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Displacements of Block Foundation Subjected to Inclined and Eccentric Impact Load

by

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Introduction

Hammers, presses, crushers and mills are typical machines which produce impact loading on the foundations. Major design criteria for designing foundations subjected to dynamic loading are amplitude and resonance criteria. But for the foundations subjected to impact loading, amplitude criterian is the main one. Based on this criterion, many investigators (Barkan 1962, Major, 1980, Novak 1983) have presented the method of analysis of the foundation subjected to impact loading. Barkan (1962) has given the fundamental equilibrium equation for a block resting on the ground surface (Fig. 1) subjected to impact load considering both translation and rotation for undamped case as:

$$m\ddot{Z} + K_z Z = 0 \tag{1}$$

$$m\ddot{X} + K_x (X - L\phi) = 0 \tag{2}$$

$$I_m \ddot{\phi} - K_x L_x + (K_\phi - WL + K_x L^2)\, \phi = 0 \tag{3}$$

Equation (1) is uncoupled and equations (2) and (3) are coupled to each other. When the impact is purely vertical and concentric, motion of the foundation will be followed by equation (1) only. Solution of eq. (1) is

$$Z = \frac{\dot{Z}c}{\omega_{nz}} \sin \omega_{nz}\, t \tag{4}$$

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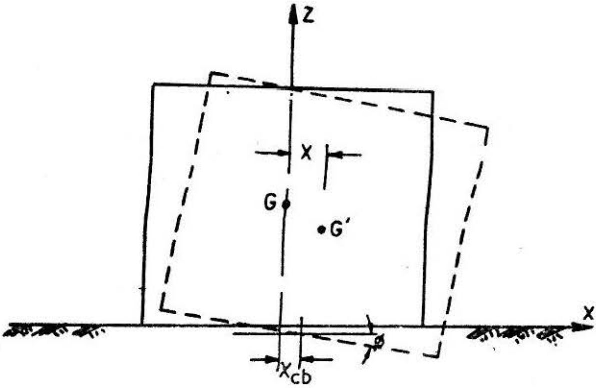

FIGURE 1(a) Equilibrium of a block subjected to an impact load with an accentricity on inclination or both in XZ-ϕ Coordinate.

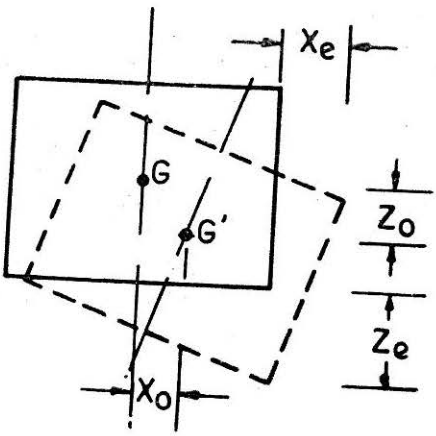

FIGURE 1(b) Various possible displacement due to impact

Maximum amplitude is

$$Z = \frac{\dot{Z}_C}{\omega_{nz}} \tag{5}$$

where $\dot{Z}_C$ = initial velocity of foundation $= \dfrac{(1+e)\,V}{1+(m/m_o)}$ (6)

V = velocity of dropping part

e = coefficient of restitution

m = mass of the foundation

m_o = mass of the dropping part

w_{nz} = natural frequency of the foundation-soil system

$= \sqrt{\dfrac{K_z}{m}}$

K_z = stiffness coefficient for vertical made = $\frac{4Gr_o}{1-\mu}$ for rigid base assumption

G = shear modulus

r_o = equivalent radius of foundation contact area

μ = Poisson's ratio.

Due to some eccentricity of impact load, the foundation is subjected to a moment at centre of gravity which causes a coupled motion following equations (2) and (3). Introducing particular solution into equations (2) and (3) and simplifying the same yields a frequency of the form

$$\omega_n^4 - \frac{(\omega_{n\phi}^2 + \omega_{nx}^2)}{\gamma}\,\omega_n^2 + \frac{\omega_{n\phi}^2\,\omega_{nx}^2}{\gamma} = 0 \tag{7}$$

where, $w_{n\phi}$ = Natural circular frequency in rocking when foundation posses infinite resistance to sliding

$= [(K_\phi - WL)/I_m]^{1/2} \approx \sqrt{K_\phi / I_m}$

K_ϕ = rocking stiffness with rigid base assumption

$= \frac{8Gr_o^3}{3(1-\mu)}$

I_m = Mass moment of inertia of the machine and foundation system about an axis passing through centroid of base contact area perpendicular to plane of vibration.

$$= I_{om} + mL^2 \tag{8}$$

where, I_{om} = mass moment of inertia of the machine foundation system about an axis passing through centre of gravity perpendicular to the plane of vibration.

$$\gamma = I_{om}/I_m \tag{9}$$

w_{nx} = natural circular fequency in sliding when footing posses infinite resistance to rocking $= \sqrt{K_x/m}$.

K_x = horizontal stiffness with rigid base assumption = $32(1-\mu)\,Gr_o/(7-8\mu)$

The two roots of the equation (7), n_1 and n_2 are the two natural frequencies for two modes i.e. sliding and rocking.

Novak (1983) presented an analytical approach for the prediction of amplitude of damped vibration of foundations for shock producing machines. In this approach the increased stiffness due to depth effect has been used. But in the above approach (Novak, 1983) only the concentric vertical impact has been considered. Impact may have some eccentricity or eccentricity and inclination both with respect to both the axis in many cases. In this paper an analysis mainly based on Barkan (1962) approach has been presented for a block resting on the ground subjected to inclined eccentric impact considering undamped case.

The embedment of the foundations to a reasonable depth increases the stiffness and damping which in turn reduces the displacement. Many investigators (Kaldjian, 1971; Anandakrishnan and Krishnaswamy, 1973; Johnson et al, 1975; Sridharan and Nagendra 1983; Gazetas et al, 1985; Gazetas and Tassoulas, 1987; Hatzikonstinow, E. et. at., 1989; Sridharan et al., 1990) have presented the analysis to estimate the stiffness increase due to embedment. In the present analysis all the displacements have been presented in terms of either concentric vertical or concentric horizontal impact case. One can use the increased stiffness due to embedment to find out displacements of concentric vertical and horizontal impact case respectively in the analysis presented herein which approximately takes into account the embedment effect. It is suggested to use increase in stiffness value due to embedment from Gazetas et al (1985) and Gazetas and Tassoulas (1987) for vertical and horizontal mode respectively.

