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Three Phase Model for Wave Propagation Through Soils

by

S. Chandra*

A.K. Jha**

R.P. Singh***

Introduction

The importance of elastic waves for determining material properties is well recognized. The application of the method in carring out non-destructive testing of concrete, determination of cracks in concrete structures and determination of dynamic elastic properties of rock materials are quite common. One of the under lying assumptions in the methodology of these tests is the elastic properties of the material. Theory of elasticity has been used to derive the wave equations relating the physical properties of the material with the wave velocities.

Soil is not a truly elastic material and in a wet state it can be idealized as a three phase medium. In the recent past, attempts have been made to correlate the soil properties such as elastic moduli, density, moisture content, void ratio, porosity and degree of saturation with the *P* and *S*-wave velocities. The wave theory to derive the theoretical relationships can not be used with much accuracy for soil. Empirical relations have been established in the past using limited data. It has been felt necessary to develop a model for three phase medium such as soil saturated with water for propagation of elastic waves through such medium.

Theories for Propagation of elastic waves in porous and granular media

The propagation of elastic waves through a porous and granular media can be understood by considering a model consisting of elastic spheres which are in contact with each other. The deformations at contact points can be computed in terms of contact pressure and elastic constants of the spheres by the theory of elasticity (Hardin and Richart 1963). Hence,

*Department of Cilvil Engineering, Indian Institute of Technology, Kanpur-208016
**National Thermal Power Corporation, New Delhi
***Department of Civil Engineering, Indian Institute of Technology, Kanpur-208216

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the stress-strain relationship for an aggregate of spheres can be determined in terms of elastic properties of the spheres themselves. This approach has been followed by Gassmann (1951), Brandt (1955). Duffy and Mindlin (1957). Duffy and Mindlin (1957) carried out complete treatment of the problem in which both the normal and tangential contact pressures were considered.

One of the approaches for the porous medium is to use the wave propagation equation for a homogeneous isotropic elastic solid with certain modifications such as consideration of degree of coupling, viscosity, texture, elasticity etc. (Patterson 1956). The porous medium was defined as a medium composed of two constituents, solid and liquid or gas. The degree of coupling between two constituents of the medium affects the inertia of the medium. In the strongly coupled medium, there constituents are displaced in phase, however, it is not true in case of weakly coupled medium. Hence, the density is affected accordingly. The difficulty involved with this approach is again with the evaluation of structure factor in case of zero coupling and specific flow resistance in case of imperfect coupling.

Biot (1962) established the relationship between stress and strain for conservative physical system which was statistically isotropic involving four distinct elastic constants. The relationships used to obtain the equations of propagation of wave involved four constants. The elastic constants were related to the coupling between fluid and solid constituents. The main problem in this approach is the determination of the constants involved (Hardin and Richart 1963).

The theories described above involve too many constants, evaluation of these constant are difficult. Hence, the need for a simple three phase model for wave propagation through soils has been used successfully for predicting degree of saturation by sonic waves (Fattohi and Rahim 1988). Figure 1 shows the idealised rock sample for propagation of transient sonic waves in the laboratory. The idea was to develop a simple method for predicting phase of saturation between water, oil and gas. This finds use in estimating the relative amount of oil and gas present in the rock body. The predicted phase of saturation using this model was found to be in good agreement with the measured values.

Proposed model

In this paper, an effort has been made to obtain a model which could be used for determining the elastic wave velocity for a three phase material. The model suggested by Fattohi and Rahim (1988) was critically examined and the tests results reported by Sudhiram (1985) were analyzed. The idea of three phase idealisation as suggested by Fattohi and Rahim (1988) was applied to the test results and the travel (t) time was computed by the expression given below :

$$t = \frac{L}{V_p} = \frac{L-L\phi}{V_g} + \frac{L-L\phi\, S_r}{V_w} + \frac{L\phi}{V_a} \tag{1}$$

where L = length of sample

ϕ = porosity,

S_r = degree of saturation and

V_a, V_g, V_w are the compressional wave velocities through air, soil grains and water respectively.

