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Technical Note

Bearing Capacity of Clay with Variable Surcharge of Finite Extent

by

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Introduction

The ultimate bearing capacity of a footing resting on or in the soil is estimated using Terzaghi's (1943) approach based on the general shear failure mechanism. This approach is generalised (Vesic, 1973) to include the effects of shape and depth of footing, the eccentricity and inclination of the load, the compressibility of the soil, etc. The expression for the ultimate bearing capacity, q_u, of the foundation, is written in the form

$$q_u = cN_c\, S_c\, d_c \,\ldots + q_0\, N_q\, S_q\, d_q \ldots + \gamma\, BN_\gamma\, S_\gamma\, d_\gamma \,.\,. \tag{1}$$

where c, ϕ, and γ are respectively the cohesion, the angle of shearing resistance, and the unit weight of the soil, N_c, N_q, N_γ -the bearing capacity factors, S_c, S_q, S_γ — shape factors, and d_c, d_q, d_γ — the depth factors, etc. Eq. (1) gets simplified as

$$q_u = c_u\, N_c + q_0 \tag{2}$$

for the undrained condition (i.e. $\phi_u = 0$, $c = c_u$) of the soil. It is implied in Eqs. 1 and 2 that the surcharge stress is uniform and extends to infinity. Also each of the terms in Eq. 1, is minimised individually to arrive at the minimum values of the bearing capacity factors. In this note, the bearing capacity of a footing on a soil in an undrained condition but with surcharge varying with distance and extending to only a finite distance beyond the footing, is estimated. Three types of variation of surcharge stress, viz., uniform, linear and exponential decay, are considered.

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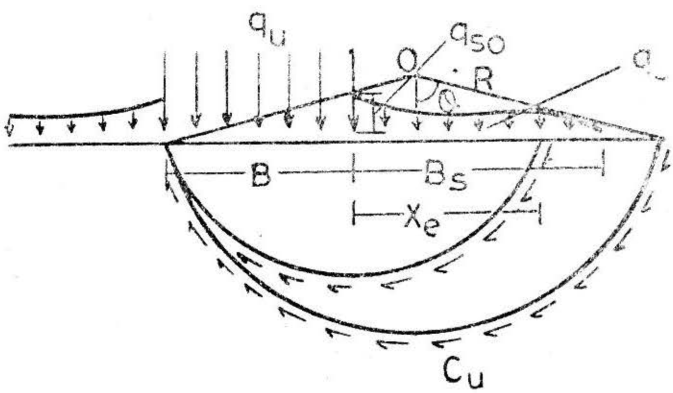
This theory is applicable for the estimation of the bearing capacity of a clay layer overlain by a stiffer layer (Meyerhof, 1974, problem) and in the case of reinforced foundation beds. The shear stresses mobilised by the upper (granular/stiff) layer get redistributed (Fig. 1) into the lower soft as surcharge stresses (Madhav and Poorooshasb, 1987). Another possible instance of surcharge of finite extent is for loads near the edges of highway pavements.

The variation of vertical stress on the interface of a two layered system, loaded by a uniform circular load, is shown in Fig. 2 (Fox, 1948), for different modular ratios (E_1/E_2). In case of a homogeneous deposit, the vertical stresses are relatively high beneath the loaded area. The stresses tend to become uniform with increasing values of the modular ratio.



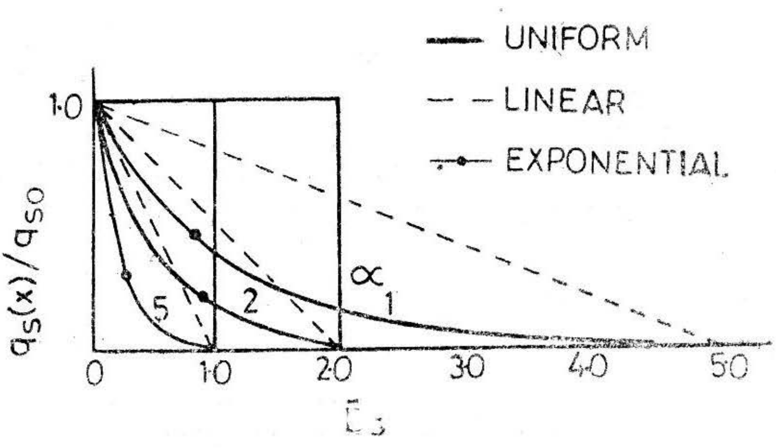


FIGURE 1 Definition Sketch : (a) Bearing Capacity Problem and (b) Types of Surcharge

