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Stress-Strain-Time Behaviour of Soils

by

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Introduction

Deformation and Shear failure of soils involves time dependnet re-arrangement of matter. Considerable attention has been directed in recent years to the study of stress-strain-time effects in soils such as creep, stress relaxation, the effects of rate of strain on shearing strength, and long term strength of soils [Push (1977), Silva et. al. (1983), Vaid et. al. (1977), Briand et. al. (1985), Singh et al. (1968)]. Attempts have also been made to characterize the creep and stress relaxation behavior of clays using rheological models composed of linear springs in combination with linear or non-linear dashpots [Komamura et. al. (1974), Abdel-Hady, M. et. al. (1966), Christensen et. al. (1964), Murayama, s. et. al. (1958)]. It is clear from these studies that the time-dependent responses of soil may assume a variety of forms depending on factors like soil type, soil structure, stress history, drainage conditions, type of loading, and other factors.

The basis of rate process theory is that particles participating in a time-dependent flow or deformation process (flow units) are constrained from movement relative to each other by virtue of energy barriers separating adjacent equilibrium positions as shown in fig : 1 (Mitchell et. al. (1976)]. The displacement of flow units to new positions requires the introduction of an activation energy, ΔE, of sufficient magnitude to surmount the barrier. The potential energy of a flow may be the same following the activation process or higher or lower than it was initially. The probability of any energy state equal to or greater than ΔE, denoted by $p(\Delta E)$ is given by;

$$P(\Delta E) = (\text{const})\ \text{expt}\ (-\Delta E/KT) \tag{1}$$

where K = Boltzmann's constant and T= Absolute temperature.
The Activation frequency (v), can be expressed as ;

$$v = (KT/h)\ \text{expt}(-\Delta E/KT) \tag{2}$$

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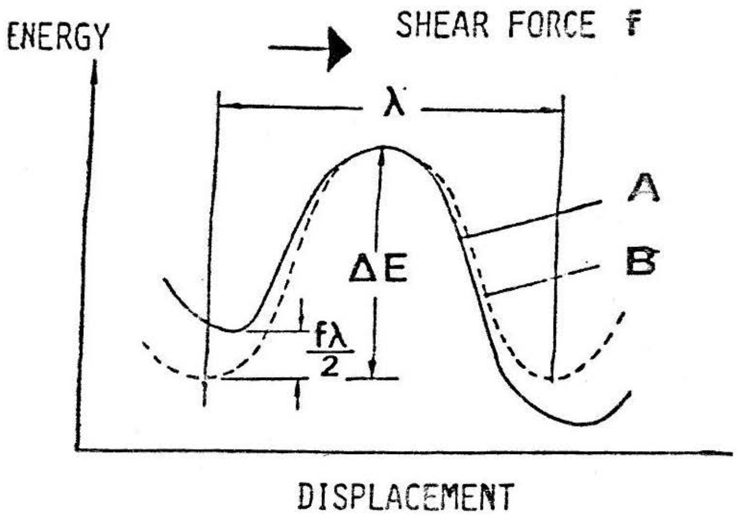


FIGURE 1 Influence of shear force on energy barriers opposing particle movement; A-force acting; B-no force acting

where h = Planck's constant.

Now, if a shear force (F) is applied as shown in fig : 1; the net activation frequency for movement (n) can be expressed as;

$$n = \overrightarrow{\nu} - \overleftarrow{\nu} = (2KT/h) \ \text{expt} \ (-\Delta E/RT) \ \sinh (F\lambda/2KT) \tag{3}$$

where R = Universal gas constant.

The idealized microstructure of a clay element under uniaxial stress conditions is illustrated in fig : 2. Now the strain-rate ($\dot{\varepsilon}$) equation can be expressed as;

$$\dot{\varepsilon} = s\,\lambda'\,n = 2\,s\,\lambda'\,(KT/h)\exp\,(-\Delta E/RT)\sin h\,(F\,\lambda/2KT) \tag{4}$$

where s = number of bonds per unit length along a chain of particles in the axial direction.

and λ' = component of particle displacement in the axial direction.

