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Prediction of Soil Behaviour
Part IV—Partly Saturated Soils

by

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Introd ion

Most of the natural soils above ground water table and invariably all the compacted soils in their initial condition are in a state of partial saturation. Partly saturated soils exist all over the world where the weather conditions are such that there is an annual excess of evaporation potential over precipitation. The heave or collapse of partly saturated soils upon saturation can cause considerable distress or damage to structure founded on them. It is estimated that damages world wide due to expansive soils exceed those due to floods, hurricanes, earthquakes and tornedos put together. (Jones and Holtz 1973). The mechanics of volume change and shear strength behaviour of unsaturated clays is of major concern to geotechnical engineers and there has been intensive research to understand and predict these behaviours. In general, these approaches attempt to define or arrive at the effective stresses which control the mechanical behaviour. But still, unanimity in understanding has not been reached as to what constitutes the effective stress in these soils.

The conventional Terzaghi's effective stress equation has been modified to account for partial saturation by including additional parameters. The most generally accepted one is that proposed by Bishop (1960) and Atchison and Bishop (1960) in the form

$$\sigma' = \sigma - u_a + X\,(u_a - u_w) \tag{50}$$

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where u_a and u_w are the pore air and pore water pressures, and X is a parameter varying from 0 to 1, depending on the degree of saturation, and defines the extent of influence of pore water tension on the effective stress. Jennings and Burland (1962) have shown that this equation cannot consistently explain the heave and collapse phenomena upon water imbibation. Atchison (1961) has brought out that the X parameter could be different for volume change and shearing resistance responses. There are a few other forms of effective stress equation (Jennings 1961, Atchison 1961, Croney *et. al.* 1958) involving σ, u_a and u_w but Blight (1983) has shown that all these are similar or can be reduced to the same form of eqn. (50) for specific conditions. Critical reappraisal of these equations (Jennings and Burland 1962, Matyas and Radhakrishna 1968, Sridharan 1968, Gulhati 1975) has indicated that analysis of behaviour of partly saturated soils by this approach is not only a complex exercise but also involves accurate measurement of u_a, u_w and X parameter which is very difficult if not impossible in practice.

More recently, Fredlund and co-workers (Fredlund and Morgenstern 1976, Fredlund 1979) have proposed basic concepts to explain the behaviours by considering independent continuous stress fields, *i.e.* $(\sigma - u_a)$, $(\sigma - u_w)$ and $(u_a - u_w)$ for different phases of the clay-water-air system. They have tried to explain the stress-strain relations by considering independent elastic modulli for each phase for the corresponding stress variables. Similarly the relation between shearing resistance and normal stress is also defined (Fredlund *et. al.* 1978) differently for each of the independent stress variable *i.e.* ϕ' for $(\sigma - u_a)$ and ϕ_b for $(u_a - u_w)$.

There is another school of thought (Lambe 1960, Sridharan 1968, Young *et. al.* 1971) which strongly feels that the interparticle physico-chemical forces have to be essentially considered to explain the behaviour of partly saturated soils. Assuming the superposition of different stress fields to be valid the equilibrium equation was written by Lambe (1960) as

$$\sigma = \overset{=}{\sigma}\, a_m + u_a\, a_a + u_w\, a_w + (R - A) \tag{51}$$

According to Bloch (1978), in an interacting system such as clay-water-air system, independent stress fields for different phases are not valid but the overall stress is the only acceptable physical stress field. The pore water and pore air pressures and the corresponding physico-chemical forces are only the consequences of the total stress variations.

Young *et. al.* 1971 have brought out that the total potential of an unsaturated soil is the sum of two components, osmotic potential which is due to the net interparticle interaction forces between charged clay particles and the matric potential which is due to the capillary force at the air water interphase. It has been shown that both osmotic and matric suctions have the same effect on the vapour pressure of the surrounding vapour and hence are of the same nature which can be added. It has also been experimentally proved (Krahn and Fredlund 1972) that the total suction is the sum of matric and osmotic suctions by independent measurements of these components using suitable techniques. Then it may be valid to write the equilibrimum equation as

$$\sigma = (R—A) + (u_a - u_w) \tag{52}$$

for the assumed case of no direct contact between particles.

