Displacement Dependent Farth Pressures in Retaining Walls

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Introduction

 \mathbf{E} valuation of stability of an earth retaining wall requires knowledge of (*i*) magnitude of earth pressure, (*ii*) distribution and point of application of earth pressure and (*iii*) displacements of the wall.

Amongst the classical theories, Coulumb's (1776) theory predicts the pressure on the wall as a force of reaction which the wall exerts to keep the soil wedge, torn off from the rest of the backfill due to wall movement, in equilibrium. The resulting pressure is hydrostatic but the theory suffers from the fact that the assumed rupture surface was plane whereas subsequently the importance of a curvilinear rupture surface was realised. Rankine (1857) used essentially the same assumptions as Coulomb but further simplified the problem by neglecting wall adhesion and wall friction. Culmann (1866) presented a graphical method for determination of magnitude of earth pressure and for locating Coulomb's most critical rupture surface. Terzaghi (1934), Row and Peaker (1965), Mackey and Kirk (1967) and Narain et. al. (1969) conducted experimental studies for active, passive and at rest pressure and the geometry of the rupture surface. Saran and Deo (1974) conducted experiments and showed that earth pressure distribution is close to parabolic as against the hydrostatic pressure distribution predicted by Coulomb (1776) and Rankine (1857). Dubrova (1963), Prakash and Saran (1971) and Joshi and Prajapati (1982) proposed analytical methods to determine earth pressure distribution.

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The criterian of displacement dependent earth pressure was not paid any attention till Dubrova (1963) developed equations which combine active and passive states into one expression for any rotation of the wall. Narain *et. al.* (1969) also predicted on the basis of experiments, dependence of earth pressure on wall displacements and parabolic distribution of earth pressure in case of rotational mode. A simplified and significant contribution in this direction is due to Reddy (1985) who proposed a mathematical model for considering continuous interaction between the backfill ard the wall using the concept of soil modulus as closely spaced independent elastoplastic springs. The analysis yields displacement dependent earth pressure when wall undergoes translation, rotation about both top and bottom in both active and passive conditions.

Problem

The limitations of the work done by Reddy (1985) are that the analysis neglected the base fliction and the side friction. Moreover, the analysis was developed for a vertical retaining wall with horizontal backfill. Present work is therefore carried out as an extension of Reddy's (1985) work and therefore considers:

- (i) Inclined backface of the wall,
- (ii) Inclined backfill, and
- (iii) Mobilisation of friction between wall and backfill.

An attempt is therefore made to predict displacement dependent earth pressures for both the modes, namely—translation and rotation about the bottom in both active and passive conditions.

Displacement Analysis Under Static Condition

There exists a continuous interaction between the backfill and rigid retaining wall over the full height of the wall. Interaction effects of the backfill are discretised using the concept of soil modulus as closely spaced independent elasto-plastic springs, ΔH apart, one end of which is fixed to the back of retaining wall at different elevations and the other end is fixed to an immovable support (Fig. 1). It is assumed that:—

- (i) spring constants, remain unchanged at all stages of movement,
- (ii) the limiting strain of each spring is proportional to its stiffness,
- (iii) the stiffness is active condition equals (K_a/K_p) times the values given by Terzaghi (1955).

Spring constants are computed from soil modulii values which vary

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FIGURE 1(b) Mathematical Model of Inclined Wall for Displacement Analysis under Static Conditions.

linearly with depth in sands and normally consolidated clays. Soil modulus, K at a depth, h from ground surface is given as:

$$K = \eta_h. h \tag{1}$$

where η_h is the constant of horizontal subgrade reaction.

Effect of Inclined Surcharge

This is taken into account by considering inclined surcharge as an equivalent surcharge of uniform height, *Heq* and obtained by equating the Coulomb's earth pressure for the inclined wall with an inclined surcharge to that for the same wall with a surcharge of uniform height acting at the horizontal ground surface.

$$P_{a} = \frac{1}{2} k_{A} \cdot \gamma' \cdot H^{2} = \frac{1}{2} K_{A} \cdot \gamma \cdot H^{2} + k_{A}' \cdot \gamma \cdot H_{ea} \cdot H$$
(2)

where

$$k_{A} = \frac{\operatorname{Cos}^{2}(\phi - \alpha)}{\operatorname{Cos}^{2}\alpha.\operatorname{Cos}(\delta + \alpha)} \times \frac{1}{\left[1 + \sqrt{\frac{\operatorname{Sin}(\phi + \delta).\operatorname{Sin}(\phi - \beta)}{\operatorname{Cos}(\alpha - \beta).\operatorname{Cos}(\delta + \alpha)}}\right]^{2}} (3)$$

and

$$k_{A}' = \frac{\cos^{2}(\phi - a)}{\cos^{2} a \cdot \cos(\delta + a)} \times \frac{1}{\left[1 + \sqrt{\frac{\sin(\phi + \delta)\sin\phi}{\cos a \cdot \cos(a + \delta)}}\right]^{2}}$$
(4)

