Indian Geotechnical Journal, 22(4) 1992

Dynamic Response of Piles under Vertical Load

by

Ashok Kumar Khan*

P.J. Pise**

Introduction

Technical and Industrial progress is accompanied by setting up of heavy machines, offshore towers, nuclear power plants etc even at sites having poor soil conditions. Invariably, under such circumstances deep foundations are adopted to transmit the load to the surrounding as well as the deeper soil strata. Use of piles may be necessary in four cases: (1) if the total pressure on the soil, both static and dynamic, is larger than the bearing capacity of the soil; (2) if the natural frequency of vibration of the foundation is to be increased; (3) if the amplitude of natural or forced vibration is to be reduced; and (4) if the residual dynamic settlement of the foundation has to be decreased.

Pile foundations subjected to dynamic loads experience various modes of vibration viz. vertical, horizontal, rocking and translational in nature and the behaviour of piles under these modes of vibrations is very complex and yet to be fully understood.

Brief Review of the Work Done

During the last decade the development of different methods of analysis of pile foundation started with the concept of elastic half-space. An analytical solution which has been widely accepted treats the foundation soil system as an oscillating body resting on a semi-infinite, homogeneous, isotropic, elastic half-space. Reissner (1936) established the theoretical basis for studying the response of a footing supported by an elastic half-space. The effects of contact pressure beneath the footing oscillating vertically were studied by Sung (1953) and Quinlan (1953). Arnold et al (1955) studied the forced vibration of a rigid body resting on a semi-infinite elastic medium. They have analyzed four modes of vibrations viz. vertical, horizontal, rocking and translational and showed that the amplitude can be obtained for any mass in terms of known constants of

*Research scholar, **Professor — Department of Civil Engineering, Indian Institute of Technology, Kharagpur-721 302, India

the system and fundamental functions $f1$ and $f2$ (Reissner's displacement functions) which depend on the exciting frequency factor and poisson's ratio of the medium. Hsieh (1962) modified the Reissner's basic differential equation for geometrical damping for vertical motion of a rigid circular footing. Baranov (1967) proposed an approximate solution to take into consideration the effect of embedment by considering the dynamic reaction below the foundation base as well as along the sides of the foundation. He observed that the effect of embedment of footing reduces the maximum amplitude and increases the resonant frequency of the foundation. Isakovich and Komarova (1968) analysed the longitudinal-flexural modes of vibration in a curved slender rod, the axis of the rod forming a plane curve. He derived the wave equation with the help of vector calculus. Novak and Baradugo (1972) extended Baranov's approach to study the effect of embedment on the vertical vibration of footings and presented a steady state solution in terms of frequency dependent spring and dash pot constants. Novak (1974) utilized Baranov's approach to propose an analysis for soil-pile interaction. He assumed that the soil is composed of a set of independent infinitesimally thin horizontal layers that extend to infinity. When a pile element undergoes a complex horizontal displacement at any depth inside the soil mass of shear modulus G_s, it is acted upon by a horizontal soil reaction. He also analyzed the dynamic soil reaction for vertical motion of pile.

The basic concept of the governing wave equations which describe coupled flexural and extensional vibration of curved beam-column in a plane has been modified and utilized for the analysis of initially curved marine conductors by Fischer (1975). He considered the external dynamic force to be repetitive in nature. The vertical soil resistance of pile has been modelled as static skin friction and assumed horizontal guides resist the horizontal movement. Novak and Grigg (1976) reported the experimental results of small pile foundations under dynamic loading conducted in the field, the soil being fine silty sand followed by gravels. The black steel pipe piles of 6cm and 9cm diameters were used as test piles. Experimental results were compared with the theoretical solution of Novak (1974).

Novak and Aboul-Ella (1978) considered the soil as a continuum and accounted for dynamic soil pile interaction, the energy dissipation being through propagation of elastic waves. Mizuhata and Kusakaba (1984) have introduced the concept of weakened zones around the piles to explain the Complex dynamic soil-pile interaction phenomenon. Wolf and Von Arx (1978) and others have utilised finite element technique to discretise soil around the pile and consistant boundary matrix to stimulate the effect of the far field. Saha (1985) investigated the straight pile taper pile and under reamed pile for vertical, horizontal and rocking mode of vibrations. He also analysed the pile supported footings. For vertical vibration he assumed only vertical displacement and lateral movement was restrained. Nogami (1986) presented a procedure for time domain analysis of axial response of single piles where a time domain soil-pile interaction force is formulated through a simple mechanical idealization of the soil medium developed from the dynamic behaviour of a plane strain continuous elastic medium. Gazetas (1984) has analysed the kinematic seismic response of single piles and pile groups. Recently an overview has also been presented by Novak (1991) using continuum as well as Winkler-type models. Ghosh et al (1992) analysed the piles with enlarged base and also reported the experimental vertical response curves for footings supported on model piles with enlarged base.

It can be seen that dynamic analysis of piles has been restricted to straight, vertical piles with either vertical or horizontal movement under either vertical or horizontal vibration. One dimensional wave propagation theory in elastic rod was adequate to analyse this problem. However, due to several reasons piles may either bend from the vertical axis or be exposed to dynamic eccentric vertical loads which may induce both longitudinal and transverse motions. A generalised theory for the analysis of piles having coupled flexural and extensional vibration is necessary to solve such a problem.

Scope of Study

The basic governing wave equation for curved beam-column in a plane has been modified and utilised for analysing the straight vertical pile in this paper. Floating as well as end bearing piles have been considered. The main objective of the study is to investigate the dynamic effect of periodic vertical loads on a single vertical pile wherein vertical as well as lateral movement of pile are allowed. The computer-aided analysis by finite difference technique has been developed which considers soil-pile interaction in a relatively simpler way. It is restricted to piles embedded in a homogeneous half-space.

METHOD OF ANALYSIS

The method as suggested by Saha (1985) to analyse the dynamic behaviour of pile and pile foundation under vertical harmonic motion assumes the pile movement along vertical direction ignoring the flexural movement of pile. The present analysis is similar to Saha's approach and it has been extended to study the vertical and horizontal movement of pile under vertical vibration. Two dimensional wave equations which describe coupled flexural and extensional motion of straight vertical piles have been presented here. It has been assumed that (1) the pile is perfectly elastic, vertical, circular in cross-section and has a perfect contact with soils, (2) the soil is a linearly elastic, isotropic, homogeneous and semi-infinite medium, and (3) the soil reaction acting on the tip is equal to that of an elastic half-space. The finite difference numerical technique has been used. This method transforms the problem of pile vibration into a computer-oriented procedure of matrix structural analysis (Salvadori and Baron (1962)).

Fig. 1 (*a*) shows the differential segment of a curved pile along with forces, moments, and displacements. An element of length dz has been considered for analysis (Fig. 1).