Equations 1-4 give the basics of rate process theory. More of the mathematical details of this is dealt with in Push (1977), Mitchell (1976).

The rate-dependent properties in clays are affected mainly by 3 elements : (1) The pore water: the higher the pore water content, the higher will be the viscosity of the clay. (2) The particle contacts: deeper the penetration of the mineral particle contacts into the adsorbed layers, the higher

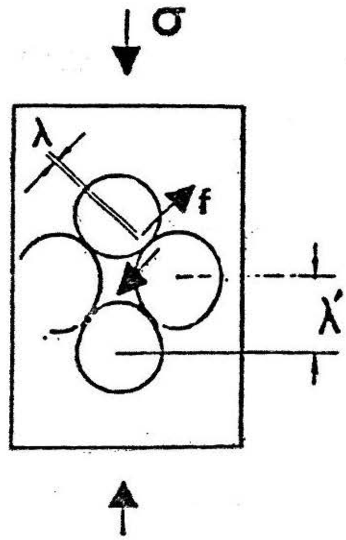


FIGURE 2 Idealized microstructure of clay element under uniaxial stress conditions

will be the viscosity of the clay. (3) Water/soil-skeleton interaction: at higher shearing rates, the particles in the soil skeleton do not have the time to find the path of least resistance and therefore exhibits higher viscosity accompanied by increased dilatency or decreased contractancy [Briand, J.L. et. al. (1985), Grawford (1959), Bjerrum et. al. (1958),].

SOIL DEFORMATION AS A RATE PROCESS

It was observed by Singh and Mitchell (1965) that for most soils (sand, clay-dry, wet, normally consolidated and over consolidated), the logarithm of strain-rate has an approximately linear relation with the logarithm of time as shown in fig: 3. Note that the slope of the curve is independent of the deviator stress. It was also observed that when the failure stage is reached at a given deviator stress level; the curve will show a reversal of slope as shown in fig: 4. The strain-rate equation (eqn: 4.) discussed earlier in this paper has been closely studied by Mitchell et. al. and concluded that soil deformation is a thermally activated process. Temperature effects on the shearing resistance of soils is dealt with later on in this paper. The variation of strain-rate with deviator stress as a function of time for a typical creep test Mitchell et. al. (1969) is shown in Figs: 5 (a) & (b). Note the variation of the curves for the drained and undrained tests.

The deformation behavior of deep sea clays is generally greater than terrestrial clays under similar loading conditions. Also the deep sea clays were found to display significant deformation under stress levels as low as the depositional nature and the marine clay structure. Recent experimental work done at the University of Rhode Island/Marine Geomechanics

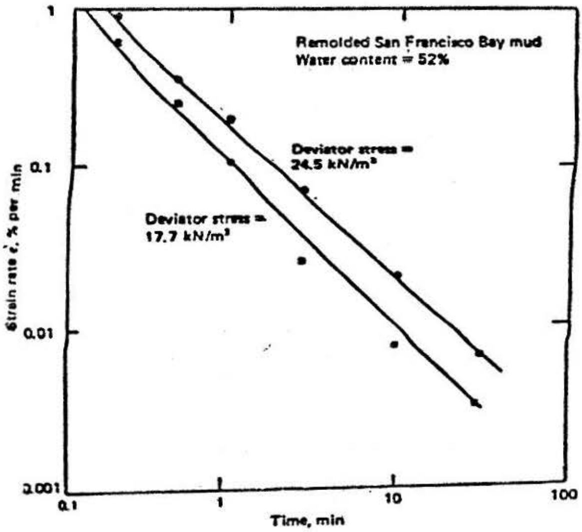


FIGURE 3 Time dependent behaviour of Strain rate during undrained Creep of remolded San Fransisco bay mud

