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Prediction of Soil Behaviour

Part II—Saturated Uncemented Soils

by

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Introduction

Design for stability in soil engineering, when reduced to the simplest form, can be visualised as a comparison between the external loading and the ability of the soil to withstand them at compatible levels of deformation. For the particulate system, soil, which requires a considerable strain (>20%) to mobilize its residual strength, it is more often the deformation or strain that controls the allowable stresses than the ultimate shear strength. Conventional method of analysis is to compute settlements treating the soil as a linear elastic material or to compute the maximum loads the soil mass could carry using limit equilibrium, as though the two are totally unconnected processes. The permeability and rate of loading conditions in the field are usually accounted for only by considering the extreme cases of undrained and drained conditions. With the emergence of the critical state concepts and the elasto-plastic constitutive models for soils, it has now been possible to unify all the above aspects into one framework which enables to predict the response of soil under any given loading knowing the basic compressibility and strength properties of the soil from conventional tests.

This paper is an attempt to proceed a step further to develop generalized models, based on the micromechanisms discussed in the earlier paper, which would enable to predict the basic properties of the soil (compressibility and shear strength) knowing only its index properties, insitu void ratio or water

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content, overburden pressure and to examine what these generalisations mean in terms of the elasto plastic models to result in a unique Cam-clay model for all fine grained soils. Also the validity and credibility of some of the currently available empirical relations for predicting these properties will be examined.

Generalisations of Compressibility Characteristics

The previous paper was culminated foreseeing a possibility of e_L serving as a parameter for generalising the mechanical behaviour of saturated soils based on the unique half space distanced versus the net repulsive force, (R-A) relation and of the existence of same order of physico-chemical potential, micro structure, stress conditions at equilibrium, shear strength and permeability coefficient at liquid limit water content for all fine grained soils. With this as reference state, subsequent changes due to further loading could be generalised. It was also seen that the generalised compression pathis obtained from the unique micro-analytical equation are of the form

$$e/e_L = 1.191 - 0.358 \log \sigma \quad (11)$$

The one dimensional compression paths of eleven natural soils from literature analysed by Nagaraj and Srinivasa Murthy (1986*a*) to examine the above mode of generalisations are shown in Fig. 9. The liquid limit of these soils has a wide variation (36%—160%) which covers the normal range encountered with natural soils. The spatially spread out e-log p curves of the different soils collapse into a narrow band in e/e_L-log p plot, when normalised with their respective liquid limit void ratios. Resulting relation has been linearised in the working stress range of 25-1000 kPa in the form

$$\left| \frac{e}{e_L} \right| = 1.122 - 0.2343 \log \sigma \quad (12)$$

with a correlation coefficient of 0.962. This generalised equation is of the same form as equation (11), but the constants differ considerably. This implies that the diffuse double layer theory is applicable to natural soils in general, but not in entirity, may be because of the deviations from the assumptions in the theory. From a more detailed investigation on this aspect (Nagaraj and Srinivasa Murthy, 1986*b*) it has been logically shown that either the orientation of the particles or the presence of coarse fraction or a possibility of contact stresses between particles cannot be a factor for such deviations, and that a possible reason could be a gradual reduction in operating specific surface resulting from grouping of particles into larger clusters with increase in stress. The phenomenon of reduction in specific surface has been further substantiated by a comparison of the computed and experimental compression paths of Na-Montmorillonite reported by Klausner and Shainberg (1967). They have computed e-log σ relations for

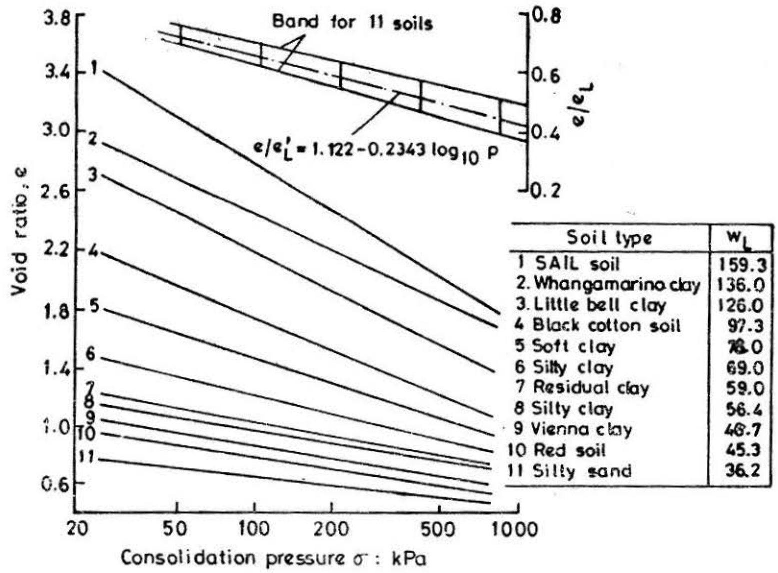

FIGURE 9 Experimental e—log σ and (e/e_L)—log σ Plots (Nagaraj and Srivanivasa Murthy, 1986a)

a given physico-chemical environment and for different number of particles in a cluster assuming interactions to be between such clusters. The experimental curve superposed on the same plot cuts across these curves indicating that the number of particles in a cluster increases during the processes of consolidation (Nagaraj and Srinivasa Murthy, 1986*b*). There is ample evidence from micro structural observations (Aylmore and Quirk, 1960, 1962; Sloane and Kell, 1966; Smart, 1967 and Delege and lefebrve, 1984) that there is aggregation of clay platelets forming domains or tactoids or clusters of oriented platelets. There is also an indication that (Dickson and Smart, 1974) the size of the domain increases with loading and that there is a breakdown of domains during the post peak deformation.

It has been further shown (Nagaraj and Srinivasa Murthy, 1983) that the reduction in specific surface at any stress level is a function of the original specific surface and hence the experimentally obtained relation (eqn. 12) wherein this reduction is inbuilt, is the true generalised compression curve of uncemented normally consolidated saturated fine grained soils.