Analysis

Depending on the nature of machinery imparting impact type of dynamic load, the angle of inclination can vary. Inclination can vary from 0° i.e. vertical impact to 90° i.e. horizontal impact. Position of application of impact load may not be on vertical line passing through the centre of gravity. Fig. 2 shows the possible inclination and eccentricity depending upon point of application of impact load. In Fig. 2, θ is the inclination of impact, E is eccentricity with respect to vertical axis, h is the half height of block. Fig. 2(a) shows a case where angle of impact is the one which passes through centre of gravity and is taken as limiting angle of impact, θ_L.

$$\theta_L = \tan^{-1}(h/E)$$

Due to elimination of eccentricity from both the axes passing through centre of gravity, two components of velocity of foundation will pass through the centre of gravity. Displacement of foundation for this case can be found out treating the block in vertical and horizontal mode separately taking vertical and horizontal component of the impact load. In Fig. 2(b) the angle of impact θ is less than the limiting angle of impact (θ_L), the line of action when extended, intersect the vertical axis passing

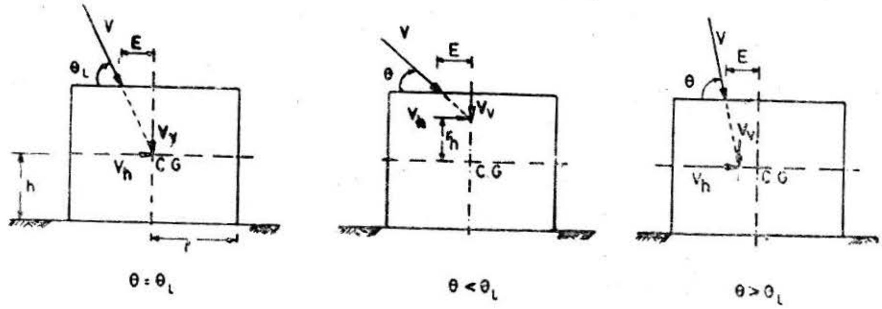

FIGURE 2 Various possible inclination of impact

through centre of gravity. Resolving the impact into vertical and horizontal components at intersection point, vertical component acts concentrically resulting in uniform vertical displacement. Horizontal component acting at eccentricity, r_h causes coupled sliding and rocking resulting in horizontal displacement of centre of gravity and rotation about centre of gravity. Rotational component causes displacement of edge in excess of centre of gravity displacement both in horizontal and vertical directions. Fig. 2(c) shows angle of impact (θ) more than the limiting angle of impact (θ_L), the line of action when extended interesects the horizontal axis through centre of gravity. Resolving the velocity at the interesecting point, horizontal component acts concentrically resulting in uniform horizontal displacement. Vertical component acting at eccentricity, r_v creates a vertical displacement and rotation about centre of gravity. Thus, it is sufficient to propose simplified methods of analysis for (a) eccentric vertical impact, r_v and (b) eccentric horizontal impact, r_h.

(a) Eccentric Vertical Impact

This case occurs due to vertical impact or due to vertical component of impact acting at an angle greater than the limiting angle of impact. Displacement at various points due to this are considered here. [Displacement at any point both horizontal, X and vertical, Z are defined by three subscripts. First one indicates the point under consideration (viz., centre of gravity as o and edge as e). Second one indicates type of loading (viz., concentric as c, eccentric as e). Third one indicates direction of loading (viz., vertical as, v and horizontal, as h) : example (*i*) Z_{oev} = vertical displacement at centre of gravity due to eccentrical vertical impact, (*ii*) X_{ech} = horizontal displacement at edge due to concentric horizontal impact. Fig 1(b) shows the diplacement at various points in general].

(i) Vertical displacement of centre of gravity

To determine the displacement of centre of gravity due to eccentric vertical impact, Z_{oev}, displacement ratio is considered as,

$$\frac{Z_{oev}}{Z_{ocv}} = \frac{\dot{Z}_{oev}/\omega_{nz}}{\dot{Z}_{ocv}/\omega_{nz}} = \frac{\dot{Z}_{oev}}{\dot{Z}_{ocv}} \tag{10}$$

where, Z_{ocv} = displacement of centre of gravity of foundation due to concentric vertical impact

$\dot{Z}_{ocv}$ = velocity of centre of gravity of foundation due to concentric vertical impact

$$= \frac{(1+e)\,V}{(1+M_R)} \tag{11}$$

where, M_R = ratio of mass of the foundation to the mass of the falling parts $= \dfrac{W}{W_o} = \dfrac{m}{m_o}$

V = velocity of falling parts just before impact.

$\dot{Z}_{oev}$ = velocity of centre of gravity of foundation due to eccentric vertical impact

$$= \frac{(1+e)\,I_m/m_o}{(1+M_R)\,(r_v^2 + I_m/m_o) - r_v^2} \quad \text{(Barkan, 1962)}$$

for a cylindrical block it can be rewritten as

$$= \frac{M_R\,I\,(1+e)\,V}{(1+M_R)\,(M_R\,I + R_v^2) - R_v^2} \tag{12}$$

where, R_v = vertical eccentric ratio $= r_v/h$ and I = nondimensional mass moment of inertia term $= [0.25 + 0.333(h/r)^2]/(h/r)^2$

Substituting $\dot{Z}_{ocv}$ and $\dot{Z}_{oev}$ into equation (10)

$$\frac{Z_{oev}}{Z_{ocv}} = \frac{M_R\,I\,(1+M_R)}{(M_R\,I + R_v^2)\;(1+M_R) - R_v^2} = V_{RRV}$$

$$\text{or, } Z_{oev} = V_{RRV} \cdot Z_{ocv} \tag{13}$$

Z_{ocv} can be determined from analysis given by Barkan for concentric vertical impact.

