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Vertical Vibration of Piles with Enlarged Bases

by

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Introduction

With rapid industrialization and due to scarcity of suitable land, Engineers are called upon to construct heavy structures and plants on sites having poor soil conditions. Generally, piles are used as foundations to support heavy structures at poor site condition. During last two decades under-reamed piles are increasingly used for foundations of structures built on expansive and other type of soil. These piles are often exposed to dynamic loads generated by operating machines, wind and earthquake.

In recent years significant progress has been made in the dynamic analysis of vertical piles under different modes of vibration. The earlier methods of analysis (Hayashi, *et.al.* 1965, Prakash and Sharma, 1968) were based on the concept of equivalent cantilever that does not take into account of energy dissipation or number of other factors. Tajimi (1966) assumed a visco-elastic stratum of Kelvin-voigt type to model the soil and account for damping and propagation of elastic waves. Penzien (1990) and Kuhlemeyer (1981) presented lumped mass solution for piles under dynamic loads. In more recent approaches (Novak, 1974, 77), Nogami & Novak (1976), Novak and Aboul-Ella (1978) considered the soil as a continuum and accounts for dynamic soil-pile interaction, energy dissipation through propagation of elastic works. Novak and Sheta (1980), Mizuhata and Kusakabe (1984) have introduced the concept of weakened zones around the piles to explain the complex dynamic soil-pile interaction phenomena. Blaney *et.al.* (1976), Wolf & Von Arx (1978), Kuhlemeyor (1979) and others have utilised finite element technique to discretise soil around the pile and a consistant boundary matrix to stimulate the effect of the far field. Nogami (1986) presented a procedure for time domain analysis of axial response of

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single piles where a time domain soil-pile interaction force is formulated through a simple mechanical idealization of the soil medium developed from the dynamic behaviour of a plane strain continuous elastic medium.

Though several methods of analysis are available for the dynamic analysis of pile foundation only a few literature are available about the dynamic analysis of piles with enlarged tips, (Saha & Ghosh, 1985). To the best of authors' knowledge no experimental investigation has been reported so far on the vertical vibration of piles with enlarged base. In the present paper, the vertical response of footings supported on model piles with enlarged base have been reported. The observed response are compared with theoretical analysis based on the extension of Novak's (1977) approach to the vertical vibration of piles with enlarged base.

Experimental Programme

An attempt was made to measure experimentally, the vertical amplitudes of vibration of a model footing supported by model enlarged based piles of different lengths and different enlarged base radius (r_b). The model enlarged based pile was made up of hollow pipe pile of aluminium alloy ALCOA-6061-T6 (Outside radius 'r_o' and inside radius 'r_i' of 0.908 cm and 0.862 cm respectively) having different lengths of 0.29 m, 0.55 m and 0.81 m. The enlarged base was modeled by aluminium disk of varying radii machined from 0.5 cm thick plate which were fixed tightly at the bottom of the piles. The ratio of radius of the disk and the outside radius of pile (r_b/r_o) used were 1.5, 2.0, 2.5, 3.0 and 3.5. The model enlarged based piles were embedded fully in a properly prepared standard Ennore sand bed (uniformity coefficient = 1.1, specific gravity = 2.67, mass density ρ_s = 1609 Kg/m^3, relative density = 0.72, $\phi = 40.5^\circ$ and velocity of shear wave = 100 m/s) in a wooden tank (70 cm $\times$ 70 cm $\times$ 110 cm deep). 2 mm thick bituminous pad was pasted inside the wooden tank to minimize wave reflection from the boundary. The model footing (made up on two thick aluminium circular disk of mass 1.45 Kg) was rigidly fixed to the top of model enlarged based pile. A clear gap of 1 cm was maintained at the bottom of the footing to avoid contact with the sand surface. The footing was excited vertically with a constant force (14 N) Phillips excite (PR 9270/01) with amplifier (GM 5535) and frequency generator (AF 1011). The exciter was properly supported from a steel frame (Fig. 2) exactly at the centre of the model footing with adjustable mounting devices for possible movement of the exciter upwards and downwards and also side ways. The amplitudes of vertical vibration of footing with increasing frequency of excitation were measured with the help of a sensor (SI 100) and a vibration indicator (SI 10M).