The process of particle grouping being irreversible (to a large extent) upon unloading, further interactions take place between such clusters with reduced specific surface and hence the rebound recompression paths are flatter. With the fact that the reduction in specific surface is a function of the original specific surface, it must be possible to generalize the rebound paths also with e_L itself. From the data of the same eleven soils (Nagaraj and Srinivasa Murthy, 1986*a*) it has been shown that the slope of the generalised average rebound-recompression path in e/e_L-log σ is unique for all soils, equal to 0.0463 with a correlation coefficient of 0.98 (Fig. 10). The position of the rebound paths depends on the value of the maximum preconsolidation pressure σ_c and hence the equation for the generalised compression path for overconsolidated states can be derived incorporating this additional parameter σ_c as: (Nagaraj and Srinivasa Murthy, 1985, 1986*b*)

$$\frac{e}{e_L} = 1.122 - 0.188 \log \sigma_c - 0.0463 \log \sigma \qquad (13)$$

A further observation is that the compression paths of soils can be linearised within the working stress range even in the log e/e_L versus log σ plot with nearly the same degree of correlation to the form

$$\log \frac{e}{e_L} = 0.1433 - 0.168 \log \sigma \qquad (14)$$

Generalisation of Shear Strength Behaviour

It is a known fact that the shear strength of a particulate system is a function of the normal stress. It has been well observed that the *e*-log *q* plot of a saturated soil is nearly parallel to its *e*-log σ plot. In fact it is

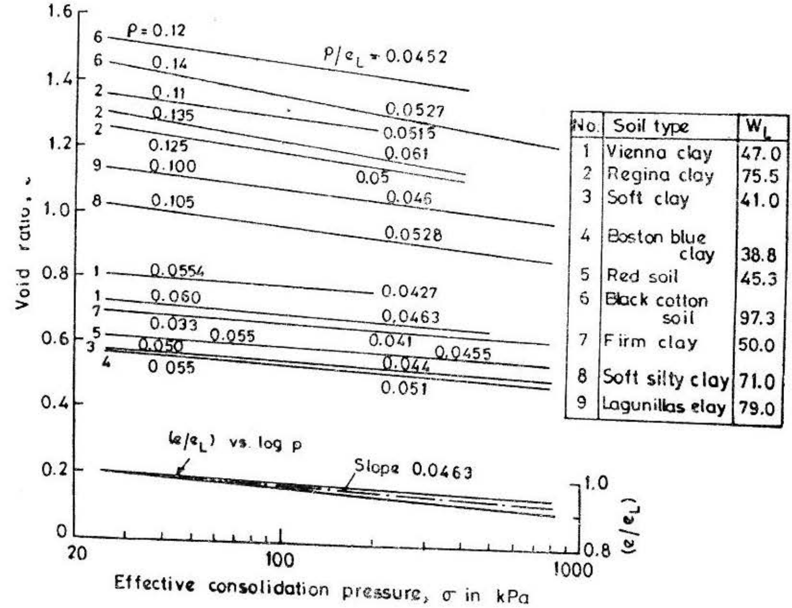

FIGURE 10 Experimental Average Rebound—Recompression Curves and Generalized (e/e_L)—$\log_{10}\sigma$ Plots

understood that at microlevel both consolidation and shearing cause re-arrangement of particles. In the constitutive modelling of soils, consolidation and shearing are considered to be two aspects of the same process. With all this, one can expect similar generalisations even for shear strength behaviour to result in a unique e/e_L-log q relation.

Nagaraj and Jayadeva (1981) have shown that the flow curves of different soils can be generalised using their respective liquid limit values to obtain unique relations of the form

$$\frac{w}{w_L} = a - b \log N \text{ (for Casagrand'e percussion method)} \tag{15}$$

$$\frac{w}{w_L} = a + b \log D \text{ (for cone penetrometer method)} \tag{16}$$

These two tests to determine liquid limit are analogous to shear tests, the number of blows, N or the depth of penetration, D being a measure of undrained shear strength at that water content. The above generalisation, is in a way, a proof for the existence of a unique e/e_L-log q relation.

To verify the possibility of such generalisations at higher working stress levels, the strength data on Weald clay and London clay by Henkel (1960), which are widely referred to in the field of geotechnical engineering and the data of two other soils generated in this laboratory, were examined in greater detail. The liquid limit of the four soils cover a good range from 43% to 106% (Srinivasa Murthy *et al.* 1988).

The isotropic consolidation plots and the void ratio versus strength plots of these four soils are shown in Figs 11 and 12. The collapse of these widely spread paths into narrow bands upon normalisation with respective e_L values can also be seen in Figs 11 and 12. It can be seen that the paths of London clay and SAIL soil which have nearly the same liquid limit closely follow each other. Statistical best fit equations for the generalised lines are of the form

$$\frac{e}{e_L} = 0.9453 - 0.1983 \log p_m \tag{17}$$

$$\frac{e}{e_L} = 0.923 - 0.195 \log p_f \tag{18}$$

$$\frac{e}{e_L} = 0.8189 - 0.1879 \log q_f \tag{19}$$

In essence these generalisations mean that in fine grained soils, the equilibrium pressure and the shearing resistance are functions of the distance between particles or interacting units.

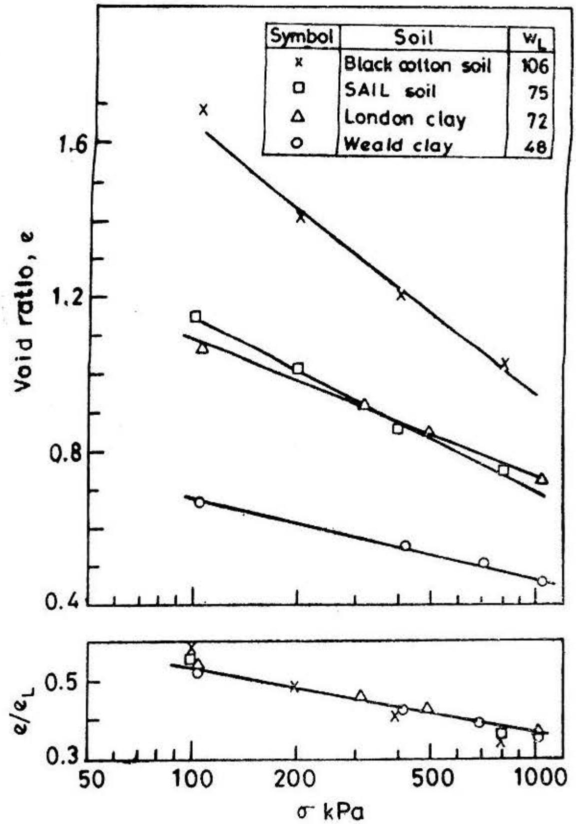

FIGURE 11 e—log p Curves for Isotropic Consolidation and their Generalisation (Srinivasa Murthy, Vatsala and Nagaraj, 1988)

Now, since e-log p_f and e-log q_f plots are parallel for a given soil, and since each of these plots is independently generalisable for all soils, it implies that the generalised e/e_L-log p_f and e/e_L-log q_f are also parallel to each other. It can be seen from the above equations (18 and 19) that these lines are parallel for all practical purposes. A direct implication of such parallelism is that $M = q_f/p_f$ (which is related to ϕ as $M = 6 \sin\phi/(3 + \sin\phi)$) is of the same order for all soils. Fig. 13 shows the Mohr's envelope for these four soils having a constant ϕ within the limits of accuracy at engineering level.

