# Three Dimensional Analysis of Embankment Dams : Effect of Valley Shape 

By

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## Introduction

Dams form an important element of multipurpose projccts. Earth and Rockfill dams are usually adopted where the foundations are not strong enough. The design of earth and rockfill dams is generally based on one or two plane sections of the dam, amenable to two dimensional analysis. While the advantages of this simplification are obvious, the degree of error associated with it is much less clear. Dams located in narrow valleys exhibit three dimensional behaviour and the external load will be transferred partly to the foundation and partly to the abutments. Load transfer to the abutments depends on the valley width and shape and is negligible for very wide valleys. It is therefore, imperative to know the limiting valley width beyond which the dam essentially acts as a two dimensional structure.

Three dimensional stress and deformation analysis of an earth and rockfill dam located in a symmetrical valley has been carried out in this paper, Sequential construction of the dam and nonlinear properties of the dam material have been considered. The valley width has been varied over a wide range in order to investigate its effect on the dam behaviour. Two dimensional plane strain analysis of the central section of the dam has also been carried out and the results are compared with those obtained from three dimensional analysis. The study leads to the conclusion that appreciable three dimensional behaviour of the dam is exhibited for valleys with valley width to height ratio of less than 4.5 . For wider valley, a two dimensional analysis of the dam is adequate for its design.

## Previous Work

Palmerton (1972) performed both plane strain and 3-D finite element

[^0]analysis of a central core earth dam located in a narrow valley with valley walls sloping at $1: 1$ and concluded that while the predicted displacements from both plane strain and three dimensional analysis were more or less similar, significant differences in stress conditions were apparent.

Eisenstein et al. (1972) studied the behaviour of a 100 m high central core dam situated in a V-shaped valley with valley walls sloping at 1:1. The study showed that in case of homogeneous dams there is no appreciable difference in the stress and movement values obtained from plane strain and 3-D analysis,. For a hetrogeneous section, however, there was enough evidence of the three-dimensonal behaviour.

Lefebvere et al. (1973) compared the results of linear plane strain and three dimensional analysis of earth dams located in V-shaped valleys with different valley wall slopes. It was concluded from the study that for valley wall slopes of $3: 1$ or flatter, a plane strain analysis of the maximum transverse section would provide reasonably accurate results, while for dams in valleys with side slopes of $1: 1$ the plane strain analysis results will not be accurate due to the effects of cross valley arching.

Eisenstein and Simon (1975) performed 2-D and 3-D linear and multilinear analysis of the 243 m high earthfill Mica Dam in Canada. A comparison of the two analysis showed that even with a structure so markedly three-dimensional as Mica Dam, the main transverse section could be studied using the plane strain model without significant loss of accuracy.

Singh, Gupta \& Saini (1985) performed a 3-D linear analysis of 260.5 m high earthfil Tehri dam in India. The dam located in a narrow gorge with valley walls sloping at $1.1 \mathrm{H}: 1 \mathrm{~V}$ exhibited a three-dimensional behaviour in that a major portion of the loads was seen to be transferred to abutements through beam action.

## Methods of Analysis

The dam has been analysed by the finite element method using 3-D as well as 2-D idealisation.

Since for the fill materials the stress strain curve is known to be non-linear the same has been taken into accunt in the analysis. The functional form as defined by Kondner's (1963) hyperbolic stress-strain model has been used for studies reported in this paper.

Expression given by Duncan and Chang (1970) for tangent modulus of elasticity, as based on this model, are given below.

$$
\begin{equation*}
E_{t}=\left(1-R_{f} m\right)^{2} E_{i} \tag{1}
\end{equation*}
$$

where $\quad E_{t}=$ the tagent modulus of elasticity at a particular stress level
$R_{f}=$ the failure ratio defined as the ratio of the deviatoric stress at failure to the ultimate deviatoric stress given by

$$
\begin{equation*}
\left.\left(\sigma_{1}-\sigma_{3}\right) f=R_{f} \cdot\left(\sigma_{1}-\sigma_{3}\right)\right) \text { ult } \tag{2}
\end{equation*}
$$