But the problem with partly saturated soils is that neither $(R—A)$ nor $(u_a - u_w)$ is easily measurable in practice. In addition, these two and the total stress are interdependent, *i.e.* changes in one of them is associated with changes in the other two and changes in any of them cannot be uniquely related with the volume changes of the soil. Hence these forms of effective stresses or bifurcating the components and applying constitutive relations to each of the phase cannot be of much use in analysis and predications.

In retrospect, what is needed may be a simple model where we can relate an easily measurable stress, say total potential with some suitable state parameter and which can define both volume change and shear strength behaviours. This paper is an attempt to examine the possibility of generating such an interrelationship for partly saturated soils.

State Parameter Approach

As stated earlier, the total potential in a partly saturated soil is the sum of osmotic and matric potentials. The osmotic pressure is reflected by the void ratio of the soil (if the soil is devoid of prestress, or void ratio plus preconsolidation pressure if the soil is overconsolidated). The matric suction obviously depends on the degree of saturation. Hence a combined parameter involving e and S_r in some form may be able to reflect the total stress $(\sigma - u_a)$.

It is well known that the capillary stress is inversely proportional to the radius of the air water interface *i.e.* $(u_a—u_w) = f(1/r)$. For a given volume of voids, the volume of air is a function of the degree of saturation and hence the radius of the air space will be a function of $(S_r)^{1/3}$. Also, since volume of voids itself is defined by, e, $(u_a—u_w)$ will be a function of e also. Now it is to be examined if there exists a defined relation between e, S_r and $(\sigma—u_a)$. For this, careful experimental programme with one dimensional compression tests under constant water content conditions were carried out. (Shashikumar 1988) To avoid the effects of stress history which would make the analysis of behaviour more complicated, all tests have been conducted on soils starting from a very loose state, void ratio being much higher than the liquid limit state. Tests were conducted with different soils and with different initial water contents. Analysis of test results has shown (Fig. 31) that there is a unique relation of the form

$$e\,(S_r)^{1/3} = a - b \log \sigma \tag{53}$$

for a given soil, and that the compression paths of all soils are generalisable as in the case of saturated soils with their respective liquid limit values as

$$(e/e_L)\,(S_r)^{1/3} = 1.19 - 0.255 \log \sigma \tag{54}$$

The above equation represents the state bounding line for unsaturated soils similar to the virgin $e - \log \sigma$ path for saturated soils. No soil can exist