The existence of a unique M value is not reflected in reality since values of ϕ ranging anywhere between 12° to 35° have been reported in literature and hence requires examination in greater detail.

The data presented by Schofield and Wroth (1968) (extracts in Table 2) for five proven normally consolidated soils at their critical state shows a variation in the value of M from 0.845 to 1.02, (ϕ from 22° to 26°) which is

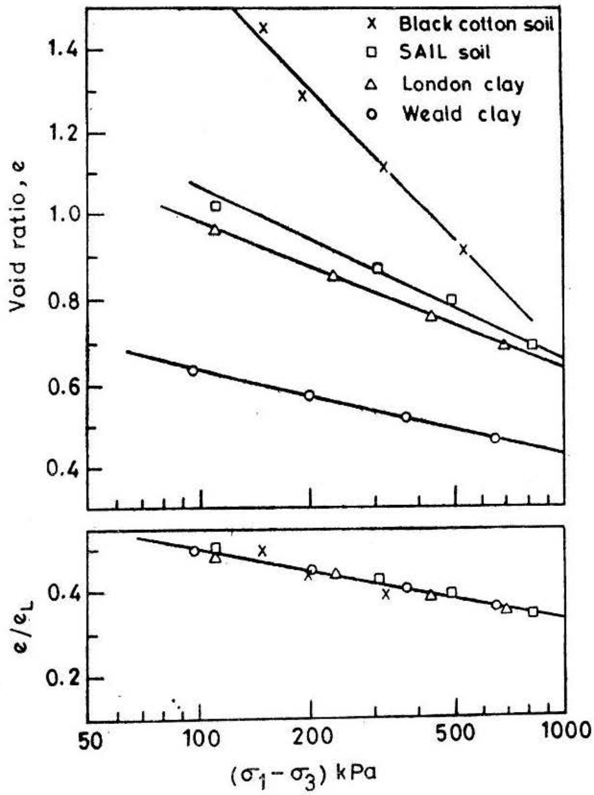

FIGURE 12 e—log (σ_1-σ_3) Curves and Generalization of Shear Strength Behaviour (Srinivasa Murthy, Vatsala and Nagaraj, 1988)

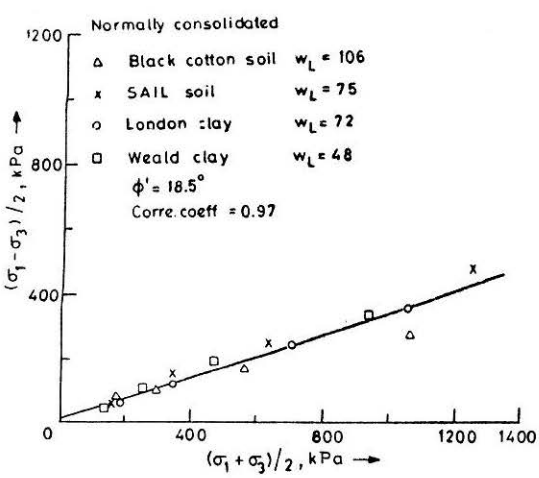

FIGURE 13 Modified Mohr—Coulomb Diagram for Normally Consolidated Clays

TABLE 2

Liquid limit, M and ϕ values of Normally consolidated clays (Data from Schofield and Wroth, 1968)

Soil	w_L	M	ϕ
Klein Belt Ton	127	0.845	21.74
Weiner Tegel	47	1.010	25.61
London clay	78	0.888	22.75
Weald clay	43	0.950	24.21
Kaolin	74	1.020	25.84

quite small for as wide a variation in liquid limits as 43% to 127%. This indicates that for all ideal conditions, a nearly constant ϕ is a possibility. Deviations from this constant value could be due to deviations from the ideal conditions such as soil being sensitive with cementation, or the state of soil not reflecting the true effective stresses as in overconsolidated and/or partly saturated soils or the soil not having reached the critical state, or variations in strain rate.

The existence of a unique frictional factor appears reasonable if the assumptions of a continuous water phase and the absence of direct contact between particles are valid. In such a case, the failure plane must pass through the fluid phase. It is then obvious that the shearing resistance is independent of surface frictional characteristics of the mineral but is a function of the internal forces mobilised. For a known physico-chemical environment, these internal forces mobilised to keep the applied pressure in equilibrium are the same for all soils and hence q_f/p_f must be a constant. shearing resistance may be regarded as 'colloidal friction , which may be analogous to the case of two magnets of like poles, where the force required to move them laterally is a function of the repulsive force between them.

Of course, it may be noted here that ϕ for all soils still is not a pin-pointed value. In fact it is generally observed that ϕ is slightly higher for massive clay minerals like Kaolinite. Frictional resistance in clay may be thought of as electrical interference, if not physical interference as in sands. Greater energy may be required to slide the particles relative to each other or to dismember the clusters, due to stronger attraction between positive edges and the negative surfaces of particles in Kaolinite minerals. But in any case, *M* cannot be very different from the range of values in Table 2, for the usual sheet type clay minerals.

A consideration of the above mode of stress transfer through physico-chemical interactions also explains the following two observed facts with fine grained soils which is otherwise difficult with the usual contact model.

(*a*) two soils can mobilise different strengths at the same void ratios.

(*b*) two soils can have the same strengths at widely different void ratios although their friction angles are nearly same.

As seen in Fig. 14, four soils equilibriate at different void ratios (A, B, C, D) under the same pressure, p but have the same generalised state ($e/e_L = A_1, B_1, C_1, D_1$), or the same '$d$' spacing between the particles. Shear strength being uniquely related to the 'd' spacing will be the same for all soils (A_1, B_2, C_2, D_2). The measured strength values (A', B', C', D') slightly vary from the constant value which may be due to experimental limitations. Similarly at a constant void ratio, different soils will have different 'd' spacings and hence different strengths (Fig. 15).