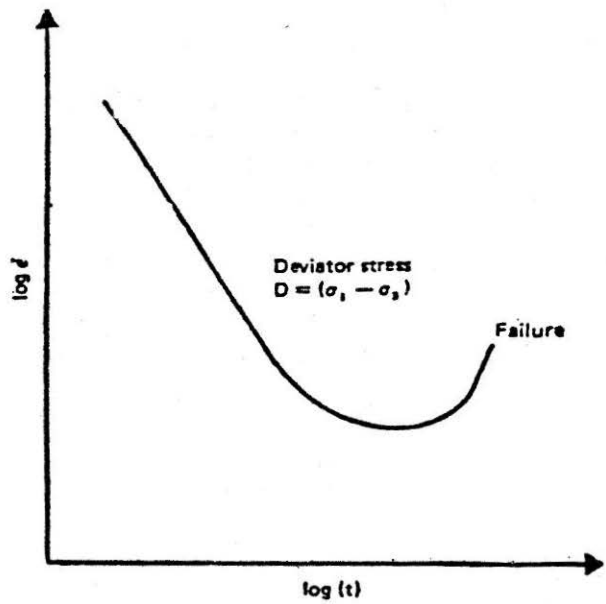
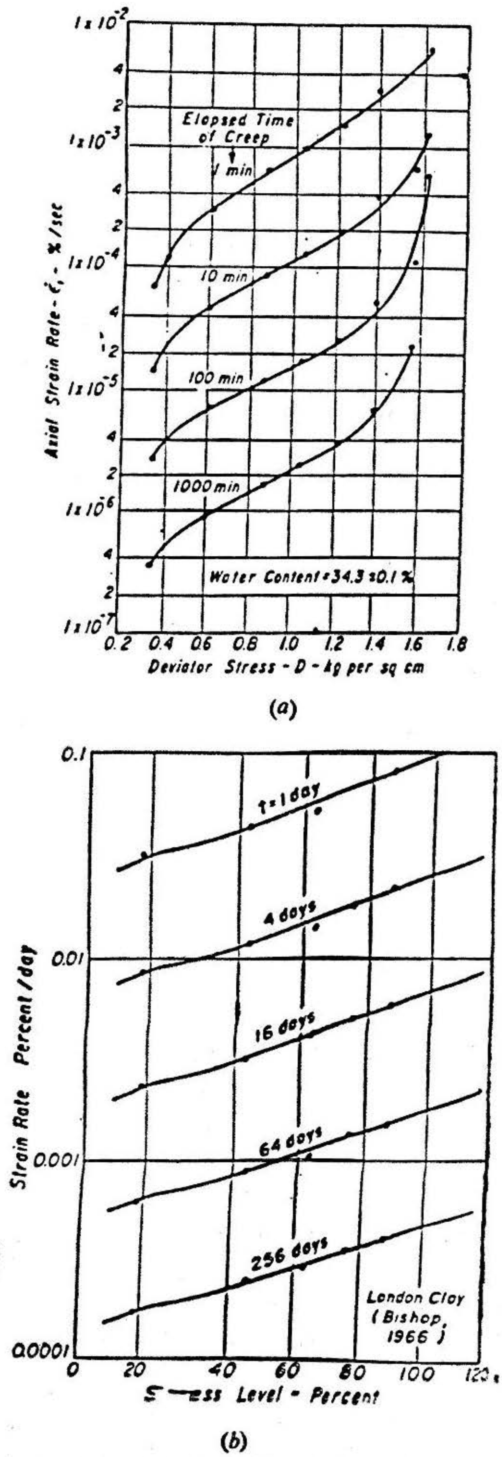


FIGURE 4 Variation of loge vs log t Showing the Reversal of Curve at Failure Stage for Larger Strains



(a)

(b)

FIGURE 5 Stress-Strain Time behaviour for : (a) Undrained Creep of Remolded illite; and (b) Drainage Creep of London Clay