F and $F + dF$ = axial forces

Q and $Q + dQ$ = shear forces

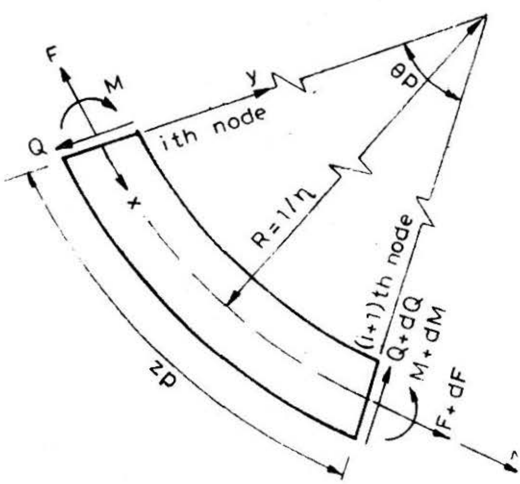

FIGURE 1 (a) Differential segment of curved pile shoiwng sign convention for forces, moments and displacements

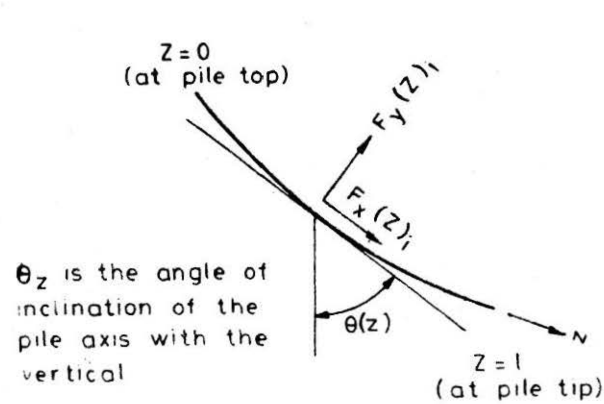

FIGURE 1(b) A portion of the planner curve of the pile axis. The stiffness shows positive directive

M and $M + dM$ = bending moment

x, y = displacement along the pile axis and normal to it respectively.

To satisfy the Compatibility condition the following equations may be written,

$$\Sigma F_{axial} = 0, \quad \Sigma Q_{normal} = 0, \quad \text{and } \Sigma M = 0$$

$$F + \frac{dF}{dZ} dZ - F \cos d\theta - Q \sin d\theta = 0 \tag{1}$$

$$Q + \frac{dQ}{dZ} dZ - Q \cos d\theta + F \sin d\theta = 0 \tag{2}$$

$$M + \frac{dM}{dZ} dZ - M - F R + (F + \frac{dF}{dZ} dZ) R = 0 \tag{3}$$

After applying curved-beam theory (Pippard and Baker (1968)),

$$F_{axial} = A E (x^1 - \eta y) \tag{4}$$

$$Q_{normal} = - E I (\eta^2 y^1 + y^{111}) \tag{5}$$

$$M = E I (\eta^2 y + y^{11}) \tag{6}$$

Substituting Eqs. 4 and 5 in Eqs. 1 & 2, the following relationships are written,

$$\rho \ddot{x} = A E [x^{11} + (a^2 \eta^3 - \eta) y^1 + a^2 \eta y^{111}] \tag{7}$$

$$\rho \ddot{y} = A E [\eta x^1 - \eta^2 y - a^2 \eta^2 y^{11} - a^2 y^1] \tag{8}$$

The basic wave equations which describe coupled flexural and extensional vibration of curved beam-column in a plane may be modified for curved pile and presented in the partial differential form as,

$$\rho \frac{\partial^2 x (z,t)}{\partial t^2} - A E \Big[\frac{\partial^2 x (z,t)}{\partial z^2} + (a^2 \eta^3 - \eta) \frac{\partial y(zt)}{\delta z} + \rho^2 \eta \frac{\partial^3 y (z,t)}{\partial z^3} \Big] + c \frac{\partial x(z,t)}{\partial t} + k_x (z,t) = 0 \tag{9}$$

$$\partial \frac{\partial^2 y(z,t)}{\partial t^2} - A E \Big[\eta \frac{\partial x(z,t)}{\partial z} - \eta^2 y(z,t) - a^2 \eta^2 \frac{\partial^2 y(z,t)}{\partial z^2} - a \frac{2\partial^4 y (z,t)}{\partial z^4} \Big] + c \frac{\delta y(z,t)}{\delta t} + k_y (z,t) = 0 \tag{10}$$

Where, ρ = linear density of pile material

$(\ )^{\cdot} = \partial (\ )/\partial t$ and $(\ )^1 = \partial (\ )/\partial z$

$a^2 = I/A$

I = central moment of inertia of pile cross section

A = cross sectional area of pile

E = Young's modulus of elasticity of pile material

η = Initial curvature of pile

c = co-efficient of pile material damping

$k_x (z.t)$, $k_y (z,t)$ = side soil reaction per unit length of pile and described by complex soil stiffness associated with vertical and horizontal displacement of pile respectively and represented as,

$$k_x (z,t) = [F_x (z) + i\, w\, c_x]\, x (z,t) \tag{11}$$

$$k_y (z,t) = [F_y (z) + i\, w\, c_y]\, y (z,t) \tag{12}$$

Where, F_x (z) and F_y (z) = Elastic stiffness functions dependent on depth for vertical and horizontal displacements respectively and are derived from Mindlin's (1936) solution (Figs. 2(*b*) & (*c*)). c_x, c_y = Viscous damping functions for vertical as well as horizontal displacements respectively; and x, y = displacement along pile axis and normal to it respectively.

When the pile element undergoes vertical as well as horizontal translation, the phenomena may be considered similar to the operation of a disk vibrating horizontally as well as vertically on a semi infinite elastic half-space. The equivalent viscous damping parameters are calculated as suggested by Hsieh (1962) for horizontal and vertical modes of vibrations and are presented in Figs. 3(*a*) & (*b*).

Differential equation of motion

When a pile is subjected to both vertical and horizontal harmonic excitations, the motion of the pile is resisted by distributed complex side soil reaction k_x (z, t) and k_y (z, t) acting along the length of the pile and a concentrated vertical soil reaction R (t) at the pile tip Fig. 2(*a*). The distributed soil reaction appears in the equation of motion for an element dz and the concentrated reaction represents the boundary condition at the tip. Assuming that the pile is undergoing complex vertical and horizontal vibrations, the complex vertical and horizontal displacements may be written as,

$$\left.\begin{aligned} x (z,t) &= x (z)\, e^{iwt} \\ y (z,t) &= y (z)\, e^{iwt} \end{aligned}\right\} \tag{13}$$