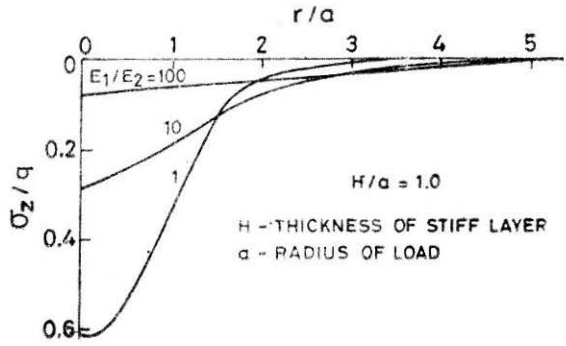


FIGURE 2 Vertical Stress on Interface of Two Layered Soil—Circular Uniform Load

In other words, the applied stresses get distributed to the soil outside of the loaded area. These stresses decrease in intensity with distance and are negligible beyond a finite distance. The vertical stress on the lower soil layer outside the loaded area can be considered as a surcharge stress extending to a finite extent.

ANALYSIS

Case A : Uniform Surcharge

Consider (Fig. 3a) a footing of width, B, resting on a soft clay with undrained cohesion, c_u. It is proposed to estimate the ultimate bearing capacity, q_u, when a uniform surcharge stress of intensity, q_{s0}, (Fig. 3*b*), exteends to a distance B_s on either side of the footing. Following Skempton's (1951) $\phi_u = 0$ analysis, the failure surface is assumed to be circular and starts from one of the edges of the footing. Depending on the location of its center and the radius, the failure surface may end within or outside the extent of the surcharge stress. The two cases correspond to :

Case A (*i*): $B + B_s > 2R \sin\theta$ and failure arc ends within the extent of the surcharge stress;

Case A (*ii*) : $B + B_s > 2R \sin\theta$ and failure are extends beyond the surcharge stress, where R and θ are the radius and half the angle subtended at the center of the trial arc. For subcase A(*i*), the surcharge stress extends beyond the point where the failure arc ends. This situation corresponds to the conventional case where the surcharge stress is assumed to extend to infinity for which case the ultimate bearing capacity, q_u, is given by Eq. 2. For subcase A (*ii*), moments of all forces about the center, 0, can be summed for equilibrium as

$$M_0 = q_u B(R \sin\theta - B/2) + q_{s0} B_s (R \sin\theta - B - B_s/3) - 2c_u R^2 \theta = 0 \quad (3)$$

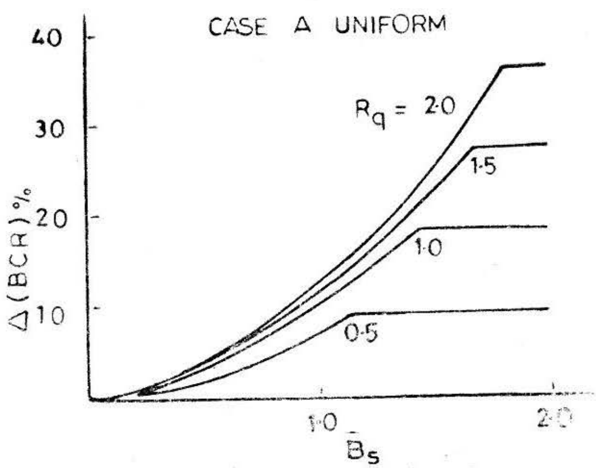


FIGURE 3 Improvement in Bearing Capacity—Uniform Surcharge

Solving for q_u.

$$q_u = c_u N_c^* = c_u \overline{N_c} + q_{s0} \overline{N_q} \tag{4}$$

where $$N_c^* = \overline{N_c} + R_q \overline{N_q} \tag{5}$$

with $$\overline{N_c} = 2\bar{R}^2\theta/(\bar{R} \sin\theta - 0.5) \tag{6}$$

$$\overline{N_q} = \bar{B}_s(-\bar{R}\sin\theta + 1 + \bar{B}_s/3)/(\bar{R}\sin\theta - 0.5). \tag{7}$$

$\bar{R} = R/B$, $\bar{B}_s = B_s/B$, and $R_q = q_{s0}/c_u$. N_c^* is a composite bearing capacity factor that depends on N_c, R_q, and N_q, and is minimised with respect to the radius R, and the angle θ. Herein the trial arc which minimises N_c^* is determined unlike the conventional analysis in which the contributions of cohesion and surcharge stress are minimised separately.