Results of Model Tests

Some typical response curves of model enlarged based piles having

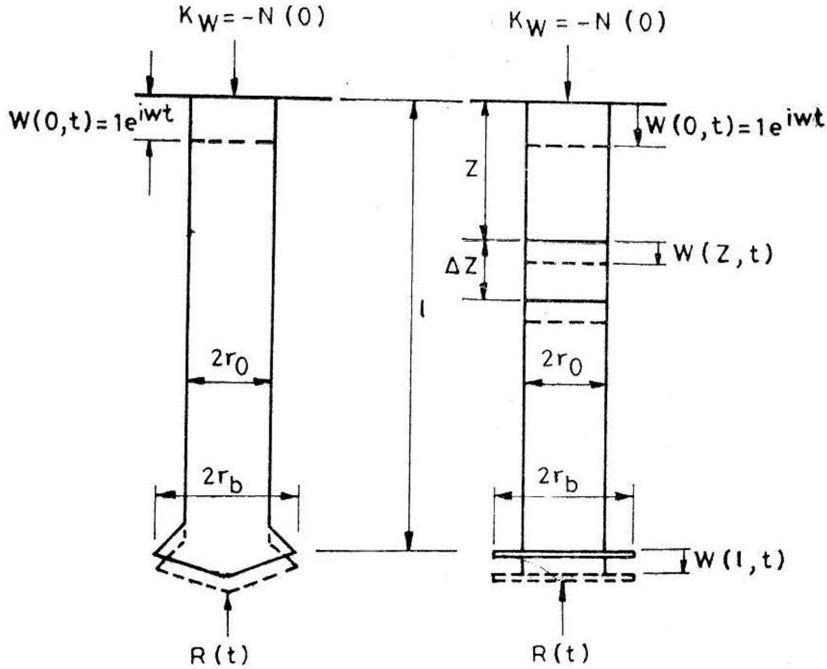


FIGURE 1 Embedded enlarged based pile under vertical vibration (a) Real pile (b) Idealised pile

r_b/r_o ratio 1, 2 and 3 are plotted in Fig. 3(*a*) & (*b*) for piles of length 29 cm and 55 cm respectively. These curves indicate that the effect of the radius of enlarged base on the response of footing becomes negligible as the length of the pile (*i.e.* slenderness ratio l/r_o) increases. Fig. 4 reveals that the resonant frequency of the footing supported on model enlarged based pile increases with the increase in base enlargement ratio but this effect decreases with the increase in l/r_o ratio. The resonant amplitudes decreases with the increase in r_b/r_o ratio. But at higher slenderness ratio of pile, the effect of base enlargement on the magnitude of resonant amplitude becomes negligible (Fig. 5).

Theoretical Analysis

Novak (1977) has given an approximate analysis of vertical vibration of floating piles. In this paper, Novak's approximate method of analysis has been extended to piles with enlarged base subjected to vertical vibration. For the present analysis, such type of pile is idealised as a pile of uniform shaft radius 'r_o' length 'l' having a weightless rigid disk of radius 'r_b' (*i.e.* the radius of enlarged base) fixed at the pile tip (Fig. 1).