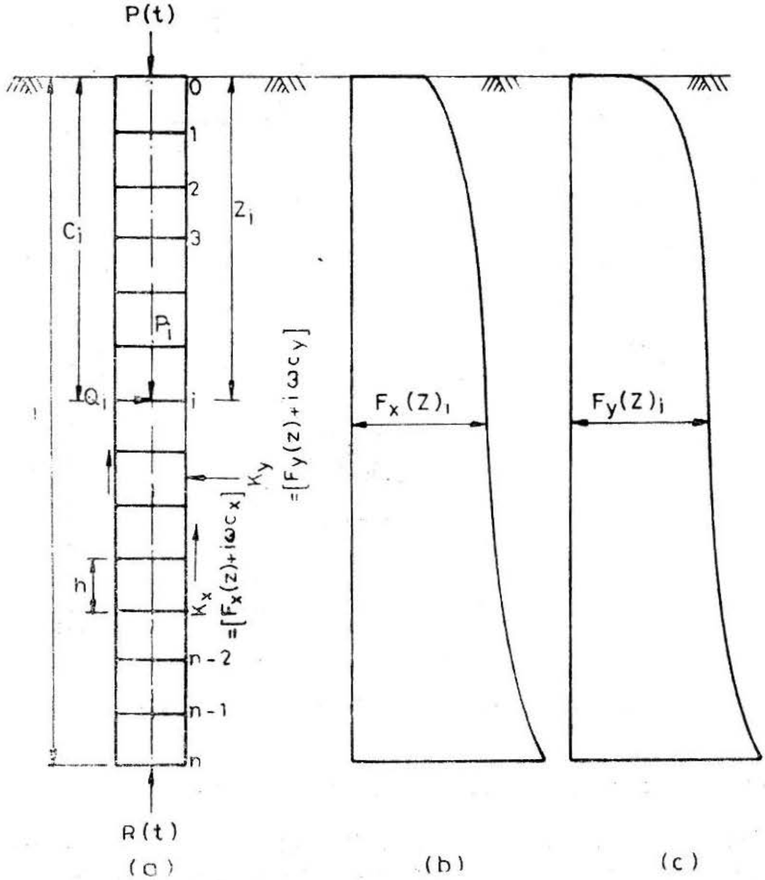

FIGURE 2 (a) **Both vertically and horizontally excited discretised pile; (b) variation of stiffness function $F_x(z)_i$ and (c) variation of stiffness Function $F_y(z)_i$**

Where, $x(z)$ and $y(z)$ are the complex amplitude in axial and lateral directions of pile at depth z.

$i = \sqrt{-1}$ and $t =$ time

Substituting the above in Eqs. 9 & 10 and assuming the initial curvature for the pile axis as infinity and neglecting the material damping, the differential wave equations of damped longitudinal-flexural modes of vibration for straight pile, are obtained as under:

$$A\ E\left[\frac{\partial^2 x(z)}{dz^2} + (a^2\eta^3 - \eta)\frac{\partial y(z)}{dz} + a\frac{2\partial^3 y(z)}{dz^3}\right] + [\sigma w^3 - F_x(z) + iwc_x(z)]\,x(z) = 0 \tag{14}$$

$$A\ E\left[n\frac{\partial x(z)}{\partial z} - a^2\eta^3\frac{3\partial^2 y(z)}{\partial z^2} - a\frac{2\partial^4 y(z)}{\partial z^4}\right] + [\sigma w^2 - A\ E\,\eta^2 - F_y(z) - i\,w\,c_y(z)]\,y(z) = 0 \tag{15}$$

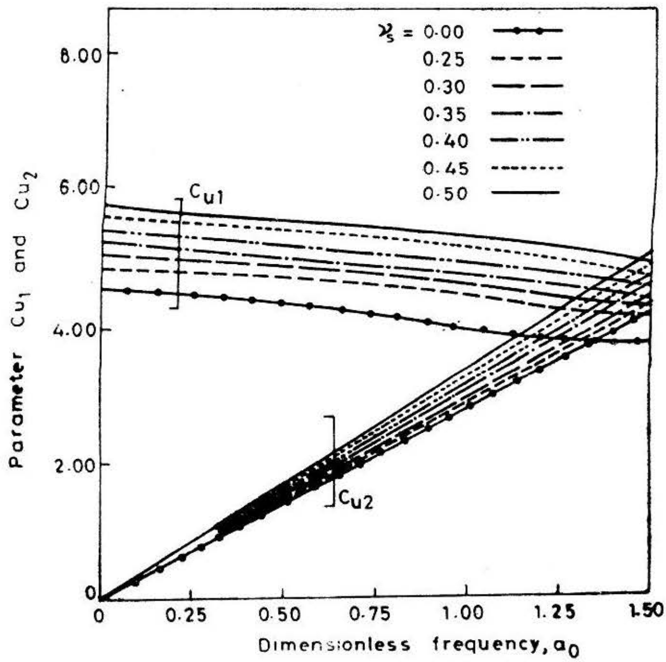

FIGURE 3 (a) Half-space stiffness (C_{u1}) and damping (C_{u2}) parameters for horizontal mode of vibration.

Analysis of single free head pile with pile tip free

This case is similar to free-head floating pile. For vertical vibration, the complex stiffness of pile at pile head is defined as the end force providing a unit displacement of the pile head. Therefore, the first boundary condition is,

$$\text{at,} \quad z = 0, \; x(z) = x_0 (0) = 1 \tag{16}$$

As the pile head is free the bending moment and shear force are zero. Therefore,

$$-EI \left\{ \eta \frac{2\,\partial\, y(z)}{\partial z} + \frac{\partial^3 y(z)}{\partial z^2} \right\} = 0 \tag{17}$$

$$EI \left\{ \eta^2 y(z) + \frac{\partial^2 y(z)}{\partial z^2} \right\} = 0 \tag{18}$$

Now at the pile tip the motion of the pile generates a concentrated reaction equal to the axial force, so,

$$F_{axial} = R(t) \; i, e;$$

$$AE \left\{ \frac{\partial\, x(z)}{\partial z} - \eta\, y\,(z) \right\} = -\, Gb\, r_0\, (c_{x1} + c_{x2})\, x(z)_{z=1} \tag{19}$$

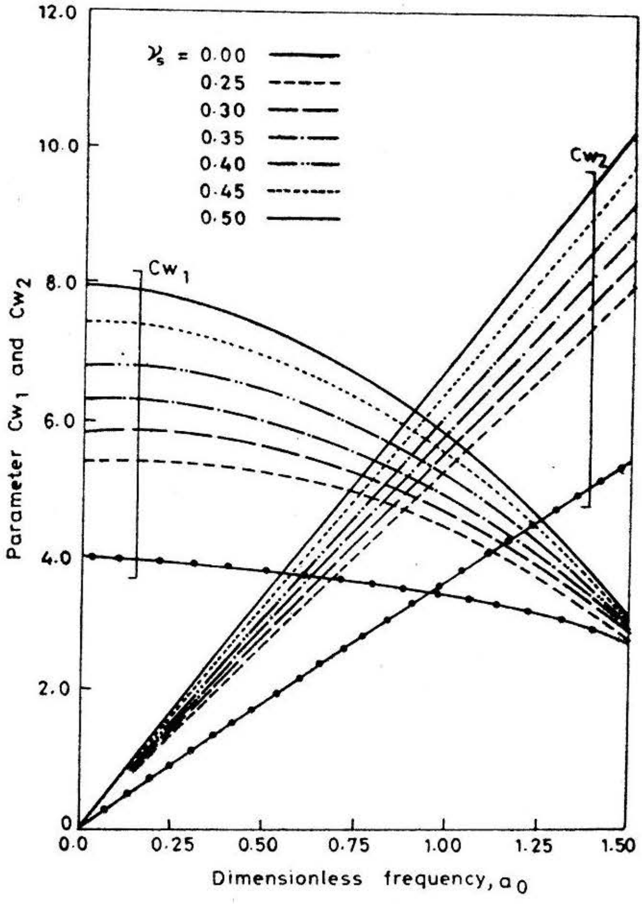

FIGURE 3 (b) Half-space stiffness (C_{w1}) and damping (C_{w2}) parameters for vertical mode of vibration

Where, G_b = shear modulus of soil below the pile tip. When G_b, tends to a very large value or infinity in comparison with the shear modulus of surrounding soil, the pile may be considered as end bearing.