Substituting k_A and k'_A in Eq. 2 and solving yields the expression for equivalent height, H_{eq} for active condition as

$$(H_{eq})_{A} = \frac{H}{2} \left[\frac{\sqrt{\cos(\alpha+\delta)} + \sqrt{\frac{\sin(\phi+\delta).\sin\phi}{\cos\alpha}}}{\sqrt{\cos(\alpha+\delta)} + \sqrt{\frac{\sin(\phi+\delta).\sin(\phi-\beta)}{\cos(\beta-\alpha)}}} \right]^{2} -1 \quad (5)$$

On similar lines, the expression for H_{eq} for the passive condition is obtained as:

$$(H_{eq})_{p} = \frac{H}{2} \left[\frac{\sqrt{\cos(\alpha-\delta)} - \sqrt{\frac{\sin(\phi+\delta).\sin\phi}{\cos\alpha}}}{\sqrt{\cos(\alpha-\delta)} - \sqrt{\frac{\sin(\phi+\delta).\sin(\phi+\beta)}{\cos(\beta-\alpha)}}} \right]^{2} - 1 \quad (6)$$

Computation of Spring Constants

Inclined retaining wall was treated as a beam simply supported at spring locations and with load intensity, η_h , h, increasing linearly with depth (Fig. 2). Inclined surcharge was replaced by a surcharge of uniform height, H_{eq} causing an additional uniform loading intensity, η_h , H_{eq} over the entire span.

Moment equilibrium equation about the point, B yields:

$$k1.\Delta H = \eta_h.H_{eq}.\Delta H. \frac{\Delta H}{2} + \eta_h \Delta H. \frac{\Delta H}{2} \cdot \frac{\Delta H}{3}$$

Form which

$$k_1 = \frac{1}{2} \eta_h$$
. $H_{eq} \Delta H + \frac{1}{6} \eta_h$. ΔH^2

Similarly taking moment about point, C gives

$$k_{2} \Delta H + k_{1} 2\Delta H = \eta_{h} H_{eq} 2\Delta H \Delta H + \eta_{h} 2\Delta H. \frac{2\Delta H}{2} \cdot \frac{2\Delta H}{3}$$

Substituting for k_1 gives

$$k_{2} \Delta H + \left[\frac{1}{2} \cdot \eta_{h} H_{eq} \Delta H + \frac{1}{6} \eta_{h} \Delta H^{2}\right] \cdot 2\Delta H$$

= $\eta_{h} \cdot H_{eq} \cdot 2 \Delta H \cdot \Delta H + \eta_{h} \cdot 2 \Delta H \cdot \frac{2\Delta H}{2} \cdot \frac{2\Delta H}{3}$

(7)





FIGURE 2 Mathematical Derivation of Spring Constants

Further simplification yields:

$$k_2 = \eta_h H_{eq} \Delta H + \eta_h \Delta H^2 \tag{8}$$

In general:

$$k_i = \eta_h H_{eq} \Delta H + (i-1) \eta_h \Delta H^2, \quad i = 2 \text{ to } (n-1)$$
 (9)

Similarly, the moment equilibrium equation about the point, E gives :

$$k_{n}\Delta H = \left[\frac{\eta_{h}H + \eta_{h}(H - \Delta H)}{2}\right] \left[\frac{2\eta_{h}H + \eta_{h}(H - \Delta H)}{\eta_{h}H + \eta_{h}(H - \Delta H)}\right] \frac{\Delta H}{3} \cdot \Delta H$$
$$+ \eta_{h}H_{eq}\Delta H \cdot \frac{\Delta H}{2}$$

which on simplification gives

$$k_n = \frac{1}{2} \eta_h H_{eq} \Delta H + \frac{1}{6} (3n-4) \eta_h \Delta H^2$$
⁽¹⁰⁾

where *n* represents the number of springs.