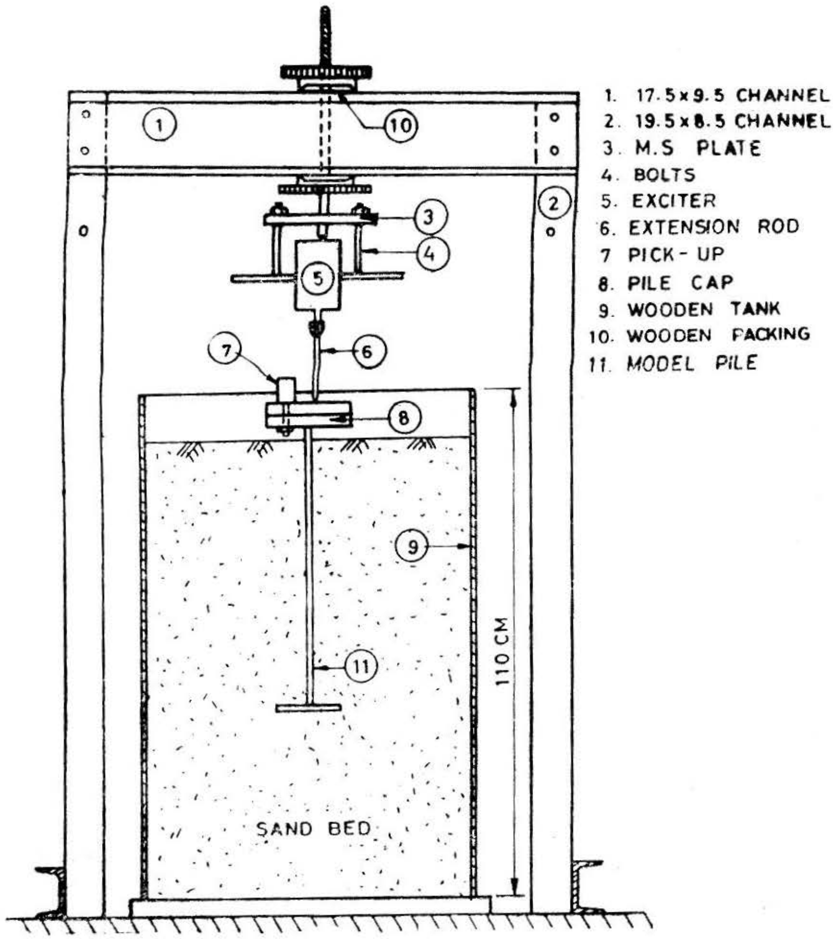


FIGURE 2 Experimental Set-up

The assumptions made are:

(*i*) Pile is vertical, elastic and circular in cross section and has perfect contact with the soil.

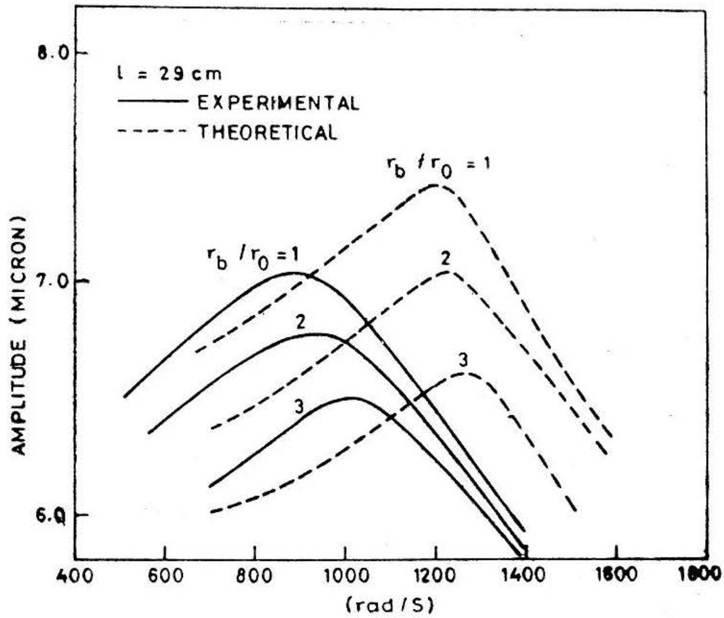
(*ii*) The soil above the pile tip is modeled as linearly elastic layer composed of infinitesimally thin independent layers.

(*iii*) The soil reaction acting at the pile tip is assumed to be equal to that of an elastic half-space.

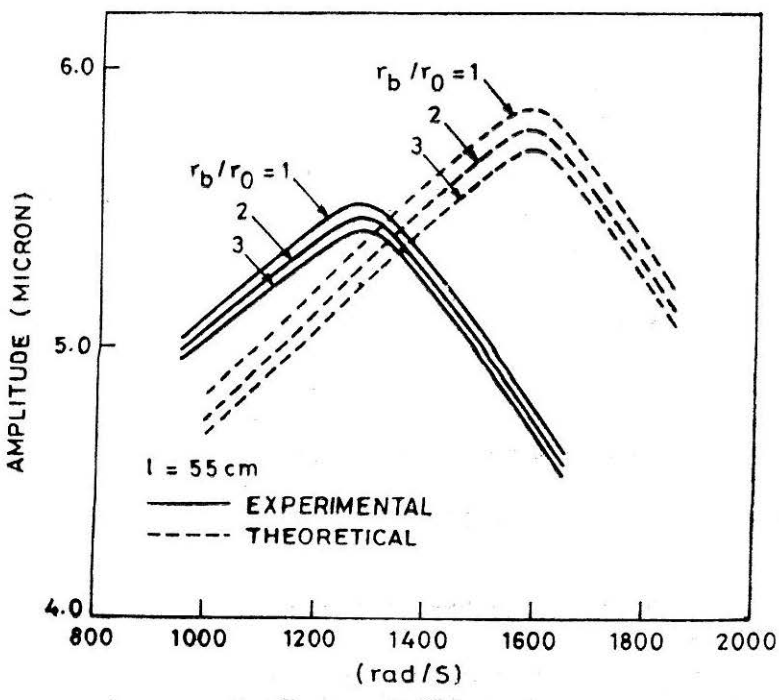
(*iv*) The motion is assumed to be small and excitation is harmonic.