Case B : Surcharge Stress decreases Linearly with Distance.

The Surcharge stress decreases linearly (Fig. 3b) as

$$q_s(x) = q_{s0}(1 - x/B_s) \tag{8}$$

The analysis follows on the same lines as in Case A except that subcase B(i) i.e. when the surcharge stress extends beyond the failure arc also needs to be considered.

Case $B(i) : B + B_s > 2R\sin\theta$

$$\overline{N_q} = \{(1 - X_e/\bar{B}_s)X_e(1 + X_e/2 - \bar{R}\sin\theta) + X_e^2(1 + X_e/3 - \bar{R}\sin\theta)/ 2\bar{B}_s\}/(\bar{R}\sin\theta - 0.5) \tag{9}$$

where $x_e = 2R\sin\theta - B$, and $X_e = x_e/B$.

Case B (*ii*) : $B + B_s < 2\ R \sin \theta$

$$\bar{N_q} = \bar{B}_s\,(1 + \bar{B}_s/3 - \bar{R}\sin\theta)/(2\bar{R}\sin\theta - 1.0) \tag{10}$$

Case C : Surcharge Stress decreases Exponentially with Distance The surcharge stress varies with distance as

$$q_s(x) = q_{s0}\exp(-\alpha' x) = q_{s0}\exp(-\alpha X) \tag{11}$$

where α. is a decay parameter, and $\alpha = \alpha' B$. Since in this case, the surcharge theoretically extends to infinity (i.e. $B_s \rightarrow \infty$). only subcase C(*i*) need be analysed. The moment equilibrium equation is

$$\bar{N_q} = [\{(1-\exp(-\alpha X_e + 1)\,(X_e + 1)\} - (\bar{R}\sin\theta - 1)\,(1 - \exp(-\alpha\, x_e)]/ (2\bar{R}\sin\theta - 1.0) \tag{12}$$

where $X_e = (2\bar{R}\sin\theta - 1)$. In all the above cases, the increase in bearing capacity, Δ(BCR), due to partial surcharge is expressed in percentage fcrm as

$$\Delta(\text{BCR}) = \{N_c^*/N_c - 1\}\ 100 \tag{13}$$

Results

The composite bearing capacity factor N_c^* is minimised with respect to the radius R and half angle, θ, subtended at the center of the trial arc. A direct search method is adopted for the minimization and the accuracy of the results ensured with a convergence criterion of 0.0001. A parametric study is carried out for the three types of surcharge stress variation : A-Uniform; B —Linear Decrease; and C-Exponential Decay, the maximum surcharge stress ratio, R_q, varying from 0 to 5, and the surcharge stress extending from 0 to 5B.

The increase in bearing capacity ratio, Δ(BCR), is evaluated for the three variations considered and presented in Figs. 3 to 5. In all these cases, the bearing capacity increases with both B_s and R_q ratios. In case of uniform surcharge (Fig 3), the ratio Δ (BCR) increases rapidly with B_s and once the maximum value of Δ(BCR) is reached corresponding to N_q of 1.0, it remains constant as surcharge extending beyond the critical failure are is equivalent to one extending to infinity. The maximum values of Δ(BCR) are 9%, 18%, 27%, and 36% (Fig 3) for R_q values of 0.5, 1.0, 1.5, and 2.0 respectively. The maximum values of Δ(BCR) corres-pond to B_s values of 1.12, 1.42, 1.68, and 1.8 for the above values of R_q. This result signifies that larger the intensity of surcharge stress, i.e. larger the value of R_q, the farther should it extend for the Δ(BCR) to reach the maximum value.