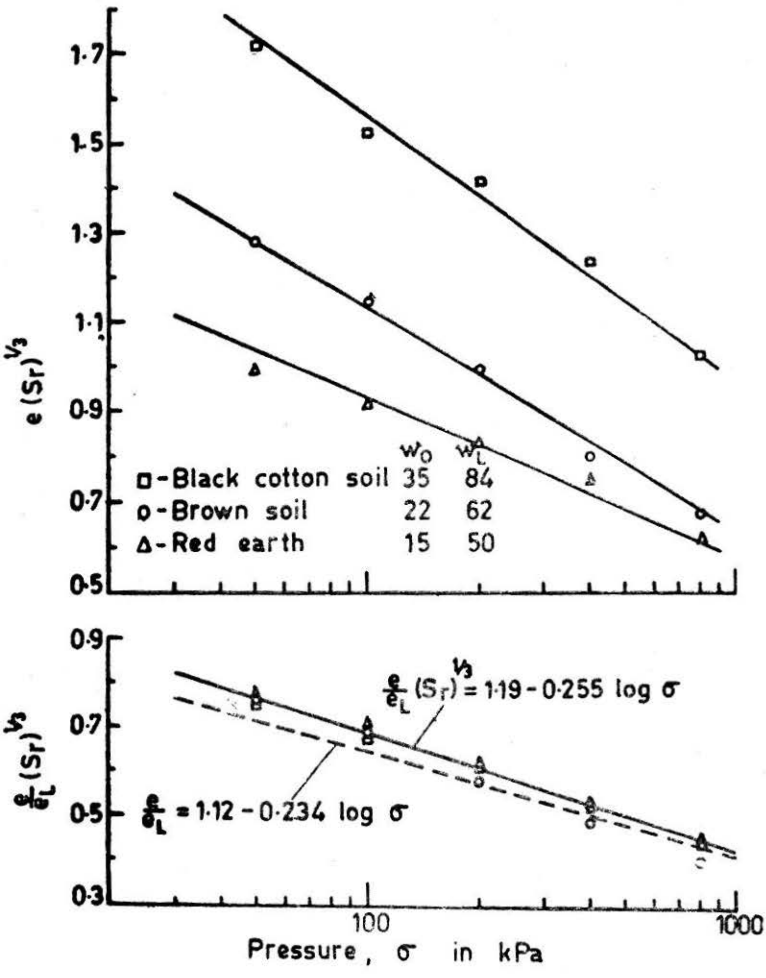


FIGURE 31. Generalization of compression paths of soils (Constant water content condition) (Data. Shashi Kumar 1988)

in a state above this line unless it is cemented. In fact, theoretically, the right hand side of this equation should be the same as that of eq. (12) for saturated soils so that for $S_r = 1$ both should give the same values. The small deviations observed may be the effect of initial fabric.

But compacted soils or the natural field soils will have invariably undergone loading and unloading (or equivalently drying and wetting) and hence will be subjected to some unknown degree of prestress. Thus they generally exist in a state much below this bounding line.

Results of the same tests (Fig. 32) indicate that the average rebound paths at different levels of preconsolidation pressures (400 kPa and 800 kPa) also follow a defined path of

$$e/e_L\ (S_r)^{1/3} = a'' - .047 \log \sigma \tag{55}$$

for all the three soils tested and for the different initial water contents. In the graph the rebound paths have been plotted from a common origin at the preconsolidation stress level, although there was some dispresion at this point for different soils, since the purpose was only to get the average slope of the rebound paths. The exact position or the intercept a'' will of course be different for different levels of preconsolidation as in the case of saturated soils. Again it can be seen that the value of the generalised slope of $\left\{\left(\frac{e}{e_L}\right)(S_r)^{1/3}\right\} - \log \sigma$ is nearly the same as obtained for $e/e_L - \log \sigma$ paths of saturated soils and hence the compressibility equation can be written similar to equation (13) in the form

$$(e/e_L)\ (S_r)^{1/3} = 1.19 - 0.208 \log \sigma_c - 0.047 \log \sigma \tag{56}$$

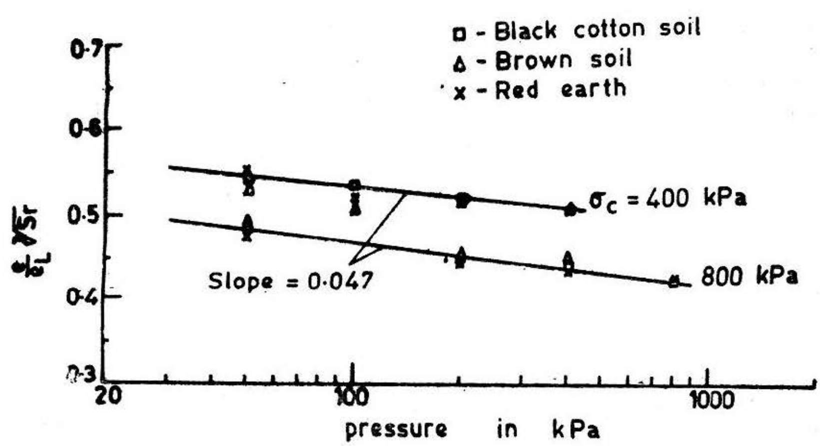


FIGURE 32 Generalization of slopes of rebound paths (constant water content condition) (Data. Shashi Kumar 1988)

These unique relations indicate that there is a viable approach to characterise the behaviour of partly saturated soils using the total stress itself but associating it with the state $e\,(S_r)^{1/3}$ instead of the usual state, e. It is to be further examined how these relations can be used to predict various aspects of soil behaviour. As noted earlier major problems with unsaturated soils are heave or collapse upon saturation and mobilisation of swelling pressure when heave is suppressed, shear strength both in existing and in swollen conditions. Compressibility under constant water content may not be a major problem as they generally exhibit negligible compressions upto sufficiently high stresses.