(ii) Vertical Displacement of the edge

Rotational component of impact creates an excess displacement of edge in vertical direction. Combining both vertical translation and rotational component,

$$Z_{eev} = Z_{oev} + r_o\,\phi_v \tag{14}$$

where, ϕ_v = rotational displacement of block due to vertical eccentric impact. For a concentric vertical impact, displacement of edge is same as displacement of centre of gravity.

i.e. $Z_{ocv} = Z_{ecv}$

Expressing the equation (14) in nondimensional form with respect to displacement of edge due to concentric vertical impact,

$$\frac{Z_{eev}}{Z_{ecv}} = \frac{Z_{oev}}{Z_{ocv}} + \frac{r_o\,\phi_v}{Z_{ocv}} = V_{RRV} + \frac{r_o\,\phi_v}{Z_{ocv}} \tag{15}$$

If the block shown in Fig. 1 is disturbed from its equilibrium position due to application of any inclined or eccentric impact type of loading, it will vibrate freely satisfying the equation (2) and (3). Solutions of equation (2) and (3) for X and ϕ using initial conditions are,

$$X = \frac{1}{(\omega_{n1}^2 - \omega_{n2}^2)} \left[\frac{(\omega_{nx}^2\,\dot{X}_{cb} - \omega_{n2}^2\,\dot{X}_{oh})}{\omega_{n1}} \sin \omega_{n1} t - \frac{(\omega_{nx}^2\,\dot{X}_{cb} - \omega_{n1}^2\,\dot{X}_{oh})}{\omega_{n2}} \sin \omega_{n2} t \right] \tag{16}$$

$$\phi = \frac{1}{L\,\omega_{nx}^2\,(\omega_{n1}^2 - \omega_{n2}^2)} \left[\frac{(\omega_{nx}^2\,\dot{X}_{cb} - \omega_{n2}^2\,\dot{X}_{oh})\,(\omega_{nx}^2 - \omega_{n1}^2)}{n1} \sin \omega_{n1} t = \frac{(\omega_{nx}^2\,\dot{X}_{cb} - \omega_{n1}^2\,\dot{X}_{oh})\,(\omega_{nx}^2 - \omega_{n2}^2)}{\omega_{n2}} \sin \omega_{n2} t \right] \tag{17}$$

where, $\dot{X}_{oh}$ = velocity of centre of gravity in forward motion

$\dot{X}_{cb}$ = velocity of centroid of base contact area

Modifying the equation (17) for eccentric vertical impact, putting $\dot{X}_{oh} = O$, the equation (17) takes the form,

$$\phi_v = \frac{\dot{X}_{cb}\,\lambda}{h(1-\beta_2/\beta_1)\,\beta_1} \left[\frac{(1-\beta_1/\lambda)\sin \omega_{n1} t}{\omega_{n1}} - \frac{(1-\beta_2/\lambda)\sin \omega_{n2} t}{\omega_{n2}} \right] \tag{18}$$

$$\beta_{1,2} = \omega_{n1,2}^2/\omega_{n\phi}^2 = \frac{(1+\lambda)+\{(1+\lambda)^2-4\lambda\gamma\}^{1/2}}{(1+\lambda)-\{(1+\lambda)^2-4\lambda\gamma\}^{1/2}}$$

$$\lambda = \frac{\omega_{nx}^2}{\omega_{n\phi}^2}$$

Now, $\frac{r_o \phi_v}{Z_{ocv}} = \frac{r_o \lambda . \dot{X}_{cb} \omega_{nz}}{h\beta_1(1-\beta_2/\beta_1) Z_{ocv} \omega_{n1}} [(1-\beta_1/\lambda) \sin \omega_{n1} t - (1-\beta_2/\lambda).$

$(\beta_1/\beta_2)^{1/2} \sin \omega_{n2} t]$ (19)

$$\dot{X}_{cb}/\dot{Z}_{ocv} = -R_v V_{RRV}/I \quad (20)$$

$$\omega_{nz}/\omega_{n1} = [(7-8\mu) \lambda/(8(1-\mu)^2 \beta_1)]^{1/2} \quad (21)$$

Using equations (20) and (21) in equation 19, the equation (19) takes the form

$$\frac{r_o \phi_v}{Z_{ocv}} = N_1 V_{RRV} R_v \quad (22)$$

where $N_1 = (r_o/h) \frac{\lambda}{\beta_1 (1-\beta_2/\beta_1) I} \sqrt{\frac{7-8\mu) \lambda}{8(1-\mu)^2\beta_1}} [-(1-\beta_1/\lambda) \sin \omega_{n1} t$

$+ (1-\beta_2/\lambda) (\beta_1/\beta_2)^{1/2} \sin \omega_{n2} t$ (23)

N_1 is a function of h/r_o, μ and t. If the boundary of t is given, the maximum value of N_1 can be obtained using optimisation theorem. In the absence of boundary of t, the maximum value of N_1 for a particular value of h/r_o and μ can be obtained considering worst possible case as,

$N_1 = \frac{\lambda}{\beta_1 (1-\beta_2/\beta_1) I} \sqrt{\frac{(7-8\mu)\lambda}{8(1-\mu)^2\beta_1}} [\text{abst} (1-\beta_1/\lambda) + \text{abst} (1-\beta_2/\lambda).$

$(\beta_1/\beta_2)^{1/2}]$ (24)

Fig. 3 presents N_1 vs. h/r_o using egn. (24) for different values of, μ. From equations. (15) and (22),

$$\frac{Z_{eev}}{Z_{ocv}} = (1+N_1 R_v) V_{RRV} \quad (25)$$

(iii) Horizontal displacement of Centre of Gravity

Rotational displacement due to eccentric vertical impact causes horizontal displacement of centre of gravity, X_{oev}.