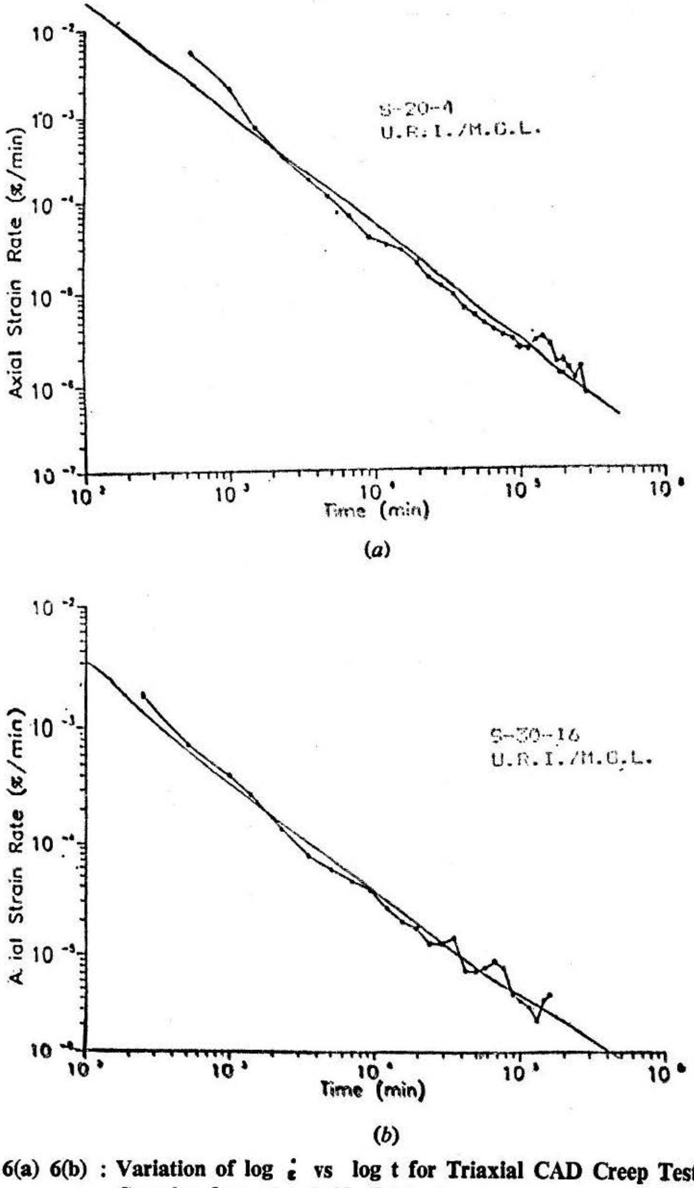


FIGURE 6(a) 6(b) : Variation of log $\dot{\epsilon}$ vs log t for Triaxial CAD Creep Tests on two Samples from the Gulf of Mexico area.

Laboratory (U.R.I./M.G.L.) also confirm to the linear behavior of the logarithmic plot of strain rate and time. Figs: 6(a) & (b) show the typical plots of two deep sea samples from the Gulf of Mexico currently being tested at the U.R.I./M.G.L. The geotechnical properties of the samples and test conditions are given in Table 1. Based on these curves a relationship of strain rate (E) and time (t) can be expressed as;

$$\dot{\epsilon} = (\text{const.})\ t^{-(const)} \tag{5}$$

Note that the tests are still in progress and that the above mentioned samples have not yet failed. Further research area of interest would be checking the validity of the logarithmic regression used in the above curves.

Although the strain-rate time relations for most clays may be represented by a simple power law [Singh and Mitchell (1965)], some undisturbed clays were found to behave as shown in fig: 7 under sustained loading. The limited instabilities which terminate the periods of steadily decreasing axial strain reflects a fundamental modification in soil structure [Bishop and Lowenbury (1969), Lo, K.Y. (1961)].

SHEARING RESISTANCE AS A RATE PROCESS

The theory of rate processes is very useful to relate the shearing resistance of soils to frictional and cohesive properties, effective soil structure, rate of strain and temperature. Several studies have shown that the undrained shear strength measured in the laboratory during the conventional shear tests increases with increase in speed of testing, particularly for clays