Assuming that the enlarged based pile is under going complex vertical vibration *i.e.*

$$W(z,t) = W(z)e^{i\omega t} \tag{1}$$



(*a*)



(*b*)

FIGURE 3 Comparison of theoretical and experimental response curves of piles with enlarged base under vertical vibration

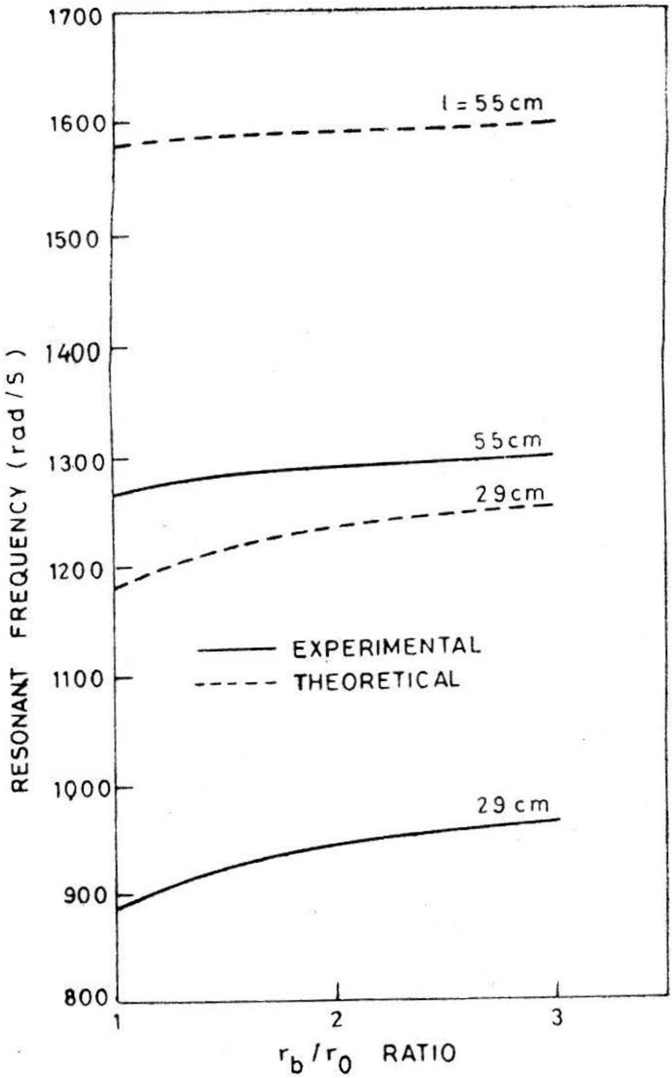


FIGURE 4 Variation of resonant frequencies with base enlargement (r_b/r_o) ratio

in which $W(z)$ = complex amplitude at depth z, $i = \sqrt{-1}$ and ω=circular frequency and t = time.

The differential equation of damped axial vibration of pile (Novak, 1977) is given as:

$$\mu \frac{\partial^2 W(z,t)}{\partial t^2} + c \frac{\partial W(z,t)}{\partial t} - EA \frac{\partial^2 W(z,t)}{\partial z^2} + P(z,t)\, dz = 0 \qquad (2)$$