Simplification of equation (1) gives

$$V_g = \frac{1-\phi}{[(1/V_p)-(\phi S_r/V_w)-(\phi/V_a)]} \tag{2}$$

Table 1 presents the values of V_g for different densities for a given value of water content. One can see from the table that compressional wave velocity through soil grains (V_g) varies with density. In case of three phase idealization of a soil as proposed by Fattohi and Rahim (1988), the velocity of compressional wave through soil grains (V_g) in highly consolidated and compacted state (*Figure* 1) should not change unless the mineralogical composition of the sample is changed. There is no justification for the variation of (V_g) in Table 2 since the measurements were conducted on one particular type of soil. Hence three phase idealization of soil samples for propagation of compressional wave velocity does not seem proper. To

TABLE 1

Results of three phase idealisation

Water content (%)	Dry density (gm/cc)	V_p (m/s)	Porosity	Degree of saturation	V_g (m/s)
	1.533	126.6	0.44	0.319	85.72
	1.600	165.3	0.41	0.351	123.75
	1.651	172.2	0.39	0.378	133.17
8.53	1.660	178.7	0.39	0.384	139.65
	1.688	181.7	0.38	0.403	144.11
	1.699	187.8	0.38	0.410	150.42
	1.710	196.2	0.37	0.416	160.55
	1.725	201.1	0.37	0.423	165.88

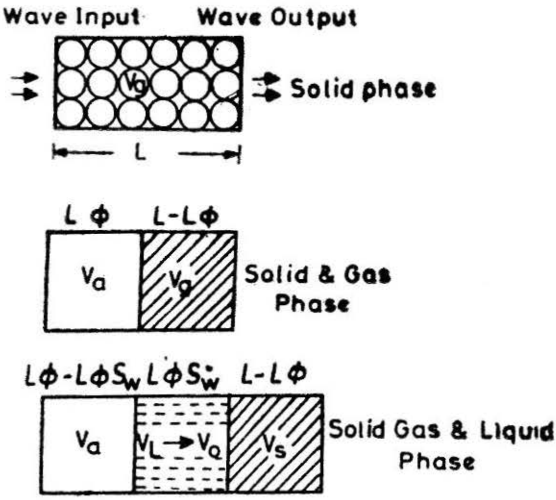

FIGURE 1 Three Phase Idealisation of rocks

overcome this difficulty, it is suggested that to study the propagation of compressional wave through soil, the soil should be idealized as consisting of soil frame and pore fluid as shown in *Figure* 2. As can be expected, the velocity of compressional wave through soil would get more affected by the arrangement of particles in a given matrix than the minerological composition of the soil particles. Hence, it is more logical to separate it into soil skeleton (soil solids completely filled with air) and water as two phases. Further, the importance of soil fabric in the behaviour of soil is incorporated better in this model. Travel time (t) for this compression wave propagation through a sample of length (L) is given by

$$t = \frac{L}{V_p} = \frac{L-L\phi S_r}{V_f} + \frac{L\phi S_r}{V_w} \tag{3}$$

where ϕ = porosity,

S_r = degree of saturation,

V_p = compressional wave velocity through soil sample,

v_f = velocity of the wave through frame and

V_w = velocity of the wave through water, for which a temperature of 25 degree centigrade is assumed.

Simplification of the above equation leads to an expression for frame velocity (V_f) as :

$$V_f = \frac{1-\phi S_r}{[(1/V_p)-(\phi S_r/V_w)]} \tag{4}$$

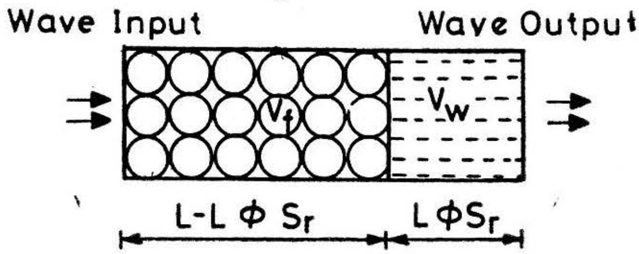

FIGURE 2 Soil modelled frame plus pore fluid.