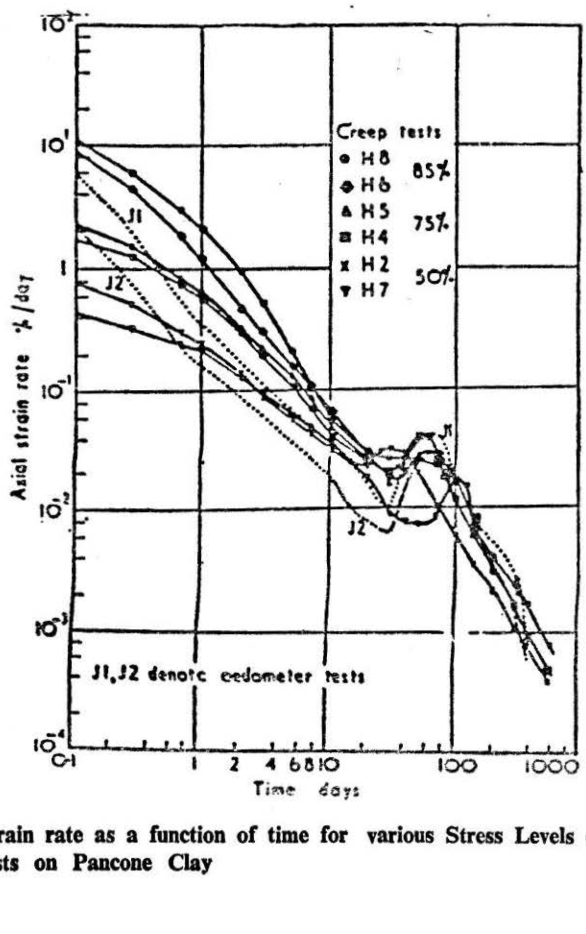


FIGURE 7 Strain rate as a function of time for various Stress Levels during drained tests on Pancone Clay

with high plasticity index [Alberro and Santoyo (1973), Berre and Bjerrum (1973), Perloff et. al. (1963)]. Aging and thixotropic hardening are other forms of time effects that influence the undrained shear behavior [Bjerrum and Lo (1963), Mitchell (1960)].

The shear strength of soils is directly proportional to the number of bonds which depends on the compressive force transmitted at the contact and this holds good for different soil types. Mitchell (1976) suggests that the effect of shearing resistance as a rate process can be expressed as;

$$\tau = \left(\frac{2a\Delta E}{\lambda N} + \frac{2aKT}{\lambda}\ln\frac{\dot{\varepsilon}}{B}\right) + \left(\frac{2b\Delta E}{\lambda N} + \frac{2bKT}{\lambda}\ln\frac{\dot{\varepsilon}}{B}\right)\sigma_f' \qquad (6)$$

where T = maximum shear stress

σ_f' = effective stress on the shear plane

a,b = constants

This is similar to Coulomb equation for strength;

$$\tau = (C) + (\text{Tan}\,\phi)\,\sigma_f' \qquad (7)$$

The unique relationship between strength and number of bonds is given in fig: 8. Now if all other factors are equal, the shearing resistance will increase linearly with the logarithm of the rate of strain as shown in fig 9. The effective cohesion and friction develop separately, with the cohesion peaking at very slow strain and friction mobilized gradually.

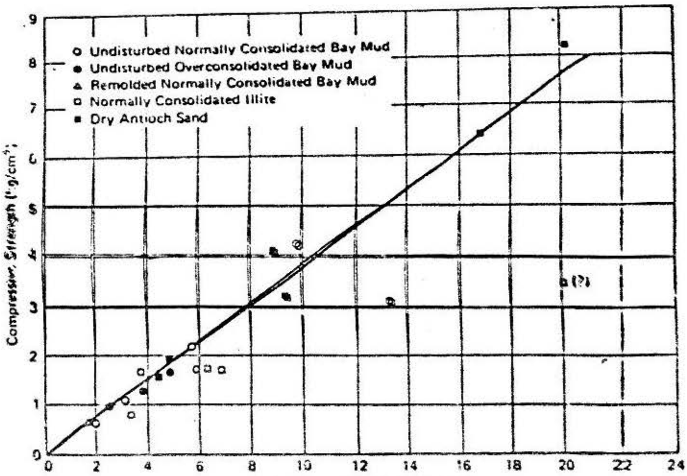


FIGURE 8 Compressive Stress as a function of Interparticle bonds for different samples