where $a 1$ and $a 2$ denote major and minor principal stresses respectively $m=\left(\sigma_{1}-\sigma_{3}\right) /\left(\sigma_{1}-\sigma_{3}\right) f$, known as the mobilisation factor and defined as the ratio of the deviatoric stress $\left(\sigma_{1}-\sigma_{3}\right)$, acting at any element to the failure deviatoric stress $\left(\sigma_{1}-\sigma_{3}\right) f$.
$E_{l}=$ initial modulus of elasticity for a particular confining pressure.
Expressing the compressive strength and confining pressure in terms of Mohr-Coulomb failure criterion and using $c$ and $\phi$ as strength parameters, the value of tangent modulus of elasticity can be expressed as

$$
\begin{equation*}
E_{t}=\left[1-\frac{R_{f}(1-\sin \phi)\left(\sigma_{1}-\sigma_{3}\right)}{2 c \cos \phi+2 \sigma_{3} \sin \phi}\right]^{2} K . P_{a}\left(\sigma_{3} / P_{a}\right)^{n} \tag{3}
\end{equation*}
$$

where $K$ and $n$ are dimensionless constants, $P_{a}$ is the atmospheric pressure in the same units as confining pressure and $\phi$ is the angle of shearing resistance for the material.

The expression for tangent poisson's ratio, has been used as given by Kulhawy and Duncan (1969) as below:

$$
\begin{align*}
V_{t} & =\frac{G-F \log \left(\sigma_{3} / P_{u}\right)}{\left(1-d \cdot \epsilon_{a}\right)^{2}}  \tag{4}\\
\text { where } \epsilon_{a} & =\frac{\left(\sigma_{1}-\sigma_{3}\right)}{E_{i}\left(1-m \cdot R_{f}\right)} \tag{5}
\end{align*}
$$

$G=$ value of initial Poisson's ratio at one atmospheric pressure
$F=$ the rate of change of initial Poisson's ratio $v_{i}$ with confining pressure
$d=$ the parameter expressing the rate of change of $v_{i}$ with strain

Incremental construction analysis as proposed by Clough and Woodward (1961) has been modified to have non-linear analysis in earth layer as used by Palmerton (1972). The residual force approach used by Sharma, et al. (1975) has been adopted in this study to analyse the dam behaviour for the construction stage.

## Numerical Data

The section of Tehri Dam has been chosen for this study. The 260.5 m
high earthfill dam is proposed to be constructed over river Bhagirathi in U.P. state of India. The dam has a moderately sloping core with filter zones supported by shell Zones on both sides of core. The dam is located in a narrow valley with valley walls sloping at approximately $1.1 \mathrm{H}: 1 \mathrm{~V}$. The gorge is about 28 m wide at the foundation level, giving a valley width to height ratio (valley width factor, $\beta$, as shown in Fig. 1) of 2.25. At the proposed dam site, the gorge is almost symmetrical and therefore, the gorge is assumed to be symmetrical with the result that only one half of the dam has been analysed. Fig 1 shows the transverse section of the dam at the deepest section and also the longitudinal section along the upstream face of core, and also the idealisation for 3-D analyses on these sections.

(a) Along Upstream Face of Core (y-z Plane)

(b) Valley Width Factor $\beta$


FIGURE 1 Idealisation and Deflected Shape for 3D Analysis ( $\beta=\mathbf{2} .25$ ) (Deflections Magnified 40 times)

In order to study the effect of valley slopes on the dam behaviour, a number of valley shapes have been analysed. For obtaining these valley shapes, the geometry of the cross-section of the dam has been kept constant and only the cross valley dimensions have been scaled by $0.5,2$ and 4 respectively giving the valley width to height ratios of $1.12,4.5$ and 9 respectively. These four shapes would be representative of a wide range of valley shapes from narrow to wide ones.

The base width and the approximate side slopes are thus as follows:

| Case No. | Base width (m) | Approx. side slopes | Valley width factor <br> $\beta$ |
| :--- | :---: | :---: | :---: |
| 1 | 14 | $0.55: 1$ | 1.12 |
| 2 | 28 | $1.1: 1$ | 2.25 |
| 3 | 56 | $2.2: 1$ | 4.50 |
| 4 | 112 | $4.4: 1$ | 9.0 |

For the 3-D analysis the dam has been idealised into 98 isoparametric 20 -noded brick elements having 724 nodal points thus possessing 2172 degrees of freedom. In an earthfill dam the foundation and abutment would generally be stronger as compared to the fill material and, therefore, the abutments and foundation have been assumed to be rigid in the present analyses.