$$X_{oev} = h \phi_v \quad (26)$$

Substituting for ϕ_v and expressing in nondimensional form with respect to vertical displacement of centre of gravity due to concentric vertical impact, Z_{ocv}, the equation (26) takes the form

$$\frac{X_{oev}}{Z_{ocv}} = N_2 V_{RRV} R_v \quad (27)$$

where, $N_2 = (h/r_o)N_1$

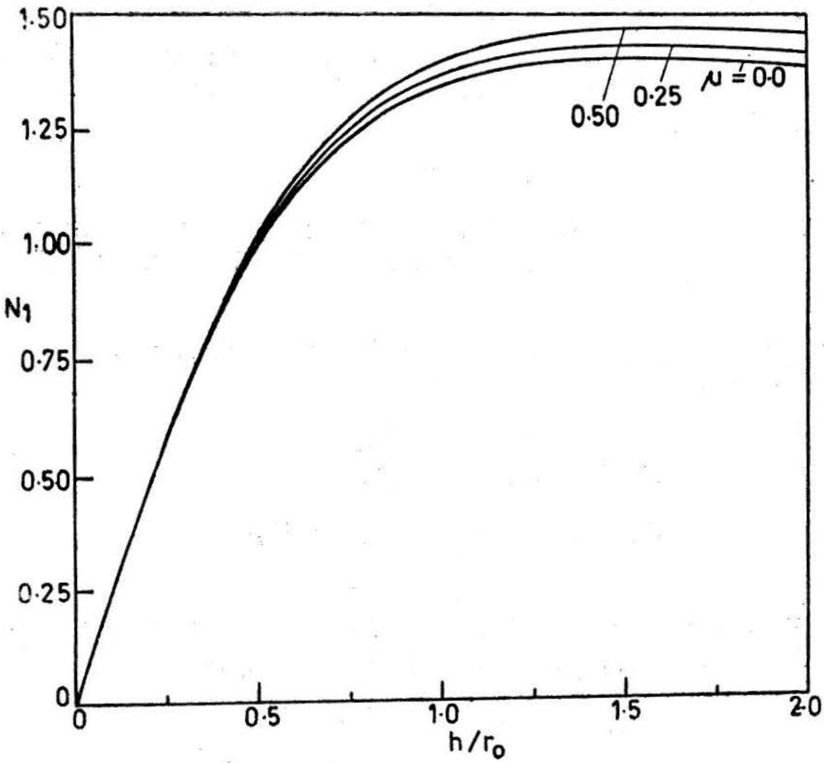

FIGURE 3 Variation of N_1 with h/r_o

(iv) Horizontal displacement of edge

Horizontal displacement of edge due to eccentric vertical impact X_{eev} is expressed as,

$$X_{eev} = X_{oev} + h\,\phi_v \tag{28}$$

From equation (26), $X_{oev} = h\,\phi_v$

Hence, $X_{eev} = 2h\,\phi_v$ (29)

Expressing equation (26) in nondimensional form,

$$\frac{X_{eev}}{Z_{ocv}} = 2N_2\, V_{RRV}\, R_v \tag{30}$$

(b) Eccentric horizontal impact

When the angle of impact is 0°, but point of application is eccentric with respect to horizontal axis or angle of impact less than limiting angle of impact, results in eccentric horizontal impact. Various possible displacements are considered as below:

(i) Horizontal displacement of centre of gravity

Horizontal displacement of centre of gravity due to eccentric horizontal load, X_{oeh}, has two components, i.e. translational and rotational, combining the two.

$$X_{oeh} = \dot{X}_{oh}/\omega_{nx} + h\,\phi_h \tag{31}$$

where, ϕ_h = rotational displacement due to eccentric horizontal impact as given in equation (17). Others are as explained earlier.

Expressing the equation *(31) in nondimensional form* with respect to concentric horizontal impact,

$$\frac{X_{oeh}}{X_{och}} = \frac{\dot{X}_{oeh}}{\omega_{nx}\,X_{och}} + \frac{h\,\phi_h}{X_{och}} \tag{32}$$

$$X_{oeh}/\omega_{nx}\,(X_{och}) = \dot{X}_{oeh}/\,\dot{X}_{och} = V_{RRH} \tag{33}$$

where,

V_{RRH} = velocity reduction ratio for eccentric horizontal impact. Similar to velocity reduction ratio in vertical impact,

$$V_{RRH} = \frac{M_R\,I\,(1+M_R)}{(M_R\,I + R_h^2)\;\;(1+M_R) - R_h^2} \tag{34}$$

R_h = horizontal eccentric ratio $= r_h/h$

and,

$$\begin{aligned} \frac{h\,\phi_h}{X_{och}} &= \frac{X_{oh}\,(\lambda/\beta_1)^{1/2}}{X_{och}\,\beta_1\,(1-\beta_2/\beta_1)}\,[(VR-\beta_2/\lambda)\,(1-\beta_1/\lambda\sin\omega_{n1}\,t \\ &\quad - (VR-\beta_1/\lambda)\,(1-\beta_2/\lambda)\,(\beta_1/\beta_2)^{1/2}\sin\omega_{n2}\,t] \\ &= V_{RRH}\,M_1 \end{aligned} \tag{35}$$

where, $\dot{X}_{cb} = \dot{X}_{oh} - L\;\dot{\phi}_{oh}$

VR = velocity ratio between velocity of centroid of base area to the velocity of centre of gravity $= \dot{X}_{cb}/\dot{X}_{oh}$

$= 1 - R_h/I$

λ, β_1, β_2 as explained earlier

and

$$\begin{aligned} M_1 &= \frac{(\lambda/\beta_1)^{1/2}\,\lambda}{\beta_1\,(1-\beta_2/\beta_1)}\,[(VR-\beta_2/\lambda)\,(1-\beta_1/\lambda)\sin\omega_{n1}\,t \\ &\quad - (VR-\beta_1/\lambda)\,(1-\beta_2/\lambda)\,(\beta_1/\beta_2)^{1/2}\sin\omega_{n2}\,t] \end{aligned}$$

Expression for optimum value of M_1 can be written similar to equation (24), as

$$M_1 = \frac{(\lambda/\beta_1)^{3/2}}{(1-\beta_2/\beta_1)} [\text{abst} \{(VR-\beta_2/\lambda)(1-\beta_1 \lambda)\} + \text{abst} \{(VR-\beta_1/\lambda). (1-\beta_2/\lambda)(\beta_1/\beta_2)^{1/2}\}] \tag{36}$$

Now, equation (32) takes the form after substituting the above,

$$\frac{X_{oeh}}{X_{och}} = V_{RRH}(1+M_1) \tag{37}$$

M_1 is a factor dependent on h/r_o, μ and eccentric ratio, R_h. Figs. 4 to 6 shows M_1 vs. h/r_o for $\mu = 0$, 0.25 and 0.5 respectively and for various values of R_h.