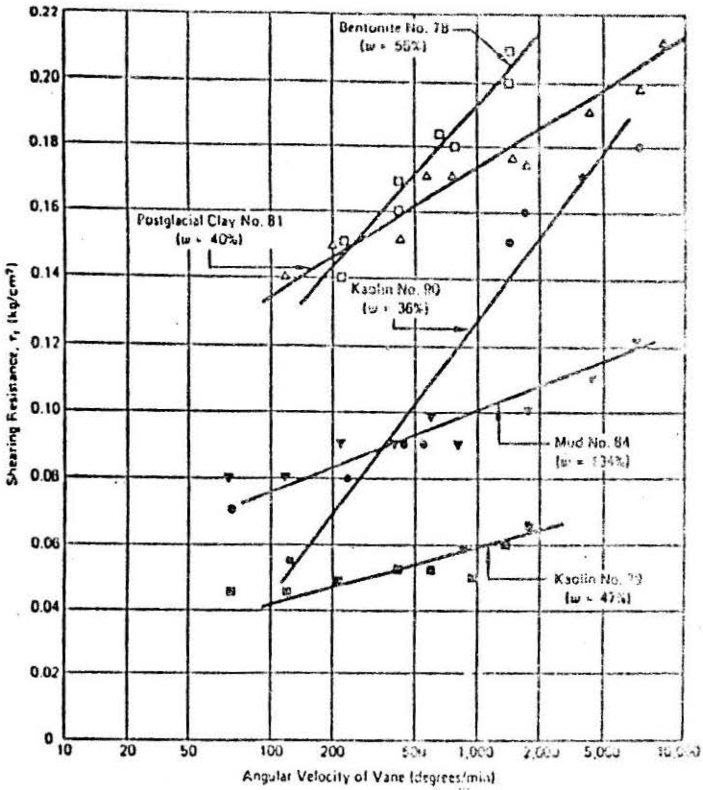


FIGURE 9 Variation of rate of Shear with Shearing resistance during Laboratory Vane Shear testing of remolded Clays

Temperature effect is yet another important effect to be considered while studying the shearing resistance of soils. All other factors being constant, an increase in temperature will cause a decrease in strength. Experimental work by Kelly (1978) has concluded that the ratio of the temperature sensitivity to shear strength is approximately the same for different clays and is approximately equal to 0.0075 C. Detailed studies on the parameters influencing the shear strength of soils is beyond the scope of this paper and can be found elsewhere, [Sharifounnasab and Ullrich (1985), Trollope (1964), Silvestri et. al. (1989)].

CREEP BEHAVIOR AS RELATED TO RATE PROCESSES

Based on the time-dependent behavior, creep can be characterized in three stages: primary creep is defined as decreasing strain-rate with time; secondary creep as constant strain-rate with time; and tertiary creep as increasing strain-rate with time. The analysis of experimental data in the past have indicated that the creep strength at constant water content and under sustained load decreases linearly with the logarithm of time when the clays are saturated [Shibate and Karube (1969), Campanella and Vaid (1974), Finna nd Sneed (1973]). Experimental work by Shibata and Karube (1969) suggests that the creep strength measured in drained condition will coincide with the normal compressive strength. If all variables such as temperature, water content, effective stress, soil structure, etc., are kept constant except deviator stress (σ_d), then the creep strain rate ($\dot{\varepsilon}_c$) may be written on the basis of rate theory as;

$$\dot{\varepsilon}_c = \text{constant}^* \exp (B\, \sigma_d) \tag{8}$$

Where B is the stress factor defined as;

$$B = 0.47\, v_f / KT \tag{9}$$

where V_f = volume of flow unit.

Thus under conditions of constant structure, temperature, water content and effective stress, the logarithm of strain-rate should be a linear functon of deviator stress.