Computation of At-Rest Pressure

Springs are assumed to be pre-compressed to develop at rest condition. This is taken as reference and any release in spring forces creates active condition and any further compression creates passive condition. The coefficient of earth pressure at rest is modified, for an inclined wall with an inclined surcharge, using a modification ratio, r given by:

$$r = \frac{KA(\alpha, \beta, \delta = 0)}{KA(\alpha = \beta = \delta = 0)}$$
(11)

where numerator is Coulomb's active earth pressure coefficient for inclined wall with inclined surcharge and the denominator represents the coefficient for vertical wall with no surcharge, friction angle being zero for both. Modified coefficient of earth pressure at rest,

$$K_{om} = K_o r = (1 - \sin\phi) r \tag{12a}$$

and modified earth pressure at rest,

$$P_{om} = \frac{1}{2} K_{om}, \gamma H^2 \tag{12b}$$

and acts normal to the wall and its component taken by the springs is :

$$P_{omi} = P_{om}. \cos a \tag{120}$$

Limiting Yield Strain of Springs

A linear variation of yield displacement, with zero at top of wall and maximum at the base was assumed. If Y_{s_i} represents yield displacement at ith spring.

$$Y_{si} = \frac{m}{100} \times h_i \tag{13}$$

where m represents the percent yield strain.

Mobilisation of Friction Angle

At any *i*th spring location, mobilised friction angle, δ_{mi} has been taken as:

$$\delta m_i = \frac{y}{y_{si}} \times \delta$$
 and $\delta m_i \ge \delta$ (14*a*)

where friction angle, $\delta = (2/3) \phi$

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(14b)

when a spring reaches its limiting displacement, full δ is mobilised in that spring at that particular displacement.

Computational Algorithm

Given: Mode of wall movement, height of wall and soil properties:

- (i) Compute limiting displacement at each spring location after assuming a certain value of limiting strain, m
- (ii) Given a known displacement to the wall, compute the displacement at the location of each spring.
- (iii) Compare this displacement with limiting displacement and determine forces P_{s1} , P_{s2} ... P_{sn} in different springs.
- (iv) If P_{ni} and P_{ti} are the normal and the tangential components of these forces in the *i*th spring.

Then for active condition:

$$P_{ni} = \frac{P_{omi} - P_{si}}{\cos(a + \delta_{mi})} \cdot \cos \vartheta_{mi}$$
(15a)

$$P_{ii} = \frac{P_{omi} - P_{si}}{\cos(\alpha + \delta_{mi})} \cdot \sin \delta_{mi}$$
(15b)

and for passive condition:

$$P_{ni} = \frac{P_{omi} + P_{si}}{\cos(\alpha + \boldsymbol{\delta}_{mi})} \cdot \cos \boldsymbol{\delta}_{mi}$$
(16a)

$$P_{ti} = \frac{P_{omi} + P_{si}}{\cos(\alpha + \delta_{mi})} \cdot \sin \delta_{mv}$$
(16b)

Therefore active or passive earth force

$$= \sqrt{(\Sigma P_{ni})^2 + (\Sigma P_{ti})^2}$$
(17)

- (v) Repeat steps (ii) to (iv) for different displacements.
- (vi) Repeat steps (i) to (v) for different values of limiting strain, m.

The procedure is repeated for different sets of parameters.

Earth Pressure Distribution

Component of earth force at *i*th spring location

$$= \frac{P_{omt} \pm P_{si}}{\cos(\alpha + \delta_{mi})}$$
(18)

knowing this force for different springs, earth pressure can be found out by dividing this force by the area equal to $(\Delta H \times 1)$ except for first and the last springs for which area equals $[(\Delta H/2) \times 1]$.

Parametric Study

(a) Back fill parameters:

TABLE 1

Backfill Parameters Considered for Diasplacement Analysis Under Static Condition

Soil Type	Density	Values of η_h (t/m ³)		Angle of
	1.5	Active	Passive 20	30°
		10		
		20	40	
		30	60	
Medium dense sand	1.6	20	40	35°
		40	80	
		60	120	
Dense sand	1.7	40	80	40°
		80	160	
		120	240	

(b) Limiting strains of springs

Maximum limiting strain at the base of wall is varied between 0-3% in active condition and 0-20% in passive condition.

(c) Wall parameters:

- (i) Height, H(m) 4, 6, 8, 10
- (ii) Angle of back face with respect to vertical, $a 0^{\circ}$, 10° , 20°
- (iii) Surcharge angle, $\beta 0^{\circ}$, 5°, 10°
- (iv) Mode of Wall Movement: (a) Translation, (b) Rotation about bottom in both active and passive conditions.

Discussion of Results

I. Active Case

Effect of wall movement

Figures 3 and 4 show the variation of earth pressure for translational and rotation about the bottom modes, of a 6m high wall retaining medium dense sand, for different values of maximum limiting strain. In both the modes, active earth pressure decreases with the increase in either displacement or rotation of the wall and attains a minimum constant value corresponding to the active condition. In either case, it has been observed that for values of limiting strain higher than the yield strain active earth pressure decreases because the more the limiting strain, more will be the displacement required to achieve a constant pressure. Also for values of limiting strain lower than the yield strain, active earth pressure increases.