Predication of Volume Change Behaviour

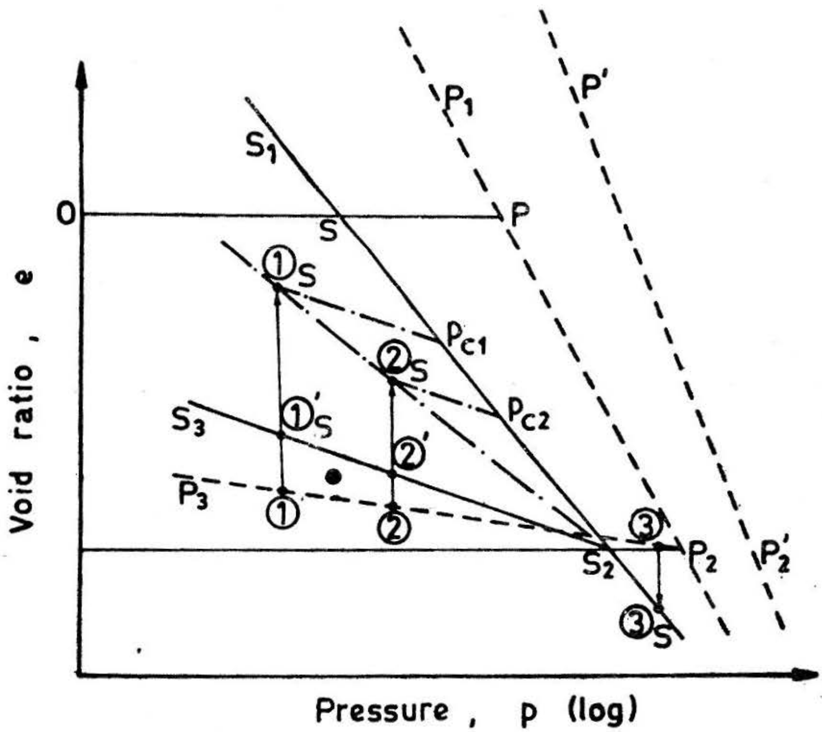
Currently Available Methods

Apart from the above basic approaches from effective stress considerations, there are a few empirical relations in literature (Komornik and David 1967, Nayak and Christensen 1971, Zacharias and Ranganatham 1972, Vijayavergiya and Ghagaly 1973, El—Sohby and Mazen 1981, 1987). In general, these relations rightly involve the parameters γ_d or water content to define the insitu state and W_L or I_P (or in one of the cases shrinkage limit) to define the type or potential of the soil. However, the lumping of parameters has been purely empirical without any basis. More recently, Nagaraj and Srinivasa Murthy (1983, 1985) and Srinivasa Murthy and Nagaraj 1987, have proposed a more reasonable or phenomenological model with diffuse double layer as the basis to predict swelling pressure and heave. From a set of known test results, they have generated and presented in the form of charts, the relation between e/e_L, p_o and p_s by a process of iteration. The computation involved solving 4 equations with 4 unknowns *i.e.* e_s, p_s, p_c and ρ (slope of the line joining the initial state with p_c) and eliminating p_c and ρ. With these charts one can readily get the values of e_s and p_s for any given values of e_o/e_L and p_o. It might be possible to refine these charts with an extensive data to use them more confidently.

Further analysis of behaviour based on the same Gouy-Chapman diffuse double layer theory, and the resulting generalisations indicates that it is possible to predict the behaviours directly without the above iterative computations and charts which will be discussed here under.