Research work by Silva et. al. (1983) suggests a drained creep relationship for deep sea clays as;

$$\log \dot{\varepsilon}_c = mts + bs + k \tag{10}$$

where t = time, s = stress level

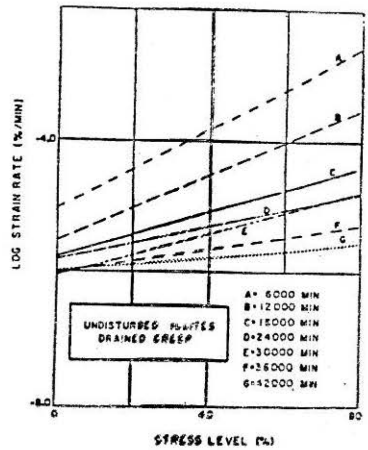
and m, b & k are constants determined from fig: 10(c).

The drained condition (shown in fig: 10) results in a deviation from the general creep deformation behavior predicted by Singh and Mitchell (1968), which relates the strain-rate versus deviatoric stress as linear function with a constant slope over time. But here, although a linear relationship exists between the axial strain-rate as a log function and stress level; the slope of the curve varies with time. More details on the stress-strain-time behavior of marine clays is presented in Akers and Silva (1980), Silva (1979), Silva and Hollister (1983), Silva et. al. (1976).

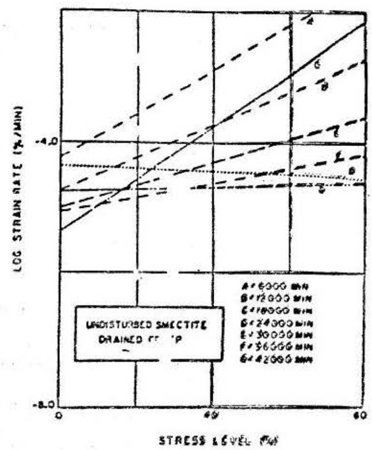
Sustained stress creep curves illustrating different forms of behavior for a range of soil types and test conditions is shownin Fig: 11(a). Note that, in spite of this apparently random behavior, time dependent deformations and stress relaxations still follow logical and often predictable patterns as discussed above.

Stress-Strain-Time Functions and Rheological Models

The uniqueness and the simplicity of the relationships discussed in the previous sections facilitates the use of simple expressions for characterization of creep. Note that the stress-strain-time function must be applicable



(a)



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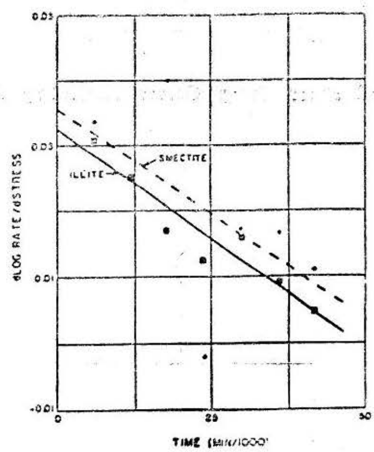

(c)

FIGURE 10(a) & 10(b); Plotof log Strain rate vs Stress level for illites and Smectites during drained Creep. 10(c): A phenomenological stress-strain-time function determined from the Slope of 10(a) & (b)

to a reasonable range of creep stresses and it must describe the behavior of a range of soil types (linear and curved relations). Singh and Mitchell (1968) defines a general stress-strain-time function as;

$$\ln \dot{\varepsilon} = \ln A + \alpha D - m \ln (t/t_1) \tag{11}$$

where $\dot{\varepsilon}$ = strain-rate at unit time

t, t = time, and reference time(say, 1 min) respectively

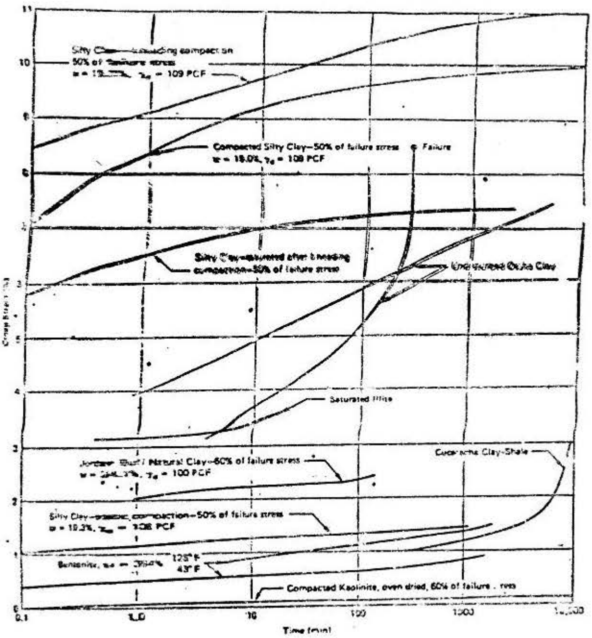

FIGURE 11(a) Sustained stress Creep Curves illustrating different forms of behavior