Where μ = mass of pile per unit length

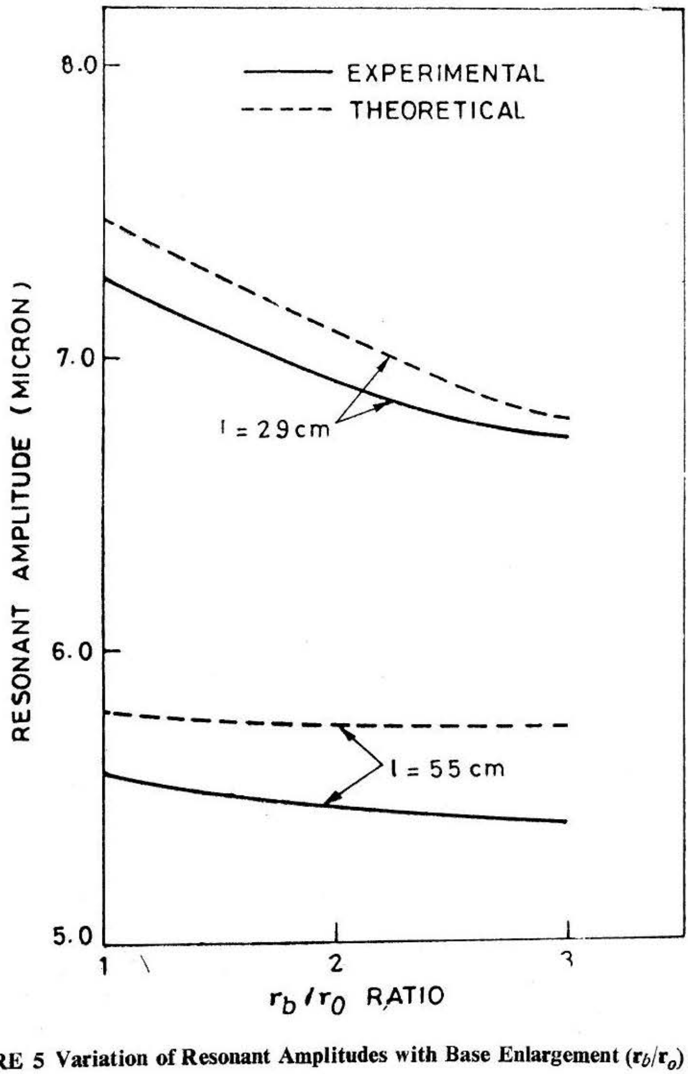


FIGURE 5 Variation of Resonant Amplitudes with Base Enlargement (r_b/r_o) ratio

c = coefficient of pile material damping

E = Young's modulus of pile material

A = cross sectional area of pile

and $P(z,t)$ = the distributed soil reaction acting on pile element dz at depth z

$$= G(S_{w1} + iS_{w2})\, W(z,t) \tag{3}$$

where G = shear modulus of soil surrounding the pile.

S_{w1} & S_{w2} = Real and imaginary parts of the side soil reation $P(z, t)$ and are function of dimentionless frequency

$a_o = r_o w / V_s$

V_s = shear wave velocity in surrounding soil.

With harmonic motion described by Eq. (1), Eq. (2) reduces to an ordinary differential equation:

$$W(z)\,[-\mu\omega^2 + ic\omega + G(S_{w1} + iS_{w2})] - E_p A \frac{d^2 w(z)}{dz^2} = 0 \tag{4}$$

The solution of eq. (4) is :

$$W(z) = B \cos \wedge \frac{z}{1} + C \sin \wedge \frac{z}{1} \tag{5}$$

in which B and C = integration constants whose value depend on boundary conditions.

$\wedge$ = complex frequency parameter

$$= 1 \sqrt{\frac{1}{E_p A} [\mu\omega^2 - GS_{w1} - i(c + GS_{w2})]} \tag{6}$$

The complex parameter can be more conveniently written as (Novak, 1977) :

$$\wedge = \wedge_1 + i\wedge_2 \tag{7}$$

Boundary Conditions

At the head of the pile, harmonic motion with a unit amplitude is assumed *i.e.* $W(0,t) = 1 \exp(i\omega t)$, since this form of excitation defines the stiffness and damping of the soil pile system at the pile head. Hence the first boundary condition is, at $z = 0$,

$$W(0) = 1 \tag{8}$$

At the pile tip (*i.e.* at $z = 1$) the motion of the pile generates concentrated half space soil reaction $R(t) = R \exp(i\omega t)$, the amplitude of which is

$$R = -G_b\, r_b\, (C_{w1} + C_{w2})\, w(1) \tag{9}$$

in which

G_b = shear modulus of soil below the pile tip

$W(i)$ = complex amplitude of the pile tip

Cw_1 & Cw_2 = dimentionless half space parameters depending on the demensionless frequency ($a_o = r_b\, \omega / V_b$) and the poisson's ratio).

The end force of the pile $\left(N(1) = E_p A \dfrac{dw(1)}{dz} \right)$ is equal to the soil reaction given by Eq. (9). Hence the boundary condition for the tip $z=1$ is

$$E_p A \frac{\wedge}{1} (-B \sin \wedge + C \cos \wedge)$$

$$= - G_b\, r_b\, (Cw_1 + i\, Cw_2)\, (B \cos \wedge + C \sin \wedge) \tag{10}$$

The first boundary condition yields:

$$B = 1 \tag{11}$$

The second boundary condition yields:

$$C(\wedge) = \frac{K^1 \wedge \sin \wedge + (Cw_1 + i\, Cw_2) \cos \wedge}{K^1 \wedge \cos \wedge + (Cw_1 + i\, Cw_2) \sin \wedge} \tag{12}$$

$$\text{in which } K^1 = \frac{E_p A}{G_b^1\; r_b} \tag{13}$$

The integration constant C is complex and can be expressed in the form

$$C(\wedge) = C(\wedge)_1 + C(\wedge)_2 \tag{14}$$

Where $C(\wedge)_1$ and $C(\wedge)_2$ are the real and imaginary part of the integration constant C.