Overconsolidated Soils

The above considerations show that the failure plane passes through the interacting fluid phase and hence there need not be any difference in the

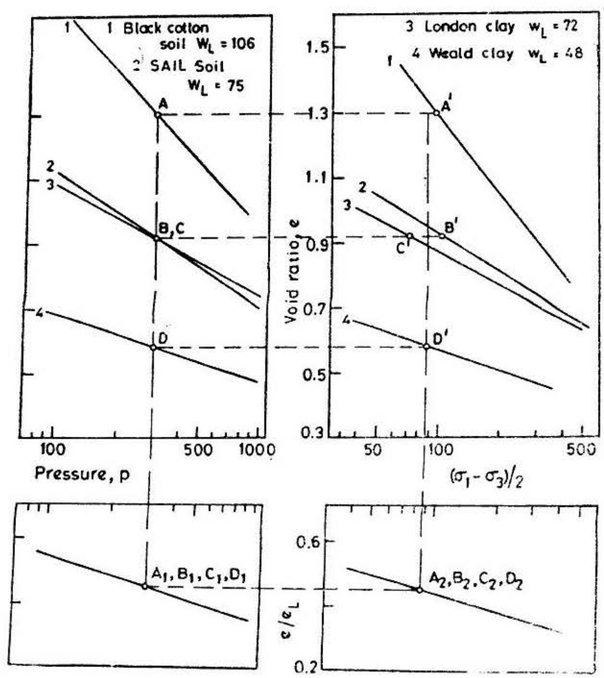

FIGURE 14 Graphical Representation of Shear Strength of Soils at the same Consolidation Pressure

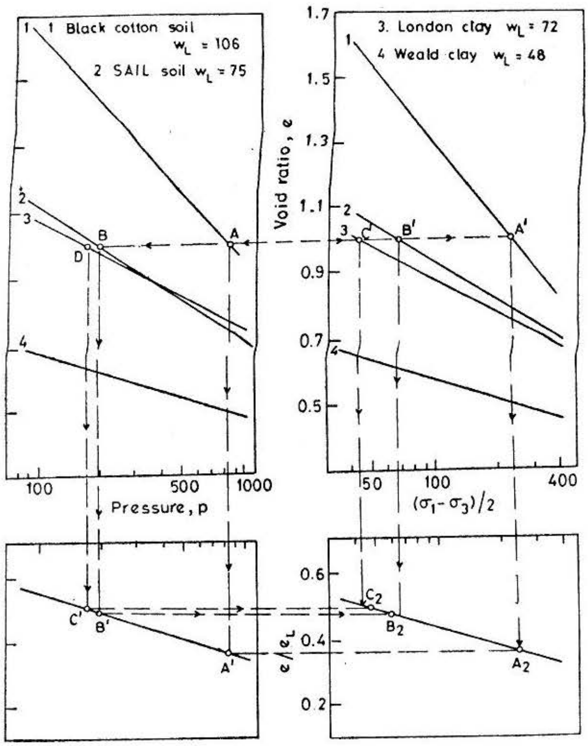

FIGURE 15 Graphical Representation of Shear Strength of Soils at the Same Void Ratio

shearing resistance of normally and overconsolidated soils, the only difference being that the interaction is between larger clusters in over consolidated soils instead of individual particles. However there may be differences in strain levels due to differences in the operating surface area in the two states as it is well known that the surface area or the liquid limits reflect the plastic modulus. In addition, it is likely that the clusters formed during loading to a higher stress get dismembered gradually with shearing, finally reaching the same state as a normally consolidated state would do at very large strains. Thus ϕ should be constant for both states at very large strains.

To verify this, results of Weald clay and London clay, overconsolidated to a maximum pressure of 846 kPa (120 psi) and those of SAIL soil and Black Cotton Soil with a p_c = 800 kpa have been examined. Fig. 16 shows the Mohr's envelope where the ϕ for overconsolidated soils is nearly the same as that for normally consolidated soils.

Further, it was also possible to generalise the peak strengths for samples rebounded from 800 kpa, in the form

$$\frac{e}{e_L} = 0.5639 - 0.076 \log q \tag{20}$$

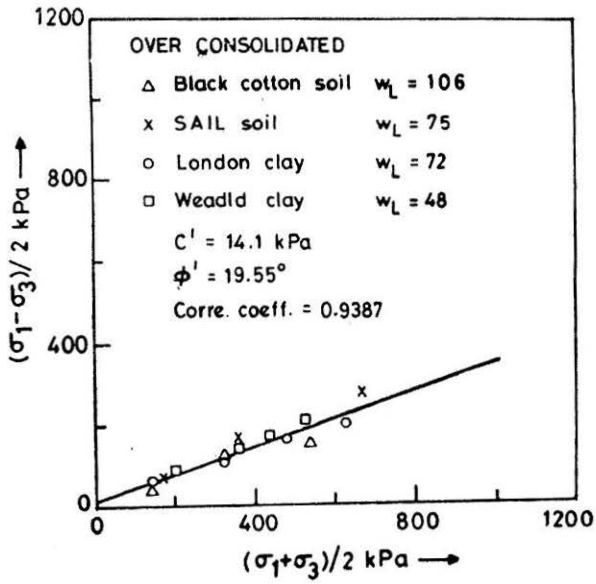

FIGURE 16 Modified Mohr-Coulomb Diagram for Overconsolidated Clays

It can be expected that similar generalised lines will be obtained for different maximum pressures all of which will be parallel to each other as was the case with e-log p plots. This aspect, however, needs further experimental verification. If this is true, a generalised equation for peak strengths can be suggested involving p_c as

$$\frac{e}{e_L} = 0.969-0.1395 \log p_c-0.076 \log q \qquad (21)$$

Prediction of Compressiblity Behaviour

With the two generalised equations (equations 12 and 13) or the alternate form of eqn. 14, it is now possible to predict the compressibility behaviour of saturated uncemented fine grained soils knowing only void ratio, over burden pressure and liquid limit. But before prediction, it is essential to identify the nature of the soil system and see whether the above generalisations are applicable to that system. For this there is no well defined and simple method in literature. But now, the same eqn. 12, which represents the generalised compression path for normally consolidated states provides a means for identifying the nature of the saturated soil system in field (Fig. 17). If the state of the soil in field (e, σ) falls along this line (zone 1), the soil is normally consolidated and if it lies above this line, the soil is cemented and soft sensitive. If the soil state lies below this line it is overconsolidated. But it could be cemented or uncemented. In that case one additional shear test will be required to differentiate between these two conditions.