(ii) Horizontal displacement of the edge

Horizontal displacement of edge due to eccentric horizontal impact,

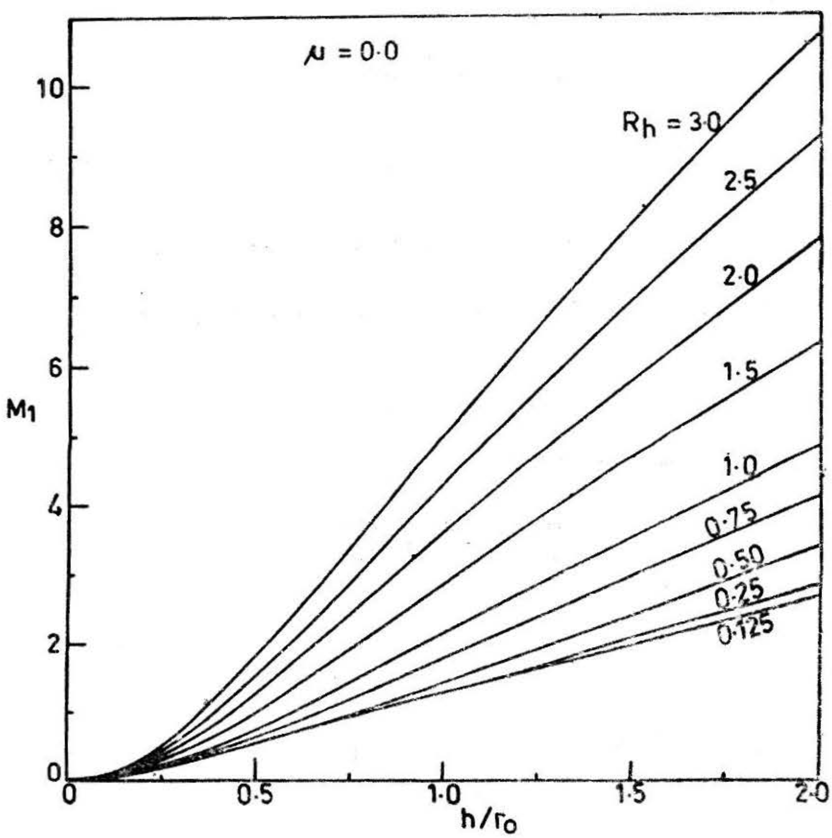

FIGURE 4 Variation of M_1 with h/r_o for Poisson's ratio, μ=0.00

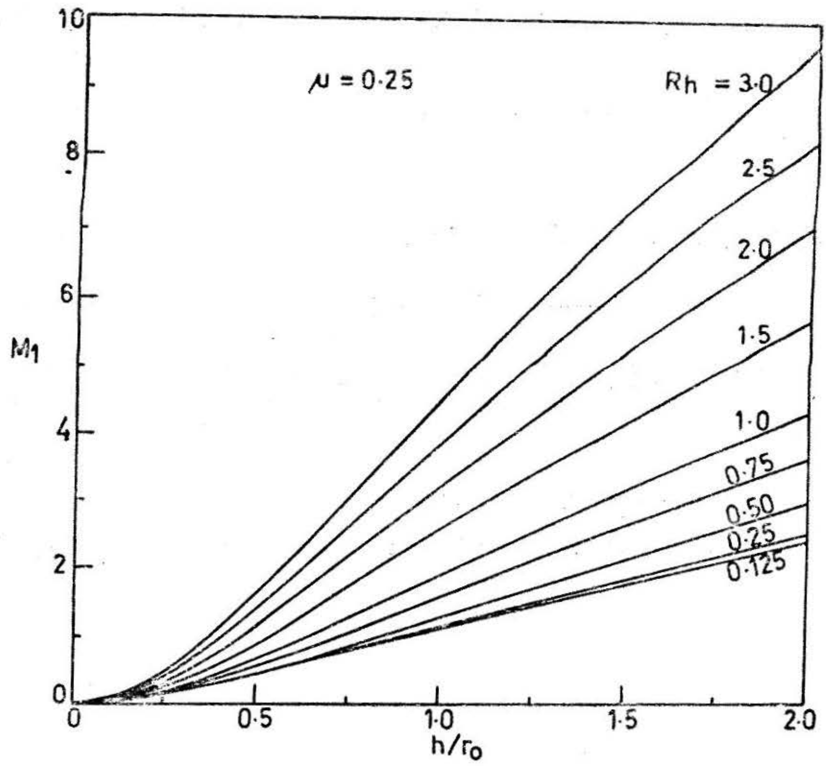

FIGURE 5 Variation of M_1 with h/r_o for Poisson's ratio, $\mu=0.25$

X_{eeh} is a combination of displacement of centre of gravity and rotational component, i.e.

$$X_{eeh} = X_{oeh}+h\,\phi_h \tag{38}$$

expressing in nondimensional form,

$$\frac{X_{eeh}}{X_{och}} = \frac{X_{oeh}}{X_{och}}+\frac{h\,\phi_h}{X_{och}} \tag{39}$$

From equation (37)