With the integration constants established, the amplitude of pile displacement is

$$W(z) = \cos \wedge \frac{z}{1} + C(\wedge) \sin \wedge \frac{z}{1} = W_1 + iW_2 \tag{15}$$

Where W_1 and W_2 are the real and imaginary part of complex displacement $W(z)$

$$W_1 = \cos(\wedge_1 \frac{z}{1}) \cosh(\wedge_2 \frac{z}{1}) + C(\wedge)_1 \sin(\wedge_1 \frac{z}{1}) .$$

$$\cosh(\wedge_2 \frac{z}{1}) - C(\wedge)_2 \cos(\wedge_1 \frac{z}{1}) \sinh(\wedge_2 \frac{z}{1}) \tag{16}$$

$$W_2 = C(\wedge)_1 \cos(\wedge_1 \frac{z}{1}) \sinh(\wedge_2 \frac{z}{1}) + C(\wedge)_2 .$$

$$\sin(\wedge_1 \frac{z}{1}) \cosh(\wedge_2 \frac{z}{1}) - \sin(\wedge_1 \frac{z}{1}) \sinh(\wedge_2 \frac{z}{1}) \tag{17}$$

The real amplitude of the motion is:

$$| W(z) | = \sqrt{W_1^2 + W_2^2} \tag{18}$$

and the phase angle is

$$\phi(z) = \tan^{-1}\left(\frac{W_2}{W_1}\right) \tag{19}$$

Stiffness and Damping Constants

The complex stiffness is equal to that force which produces unit dynamic displacement at the pile head at certain frequency.

That is $K_w = -N(0)$ in which $N(z) = EpA\ dw(z)/dz$. Differentiating Eq (15) and substituting $Z = 0$ yields the complex stiffness

$$K_w = \frac{E_pA}{1} F_w(\wedge) \tag{20}$$

in which $F_w(\wedge) = -\wedge C(\wedge) = F_w(\wedge)_1 + iF_w(\wedge)_2$ (21)

Subscript 1 denotes the real part of Fw which defines the stiffness and subscript 2 indicates the imaginary (out of phase) part of Fw which relates to damping.

The stiffness constant of one pile can be re-written as:

$$K_w^1 = \frac{E_pA}{r_o} f_{w1} \tag{22}$$

in which $f_{w1} = F_w(\wedge)_1/(1/r_o)$ (23)

The equivalent viscous damping constant can be expressed as:

$$C_w^1 = \frac{E_pA}{v_s} f_{w2} \tag{24}$$

in which $f_{w2} = F_w(\wedge)_2/\left(\frac{a_o 1}{r_o}\right)$ (25)

Here f_{w1} and f_{w2} are the dimensionless parameters characterizing stiffness and damping constants respectively.

Vertical Response of Footing

With the stiffness and damping constants determined using Eq. (22) and Eq (24), the vertical amplitude of a footing supported on the under-reamed pile can be obtained as that of a shallow footing. Hence, the amplitude of vertical displacement Wo of footing may be expressed as

$$W_o = \frac{P_o}{\sqrt{(K^1_w - M\omega^2)^2 + (C^1_w \omega)^2}} \tag{26}$$

Where M = mass of footing

P_o = Maximum amplitude of exciting force.

Comparison with Experimental Results

The theoretical response of footing (mass 1.45 Kg) supported on model enlarged based piles of varying lengths and base enlargement ratio (r_b/r_o) are shown in Fig. 3(*a*) & (*b*). The general nature of theoretical and experimental response curves are quite similar. However theoretical analysis predicts somewhat higher resonant amplitudes and frequencies. This difference between the observed and theoretical resonant frequencies and amplitudes may be due to the small size of model piles used in the test and