Phenomenological Approach

Fig. (33) schematically shows the compression path (P_1P_2) of a partly saturated soil under constant water content condition, (starting from very loose state without prestress) in relation to the compression path (S_1S_2) of a saturated soil. The position of the line depends on the initial degree of saturation. *i.e.* at any given void ratio, higher the degree of saturation smaller



P_1, P_2 & S_1, S_2 - Normally consolidated compression paths of partly saturated and fully saturated states

P_2, P_3 & S_2, S_3 - Rebound lines of partly and fully saturated states

$①_S$ - $②_S$ - Inundated line of slope, P

FIGURE 33 Schematic diagram of volume change behaviour of partly saturated soil

will be the deviation from the saturated line. At any void ratio, the stress OS on the saturated line would be the osmotic component and the difference SP would be the matric component. Thus when the soil is unloaded from P_2 to P_3, the actual preconsolidation pressure on the soil is only S_2, because the capillary component S_2P_2 does not cause a change in the structure of the soil. The rebound path P_2P_3 would be much flatter than and would cut across the rebound path S_2S_3 of the saturated soil. If now the soil is anywhere along the line P_2P_3 and is inundated keeping the stress constant, one can logically expect it to heave (Point 1) or collapse onto the saturated path S_2S_3. Of course, if the partly saturated state lies to the right of virgin path S_1S_2 (Point 3), it would always collapse, to reach this saturated path S_1S_2S. But experimental results have shown that it is not so. The final swollen points upon imbibation under different pressures do lie on a defined line ($(1)_s$ $(2)_s$ (Fig. 33)) which is much steeper than the swelling line of the saturated line. Probably, during the process of heave, the soil is being partially remoulded, which in effect is reducing the actual preconsolidation pressure, thereby making the soil to heave further corresponding to a swelling line of lower p_o. The amount of remoulding may be higher for higher magnitudes of initial heave (*i.e.* the remoulding and hence the additional heave is more for (1) than for (2) and $(1)_s$ corresponds to a p_c lower than $(2)_s$ thereby making this inundated line $(1)_s$ — $(2)_s$ steeper. It has been further found that the slope of this inundated line is also unique having a value of about 0.14 independent of soil type. (Fig. 34)

With this analysis, it would now be possible to predict the complete

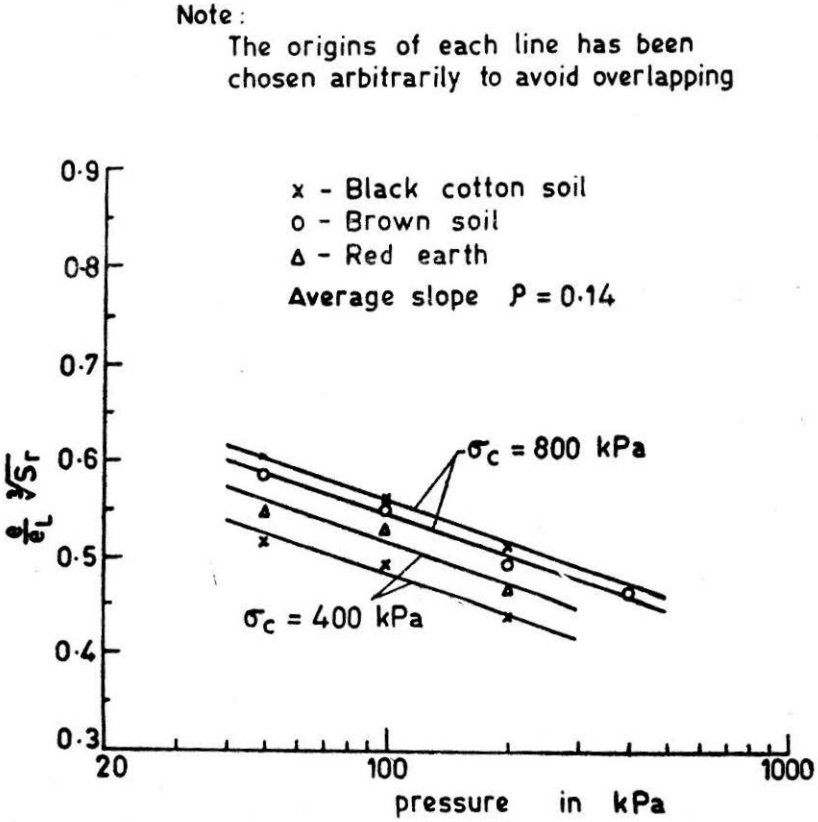


FIGURE 34 Paths of equilibrium states under inundated conditions (Data. Shashi Kumar 1988)