$$\frac{X_{oeh}}{X_{och}} = (1+M_1)\;V_{RRH}$$

and also from equation (35) $h\,\phi_h/X_{och} = V_{RRH}\,M_1$

Hence,

$$\frac{X_{eeh}}{X_{och}} = V_{RRH}\,(1+2\,M_1) \tag{40}$$

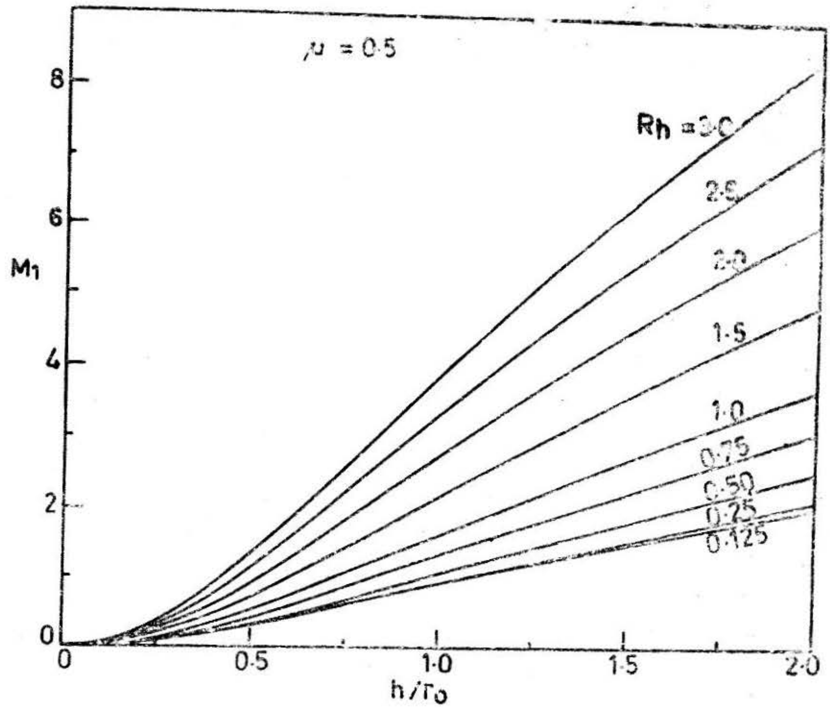

FIGURE 6 Variation of M_1 with h/r_o for Poisson's ratio, $\mu=0.5$

(iii) Vertical displacement of centre of gravity

Centre of gravity of block goes down by horizontal displacement times the rotation. Since, both the quantities are small, the resulting displacement is of small order.

Z_{oeh} = vertical displacement of centre of gravity due to eccentric horizontal impact

$= X_{oeh}\,\phi_h$

Substituting for X_{oeh} and ϕ_h from equiations (37) and (17) respectively and simplifying

$$\frac{h\,Z_{oeh}}{(X_{och})^2} = V^2_{RRH}\,(1+M_1)\,M_1 \tag{41}$$

(iv) Vertical displacement of edge

Vertical displacement of edge due to eccentric horizontal impact, Z_{eeh} is caused by rotational displacement. Mathematically,

$$Z_{eeh} = r_o\,\phi_h \tag{42}$$

Substituting for ϕ_h from equation (17) and expressing in nondimensional form with respect to concentric horizontal impact,

$$\frac{Z_{eeh}}{X_{och}} = \frac{\lambda\, r_o\, V_{RRH}\, (\lambda/\beta_1)^{1/2}}{h\, \beta_1\, (1-\beta_2/\beta_1)} \left[(VR-\beta_2/\lambda)\,(1-\beta_1/\lambda) \sin \omega_{n1}\, t \right.$$

$$\left. - (VR-\beta_1/\lambda)\,(1-\beta_2/\lambda)\,(\beta_1/\beta_2)^{1/2} \sin \omega_{n2}\, t \right]$$

$$= M_2\, V_{RRH} \tag{43}$$

where, $M_2 = (r_o/h) M_1$.

Results and Discussions

Analysis has been presented in this paper for eccentric vertical and horizontal impact. From this analysis the displacement of block foundation at various probable points can be found out for eccentric vertical and horizontal impact. Combining the analysis presented herein with analysis presented by Barkan for concentric, vertical and horizontal impact, one can analyse all the probable types of impacts. Combining both the analyses, the displacement at various probable points are summarised for various cases as below.

(1) *Inclined Concentric Impact*: ($\theta = \theta_L$) If V be the velocity of inclined impact, the two component V_v and V_H will pass through the centre of gravity. Then, analysis will follow the concentric vertical and horizontal impact. Vertical displacement at any point for concentric vertical impact is (Barken 1962)

$$Z_{ocv} = Z_{ecv} = \frac{\dot{Z}_C}{\omega_{nz}} = \frac{(1+e)\, V_v}{(1+m/m_o)} \sqrt{\frac{m}{K_z}}$$

$$= \frac{(1+e)\, V_v}{1+M_R} \sqrt{\frac{(1-\mu)\, m}{4Gr_o}} \tag{44}$$

Similarly one can get displacement for concentric horizontal impact as

$$X_{och} = X_{ech} = \frac{(1+e)\, V_H}{(1+M_R)\omega_{nx}} = \frac{(1+e)\, V_H}{(1+M_R)} \sqrt{\frac{m}{K_x}}$$

$$= \frac{(1+e)\, V_H}{1+M_R} \sqrt{\frac{(7-8\mu)\, m}{32\,(1-\mu)\, Gr_o}} \tag{45}$$

(2) *Inclined eccentric vertical impact* ($\theta > \theta_L$) : Extension line of impact intersects horizontal axis passing through centre of gravity. This case is a combination of case as detailed in section (a) and concentric horizontal impact. Distance of intersection position from vertical axis through centre of gravity.

$$= r_v = E - h/\tan\theta$$

$$R_v = \frac{r_v}{h} = E/h - 1/\tan\theta$$

Knowing the R_v, h/r_o, μ, M_R one can find out the parameter V_{RRV} and N_1, N_2 following the analysis detailed in Section (*a*).

Displacements for various possible points are:

(*i*) *Vertical displacement of centre of gravity*: Vertical displacement for this case is caused by eccentric vertical component.

Hence, from eqn. (11) vertical displacement of centre of gravity

$$= Z_{oev} = V_{RRV} . Z_{ocv} \tag{46}$$

(*ii*) *Horizontal displacement of centre of gravity*: Horizontal displacement of centre of gravity is caused by concentric horizontal component and eccentric vertical component.

Horizontal displacement of centre of gravity due to concentric horizontal impact, X_{och} is as given in equation (45). Horizontal displacement of centre of gravity due to eccentric vertical impact, X_{oev}, can be obtained from equation (27) as,

$$X_{oev} = N_2 \, V_{RRV} \, R_v \, Z_{ocv} \tag{47}$$

Hence, total horizontal displacement of centre of gravity =

$$X_{och} \pm N_2 \, V_{RRV} \, R_v \, Z_{ocv} \tag{48}$$

$\pm$ sign for clockwise and anti clockwise rotational velocity respectively.

(*iii*) *Vertical displacement of edge*: Vertical displacement of edge is caused by eccentric vertical component.

