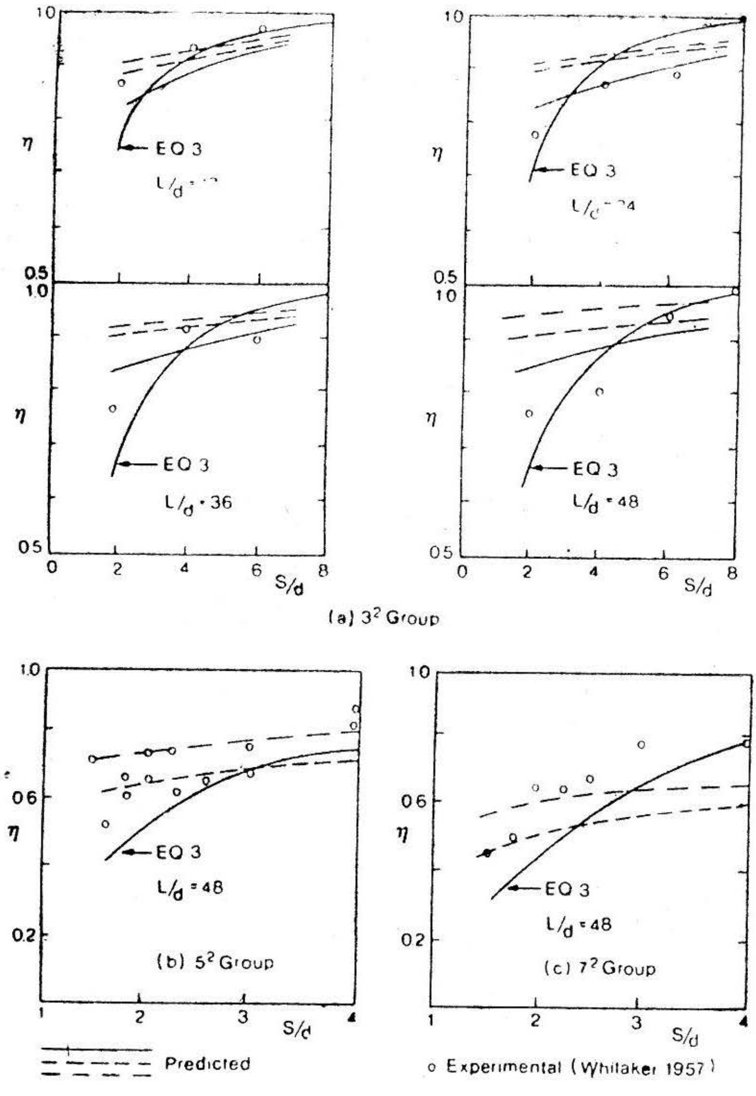


FIGURE 6 Effect of Pile Lengh and Group Size on Effiency,

If the rigorous method of estimating group efficiency (Eq. 25) has been adopted for the calculation of individual pile loads, possibly the agreement could be better.

Conclusions

The ultimate load carrying capacity of a pile group in clay, is estimated based on the capacity of a single pile and through a reduction factor. While reliable methods are available in literature for evaluating the former, only

TABLE 2

Comparison of Loads Carried by Piles in a Group

(a) Three Square Group L/d = 16

Pile	s/d = 2		s/d = 4				
	Expt*	Theory	Expt*	Theory Expt*			
A	13.0	11.55	14.1	11.36	X^A	X^B	X
B	10.6	10.89	9.7	10.99	X	X^C	X
C	9.1	10.22	7.1	10.58	X	X	X

(b) Five Square Group L/d = 24

Pile	s/d = 2		s/d = 4						
	Expt*	Theory	Expt*	Theory					
A	5.12	4.50	4.50	4.33	X^A	X^B	X^C	X	X
B	4.50	4.12	4.17	4.08	X	X^D	X^E	X	X
C	3.83	4.03	4.17	4.02	X	X	X^F	X	X
D	3.50	3.72	4.32	3.81	X	X	X	X	X
E	3.30	3.13	3.89	3.74	X	X	X	X	X
F	2.10	3.54	3.73	3.67					

*Experimental Results from Whitaker (1957).

few empirical methods are used for estimating the latter. A new concept called the extended Terzaghi-Peck concept, is presented herein to quantity the influence of a pile on the load carrying capacity of an adjacent pile through a reduction factor. The reduction factor is a function of the spacing between and the length of the piles, and the variation of the strength of the soil with depth. The method of superposition is valid for estimating the total reduction in pile capacity due to many piles in a group. A rigorous and a simpler approximate methods of evaluating pile group efficiencies, are presented. The predicted efficiencies of the groups and the percentage loads in individual piles in a group, compare well with the measured quantities, validating the method proposed.

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Notation

$\{A\}$	— vector of soil strength;
B_G nd L_G	— width and length of pile group in plan;
C_b	— cohesion at pile base;
C_α	— adhesion of soil/pile interface;
c	— depth of point force;
d	— diameter;
i, j, k, l	— indices;
L	— length of pile;
m, n	— number of rows and columns in pile groups;
N_C	— bearing capacity factor;
P	— load on pile;
P_P	— percentage load carried by pile;
P_B	— ultimate load on pile group by block failure;
P_G	— pile group capacity;
p	— stresses on pile shaft and base;

q — normal stress on pile base;

R_z — reduction factor;

s — spacing;

$[U]$ — unit matrix;

x, y, z — coordinates;

α_{ij} — influence coefficient;

η, η_B — efficiencies;

υ — Poisson's ratio;

τ — shear stresses;

ξ — parameter