$x(z)_{z=1}$ = complex amplitude of the pile tip.

$c_{x1}\,;\,c_{x2}$ = half-space parameters in vertical mode of vibration depending on dimensionless frequency,

$a_0^1 = r_o\, w/v_b$.

v_b = shear wave velocity below pile tip and

w = circular frequency.

Finite difference solution

For simplicity and convenience the finite difference numerical technique is adopted for the solution of fourth order differential wave equations. For this purpose the pile length is divided into n number of elements with 0 to n nodes. In formulation of the set of finite difference equations, the boundary conditions are also satisfied.

A set of $2 \times n$ simultaneous equations so obtained may be written in the form,

$$[A]\{W\} = \{B\} \tag{20}$$

Where, the complex co-efficient matrix $[A]$, the complex displacement vector $\{W\}$ and load vector $\{B\}$ are determined from the respective finite difference equations.

Suitable computer programs in fortran *iv* have been developed to generate the matrices for the equation (20) and for subsequent solution of the complex simultaneous equations on a high speed computer (CYBER 840) for complex nodal displacements.

The nodal deflections at any depth z are,

$$\left.\begin{aligned} x(z) &= x_1 + ix_2 \\ y(z) &= y_1 + iy_2 \end{aligned}\right\} \tag{21}$$

Where, x_1, y_1 and x_2, y_2 are the real and imaginary parts of complex displacement. Therefore, the real amplitude of motion is,

$$\left.\begin{aligned} x(z) &= [x_1^2 + x_2^2]^{1/2} \text{ and,} \\ y(z) &= [y_1^2 + y_2^2]^{1/2} \end{aligned}\right\} \tag{22}$$

and the phase angle in X-direction is,

$$\phi(z) = \tan^{-1} \frac{x_2}{x_1} \qquad \text{and in } Y \text{ direction is,}$$

$$\theta(z) = \tan \frac{-1\, y_2}{y_1}$$

Now, the complex vertical and horizontal pile stiffness may be written as,

$$\left.\begin{aligned} k_x^c &= k_x + iwc_x \text{ and} \\ k_y^c &= k_y + iwc_y \end{aligned}\right\} \tag{23}$$

Where the real part of the complex stiffness is related to the stiffness co-efficient and imaginary part to damping. Where, k_x, k_y, c_x and c_y may be expressed as,

$$\left.\begin{aligned} k_x &= \frac{E\,A}{r^0}\, f_{x1} & \quad c_x &= \frac{E\,A}{v_s}\, f_{x2} \\ &\text{and} & & \\ k_y &= \frac{E\,I}{r_0^3}\, f_{y1} & \quad c_y &= \frac{E\,I}{r_0^2\, v_s}\, f_{y2} \end{aligned}\right\} \tag{24}$$

Here f_{x1}, f_{y1}, and f_{x2}, f_{y2} are the dimesionless pile stiffness and damping parameters respectively under coupled modes of vibration and are presented in Figs. (4) and (5).

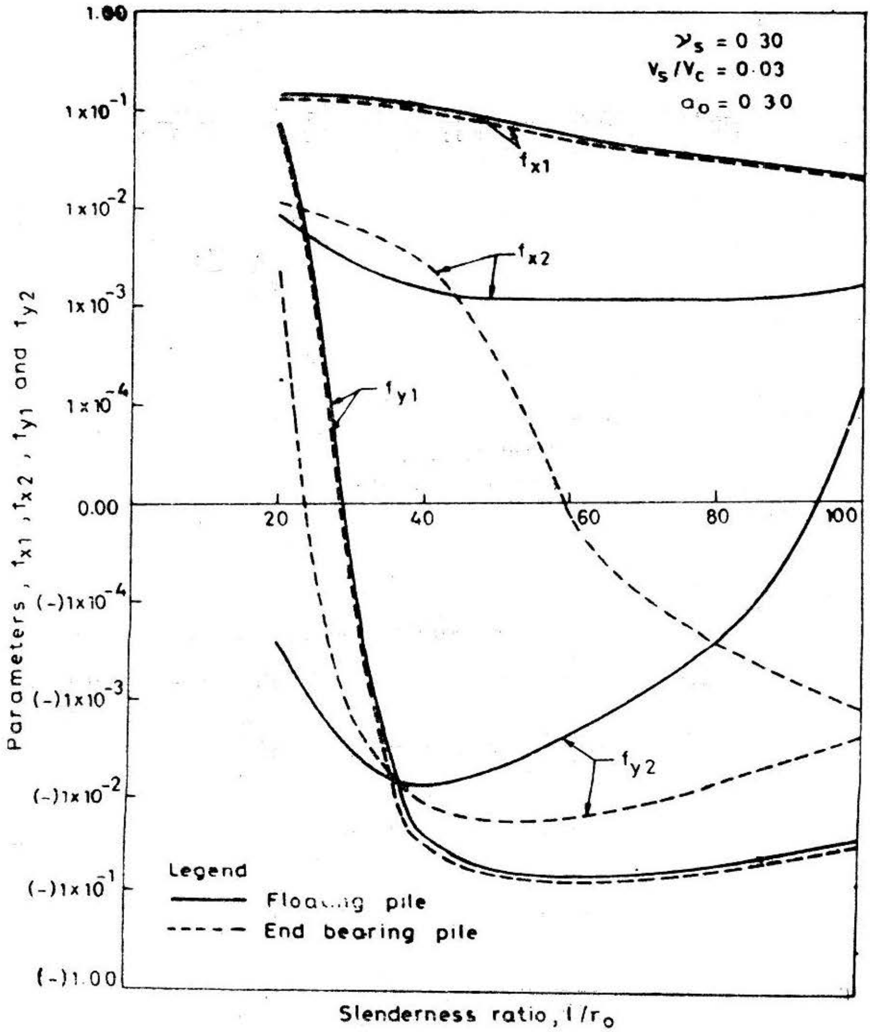

FIGURE 4 Variation of stiffness and damping parameters of piles with slenderness ratio