volume change behaviour of partly saturated soils at any given state. The only input parameters required are liquid limit, the insitu void ratio e, degree of saturation, S_r, and the overburden pressure, σ, of the soil, Degree of saturation is the only additional parameter over those required for the saturated systems. The step by step procedure for prediction is as follows: Compute $e_o\,(Sr_o)^{1/3}$. If the point $(e_o(Sr_o)^{1/3}, \sigma_o)$ lies along the bounding line (eq. 54) the soil can be considered to be without prestress. Its compressibility under constant water content can be obtained using the same equation (54). *OR* upon inundation under constant stress, the soil will collapse to reach the saturated line (Eqn. 12)

$$e/e_L = 1.122 - 0.234 \log \sigma$$

The magnitude of collapse can be computed as the difference in void ratios from eq. (54) and eq. (12) under the same given, σ.

If the field point $(e_o\,(Sr_o)^{1/3}, \sigma_o)$ lies below the bounding line the soil is prestressed. Then compute p_c of the partly saturated state using equation (56) for the known initial values of e_o, Sr_o, σ_o. The compressibility of the soil can be obtained by the same equation upto the pc level and by the eq. (54) beyond the level of p_c. To predict heave or collapse and swelling pressure, compute the state $e/e_L(S_r)^{1/3}$ corresponding to the state p_c of the partly saturated state, compute e and S_r of this state from the known initial values e_o, Sr_o and for constant water content condition. Compute the pressure corresponding to this void ratio on the saturated line (eq. 12) which would be the preconsolidation pressure $p_{c(sat)}$. Pass the inundated line with a slope of 0.14 through the point $(p_c)_{(sat)}$ on the saturated line. The pressure corresponding to the insitu void ratio on this inundated line gives the swelling pressure. And under any pressure, the magnitude of heave or collapse is given by the difference in the void ratio on this line and the initial void ratio.

Shear Strength

In partly saturated soils, with sufficiently low degree of saturation ($<75\%$) it is generally known that air permeability will be high and water permeability will be low. In such cases there may not be much of a difference in drained or undrained shearing, only air being compressed in both the cases. In any case, the unconfined compression strength without any measurement of pore pressure may itself be a good reflection of the strength of the sample. Hence for a first level examination, unconfined compression tests conducted at considerably fast rate of shear, were examined. For a large number of data (Sivakumar Babu 1990) it has been observed (Fig. 36) that there is a general relation of the form