Effect of yield strain

Effect of limiting strain on active earth pressure is plotted for different heights, in Figs. 5 and 6 for the two modes of translation and rotation about toe. The curves are plotted for the case of loose sand backfill. For a particular case, the curve was used to determine yield strain at these heights.

Translation mode

Plots have been made between yield strain, m (%) and height of wall for different values of backface inclination a and for all the three backfill types. Figures 7, 8 and 9 show representative plots for the medium dense



FIGURE 3 Translation-Active Case-Wall Displacement Vs Pressure



FIGURE 4 Rotation about Bottom - Active Case Rotation Vs Pressure

sand for three different values of modulus of horizontal subgrade reaction, $\eta_h = 20, 40$ and 60 t/m.³ These reveal that:

- (i) Yield strain was independent of the surcharge angle,
- (ii) Other parameters held constant yield strain decreases with increase in wall height and increases with increase in wall angle, β
- (iii) Yield strain decreases with increase in value of soil surgrade modulus, β
- (iv) Value of yield strain in case of medium dense sand was found to be less than that in loose sand and in case of dense backfill, to be less than that in case of medium dense sand. This is obvious because the looser the backfill, the less restrained it is and hence more the yield strain.









Rotation mode

Figures 10, 11 and 12 show similar plots for the rotation about bottom mode. It is observed that:

- (i) Qualitatively the results are similar to those in translation mode
- (ii) Qualitatively, value of yield strain for rotation of wall about bottom was more compared to the corresponding case of translation mode
- (iii) As the density of the backfill increases, the values of yield strain decrease.

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FIGURE 7 Translation-Active Case Wall Height' Vs Yield Strain















FIGURE 11 Rotation about Bottom-Active Case Wall Height Vs Yield Strain





Earth pressure distribution

Classical earth pressure theories consider the pressure distribution as hydrostatic. However, many experimental investigations have revealed that it is nonlinear and depends on type of wall movement.

Figure 13 shows the active earth pressure distribution for a 4.0m high wall with loose sand at limiting strain equal to yield strain. The pressure distribution is parabolic thereby moving the point of application of the earth pressure higher than the conventional notion of one third the wall height from the base.

Figure 14 shows the distribution of pressure for a 6.0m high wall in rotation mode. The pressure distribution is non-linear but not parabolic. In





both the figures, the line corresponding to Coulombs pressure distribution is also shown.

Variation of earth with limiting strain

As limiting strain for the spring increases, larger displacements are required for full active condition and hence springs are released more from the at rest condition. Active earth force therefore decreases as the limiting strain increases.

If p is the earth at any limiting strain, m, p_c the Coulumb's earth force which occurs at yield strain, m_c , then Fig. 15 shows the plot of (p/p_c) versus (m/m_c) . It is evident that initially the earth force is equal to p_o and the







FIGURE 15 Translation And Rotation Bbout Botton Active Case Variation of Earth Force Vs Limiting Strains

ordinate is (p_o/p_c) . As limiting strain increases, (m/m_c) increases and earth force decreases and hence ratio (p/p_c) decreases. At $(m/m_c) = 1$, earth force, p equals p_c and hence $(p/p_c) = 1$. This variation has been found to be independent of the mode of wall movement.

II. Passive Case

Effect of wall movement on passive earth pressure

Figures 16 and 17 show the variation of passive earth pressure for Translation and rotation at bottom modes of a 6.0m high retaining wall in medium dense sand for different values of maximum limiting strain. It can be observed that in both the cases, the earth pressure increases as displacement increases and finally attains a constant value known as passive earth pressure. The plot also indicate that passive earth pressure increases for values of limiting strain higher than the yield strain. For values of limiting strain lower than the yield strain, passive earth pressure reduces. It has also been noted that the limiting strain needed for full passive condition to be mobilised is much more than that needed for full active condition to develop (Figs. 3 and 4).

Effect of yield strain

The effect of yield strain for passive earth pressure is plotted for different wall heights in Fig. 18 for translation mode and in Fig. 19 for rotation about bottom mode. The sand is considered to be in medium dense condition. For a particular case, the curve was used to determine the yield strain at different heights.