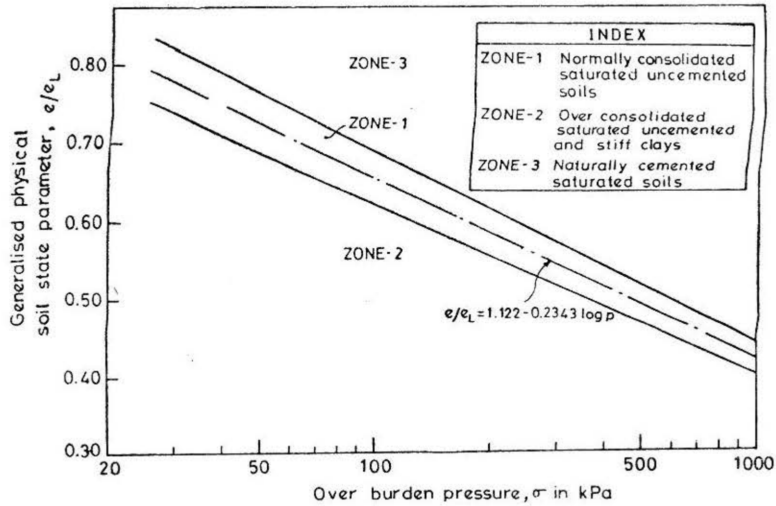

FIGURE 17 Different Zones to Identify Soil States from Generalised State Parameter—Pressure diagram

The discussions in this paper are restricted to prediction of uncemented saturated soils only. Prediction of cemented soils will be discussed in the next paper.

The compressibility of a soil is usually defined in terms of the slope of the *e*-log *p* curve, C_c=de/d (log *p*). If the soil is identified to be normally consolidated then from equation 12,

$$C_c = \frac{de}{d\,(\log p)} = 0.234\, e_L \tag{22}$$

which predicts a constant slope for compression line with pressure and it is a function of the liquid limit of the soil.

Alternately, using the other form of eqn. 14, C_c=de/d (log) *p* works out to be 0.39 *e*, which gives a path with variable slope dependent on the stress level.

The two equations yield the same value at a stress of about 200 kpa at which the void ratio is about 0.6 e_L.

If the soil is identified to be overconsolidated, then the preconsolidation pressure can be predicted using the eqn. 13 and the known values of *e* and *p* in the field. Then the compressibility of the soil can be predicted using the same eqn. 13, for stress levels upto the p_c, and using eqn. 12 for stress levels beyond the level of p_C.

Currently Available Methods for Prediction of Compressibility

A direct method for assessing compressibility is to run laboratory consolidation tests on undisturbed soils. Great care and time are required to obtain reliable data where a large number of samples are to be tested. This also involves considerable cost. Hence there have been several attempts in literature to derive simple empirical relations to correlate compression index with inferential and or state parameters based on experimental results of local soils. Table 3 lists a few commonly used relations, amongst which Skempton's (1944) relation $C_c = 0.007\,(w_L - 10)$ is the most widely accepted one. This equation was derived based on the test results of normally consolidated and moderately sensitive soils whose initial water contents were at their liquid limits. It has been possible to show that (Nagaraj and Srinivasa Murthy, 1986*a*) the expression $C_c = 0.234\ e_L$ can be reduced to

$$C_c = 0.0075\,(w_L - 9.46) \tag{23}$$

which is of the same form as Skempton's equation by suitable substitutions using the known relations of $e/e_L = d/d_L = d/82A°$ and $e = G\,\gamma_w\,S\ \ d$ and specific surface, $S = (-14 + 1.48\ w_L)$. Thus it was concluded that Skempton's relation follows Gouy Chapman's theory and is applicable to normally consolidated uncemented soils.

The relation, $C_c = 0.009\,(w_L - 10)$ proposed by Terzaghi and Peck (1948), is of a similar form but gives slightly higher values. Hence it may be applicable to moderately sensitive soils which exhibit steeper compression curves beyond the level of quasi-preconsolidation pressure.

The other relation, $C_c = 0.0046\,(w_L - 9)$ which yields lower C_c values was similarly concluded to be following double layer theory and to be applicable for overconsolidated soils.

The relations in Table 2 can in general be grouped into two categories

(*a*) in which C_c is related to the liquid limit

(*b*) in which C_c is linked with natural water content or insitu void ratio of soil.

Nagaraj and Srinivasa Murthy (1986*a*) have critically examined each of these equations for their validity comparing them with the generalised model developed earlier and have identified the nature of soil system for which it is applicable in each case.

Prediction of Shear Strength

Shear strength of a soil is conventionally defined in terms of the two components cohesion, c and angle of internal friction, ϕ. But now it is fairly accepted that there is truly nothing like cohesion unless the soil is chemically cemented. The cohesion component observed with overconsolidated or partly saturated soils is due to our inability of assessing the

TABLE 3

Compression Index Equations (Nagaraj and Srinivasa Murthy, 1986a)

Equation	Reference	Regions/Conditions of Applicability: As indicated in reference	Regions/Conditions of Applicability: Inferred by authors
$C_c=0.007\ (W_L-10)$	Skempton (1944)	Remoulded clays	Normally consolidated, $S_t<1.5$
$C_c=0.009\ (w_L\text{-}10)$	Terzaghi & Peck (1948)	Normally consolidated, Moderately sensitive	Moderately sensitive $S_t<5$
$C_c=0.01\ w_n$	Koppula (1981)	Chicago and Alberta	Normally consolidated $S_t<1.5$
$C_c=0.0115\ w_n$	Bowles (1979)	Organic silts and clays	Normally consolidated $S_t<1.5$
$C_c=1.15\ (e\text{-}e_o)$	Nishida (1956)	All clays	-do-
$C_c=1.15\ (e\text{-}0.35)$	-do-	-do-	-do-
$C_c=0.54\ (e_n\text{-}0.35)$	-do-	Natural soils	-do-
$C_c=0.75\ (e_o\text{-}0.50)$	Bowles (1979)	Soils with low plasticity	Moderately sensitive $S_t<5$
$C_c=0.0046\ (w_L\text{-}9)$	Bowles (1979)	Brazallian clays	Moderately overconsolidated
$C_c=1.21+1.055 \times (e_o\text{-}1.87)$	-do-	Motley clays from Sao-paulo city	Highly sensitive $S_t>5$
$C_c=0.30\ (e_o\text{-}0.27)$	Hough (1957)	Inorganic silty sand-silty clay	Overconsolidated
$C_c=0.208 \times (e_o+0.0083)$	Bowles (1979)	Chicago clays	Moderately overconsolidated
$C_c=0.156\ e_o +0.0107$	-do-	All clays	-do-
$C_c=0.5(\gamma w/\gamma a^2)^{1.2}$	Oswald (1980)	Soil system of all complexities and types	Not applicable to any condition

Note: C_c—Compression index
w_L—liquid limit water content
γ_d—dry density of soil at which C_c is required
e_o—initial or insitu void ratio
w_n—natural water content
γ_w—unit weight of water
S_t—sensitivity of the clay

true effective stresses. Also, it is known that these two parameters are not constants for a given soil but are influenced by several factors. A more convenient way to assess strength may be through the state parameter, since it is established that shear strength is uniquely related to the void ratio of the soil at very large strains. From the discussions in the earlier sections there are two generalised equations (19) and (21) for failure shear strength. Strictly speaking, the overconsolidated samples also should reach the same critical state line as normally consolidated states at very large strains but it is felt that the experimental values will become unreliable (Atkinson and Bransby, 1978) at such large strains in overconsolidated soils due to the formation of thin shear zones and nonuniformity in the sample.