Verification of Model

The data obtained by Sudhiram (1985) have been used for verification of the proposed model. Sudhiram (1985) conducted ultrasonic tests on 163 reconstituted soil samples. These samples were reconstituted from disturbed soil samples collected from 0.25—1.0 m depth from Golf course area at the AIT Bangkok campus. The physical properties of the soil are given in *Table* 2. The soil samples are compacted in three different sizes using triaxial mould, CBR mould and proctor mould. On each sample P and S wave velocity measurements are made. Water content at which soil samples are compacted varies from 6.6—19.25%. The density for each water content varies between 1.3—1—8 gm/cc. Unconfined compression tests are also conducted on each soil sample. Finally, V_p, V_s, static modulus and unconfined compression strength alongwith other relevant quantities have been reported. Water content in the soil samples are limited only about 20% to avoid difficulties in handling soft samples in terms of making proper contacts of probes with soil samples. We have also found that with the increase of water content more than 20%, it was difficult to measure shear wave velocity.

TABLE 2

Various Soil Properties

Parameter	Values
Liquid Limit (%)	60.00
Plastic Limit (%)	25.21
Plasticity Index (%)	35.49
Specific Gravity	2.88
Optimum moisture content (%)	15.00
Soil classification	MH

Suggested relationship among V_p, V_f and water content

In Table 3, the measured values of V_p for different water contents and dry densities are given alongwith the theoretical values calculated using equation (4) and the ratio of V_f/V_p. As one can see from the table, the velocity of compressional wave (V_p) through frames (V_f) varies with density for a particular value of water content. This is quite expected, as any increase in dry density for a given water content is due to closer packing of soil grains and a result the velocity through frame (V_f) increases.

Another interesting observation which can be made from Table 3 is that ratio V_f / V_p is fairly constant for a particular values of water content in the lower ranges of water content values. For higher water content it shows slight variation and an average value of V_f/V_p for a particular water content is computed. The plot of V_f/V_p against water content is shown is Figure 3, which is a straight line. The equation of the best fit with data is given as :

$$\frac{V_f}{V_p} = -0.014331\,w + 0.986954 \qquad (5)$$

Suggested equation for prediction of V_p.

Combining equation (4) and (5) and solving for V_p, one gets

$$V_p = \frac{V_w}{\phi S_r}\left[1 - \frac{1-\phi S_r}{(0.014331w + 0.98695)}\right] \qquad (6)$$

The above equation can be used to calculate V_p of a soil sample for which ϕ, S_r and w are known. In Figure 4, we have shown the predicted values of V_p using equation (6) against measured V_p values. One can see from the Figure 4 that in almost all the cases the predicted values are close to the experimental values. The maximum error in prediction of V_p has been found to be about 15 percent. However, in most of the cases the error is below 10 percent. Hence, the suggested relationship can be used with reasonable amount of accuracy.

Suggested equation for the prediction of degree of saturation

Simplification of equation (3) for S_r, yields:

$$S_r = \frac{[V_p/V_f)-1]}{V_p\phi[(1/V_f)-(1/V_w)]} \qquad (7)$$

Now, from equation (5), V_f/V_p can be computed for given value of water content and using equation (7), degree of saturation can be computed. Figure 5 shows the plot of predicted degree of saturation (S_r) against

TABLE 3

Results of the Model proposed

Water content (%)	Dry Density (gm/cc)	V_p experimental (m/s)	V_f (m/s)	V_f/V_p
	1.615	112.4	100.79	0.897
	1.653	120.2	107.40	0.893
	1.656	122.5	109.47	0.893
6.40	1.660	126.1	112.72	0.894
	1.700	138.9	123.91	0.892
	1.705	138.8	123.82	0.892
	1.602	135.8	120.61	0.888
	1.627	127.4	112.78	0.885
	1.681	145.5	128.43	0.882
	1.685	155.7	137.52	0.883
	1.728	163.2	143.79	0.881
7.24	1.730	161.2	142.00	0.881
	1.744	164.0	144.36	0.880
	1.746	173.4	152.76	0.880
	1.769	176.4	155.13	0.879
	1.533	126.6	110.34	0.871
	1.600	165.3	143.86	0.870
	1.651	172.2	149.18	0.866
	1.660	178.7	154.80	0.866
8.53	1.688	181.7	156.90	0.863
	1.699	187.8	162.11	0.863
	1.710	196.2	169.52	0.864
	1.725	201.1	173.66	0.863
	1.379	104.0	89.75	0.863
	1.485	121.4	103.55	0.853
	1.569	152.5	129.32	0.848
	1.631	168.9	142.55	0.844
	1.694	191.3	160.56	0.839
10.03	1.722	188.7	157.56	0.835
	1.743	204.0	170.34	0.835
	1.764	213.5	178.34	0.835
	1.785	220.4	178.27	0.834