FIGURE 16 Translation-Passive Case Wall Displacement Vs Pressure



FIGURE 17 Rotation About Bottom-Passive Case Wall Rotation Vs Pressure

Plots have been made between yield strain and height of wall for different backface inclinations a and for all the three backfill types. Figures 20, 21 and 22 show representative plots for three different values modulus of horizontal subgrade reaction, $\eta_h = 40$, 80 and 120 t/m^3 . These reveal that for:

Translation mode

- (i) Yield strain is independent of the surcharge angle β . This was observed by considering three different values of β , namely, 0°, 5° and 10°.
- (ii) Yield strain decreases with increase in wall height. This rate of decrease is more for smaller values of wall angle.
- (iii) Yield strain decreases as wall angle, β increases, other parameters being constant.
- (*iv*) For the same height of wall, the yield strain decreases as the value of coefficient of horizontal subgrade reaction increases, the other parameters remaining constant.
- (v) Value of yield strain in dense sand is less than that for medium dense sand which in turn is less than that for loose sand.





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FIGURE 19 Rotation About Bottom - Passive Case Limiting Strains Versus Pressure

Rotation about bottom mode

The representative plots between yield strain and wall height are presented in Figs. 23, 24 and 25 for values of $\eta_h = 40$, 80 and 120 t/m^3 for medium dense sand:

- (i) Qualitatively these are similar to the plots for translation mode.
- (ii) The yield strain for rotation of wall about bottom is found to be more compared to the corresponding yield strain in the translation mode.
- (iii) The level of yield strain decreases with increase in density of backfill.



FIGURE 20 Translation-Passive Case Wall Height Vs Yield Strains







FIGURE 22 Translation-Passive Case Wall Height Vs Yield Strains







FIGURE 24 Rotation About Bottom - Passive Case Wall Height Vs Yield Strains



FIGURE 25 Rotation About Bottom - Passive Case Wall Height Vs Yield Strains

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Earth Pressure Distribution

Contrary to the consideration of the classical theories, experimental observations suggest a nonlinear distribution of passive earth pressure which was found to be dependent on type of wall movement.

Figure 26 shows the passive pressure distribution for a 6.0m high wall due to loose sand backfill for translational mode. The pressure distribution is given for limiting strain equal to the yield strain and for limiting strain equal to half the yield strain. The pressure distribution, though not strictly hydrostatic, shows a tendency to become so. Such a tendency was noted by Narain *et. al.* (1969). Coulomb's pressure distribution is also shown in the same plot for comparison.

The passive earth pressure distribution against a 6.0m high wall due to loose sand backfill for rotation about bottom mode is presented in Fig. 27,



FIGURE 26 Pressure Distribution On Wall-Passive Translation



FIGURE 27 Rotation About Bottom - Passive Case Pressure Distribution On Wall



FIGURE 28 Translation And Rotation About Bottom, Passive Case, Variation of Earth Force With Limiting Strains

for full mobilisation and for half the mobilisation. Comparison with Coulomb's pressure distribution can also be seen in the same plot. It is clear that pressure distribution is nonlinear.

Variation of earth pressure with limiting strain

Figure 28 shows the plot of passive earth pressure (p/p_c) versus limiting strain (m/m_c) . The parameters p, p_c , m and m_c are defined earlier. It is clear that initially the earth force equals p_o and hence the initial ordinate is p_o/p_c . As p_c is more than at rest pressure, p_o , the ratio, p_o/p_c is less than 1. With the increase in limiting strain, the ratio m/m_c increases and correspondingly the passive or the ratio p/p_c also increases. At limiting strain equal to yield strain, m/m_c equals 1, the passive pressure, p equals the Coulomb's passive earth pressure, p_c and hence the ratio, p/p_c also equals 1. It is also noticed that this plot is independent of the mode of wall movement, whether traslation or rotation about the bottom.

Conclusions

On the basis of the above analytical investigations on rigid retaining walls under static conditions, the following conclusions can be drawn:

Active State

(i) Under both the modes of wall movement, earth pressure decreases with increase in wall displacement and finally attains a constant value. (ii) Yield strain decreases with increase in wall height, increases with increase in wall angle, and decreases with improvement in backfill soil properties. ¥

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- (iii) In case of translation, pressure distribution is parabolic at limiting strain equal to yield strain and of nonlinear type for the rotation mode.
- (iv) Variation of earth force with limiting strain is identical for both the modes.

Passive State

- (i) Under both the modes of wall movement, the total passive earth pressure increases with increase in wall displacement and finally attains a constant value.
- (ii) The yield strain decreases with:
 - —increase in height and also with increase in inclination of backface, improvement in backfill soil properties *i.e.* with increase in either soil subgrade reaction or internal angle or density.
- (iii) The pressure distribution can be treated as nearly hydrostatic for translation mode but is nonlinear for the case of rotation about bottom.
- (iv) The variation of earth pressure with limiting strain is identical for both the modes.

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