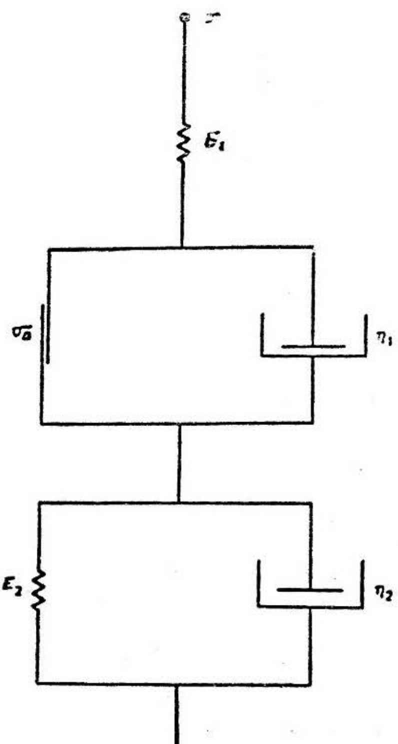

FIGURE 11(b) Modified Komamura-Huang Rheological model for characterization of the stress-strain-time behaviour of soils

D = stress density

m = absolute value of slope of the straight line on log strain-rate versus log time plot

a = slope of the linear part of log strain-rate versus log time plot

A = value of strain-rate obtained by projecting the straight line portion of the curve between log strain rate and deviator stress at unit time to value at $D = O$

Note that the relation given in eqn: 11 can be used to describe the creep behavior of a variety of soils.

Several rheological models were developed in an effort to duplicate the stress-strain-time response of a soil in terms of linear springs, linear and non-linear dashpots, and sliders [Komamura and Huang (1974), Abdel-Hadi and Herrin (1966), Christensen and WU (1964)]. Of these, the modified Komamura and Huang model (Fig: 12(*b*).) can account for the creep behavior of several soils. The complete stress-strain-time equation for this model can be expressed as;

$$\varepsilon = (\sigma/E_1) + \left(\frac{\sigma - \sigma_0}{\eta_1}\right) t + (\sigma/E_2)\,(1 - \exp\,(E_2\, t/\eta_2) \qquad (12)$$

For low stress intensities, the second term on the right hand side vanishes and creep ultimately ceases; for $\sigma < \sigma_o$, creep continues indefinitely. For sufficiently high water content, σ_o, E_1 and E_2 approaches zero and the model predicts essentially viscous behavior. The future area of research interest is the numerical modelling of creep processes and research is being done at the U.R.I./M.G.L. to numerically predict the creep processes of the continental slope and rise sediments.

Conclusions

The theory of rate processes can be used as a basis for the analysis of the stress-strain-time behavior of several soils. However, the validity of the basic assumptions made in rate process theory needs to be reviewed, especially for clays. The following drawbacks can be pointed out while applying rate proces theory for clays :

(1) The application of rate process theory implies that all particles are equally stressed, which is not true for natural, aggregated clays.

(2) Due to the heterogenious character of clay particles, only a small fraction of the total number of particle bonds are fully activated in the flow process even at failure stresses [Push (1976)].

(3) At subfailure stresses, the local strain is entirely determined by the local stress fields which are mainly governed by particle aggregate and pore geometry.

(4) A close approximation to reality would be the assumption that hydrogen bonding exists through thin interparticle water films and this in turn is responsible for shear resistance and the creep properties.

It follows from the above discussions that there is a need for additional research on the basic assumptions and application of rate process theory for non-cohesive soils.

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