$$(e/e_L)\,(S_r)^{1/3} = 1.034 - 0.215 \log q \qquad (57)$$

irrespective of the type or amount of compaction used for the preparation

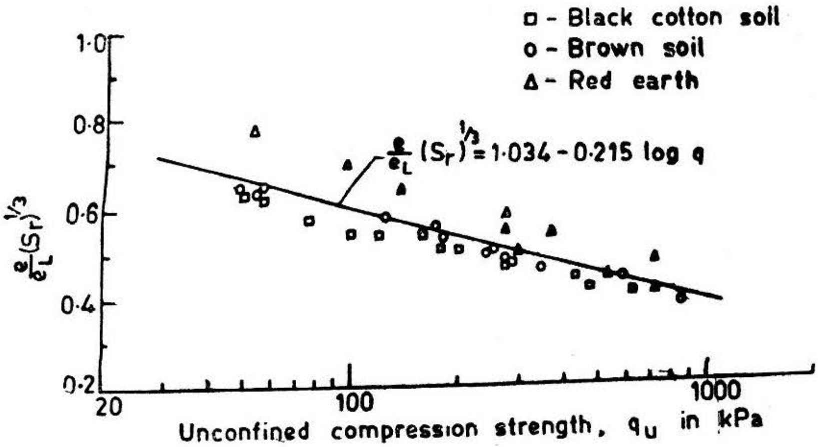


FIGURE 35 e/e_L $(S_r)^{1/3}$ **versus log** q_u **relation**

of the samples. This confirms further that (e/e_L) $(S_r)^{1/3}$, or correspondingly the total stress, reflects even the shearing resistance of partly saturated soils. Kirby (1989), has also shown that the shear behaviour of partly—saturated soils is very much similar to that of saturated soils in the total stress space, *i.e.* q versus p. They also exhibit similar forms of yield curve , critical state line, compression during shearing for states wet of critical and dilation for states dry of critical. Thus it appears that it is possible to develop constitutive models with e $(S_r)^{1/3}$, q, p coordinates (p being the total stress itself or $p—u_a$ if air pressure is not zero). This requires extensive study to arrive at appropriate relations.

For practical stability or bearing capacity solutions, the failure strength required may be obtained using eq. (57). This is on the assumption that for a given e $(S_r)^{1/3}$ the strength of the soil is the same at critical states whether normally consolidated or overconsolidated. Usually this may not be realised due to experimental difficulties and more so for partly saturated soils where it would not be possible to maintain e $(S_r)^{1/3}$ constant or its changes during shearing can be estimated. In any case, the unconfined compression test will give the conservative value.

Considering again Fredlund's (1978) work where it has been brought out that ϕ' for the osmotic component could be different from ϕ_b for the matrix component of normal stresses, it may be surprising as to why there exists a defined relation between total stress p (or e/e_L $(S_r)^{1/3}$) with q. It is the opinion of the authors that contribution of matrix component (which is pure tension) to shearing resistance is always $(u_a - u_w)/_2$ in which case, for tan ϕ_b to be equal to 0.5, ϕ_b should be 26.5°. For most soils, ϕ' also would be in the range of 20—26° and hence the average value reflected appears to be reasonable.

Concluding Remarks

Often, analysis and prediction of behaviour of partly saturated soils using modified effective stress equations tends to be complicated and unreliable due to the difficulty in measuring the involved parameters. The discussions presented in this paper point to the possibility of generalising the behaviours in terms of the total stress, which is uniquely related to a lumped state parameter $e\ (S_r)^{1/3}$. The resulting interrelationships can be used to predict the volume change and shear strength behaviours of partly saturated soils.

References

AITCHISON, G.D. (1961). Effective stresses in multiphase systems. Proc. III Australian-Newzealand Conf. on SM & FE: 290-212.

AITCHION, G.D. and BISHOP, A.W. (1960). Discussions, *Proc. of London Conf.* on Pore Pressure and Suction in Soils.

BISHOP, A.W. (1960). Discussions, *Proc. of London Conf.* on pore pressure and suction in soils.

BLIGHT, G.E. (1983). Aspects of the capillary model for unsaturated soils.

BLOCH, P. (1978). Discussion on stress state variations for unsaturated soils, By Fredlund and Morgenstern (1977) ASCE, 104 : GT2, 303-304.

CRONEY, D., COLEMAN, J.D., and BLACK, W.P. (1958). The movement and distribution of water in soil in relation to highway design and performance, Highway Research Board, Spl. Rep. No. 40, Washington.

EL-SOHBY, M.A. and MAZEN, O.M. (1987). On the prediction of swelling pressure and deformational behaviour of expansive a soils. 9th Reg. Conf. for Africa on SM & FE, Lagos, 1 : 129-133.

FREDLUND, D.G. and MORGENSTERN, N.R. 1(1976). Constitutive relations for volume change in unsaturated soils. *Can Geotech. J.*, 13 : 3, 261-276.

FREDLUND, D.G. (1979).Appropriate concepts and technology for unsaturated soils. *Can. Geotech. J.* 116 :1 , 121-129.

FREDLUND, D.G, MORGENSTERN, N.R. and WIDGER, A. (1978). Shear strength of unsaturated soils *Can. Geotech. J.* 15:3, 313-321.

GULHATI, S. (1975). Shear behaviour of partly saturated soils. Proc. 5th Asian Reg. Conf. on SM & FE. 1, Bangalore , 87-90.