From equation (25), vertical displacement of edge

$$\begin{aligned} &= Z_{eev} = Z_{ecv} \, (1+N_1 \, R_v) \, V_{RRV} \\ &\qquad\quad = Z_{ocv} \, (1+N_1 \, R_v) \, V_{RRV} \end{aligned} \tag{49}$$

Since for concentric vertical impact, $Z_{ecv} = Z_{ocv}$

(*iv*) *Horizontal displacement of edge*: Horizontal displacement of edge is caused by concentric horizontal component and eccentric vertical component. Horizontal displacement of edge due to concentric horizontal impact is as given in equation (44). Horizontal displacement of edge due to eccentric vertical impact is (equation 30)

$$X_{eev} = 2 \, Z_{ocv} \, N_2 \, V_{RRV} \, R_v \tag{50}$$

Total horizontal displacement of edge

$$\begin{aligned} &= X_{ech} \pm 2 \, N_2 \, R_v \, V_{RRV} \, Z_{ocv} \\ &= X_{och} \pm 2 \, N_2 \, R_v \, V_{RRV} \, Z_{ocv} \end{aligned} \tag{51}$$

Since for concentric horizontal impact, $Z_{och} = X_{ech}$

(3) *Inclined Eccentric Horizontal Impact*: $\theta < \theta_L$ Extension of line of impact intersect vertical axis passing through centre of gravity.

$$r_h = h - E \tan \theta$$

$$R_h = r_h/h = 1 - (E/h) \tan \theta$$

Resolving velocity into components at intersection point,

$$V_v = V \sin \theta, \; V_H = V \cos \theta$$

This case is a combination of section (*b*) and concentric vertical impact. Now, knowing R_h, μ, h/r_o, V one can findout the parameter V_{RRH}, M_1, M_2 following Section (*b*).

Displacements are:

(*i*) *Vertical displacement of centre of gravity*

The vertical displacement of centre of gravity is combination of displacement due to concentric vertical component and eccentric horizontal component. Displacement due to concentric vertical component, $Zocv$ is as given in equation (44) and displacement due to eccentric horizontal component, Z_{oeh} can be obtained from eqn. (41) as

$$Z_{oeh} = \frac{(X_{oeh})^2}{h} (1 + M_1) M_1 \; V^2_{RRH} \tag{52}$$

Hence, total displacement of centre of gravity

$$= Z_{ocv} + Z_{oeh} \tag{53}$$

(*ii*) *Horizontal displacement of centre of gravity*?

This displacement is same as the displacement due to eccentric horizontal impact, i.e. from equation (37).

$$X_{oeh} = V_{RRH} \, X_{och} (1 + M_1) \tag{54}$$

(*iii*) *Vertical displacement of edge*

Total vertical displacement of edge is the combination of displacement due to concentric vertical impact and displacement of edge due to eccentric horizontal impact

$$= Z_{ecv} + Z_{eeh} \tag{55}$$

From equation (43),

$Z_{eeh} = X_{och} \, M_2 \, V_{RRH}$

and Z_{ecv} as in equation (44)

(iv) Horizontal displacement of edge

The horizontal displacement of edge is the displacement due to eccentric horizontal component, i.e. from eqn. 37,

$$X_{eeh} = V_{RRH}\,(1+2M_1)\; X_{och} \tag{56}$$

(4) *Eccentric vertical impact* ($\theta = 90°$ $E \neq 0$) : The case as detailed in Section (*a*).

(5) *Eccentric horizontal impact* ($\theta = 0°$ $E \neq 0$) : The case as detailed in Section (*b*).

Conclusions

Displacement amplitude is the most effective design criterion for the foundations subjected to impact type of dynamic loading. In this paper, a simplified analysis has been presented to arrive at displacement amplitude at various possible points for the foundation subjected to inclined eccentric impact load. Using the foundation data (i.e. half height of foundation, h; radius, r_o; eccentricity, E; and horizontal eccentricity, r_h or vertical eccentricity r_v), soil data (i.e., Poisson's ratio, μ and shear modulus, G) and charts presented (i.e. Figs. 3-6) one can arrive at the displacement at various points easily of foundation block subjected to impact load at any inclination and at any eccentricity. A numerical example has been given in *A-I* to illustrate the design procedure using the equations and charts presented in the analysis.

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APPENDIX—I

Numerical example

Calculate displacement amplitude at various possible points when the angle of impact is (*i*) 40° and (*ii*) 75° based on the following data.

$G = 4500\ t/m^2$, $\mu = 0.25$, size of foundation $= 2.5m \times 2.5m \times 1.5m$, eccentricity, $E = 0.5m$, weight of falling parts, $w_o = 0.5t$. Velocity of impact, $V = 7.5m/\text{sec}$, Coefficient of restitution, $e = 0.5$

Solution

Weight of block, $W = (2.5 \times 2.5 \times 1.5 \times 2.4)t = 22.5$ tons:

$$M_R = \frac{m}{m_o} = \frac{W}{W_o} = \frac{22.5}{0.5} = 45,\ r_o = \sqrt{\frac{2.5 \times 2.5}{\pi}} = 1.41\ m$$

$$h/r_o = 0.75/1.41 = 0.53$$

$$\theta_L = \tan^{-1}(0.75/0.5) = 56°\ 31$$

Case (*i*) $\theta = 40°$

$\theta < \theta_L$ i.e. inclined eccentric horizontal impact

$$r_h = 0.75 - 0.5 \tan 40° = 0.33\ R_h = r_h/h = 0.4406$$

$$I = [0.25 + 0.33\,(h/r_o)^2]/(h/r_o)^2 = 1.22$$

$$V_H = V \cos 40° = 5.745\ m/\text{sec},\ V_v = V \sin 40° = 4{\cdot}82\ m/\text{sec}$$

$$Z_{ocv} = \frac{(1+e)\,V_v}{(1+M_R)} \sqrt{\frac{(1-\mu)\,W}{g\,4Gr_o}} = 1.294 \text{ mm (equation 44)}$$

$$X_{och} = \frac{(1+e)\,V_H}{(1+M_R)} \sqrt{\frac{(7-8\mu)\,m}{32\,(1-\mu)\,Gr_o}} = 1.626 \text{ mm (equation 45)}$$

$$V_{RRH} = \frac{M_R\, I\,(1+M_R)}{(M_{RI}+R_h^2)\,(1+M_R)-R_h^2} = 0.9965 \text{ (equation 45)}$$

(*a*) Vertical displacement of center of gravity

$$= Z_{ocv} + \frac{(X_{och})^2}{h}\,(1+M_1)\,M_1\,V_{RRH}^2 \text{ (equation 53)}$$

$= 1.296$ mm

[$M_1 = 0.45$ for $h/r_o = 0.53$, $\mu = 0.25$

and $R_h = 0.4406$ from equation (36)

same can be obtained from Fig. 5.]