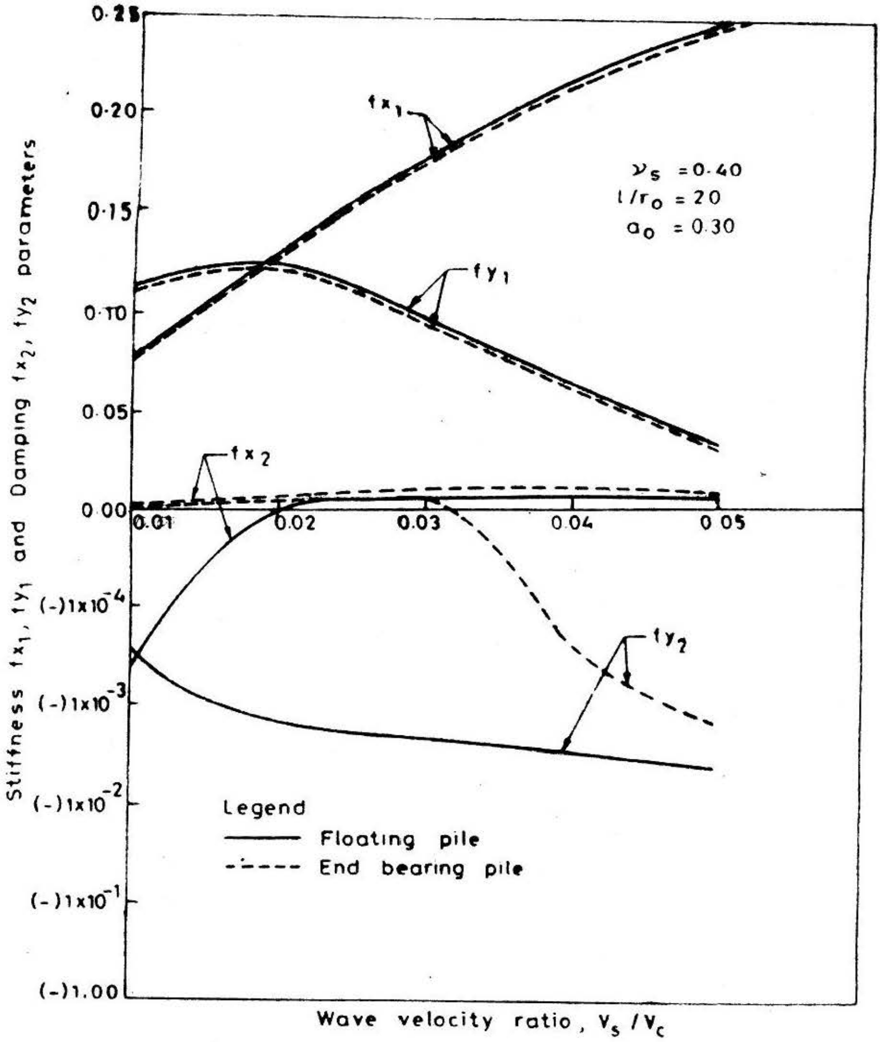

FIGURE 5 Variation of stiffness and damping parameters of piles with wave velocity ratio

$$\left.\begin{aligned} f_{x1} &= -\frac{r_0}{2h}[-3\,x_{01} + 4\,x_{11} - x_{21}] \\ f_{x2} &= -\frac{v_s}{2h\omega}[-3\,x_{02} + 4\,x_{12} - x_{22}] \\ f_{y1} &= -\frac{r_0}{2h}[-3\,y_{01} + 4\,y_{11} - y_{21}] \\ y_{y2} &= -\frac{v_s}{2h\omega}[-3\,y_{02} + 4\,y_{12} - y_{22} \end{aligned}\right\} \tag{25}$$

Where, x_{01}, x_{11}, x_{21} and y_{01}, y_{11}, y_{21} are the real part of complex displacement in vertical as well as lateral direction respectively at nodes 0,1,2, etc. and x_{02}, x_{12}, x_{22} and y_{02}, y_{12}, y_{22} are the imaginary parts of complex displacement at nodes 0,1,2 etc.

Equivalent Stiffness and Damping co-efficient of Pile Supported Footing:

The stiffness and damping co-efficient of individual piles are used to determine equivalent stiffness and damping co-efficient of a footing supported on piles. For footing supported on piles subjected to complex vertical and horizontal excitation,

$$P(t) = P_0\, e^{iwt} \text{ and } Q(t) = Q_0\, e^{iwt} \tag{26}$$

The steady state responses are,

$$x(t) = x_0 \cos (wt + \phi) \text{ and } y(t) = y_0 \cos (wt + \phi) \tag{27}$$

The real force amplitudes for rotating mass type excitation for both the cases are,

$$P_o = m_{ex}\, e_x\, w^2 \text{ and } Q_o = m_{ey}\, e_y\, w^2 \tag{28}$$

$m_{ex}ex$ and $m_{ey}ey$ being the eccentric mass moments in both the directions. The amplitudes of vertical and horizontal displacement of the footing are given as,

$$\left.\begin{aligned} x_o &= \frac{P_o}{[(k_x - Mw^2)^2 + (wc_x)^2]^{1/2}} \text{ and} \\ y_o &= \frac{Q_o}{[(k_y - Mw^2)^2 + (wc_y)^2]^{1/2}} \end{aligned}\right\} \tag{29}$$

where, M = total mass of footing.

The dimensionless amplitude, A_{xx} and A_{yy} at frequency w is written as,

$$\left.\begin{aligned} A_{xx} &= \frac{Mx_o}{m_{ex}\, ex} = \frac{w^2}{\left[\left\{\frac{K_x}{M} - w^2\right\}^2 + \left\{\frac{wc_x}{M}\right\}^2\right]^{1/2}} \text{ and} \\ A_{yy} &= \frac{My_y}{m_{ey}\, ey} = \frac{w^2}{\left[\left\{\frac{k_y}{M} - w^2\right\}^2 + \left\{\frac{wc_y}{M}\right\}^2\right]^{1/2}} \end{aligned}\right\} \tag{30}$$

DISCUSSION OF THE RESULTS

Using Hsieh's (1962) solution the variation of half-space parameters C_{u1}, C_{u2}, C_{w1} and C_{w2} with dimensionless frequency, a_0 have been generated and presented through Figs. 3(*a*) and (*b*) for various values of Poisson's ratio. They are used to determine the viscous damping functions in horizontal and vertical modes of vibration. For horizontal mode of vibration, stiffness parameter, C_{u1} decreases whereas damping parameter C_{u2} increases with increase in dimensionless frequency, a_0 (Fig. 3(*a*)). For vertical mode of vibration, the stiffness parameter, C_{w1} decreases and C_{w1} increases with increase in a_0 (Fig. 3(*b*)).

Using appropriate Eqs. f_{x1}, f_{x2}, f_{y1} and f_{y2} have been evaluated and shown in Fig. 4 for a typical case of $v_s=0.30$, $Vs/Vc=.03$, $a_0 = 0.30$ for floating and end-bearing piles. f_{x1} and f_{y1} are found to be independent of the end condition of the pile. f_{x1} decreases with increase in slenderness ratio, $1/r_0$. Stiffness function f_{y1} decreases upto $1/r_0=40$ and threeafter it increases with increase in slendezess ratio. The damping function f_{x2} for floating pile decreases gradually upto $1/r_0=40$ and remains practically constant thereafter and f_{y2} decreases steeply upto $1/r_0=40$ and subsequently increases. In case of end bearing pile the damping function f_{x2} decreases with increase in $1/r_0$ and the damping function f_{y2} decreases very steeply upto $1/r_0=40$ and subsequently increases gradually with increase in $1/r_0$.

Fig. 5 shows the variation of stiffness and damping parameters in vertical and horizontal directions, f_{x1}, f_{x2} and f_{y1}, f_{y2} respectively with Vs/Vc. In case of floating and end bearing piles for typical values of $v_s=0.4$, $1/r_0=20$ and $a_0=0.30$, f_{x1} and f_{y1} are almost independent of tip conditions. The stiffness parameter f_{x1} increases with increase in velocity ratio, Vs/Vc. Stiffness parameter, f_{y1} in horizontal direction marginally increases upto $Vs/Vc=0.02$ and thereafter decreases gradually. End conditions significantly influence damping parameters f_{x2} and f_{y2}. For floating pile f_{x2} increases upto $Vs/Vc=0.02$ and thereafter practically remais constant. The damping parameter f_{y2} for floating pile decreases gradually with increase in Vs/Vc. For end bearing pile, the damping parameter f_{x2} is almost constant and has negligible value for the range of Vs/Vc considered and the damping parameter f_{y2} is almost constant upto $Vs/Vc=0.03$ and thereafter it decreases steeply.