The following material properties have been adopted in this study:
TABLE 1
Material Properties

| Sl. No. Parameters | Shell | Transition | Core |  |
| :--- | :---: | :---: | :---: | :---: |
| 1. $\quad$ Unit wt. in tonnes $/ \mathrm{m}^{3}$ | 1.80 | 1.99 | 1.96 |  |
| 2. $\quad$ Cohesion, C , tonnes $/ \mathrm{m}^{2}$ | - | - | 1.00 |  |
| 3. Friction angle, $\phi(\mathrm{o})$ | 38 | 32 | 27 |  |
| 4. $\quad$ Modulus number K | 2500 | 3000 | 500 |  |
| 5. Modulus exponent n | 0.25 | 0.30 | 0.60 |  |
| 6. Failure ratio Rf | 0.76 | 0.76 | 0.90 |  |
| 7. Poisson's ratio parameters |  |  |  |  |
|  | G | 0.43 | 0.43 | 0.48 |
|  | F | 0.19 | 0.19 | 0.0 |
|  | d | 14.80 | 14.80 | 0.0 |

For 2-D analysis the finite element mesh has been kept the same as that on the maximum transverse section for the 3-D analysis. The mesh consists of 54 elements (8-noded isoparametric) with 197 nodal points possessing 394 degrees of freedom.

The dam for both 3-D and plane strain analysis has been assumed to be constructed in 6 layers each of 40 m height. For convergence of iterations the norm of residuals has been restricted to less than $4 \%$ of the norm of applied loads.

## Results and Discussions

The finite element analysis yields deformations and normal stresses (transverse, longitudinal and vertical), shear stresses and the three principal stresses in the dam. However due to limitations of space only the five parameters, horizontal displacement- $u$, vertical movement- $w$ and the three normal stresses, $\sigma_{x}, \sigma_{y}$ and $\sigma_{z}$ in the transverse, longitudinal and vertical directions have been studied in this presentation only at the central section of dam. The results of the four shapes have been compared with those of a plane strain analysis.

The following sign convention has been used in this presentation:
horizontal transverse movement-positive towards downstream
vertical movement-positive upward
normal stresses-negative stress is compressive.
The displacements and stresses have been plotted along 7 different locations (1-1 through 7-7 shown in Fig. 1) to study the variation along height and along the 6 horizontal planes, namely at base and at $40,80,120,160$ and 200 m above the base to study the behaviour at constant elevations.

## Horizontal Movements $U$

Fig. 2 shows the variation of horizontal movement $u$ along horizontal planes. It is seen that the movements are practically zero at the core centre line. From the centre line of core the horizontal movement values increase sharply in the core and transitions on both sides. The variation of $u$ along the horizontal planes in shells is gradual and small.

For a narrow valley with valley width factor of 1.12 the maximum value of movement $u$ on a horizontal plane occurs at the shell/transition interface at all elevations. For other valley shapes the maximum $u$-values occur in the shells except at elevations 40 m in the upstream portion where the maximum values of $u$ occur at the upstream shell/transition interface.

For valley width factor of 1.12 the movement values gradually reduce from the core faces towards the exteriors. For other valley shapes (sides slopes of $1.1 \mathrm{H}: 1 \mathrm{~V}, 2.2 \mathrm{H}: 1 \mathrm{~V}, 4.4 \mathrm{H}: 1 \mathrm{~V}$ ) the $u$ values gradually increase from the core towards the exteriors. In the downstream shell the gradual increase of $u$ continues up to the downstream face. In the upstream shell, the increase of $u$ continues only upto vertical no. 1 beyond which the $u$ values gradually decrease towards the face. The decrease in $u$ values


FIGURE 2 Horizontal Movements $u$ at Constant Elevations over Central Section
towards the upstream face from the maximum values near vertical no. 1 is sharp in the lower third height, small in the middle third portion and practically nil in the upper third height.

In the downstream portion for a narrow valley (valley width factor $\beta=1.12$ ) only, the maximum movement values occur in transition at lower levels. In the upper two third height the maximum values occur some where near the toe. For wide valleys the maximum movement points occur always near the toe. The fact that in the upstream shell the movements are maximum in the centre of shell and on the downstream shell the movements continue to increase upto the faces may be due to the downstream slope being steeper than the upstream slope. The shells exert compression on the core. The downstream shell is in active state while the upstream slope starts coming into the passive state.