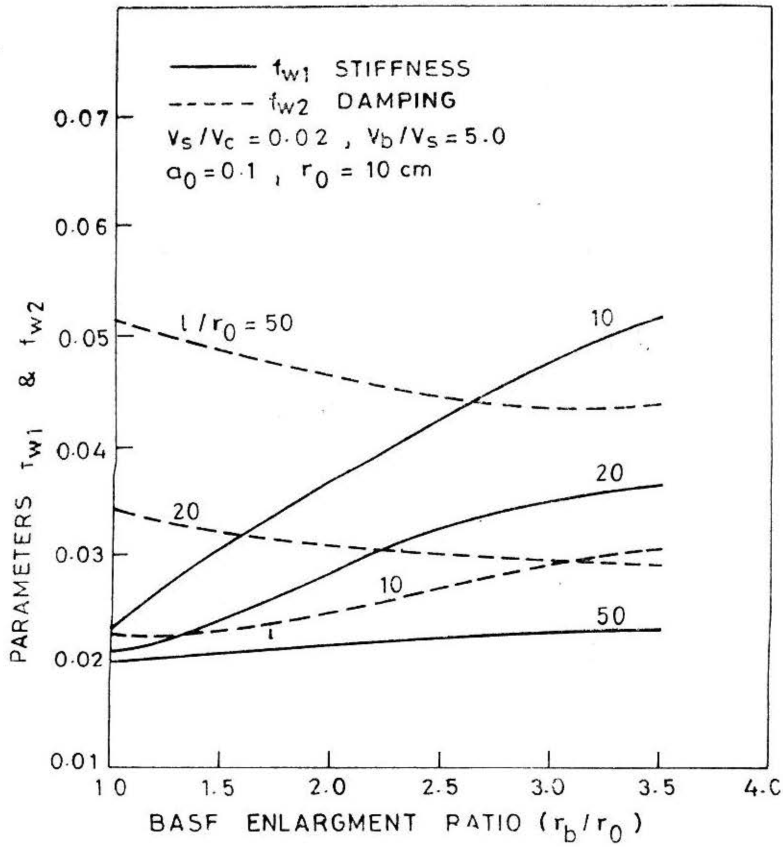


FIGURE 6 Variations of stiffness and damping parameters of piles with base enlargement ratio (r_b/r_o)

may also be due to the reflection of waves from the walls of the test tank. However the nature of theoretical variations of resonant amplitudes and frequencies with base enlargement ratio (r_b/r_o) are quite with those from the experiments (Fig 4 and Fig 5).

Theoretical Results & Discussion

The effect of base enlargement ratio (r_b/r_o) of concrete pile of shaft radius 10 cms on stiffness and damping parameters fw_1 and fw_2 are shown in Fig. 6. Generally the stiffness parameter fw_1 increases with the increase in ratio where as the damping parameter fw_2 decreases with the increase in base enlargement ratio except for piles with small slenderness ratio.

The magnitude of stiffness and damping of a pile with enlarged base at a given frequency depends on the slenderness ratio and r_b/r_o ratio. The variation of stiffness and damping parameters with slenderness ratio are shown in Fig. (7). It is observed that at a given frequency both stiffness

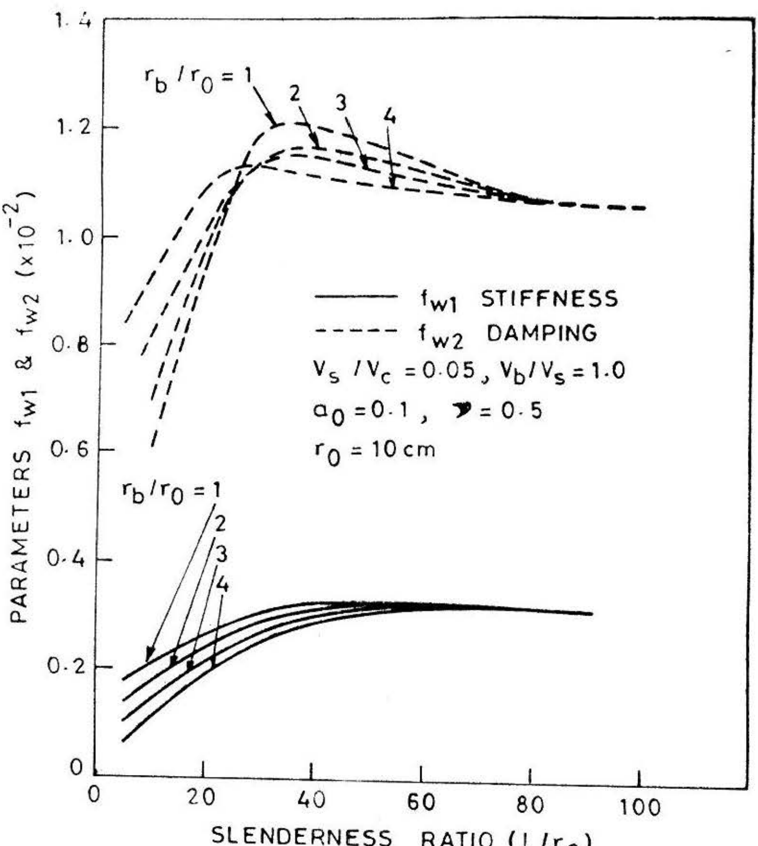


FIGURE 7 Variations of stiffness and damping parameters of piles with enlarged base with slenderness ratio