With the equations (19) and (21), the undrained strength of any soil at any given void ratio can be predicted knowing only its liquid limit and p_C if the soil is over consolidated. However while using equation (21), caution is to be excercised since the slope of the line is too small and the strength is in logarithamic scale, a small error in e/e_L can lead to unrealistic predictions. The purpose of giving this equaion was only to indicate the possibility of generalisation.

Alternately, since it has been shown that ϕ (or M) varies within a very narrow range for all soils exhibiting particulate behaviour and devoid of any stress history effects and cementaton, the failure strength can be predicted with $c = 0$ and $\phi \simeq 20°$. But for engineering applications, the behaviour at strains with in the allowable limits will be of interest. In that case, one has to adopt the appropriate stress-strain relations. A discussion on the generalised Cam-clay model is presented later.

Currently Available Relations for Predicting Shear Strength

The two empirical relations which are most commonly used for strength predictions are:

(1) For normally consolidated undisturbed clays (Skempton and Henkel 1953; Skempton 1954)

$$C_u/p = 0.11 + 0.0037\, I_p \qquad (24)$$

Where c_u is the undrained shear strength, p is the overburden pressure and I_p is the plasticity index.

(2) I_L vs remoulded strength relation in graphical form (Houston and Mitchell, 1967; Schofield and Wroth, 1968)

The latter is not used as much as the former, since what one needs in field is the undisturbed strength and not the remoulded strength. The validity of these two relations from micro mechanistic considerations merits examination.

C_u/p vs I_L Relation

There have been several studies (Bjerrum, 1954; Grace and Henry, 1957; W_u, 1958; osterman, 1960; Metcalf and Townsend, 1960; Leonards 1962, Bishop and Henkel, 1962; Lumb and Holt, 1968; Cox, 1970) in literature to examine the validity of Skempton's relation. A large number of data agree with this relation and there is an equally large volume of data that contradicts it. Most of the above examinations have been only experimental. Only a few (Kenny, 1959, 1960) have concluded that the C_u/p ratio for undisturbed soils is strongly dependent on geological and time history of the soil and cannot be reflected by two inferential parameters ($I_L = w_L - w_P$) determined on completely remoulded soil. Sridharan and Rao (1973) have concluded that no linear increase in C_u/p occurs with increase in I_p and instead both theoretical and experimental results tend to show a decrease in C_u/p with increase in I_P. But no attempt has been made to understand the basic considerations.

Plasticity index, being the numerical difference between liquid and plastic limits, can be expressed in terms of the 'd' spacings corresponding to these states as

$$I_P = kd_L - kd_P \qquad (25)$$
$$= k(d_L - d_P)$$

where k is the functional relationship between 'd' spacing and the water content.

Since d_L and d_P are uniquely related to specific values of equilibrium pressures and shear strengths, ($d_L - d_P$) represents a constant range of change of pressure or shear strength for all soils, although there can be a wide range of I_P value ($w_L - w_P$) depending on the surface area of the soil. Hence I_P cannot reflect any variation of mechanical properties.

Alternately, as discussed earlier, the soil behaviour can be generalised to yield unique relations for normally consolidated uncemented soils in the form

$$d = a - b \log p \qquad (26)$$

$$d = c - b \log q \text{ or } d = c - b \log C_u \qquad (27)$$

where the two equations are parallel with the same slope b. Subtracting one from the other

$$\log \frac{C_u}{p} = 10^{(a-c)/b} \qquad (28)$$

In these equations, a, b and c being constants irrespective of the soil type, C_u/p is also a constant *i.e.* C_u/p vs I_P should be a horizontal line. Prediction of C_u/p magnitudes from index properties can be further examined. Schofield and Wroth (1968) have derived an expression for C_u/p, based on critical state concepts in the form

$$\frac{C_u}{p} = \frac{1}{2} M \exp-\{(\lambda-\kappa)/\lambda\} \tag{29}$$

Values of C_u/p computed from this equation and the corresponding values using Skempton's equation for the five soils reported by them are indicated in Table 4. It can be seen that the values of C_u/p computed from eqn. (29) using the actual values of λ, κ and M are fairly constant with an average of 0.24 as against a variation of 0.2025 to 0.4467 given by Skempton's equation, for the reported values of λ, κ, M, w_L and w_P. In eqn. (29), p is the isotropic consolidation pressure. For k_o consolidation pressures values of C_u/p would be slightly smaller but still would be of the same range for all soils. Further, Skempton's relation predicts higher strengths for highly plastic soils than for low plastic soils at the same confining pressure which is contrary to the generally observed behaviour.

From the above discussions it can be concluded that C_u/p vs I_P relation is not tenable for uncemented normally consolidated soils. From a thorough examination of Skempton's work, Srinivasa Murthy *et al* (1986) have shown that the data used to arrive at this relation were of sensitive soils, and have also shown that the possibility of such a relation is much less reasonable for sensitive soils. This is because in sensitive soils, C_u is nearly constant for all confining pressures upto the yield stress p_c, and hence C_u/p varies with the level of p, even for a single soil. Despite the above

TABLE 4

Comprarison of C_u/p values (Data after Schofield and Wroth, 1968)

Soil	Klein Belt ton	Wiener Tigel. V	London clay	Weald clay	Kaolinite
w_L (%)	127	47	78	43	74
w_p (%)	36	22	26	18	42
λ	0.356	0.122	0.161	0.093	0.260
k	0.184	0.026	0.062	0.035	0.050
M	0.85	1.01	0.89	0.95	1.02
C_u/p (Eqn. 29)	0.261	0.23	0.240	0.245	0.227
C_u/p (Eqn. 21)	0.447	0.203	0.271	0.203	0.228

limitations, this relation is being used extensively by geotechnical engineers even to this day, this being the only available simple tool.

I_L Versus Remoulded Strength

Based on the observation of a definite trend in the relation between liquidity index and remoulded strength of several clays (Skempton and Northey, 1953; Schofield and Worth, 1968; Houston and Mitchell, 1969; Wroth and Wood, 1978) have linearly idealised this relation. However, no scientific basis for such a relation was identified.