TABLE 3 (*Contd.*)

	1.412	126.3	106.95	0.846
	1.491	150.6	126.48	0.839
	1.486	151.4	127.44	0.841
	1.629	202.5	168.18	0.830
11.12	1.687	226.5	187.08	0.826
	1.704	234.2	194.06	0.828
	1.751	257.8	211.55	0.820
	1.787	288.0	237.38	0.824
	1.807	288.0	236.37	0.820
	1.268	114.9	92.81	0.808
	1.408	147.5	116.22	0.788
	1.429	150.1	117.18	0.780
15.50	1.519	172.8	133.80	0.774
	1.603	196.2	150.15	0.765
	1.686	226.7	170.90	0.754

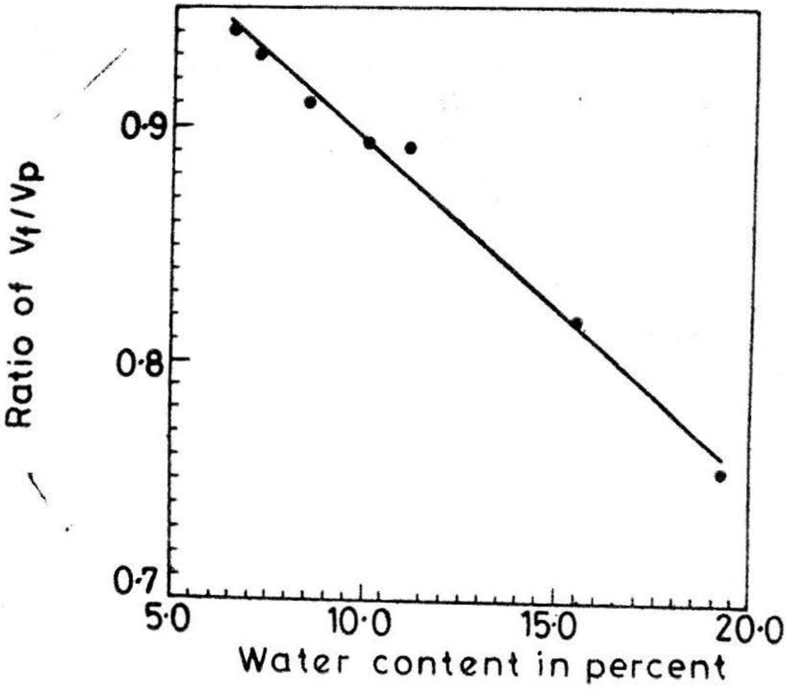

FIGURE 3 Variation of V_f/V_p with water content

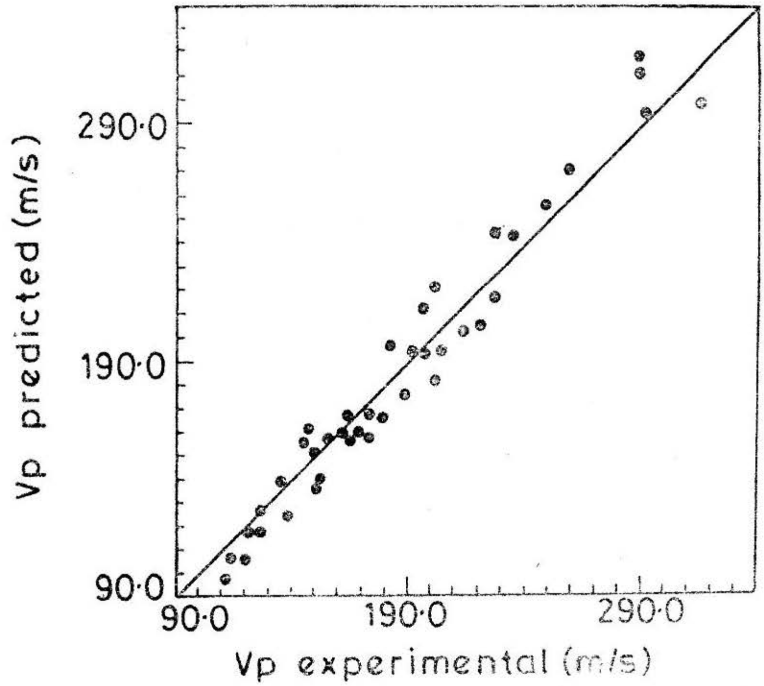

FIGURE 4 Variation of V_p predicted based on theoretical calculation with experimental values

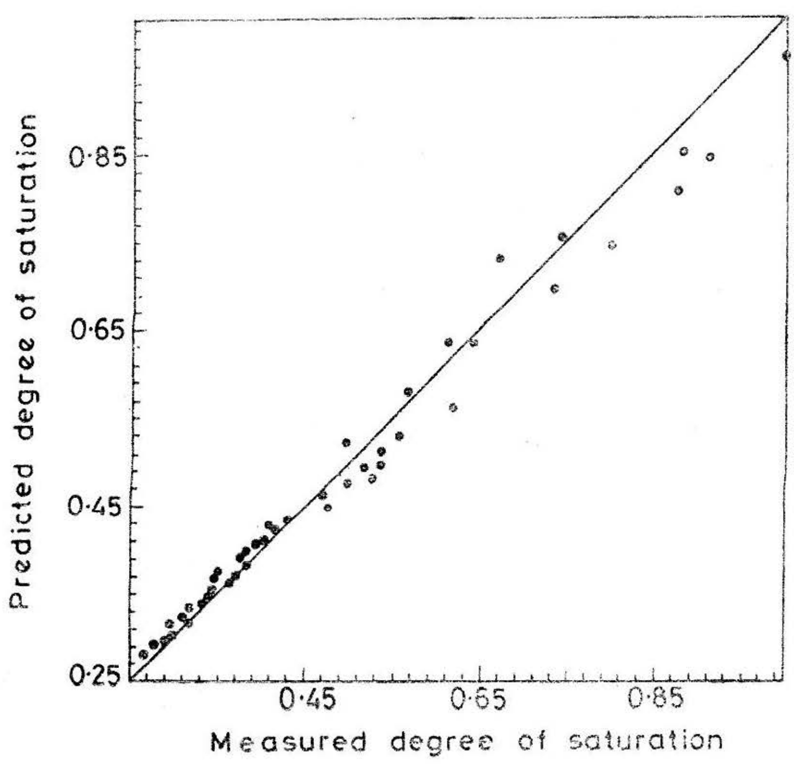

FIGURE 5 Variation of saturation (S_r) predicted based on theoretical calculation with experimental values

measured values. The predicted values are in good agreement with the experimental values. The maximum error in prediction of degree of saturation (S_r) has been found to be about 10 percent. The model discussed in this paper has been proposed on the basis of the measurements performed on one particular soil sample. However, 165 samples of the particular soil has been taken for measurement.

Conclusions

In this study, a new model has been suggested for propagation of compressional wave through soils. This model is simple and takes care of importance of soil fabric. Using the model proposed one can predict the compressional wave velocity (V_p) and degree of saturation (S_r) with reasonable amount of confidence. The maximum error in prediction of V_p and S_r is about 15 percent and 10 percent respectively. The present model has been proposed based on the results of tests performed on only one particular soil, but the idea is fairly general and should hold good for most of the soils. The proposed model will give an insight in understanding the physics of wave propagation through soil of different porosity and water content.

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