(*b*) Horizontal displacement of C.G. =
$V_{RRH}\ X_{och}\,(1+M_1)$ (equation 59)
$= 2.349$ mm

(*c*) Vertical displacement of edge $= Z_{ecv} + Z_{ech}$
[$M_2 = 0.45/0{\cdot}53 = 0.85$] $= Z_{ocv}+X_{och}\ M_2\ V_{RRH}$ (equation 55)
$= 2.671$ mm

(*d*) Horizontal displacement of edge =
$V_{RRH}\,(1+2M_1)\ X_{och}$ (equation 56)
$= 3.078$ mm

Case (*ii*) $\theta = 75°$

$\theta > \theta_L$ i.e. inclined eccentric vertical impact

$r_v = 0.5-0.75/\tan 75° = 0.3m$, $R_v = r_v/h = 0.3987$

$V_H = 1.941\ m/sec,$ $\qquad V_V = 7.244m$

$Z_{ocv} = 1.945m,$ $\qquad X_{och} = 0.549\ mm$

$$V_{RRV} = \frac{M_R\, I\,(1+M_R)}{(M_R\, I+R_v^2)\,(1+M_R)-R_v^2} = 0.9972$$

(*a*) Vertical displacement of C.G. $= V_{RRV}.\ Zocv$
$= 1.939\ mm$ (equation 46)

(*b*) Horizontal displacement of C.G. $= X_{och} + N_2\ V_{RRV}\ R_v\ Z_{ocv}$
(equation 48)

$= 0.974$mm

[$N_2 = N_1 \times h/r_o$

$N_1 = 1.04$ for $h/r_o = 0.53$ and

$\mu = 0.25$ from equation (24)

Same can be obtained from Fig. (3)

Hence, $N_2 = 1.04 \times 0.53 = 0.55]$

(*c*) Vertical displacement of edge $= Z_{ocv}\ (1+N_1\ R_v)\ V_{RRV}$ (equation 49)

$= 2.743$mm

(*d*) Horizontal displacement of edge $= X_{och} + 2N_2\ R_v\ V_{RRV}\ Z_{ocv}$ (equation 51)

$= 1.401$ mm.

APPENDIX—II

Notations

E	distance of impact position from vertical axis passing through centre of gravity (eccentric position)
e	coefficient of restitution
G	dynamic shear modulus of soil
h	half height of the foundation block
I_{om}	mass moment of inertia of machine foundation system about an axis passing through centre of gravity perpendicular to plane of vibration
$I = \{0.25 + 0.333\,(h/r_o)^2$	nondimensional mass moment of inertia term
K	stiffness coefficient
K_z, K_x, K_ϕ	stiffness coefficient of soil under foundation for vertical, horizontal and rocking mode respectively
L	distance of centre of gravity of machine foundation system from base of the block
I_m	mass moment of inertia of machine foundation system about an axis passing through centroid of base contact area perpendicular to plane of vibration
$M_R = \dfrac{m}{m_o} = \dfrac{W}{W_o}$	mass ratio—ratio of mass of system to mass of falling parts

M_1, M_2	nondimensional displacement functions
m	mass of the foundation
m_o	mass of the hammer (falling part)
N_1, N_2	nondimensional displacement functions
r_o	equivalent radius of the block
r_h, r_v	distance from horizontal and vertical axis passing through centre of gravity to eccentric position
R_h, R_v	nondimensional eccentric ratio for horizontal and vertical eccentricty
t	time variable
V	velocity of falling part just before impact
V_h and V_v	horizontal and vertical component of impact
V_{RRH}, V_{RRV}	velocity reduction ratio for horizontal and vertical impact
V_R	velocity ratio, ratio of velocity of centre of gravity in x direction to velocity of centroid of base contact area
$\dot{X}_{cb}$	velocity of centroid of base contact area
$\dot{X}_{oh}$	velocity of centre of gravity in forward direction due to coupled motion
$\dot{X}_{och}$	velocity of centre of gravity due to concentric horizontal impact
X_{och}	horizontal displacement of centre of gravity due to concentric horizontal impact
X_{oev}, X_{eev}	horizontal displacement of centre of gravity and edge respectively due to eccentric vertical impact
X_{oeh}, X_{eeh}	horizontal displacement of centre of gravity and edge respectively due to eccentric horizontal impact
Z	vertical displacement amplitude
$\dot{Z}_{ocv}$	velocity of centre of gravity due to concentric vertical impact

Z_{ocv}	vertical displacement of centre of gravity due to concentric vertical impact
Z_{oev}, Z_{eev}	vertical displacement of centre of gravity and edge respectively due to eccentric vertical impact
Z_{oeh}, Z_{eeh}	vertical displacement of centre of gravity and edge respectively due to eccentric horizontal impact
$\beta_{1, 2}$	$= \omega^2_{1, 2} / \omega^2_{n\phi}$
θ	= inclination of impact
θ_L	= limiting angle = $\tan^{-1}$ (h/E)
	$= I_{om}/I_m$
λ	$= \omega^2_{nx} / \omega^2_{n\phi}$
μ	= Poisson's ratio
	= mass density of soil
ϕ	= rotational amplitude
$\phi, \ddot{\phi}$	= rotational velocity and acceleration respectively
ϕ_{oh}, ϕ_{ov}	= rotational velocity of centre of gravity of block due to horizontal and vertical eccentric impact respectively
$\phi_v, \phi_h,$	= rotational amplitude of block due to vertical and horizontal eccentric impact respectively
ω_n	= circular natural frequency
$\omega_{nz}, \omega_{nx}, \omega n\phi$	= circular natural frequency of vertical, horizontal and rocking mode respectively
ω_{n1}, ω_{n2}	= circular natural frequency of vibration in I and II mode respectively.