Liquidity index being a ratio of the difference in natural water content and plastic limit to the plasticity index, can be expressed in terms of the 'd' spacing between particles as

$$I_L = \frac{w_n - w_p}{w_L - w_p} = \frac{kd - kd_P}{kd_L - kd_P} = \frac{d - d_P}{d_L - d_P} \tag{30}$$

Since the functional form 'k' which reflects the soil type gets erased out, and d_L and d_P are constants for all soils, I_L reduces to be a function of 'd' alone.

As seen earlier, the generalised state parameter, e/e_L is also a function of 'd' and hence there must be a relation between I_L and e/e_L. From the data of consistency limits as reported by various sources and at different assumed water content values, Srinivasa Murthy *et al.* (1986) have shown that the relation between the two can be expressed by the equation

$$I_L = 1.548 \left[\frac{w}{w_L} \right] - 0.559 \tag{31}$$

with a correlation coefficient of 0.992. As it has been proved based on Gouy-chapman theory that there is a unique relation between e/e_L and q, I_L versus remoulded strength relation should also be tenable and will be applicable for normally consolidated states. But, then e/e_L is a better parameter to correlate the mechanical properties than I_L, because this does not involve the additional parameter plastic limit. It has been shown (Nagaraj and Jayadeva, 1983; Tamas Paul, 1984) that plastic limit is only a function of liquid limit and not an independent parameter for usual inorganic soils with sheet type minerals and hence liquid limit alone will be sufficient to reflect the soil type.

Generalised Cam-clay Model

It is known that the popularity of the elasto-plastic Cam-clay model is because that it combines all loading processes, whether consolidation-isotropic or anisotropic, or shearing into one framework and enables a complete description of the stress-strain behaviour for any loading. In this section, it is attempted to extend the generalisation obtained earlier to stress-strain relations to result in a unique Cam-clay model. Before that, a brief description of the model is presented here.

Basic Plasticity Theory

A perfectly plastic material deforms continuously at constant stress with zero volume change once the yield stress has been reached. Engineering materials are rarely perfectly plastic but are either work hardening or work softening. Especially soils and other particulate systems undergo plastic volume changes with stress, and the material with changed volume behaves altogether as a different material with its own yield properties. The theory of perfect plasticity has been extended to soils with work hardening using Drucker's (1964) stability criteria.

The essentials of a plasticity model are

(1) an yield surface which defines the stress states at which the material experiences plastic strains,

(2) a flow rule which defines the direction of the plastic strain increments. A potential surface in stress space is defined and the partial derivative of this function with stress in any direction gives the strain increment in that direction. A direct consequence of Drucker's stability criteria is an associated flow rule by which the potential surface is the same as the yield surface, so that the strain increment is normal to the yield surface at the current stress state.

(3) a hardening rule which describes the growth or propagation of the yield surface with plastic work done.

(4) a failure criteria which defines the limiting or ultimate stresses.

The basic Cam-clay model was developed for axi-symmetric stress coordinates *i.e* $\sigma_2 = \sigma_3$, with spherical and deviatoric components given by $p = (\sigma_1' + \sigma_2' + \sigma_3')/3$ and $q = (\sigma_1 - \sigma_3)/2$ and corresponding strain components $\varepsilon_v = \varepsilon_1 + \varepsilon_2 + \varepsilon_3$ and $\varepsilon_s = 2(\varepsilon_1 - \varepsilon_3)/3$. The formulation has been later extended to plain strain and general stress conditions. The yield surface for the Cam-clay model was derived from the work done or energy considerations on the assumption of pure frictional dissipation of plastic energy of the form $\delta W = Mp\,\delta\varepsilon^p$ (where M is the frictional factor) and using the normality flow rule and critical state concepts (which define the failure states). The yield surface was assumed to expand isotropically and the hardening rule was obtained from the consolidation behaviour of the soil noting that each point on the e-log p curve corresponds to states with $q = o$ on successively hardening yield surfaces.

Soil, if continuously distorted to very large strains, will reach a state when it flows like fluids without any further changes in stresses or volume and this state is referred to as the critical state. All such critical state points for different initial conditions of the soil lie along a curve in $p - q - v$ space defined by

$$v = \Gamma - \lambda \ln p \tag{32}$$

$$q = M p \tag{33}$$

Where Γ and λ are the intercept at unit pressure and slope respectively of the critical state line and M is the slope of this line in $q - p$ space. *i.e.*, the curve is parallel to the virgin compression curve ($v = N - \ln p$) in $v - p$ plane and is a straight line passing through the origin in $q - p$ plane. Soils in a state with $q/p < M$ (wet state) compress upon yielding and dilate if $q/p > M$ (dry states). The model assumes purely elastic behaviour for stress states within the yield surface and no recoverable shear strains. The elastic bulk modulus was obtained from the rebound part of the consolidation curve as $K = p/\kappa$ where κ is the slope of the rebound path in $v - \ln p$ plot. With all this, the expression for the yield surface was of the form

$$q = \frac{M p}{\lambda - \kappa} (\Gamma + \lambda - \kappa - v - \lambda \ln p) \tag{34}$$

A differentiation of the above equation with respect to v and separation of elastic strain would gives the expression for plastic volumetric strain increments as

$$d\varepsilon_v^p = \frac{\lambda - \kappa}{Mpv} \{(M - q/p)\, \delta p + \delta q\} \tag{35}$$

and from the normality rule, the shear strain component would be

$$d\varepsilon_s^p = \frac{d\varepsilon_v^p}{(M - q/p)} \tag{36}$$

With these three equations together with the elastic strain components, the entire stress-strain behaviour for any loading can be obtained from an incremental working procedure.

A modified version of Cam-clay which is more widely used has been derived (Roscoe and Burland, 1968) assuming a slightly different form of energy dissipation, where the expression for yield surface results in an ellipse given by

$$(p - p_x)^2 + \frac{q^2}{M^2} = p_x^2 \tag{37}$$

where $p_x = p_c/2$

Here again, the hardening law is derived from the consolidation curve and there are not many changes in the two forms of Cam-clay model.

As can be seen from the above expressions, the only soil parameters appearing are λ, κ, Γ, M where λ and κ are the slopes of the compression and rebound paths of v—$ln\, p$ curve and Γ is the intercept at unit pressure of the v—$ln\, p$ curve and M is the friction factor given by the slope of the failure envelope in q—p space.

Since it has been possible to generalise the consolidation curve to encompass all fine grained soils and since it has been shown that M ($M = 6\ \mathrm{Sin}\phi/(3 + \mathrm{Sin}\phi)$) can be regarded as constant within the narrow range of variation for uncemented fine grained soils, it must be possible to generalise the entire stress-strain behaviour described by the Cam-clay model (Srinivasa Murthy *et al.* 1988).