Fig. 6 shows the comparison between author's solution with the lumped mass solutions published by Kuhlemeyer (1981) for vertical response of floating piles driven into a soil system that can be approximated as a homogeneous soil half-space. The curves are for three different values of mass ratios, B and for $1/r_0=40$ and $MR=Ep/Es=2000$, (The mass ratios has been defined as $B=Wf_s/\gamma_s r_0^3$, Where, W=applied static load on pile, γ_s=unit weight of soil and f_s is a function of Ep/Es and $1/r_0$). The agreement between author's solution and Kuhlemeyer's analysis is quite satisfactory. However, the predicted values of resonance amplitudes are consistently more than those given by Kuhlemeyer.

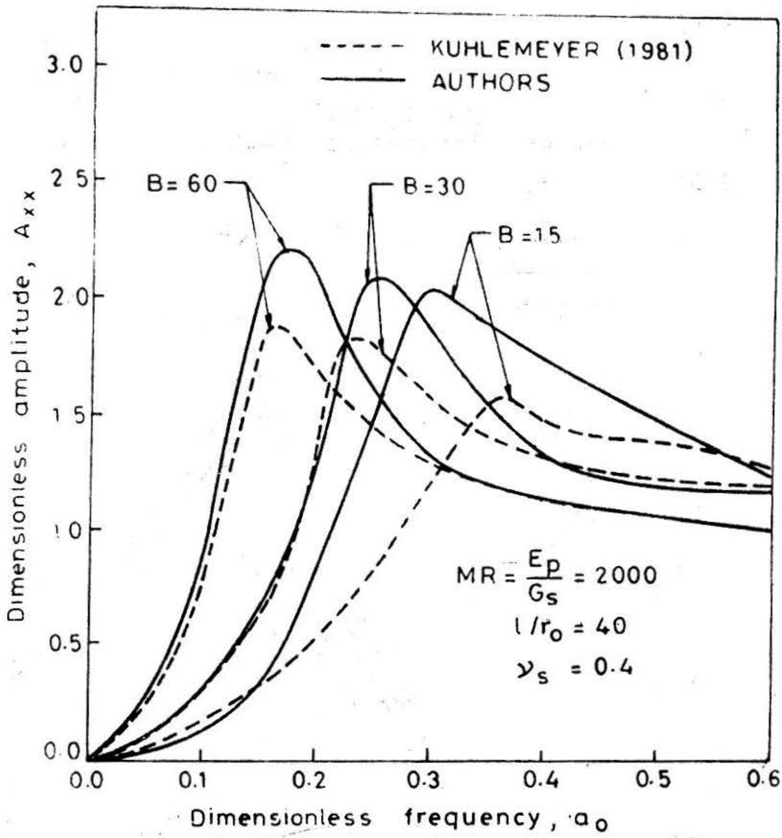

FIGURE 6 Comparison of theoretical solution with experimental results

Fig. 7, reports three vertical response curves of a footing (1260.4kg) supported by single black steel pipe pile (r_0=4.5cm, A=14.4cm^2 and l/r_0=50.6) *fully embedded in soil* (σ_s=1792kg/m^3, ν_s=0.25 and V_s=175m/sec). These vertical response curves were obtained by Novak and Grigg (1976) experimentally by varying the magnitude of exciting force and frequency. The dotted lines represent the analytical response curve by Saha (1985). The continuous line with circular points represent the authors response curve considering only vertical movement of pile shaft. It is observed that the agreement between experimental and analytical solution of Saha and authors' response is satisfactory at all frequency ranges, except near resonance. The theoretical vertical resonant amplitude is somewhat less than that obtained from the experiment. This may be due to damping produced by transverse movement. The analytical resonant amplitude obtained from author's solution is 82%, 76% and 88% of the corresponding experimental resonant amplitudes in case of curve I, curve II, and curve III respectively. The analytical

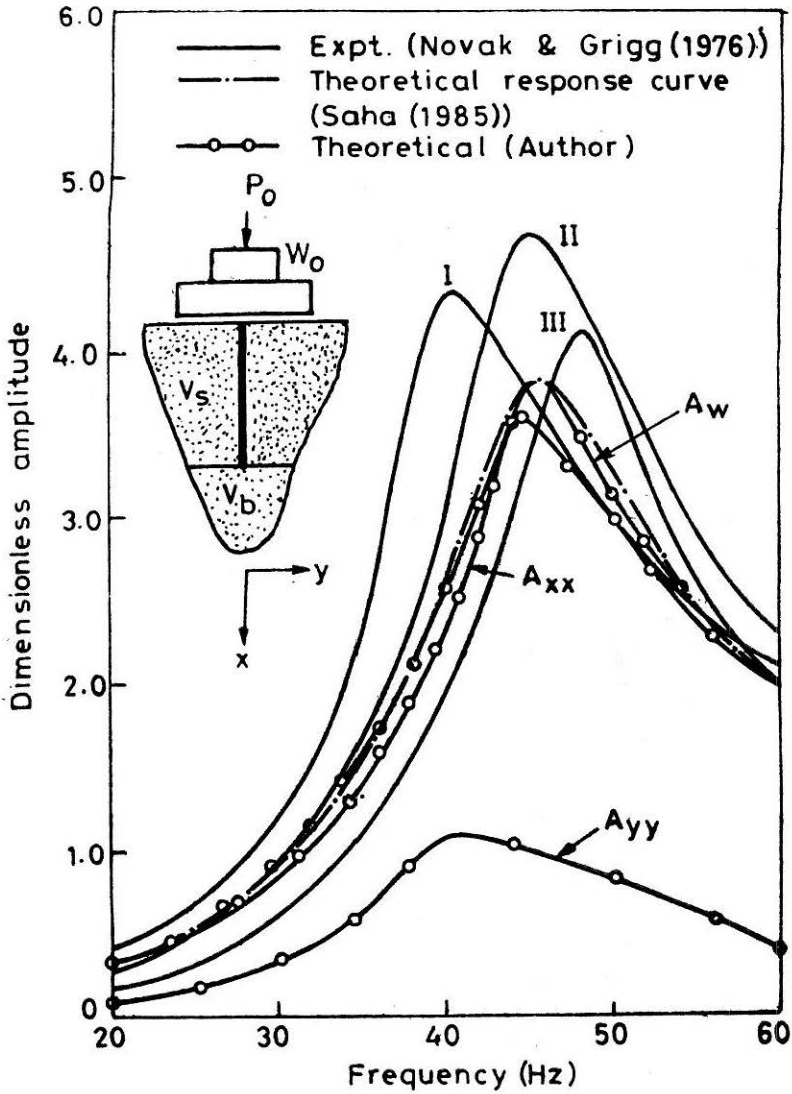

FIGURE 7 Comparison of experimental and theoretical response (measured along pile axis) curve for vertical vibration of pile

results obtained by the author is 92% of the value reported by Saha. The lateral amplitude frequency curve is also shown in Fig. 7. After superimposing the lateral amplitude, on the existing vertical amplitude, the resultant amplitude A_w is also plotted. The analytically obtained resultant amplitude is 98% of the value reported by Saha.