If the surcharge stress decreases linearly with distance (Eq. 8), the

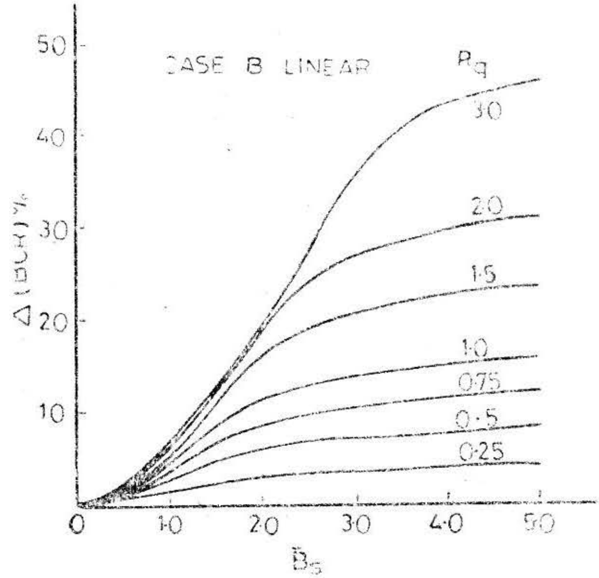


FIGURE 4 Improvement in Bearing Capacity—Linear Decrease of Surcharge

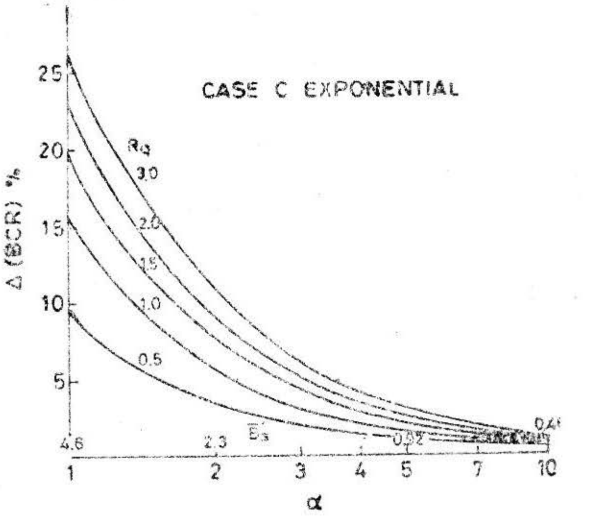


FIGURE 5 Improvement in Bearing Capacity—Exponential Decrease of Surcharge

Δ(BCR) values increase gradually with B_s (Fig 4) and attain their maxima asymptotically. The maximum Δ(BCR) values are 8.3%, 15.5% and 31% respectively for Rq values of 0.5, 1.0 and 2.0. Even in this case, the distance B_s, beyond which Δ(BCR) remains nearly constant increases with the intensity of maximum surcharge stress, R_q.

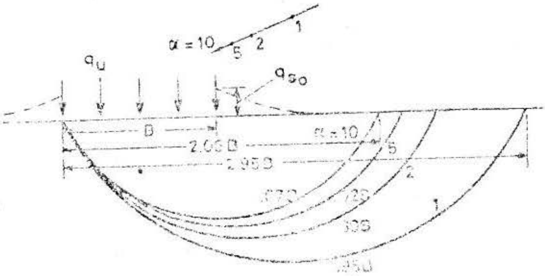


FIGURE 6 Critical Circles for Case C-Exponential Decrease of Surcharge

Case C corresponds to exponential decrease of surcharge stress with distance. The parameter, α, signifies the decay of surcharge stress with distance. Smaller the value of α, slower is the decrease in surcharge stress and vice versa. If B_s is defined as the distance at which the surcharge stress is one percent of the maximum value, α values of 1, 2, 5, and 10 correspond to B_s values of 4.6, 2.3, 0.92, and 0.46 respectively. The surcharge stress values beyond these distances are negligible. The Δ(BCR) values increase with the surcharge stress ratio, R_q, (Fig. 5), and decrease rapidly with increasing values of α. The critical circles which give the minimum values of N_c^* for different values of α (Case C) are depicted in Fig. 6. The surcharge stress intensity at any distance is higher for decreasing values of α. The critical circles are deeper and extend over larger widths with decreasing α—values. For α decreasing from 10.0 to 1.0, the maximum depth of the critical arc increases from 0.67B to 0.95B and the lateral extent increases from 2.05B to 2.95B from the left edge of the arc. Thus, if the surcharge stress extends over larger width, the critical arc becomes significantly larger and deeper, and contributes to larger increase in bearing capacity. The improvement in bearing capacity could be much more in case of nonhomogeneous soils in which the strength increases with depth. The increase of bearing capacity of reinforced foundation beds, in part, is contributed by the stiff granular layer spreading the load over wider area on the soft soil below which, in turn, acts as a surcharge stress.

Conclusions

The conventional approaches for the estimation of bearing capacity of foundations on soil, consider the surcharge stress to be uniform and of infinite extent. This note presents a simple theory for the bearing capacity of a footing on a cohesive soil in an undrained condition with surcharge stress of finite extent and whose variation with distance is constant, linear or an exponential decrease. The composite bearing capacity factor, N_c^*, is minimised and obtained as a function of the maximum value, R_q, and relative extent, B_s, of the surcharge stress. Results are presented quantifying the improvement in bearing capacity Δ(BCR). The critical failure circle is shown to extend deeper into the ground with increasing extent of the surcharge stress.

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