and damping parameter increase with slenderness ratio and gradually attain constant value at $l/r_o > 80$. It is further observed that both stiffness and damping parameter increase with base enlargement ratio for a given slenderness ratio. But the difference in the magnitude of parameters due to r_b/r_o ratio gradually vanishes at higher slenderness ratio ($l/r_o > 80$).

With the decrease of relaxation of pile tip (*i.e.* with the increase in V_b/V_s ratio) the stiffness parameter increases but the damping parameter decreases (Fig. 8). But both the parameters fw_1 and fw_2 attain steady values at $V_b/V_s > 50$. At any V_b/V_s ratio less than 50, both stiffness and damping parameters increase with base enlargement ratio. This effect of base

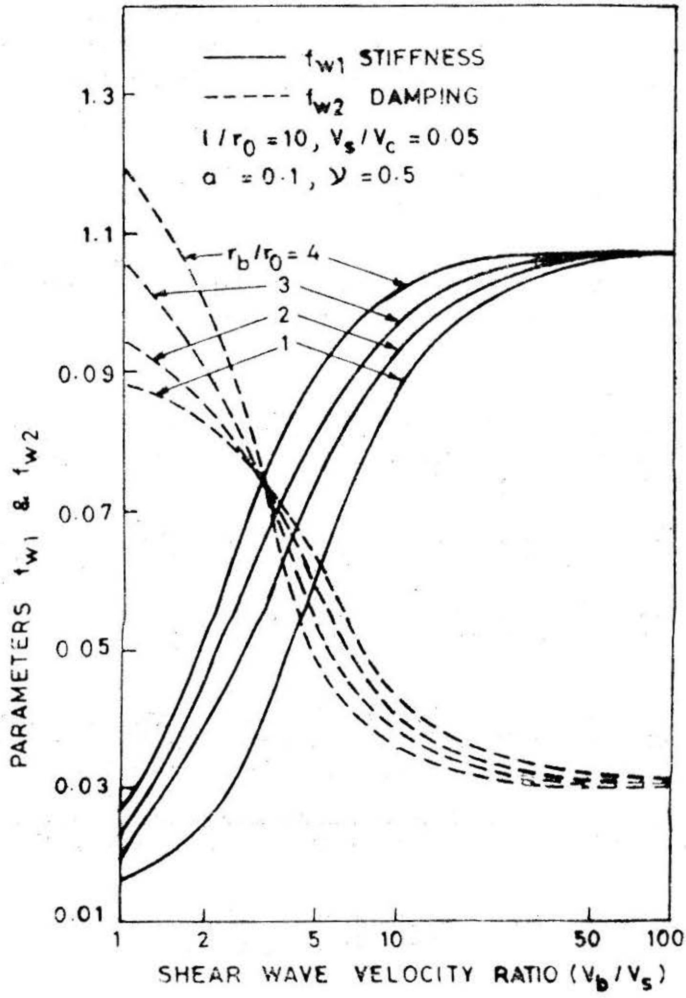


FIGURE 8 Variations of stiffness and damping parameters of piles with enlarged base with shear wave velocity ratio (V_b/V_s)

enlargement vanishes when the pile tends to become and baring (*i.e.* when $V_b/V_s > 50$).

Conclusions

From the present experimental and theoretical investigation on the dynamic behaviour of piles with enlarged base, the following conclusions may be tentatively drawn:

1. Novak's (1977) approximate method of analysis can be extended for predicting dynamic response of footing resting on piles with enlarged base.
2. The resonant frequency increases but the resonant amplitude decreases with the increase in base enlargement ratio. This effect is more pronounced for piles having smaller slenderness ratio.
3. Both stiffness and damping parameters of floating pile with enlarged base increase with the increase in base enlargement ratio. But this effect vanishes for piles with higher slenderness ratio.
4. The magnitudes of stiffness and damping parameters of an enlarged based pile increase with slenderness ratio but attains a constant value of higher slenderness ratio.

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