Figure 18 shows the normalised stress paths of several drained and undrained tests on Weald clay and London clay together with the average path of several tests on kaoline clay (Atkinson and Bransby, 1978) in q/p_e vs p/p_e plane. It can be seen that the path is unique for all the three soils. This indicates that normalisation with equivalent pressure p_e not only accounts for the stress level but it even encompasses the soil type. The generalisation of effective stress-water content path may be more evident from Fig. 19 which shows the collapse of the stress paths of Weald clay and London clay in e—p plane when normalised with respective liquid limit void ratios. The possible

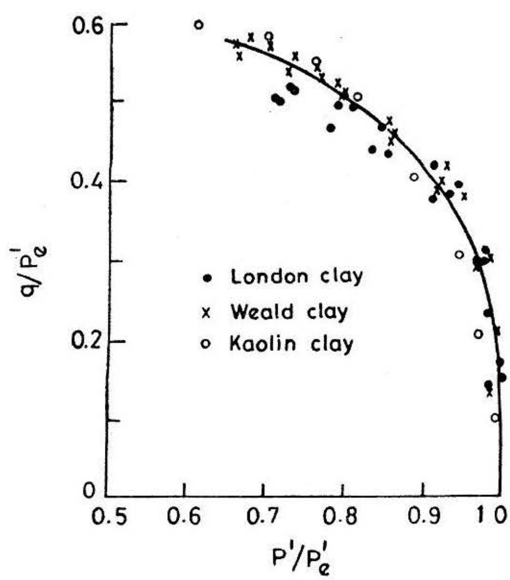

FIGURE 18 Normalized Roscoe Surface (Srinivasa Murthy, Vatsala and Nagaraj, 1988)

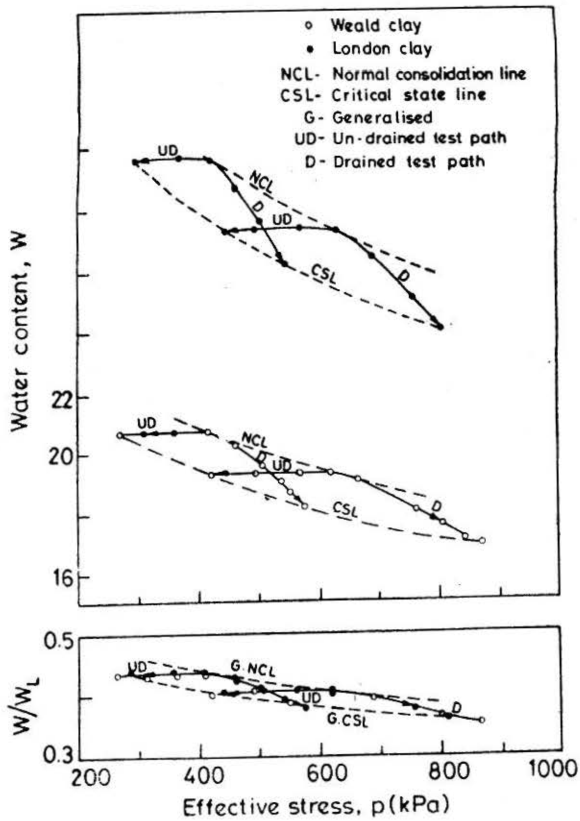

FIGURE 19 Generalized Stress Paths (Srinivasa Murthy, Vatsala and Nagaraj, 1988)

unique Cam-clay yield surface is shown in Fig. 20 in $p - q - e/e_L$ space for the data of the four soils analysed before.

For predicting the stress-strain behaviour of a given soil using the Cam-clay model, the parameters λ, κ, Γ can be predicted knowing the liquid limit of the soil using the generalised compression equation as

$$\lambda = \frac{0.234 e_L}{2.303}, \quad \kappa = \frac{0.0463 e_L}{2.303} \text{ and } \Gamma = 0.945 e_L + 1$$

and taking M to be constant the Cam-clay equations can be written in suitable generalised form as

$$q = \frac{Mp}{\lambda' - \kappa'} \{\Gamma' + \lambda' - \kappa' - (1 + e) - \lambda' \ln p\} \tag{38}$$

Where λ', $'\kappa$, Γ' are the corresponding values obtained from the generalised compression curve in e/e_L — $ln\ p$ plot.

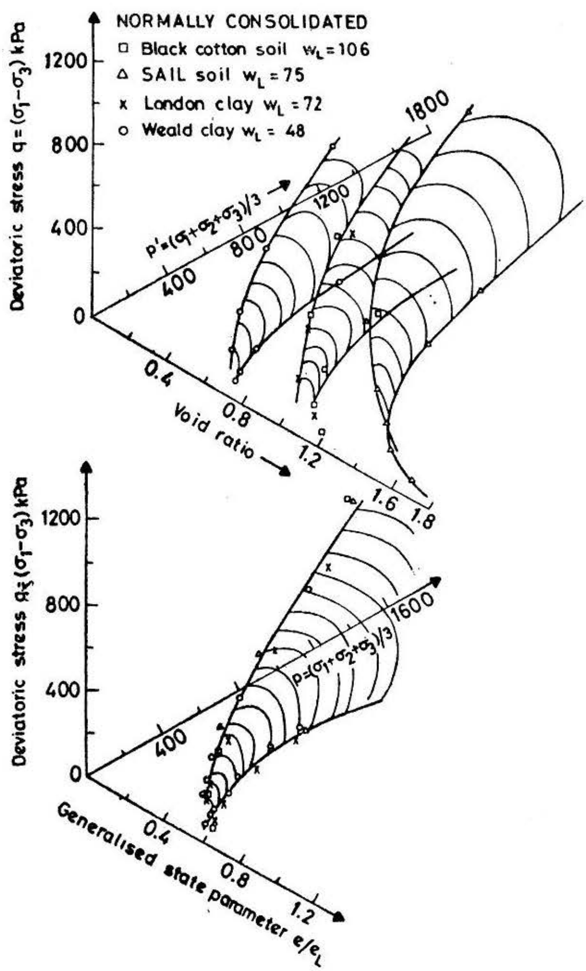

FIGURE 20 P-q-e Diagram for Clays and Generalization in P-q-(e/e_L)

Concluding Remarks

In this paper, it has been attempted to provide the macro-level relations of the micro-analytical model developed in the earlier paper. It has been possible to develop unique compressibility and shear strength equations for both normally and overconsolidated saturated uncemented states. In the light of these generalised equations, the available forms of empirical equations for predicting mechanical properties have been re-examined to identify their base. Finally the possibility of developing a unique Cam-clay model with generalised parameter has been indicated.

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