JENNINGS, J.E. (1961). A revised effective stress law for the use in the prediction of the behaviour of unsaturated soils, *Proc. of London Conf.* on Pore Pressure and Suction in soils, Butter-worth, 1961, 26-30.

JENNINGS, J.E. and BURLAND, J.B. (1962). Limitations to the use of effective stresses in partly saturated soils. *Geotechnique*, 12-:2 , 125-144.

JONES and HOLTZ (1973) Expansive soils—The hidden diaster, *Civil Eng.. Dn.* ASCE, 43, CE-8, 49-57.

KOMORNIK, A. and DAVID, D. (1969). *Prediction of swelling pressure of clays.* ASCE, 95 : SMI, 209-225.

KRAHN, J. and FREDLUND, D.G. (1972). On total, matric and osmotic suction. *Soil Science*, 114 : 5,

KIRBY, J.M. (1989). Measurements of yield surfaces and critical state of some unsaturated agricultural soils. *J. of Soil Science*, 40, 167-182.

LAMBE, T.W. (1960). Mechanistic picture of shear strength in clay. Res. Conf. on Shear Strength of Cohesive Soils. Univ. of Colorado, Boulder, 555-580.

MATYAS, E.L. and RADHAKRISHNA, H.S. (1968). Volume change characteristics of partially saturated clays. *Geotechnique*, 18, No 4, 432-448.

NAGARAJ, T.S. and SRINIVASA MURTHY, B.R. (1983). An approach for prediction of swelling soil behaviour. 7th Asian Regional Conf. Haifa, August. 1.

NAGARAJ, T.S. and SRINIVASA MURTHY, B.R. (1985a). Compressibility of partly saturated soils, *Jl. of Geotech.*. Dn. ASCE, 111 : 7, 937-942.

NAGARAJ, T.S. and SRINIVASA MURTHY, B.R. (1985b). Rational approach to predict swelling soil behaviour. Transportation Res. Rec. No. 1032, TRB : 1-7.

NAGARAJ, T.S., SRINIVASA MURTHY, B.R. and VATSALA, A. (1990): Prediction of soil behaviour-Development of generalised approach, *Ind. Geotech. Jl*, 20 : No. 4.

NAGARAJ, T.S., SRINIVASA MURTHY, B.R. and VATSALA, A. (1991a) : Prediction of soil behaviour-Saturated Uncemented soils, *Ind. Geotech. Jl*, 21 : 1 .

NAGARAJ, T.S., SRINIVASA MURTHY, B.R. and VATSALA, A. (1991b) : Pre: diction of soil behaviour-Cemented Saturated Soils, *Ind. Geotch. Jl*, 21: 2.

NAYAK, N.V. and CHRISTENSEN, R.W. (1971). Swelling characteristics of compacted expansive soils. Clays and Clay Minerals. 19, 251-261.

SHASHIKUMAR, B.M. (1988). Analysis of volume change behaviour of partly saturated soils. M. Sc. thesis, Indian Institute of Science, Bangalore, India.

SIVAKUMAR BABU, G.L. (1990). Personal Communication, I.I.Sc., Bangalore.

SRIDHARAN, A. (1968). Some studies on the strength of partly saturated soils. Ph.D. thesis, Purdue University, Lafayette, Indiana.

SRINIVASA MURTHY, B.R. and NAGARAJ, T.S. (1988). Prediction of heave in swelling soils. 6th Int. Conf. on Expansive Soils. New Delhi, India, 1, 201-206.

VIJAYAVERGIYA, V.N. and GHAZZALEY, O.I. (1973). Prediction of swelling potential for natural clays. Proc. 3rd Int. Conf. on expansive soils. Haifa, Isreal, 1 : 227-236.

YONG, R.N., JAPP, R.D., and HOW, G.C. (1971). Shear strength of partially saturated clays. *Proc. Asian Reg. Conf.* on SM & FE, Bangkok, 1, 183-187.

ZACHARIAS, G. and RANGANATHAM, B.V. (1972). Swelling and swelling characteristics of synthetic clays. Proc. Symp. on Strength and Deformation Behaviour of Soils. Bangalore, 1 : 129-135.