Figs. 8(*a*) and (*b*) show the variation of dimensionless amplitude A_{xx} and A_{yy} in vertical as well as lateral directions, respectively with frequency for typical cases for floating pile and Figs. 9(*a*) and (*b*) for end bearing pile. For floating pile, it is observed that as the slenderness ratio, l/r_0 increases the resonant dimensionless amplitude in both vertical and horizontal directions decrease significantly. With increase in wave velocity ratio, Vs/Vc the resonant amplitude for both the directions decrease significantly and they occur at lower frequency values. For example the resonant amplitude in vertical direction and frequency for $Vs/Vc=0.01$ for $l/r_0=60$ are 6.8 and 426 rad/sec and for $Vs/Vc=0.05$ are 2.19 and 61.89 rad/sec.

For end bearing pile, (Figs. 9(*a*) and (*b*)) it is observed that A_{xx} and A_{yy} vary with frequency randomly. Therefore it is very difficult to draw proper conclusions. However, it is also observed that the resonant amplitude is generally low in case of end bearing pile in comparison with floating pile.

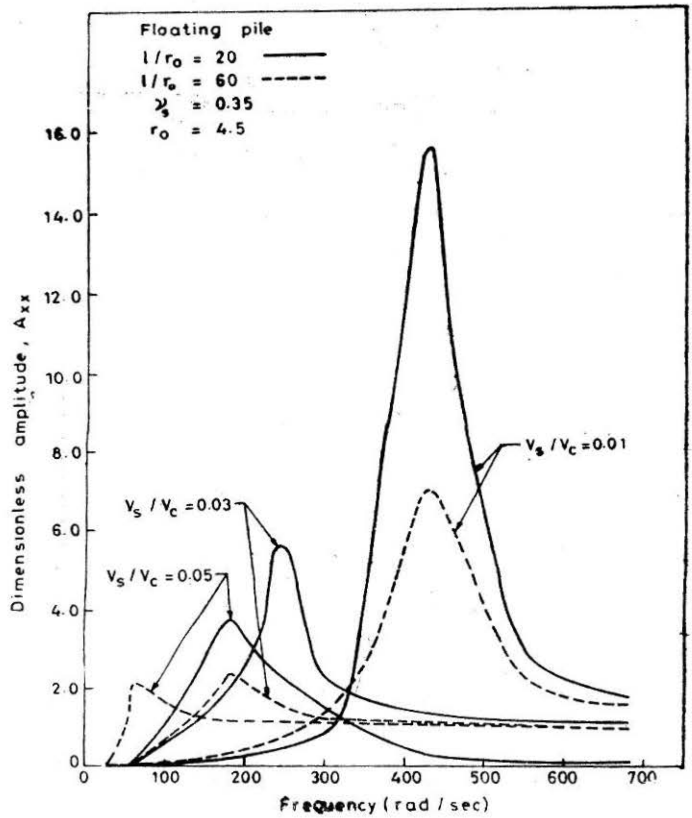

FIGURE 8 (a) Variation of A_{xx} with frequency for various wave velocity ratics

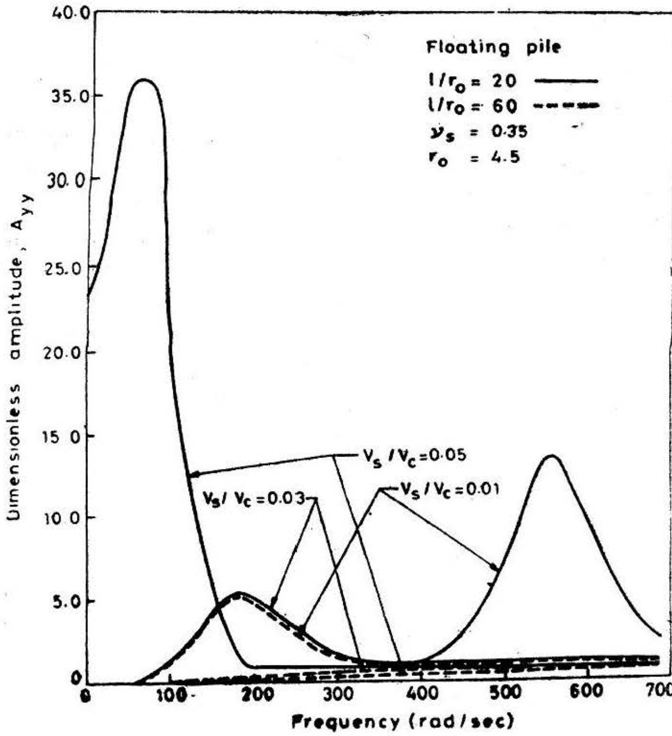

FIGURE 8 (b) Variation of A_{yy} with frequency for different wave velocity ratics

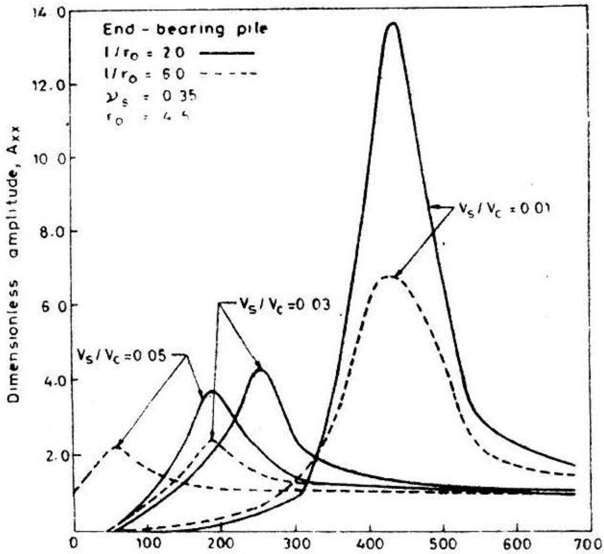

FIGURE 9 (a) Variation of A_{xx} with frequency for various wave velocity ratios

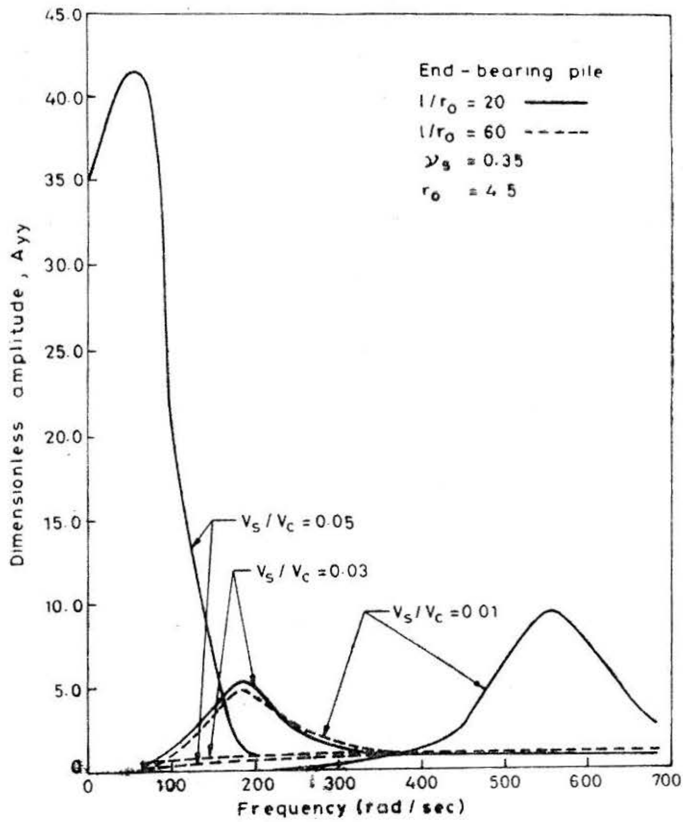

FIGURE 9 (b) Variation of A_{yy} with frequency for various wave velocity ratios

Conclusions

An analysis to predict the dynamic response of a pile under vertical harmonic motion has been presented here. It is capable of predicting the response of a curved as well as straight vertical pile, wherein both the longitudinal and flexural movement of a pile have been considered.

The half-space parameters C_{u1}, C_u^2 and C_{w1}, C_w^2 in horizontal and vertical modes of vibration respectively depend on dimensionless frequency, a_0 and Poisson's ratio of soil.

Stiffness functions f_{x1} and f_{y1} in vertical and horizontal directions are independent of pile tip conditions, but, they vary with the slenderness ratio of a pile and wave velocity ratio *i.e.*, ratio of shear wave velocity through soil and pile. The damping functions f_{x1} and f_{y2} in vertical as well as horizontal directions depend on pile tip conditions, slenderness ratio and wave velocity ratio.

The agreement between the author's results with the lumped mass solution given by Kuhlemeyer (1981) for vertical response of floating piles driven into a soil system that is approximated here as a homogeneous soil half-space is quite satisfactory. However, the author's estimated values of resonant amplitudes are consistently higher than those given by Kuhlemeyer.

The predicted lateral amplitude is considerably less than that in the longitudinal direction. Author's estimated value of the resonant amplitude in the vertical direction is less than that estimated by Saha (1985) and observed by Novak *et al* (1976). However, the predicted resultant amplitude, A_{uv}, is 98% of that obtained by Saha.

In general, the dimensionless amplitudes in vertical and horizontal directions depend on pile tip conditions, velocity ratio and slenderness ratio. For floating pile, as the slenderness ratio increases, the resonant dimensionless amplitude in both vertical and horizontal directions decrease significantly. As wave velocity ratio increases, the resonant amplitudes in both the directions decrease and occur at lower frequency values. In general, for end bearing pile the resonant amplitudes in both the directions are less than those noted for floating pile.

REFERENCES

ARNOLD, R.N., BYCRAFT, G.N. and WARBURTON, G.B. (1955) : "Forced vibration of a body on an infinite elastic solids", *Journal of Applied Mechanics*, ASME, Vol. 22, 3 : 391-400.

BARANOV, V.A. (1967), "On the calculation of excited vibrations of an embedded foundation". translated from Russian, Voprasy Dinamiki Prochnosti, 14 : Polytechnical Institute of Riga, 195-209.

FISCHER, F.J. (1975), "Driving analysis for initially curved marine conductors", Off-shore technology conference, OTC 2309 : 633-644.

GAZETAS, G. (1984) : "Seismic Response of End Bearing piles" *Int. Journal of Soil Dynamic and Earthquakes Engineering*, Vol. 3, No. 2, pp 82-93.

GHOSH, D.P. and SHARMA, B.B. (1992), "Vertical Vibration of Piles with Enlarged Bases". *Indian Geotechnical Journal*, Vol. 22, No. 2, pp 85-99.

HSIEH, T.K. (1962), "Foundation vibrations" *Proc. Inst. of Civil Engineers*, London, Vol. 22, 211-255.

ISACOVICH, M.A. and KOMAROVA, L.N. (1968), "Longitudinal—Flexural modes in a slender rod", Soviet Physics-Acoustics, No. 13, 491-494.

KUHLEMEYER, R.L. (1981), "Dynamic Response Curves for loaded Floating pile Foundation" *Canadian Geotech Journal*, Vol. 18, No. 2, pp 300-312.

MINDLIN, R.D. (1936), "Force at a point in the interior of a semi-infinite solid", *Journal of Physics*, Vol. 7, 195-202.

MIZUHATA, K. and KUSAKABE, K. (1984), "Comparison of Experimental and Analytical Results of Vibration of a Full Scale Pile". *Proc. 8th World Conf. on Earthquake Engg., Sanfransisco*. Vol. III, pp 633-640.

NOGAMI, T. (1986), "Time Domain Axial Response of Dynamically loaded Single Piles", *Journal of Engg. Mech.* ASCE, Vol. 112, No. 11, pp 1241-1252.

NOVAK, M. and BEREDUGO, Y.O. (1972), "Vertical Vibration of Embedded Footings", *Journal of SM & FE,* ASCE, Vol. 98, SM 2 : 1291-1309.

NOVAK, M. (1974), "Dynamic Stiffness and Damping of Piles" *Canadian Geotechcal Journal,* Vol. 11, 4: 574-598.

NOVAK, M. and GRIGG, R.F. (1976), "Dynamic experiments with small pile foundations", *Canadian Geotech. Journal,* Vol. 13, 372-385.

NOVAK, M. and ABOUL-ELLA, F. (1978), "Impedence Function for Piles Embedded in Layered Medium". *Journal of Engg. Mech.* ASCE, Vol. 104, No. EM-3, pp 643-661.

NOVAK, M. (1991), "Piles Under Dynamic loads". Proc. 2nd Int. Conf. on Recent Advances in Geotech. *Earthquake Engg. and Soil Dynamics,* Vol. III Univ. of Missouri-Rolla.

PIPPARD, A.J.S. and BAKER, S.J. (1968), "The Analysis of Engineering Structures", Edward Arnold (Publishers) Ltd. LONDON, Fourth Edition.

QUINLAN, P.M. (1953), "The elastic theory of soil dynamics", Symp. on dynamic testing of soils, ASTM, STP. No. 156, 3-34.

REISSNER, E. (1936), "Stationare axial symetrische durch eine Schuttelnde masse erregte Schwingungan eines homogonen elstichen Halbraumes", *Ingenieur Archir.*, Vol. 7, part-6 381-396.

SUNG, T.Y. (1953), "Vibrations is semi-infinite solids due to periodic surface loading", *Symp. on Dynamic Testing of Solids ASTM,* STP No. 156: 35-63.

SAHA, S. (1985), "A Study on the dynamic behaviour of pile foundations", Ph. D. *Thesis submitted to Department of Civil Engineering,* I.I.T., Kharagpur, India.

SALVADORI, M.G. and BARON, M.L. (1962), "Numerical methods in Engineering", *Prentice-Hall,* Inc. Second Edition.

WOLF, P.J. and VON ARX, G.A. (1978), "Impedence Function of a Group of Vertical Piles". *Proc. Specially Conf. of Geotech. Engg.* ASCE, Pasadena, pp 1024-41.