It is also seen that the magnitude of movements increase with valley width factor and that the $u$-curve for $\beta=9$ lies distinctly away from those for other valley shapes. The maximum upstream horizontal movements are $15.25,20.48,31.63$ and 46.46 cms for valleys with $\beta=1.12,2.25,4.5$ and 9 respectively. The corresponding maximum downstream movements are $12.81,16.09,25.51$ and 41.81 cms . The values of $u$ for valleys with valley width factors $\beta$ of $1.12,2.25,4.5$ are approximately $30 \%, 40 \%$ and $65 \%$ of the corresponding $u$ values for a valley with $\beta=9$.

The variation of horizontal movements along height is shown in Fig. 3. It is seen from the figure that the variation of $u$ along height is parabolic with the maximum values occuring at about mid-height in the shells and about $1 / 3$ rd height above base in the core and transitions. The values of $u$ at the faces are very small. It is also evident that the movement values increase with valley width factor, with the values for the valley with $\beta=9$, being very close to those of plane strain analysis. The values of $u$ obtained by 3-D analyses for valleys with valley width factors of $1.12,2.25,4.5$ and 9.0 respectively are about 25 to $35 \%, 30$ to $50 \%, 60$ to $75 \%$ and 70 to $100 \%$ of the plane strain values.


FIGURE 3 Horizontal Movements $u$ along Height at Different Locations over Central Section

## Vertical Settlement

The variation of vertical settlement $w$ along height is shown in Fig. 4. It is seen from the figure that at all sections, the magnitude of vertical settle-ment- $w$ increases with valley width and that the $w$ values for a valley with valley width factor, $\beta=9$ are more or less comparable to those of plane strain analysis.

The distribution of $w$ along height is parabolic for all verticals with maxima at about two third the corresponding height above the base and then decreasing towards the top for all valley shapes. The maximum settlements occur at the vertical in the middle of the core. Typical maximum settlement values for different analysis are shown in table 2.

It is obvious from table 2 and figure 4 that the settlements in downstream shell are more than the corresponding settlements in the upstream shell. The 3-D settlements are in the range of 35 to $45 \%, 50$ to $60 \%, 60$ to $75 \%$ and 90 to $98 \%$ of the plane strain settlement values for valleys with valley width factors $\beta=1.12,2.25,4.5$ and 9 respectively.

The variation of settlements $w$ on horizontal planes is shown in Fig. 5 for all analyses. It is seen that for all analyses, at all elevations the settlements are more in the interior as compared to those in the outer zones of the dam. In the lower one third height the maximum settlement along horizontal planes are observed in the down stream shell, whereas in the remaining upper two third portion the maximum settlements are observed in the core. The settlement curves for valley with valley width factor $\beta=9$ are away from the group of curves for valleys with $\beta=1.12,2.25$ and 4.5 indicating that the effect of valley width is evident for valleys with valley width factor of 4.5 or less and that it is negligible for a valley with valley width factor equal to 9 or more.

## Horizontal Transverse Normal Stress $\boldsymbol{\sigma}_{x}$ :

The distribution of horizontal normal stress $\sigma_{x}$ along the height is shown in Fig. 6. It is seen that near the top the $\sigma_{x}$ values are small at all verticals. In shells and transitions for all valley shapes the $\sigma_{x}$ gradually increases with height of overburden till at about 40 m elevation when it starts decreasing. In core, for all analyses the $\sigma_{x}$ increases at a gradually increasing rate with the overburden depth. At the core faces, however, in the core trench the stress distribution is zig-zag, the stress values first decreasing but increasing again at the foundation contact. On the upstream face the maximum $\sigma_{x}$ values are at the core trench base but at the down stream face at the general gound level. The stress in the interior of the core is more than those at the faces.

In the shells the $\sigma_{x}$ values vary with valley width factor all along the


VERTICAL NO. 2

VERTICAL NO. 1


VERTICAL NO. 4


VERTICAL NO. 3


VERTICAL NO. 6


$$
\begin{array}{ll}
\because & 2=0 \\
\longleftrightarrow & \beta=1.12 \\
\hdashline & \beta=2.25 \\
\hdashline & \beta=4.5 \\
\hdashline & \beta=9
\end{array}
$$

FIGURE 4 Vertical Movements $w$ along Height at Different Locations over Central Sections

TABLE 2
Maximum Settlement (cms)

| Vertical <br> No. | 3-D Analysis |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  | $B=1.12$ | $B=2.25$ | $B=4.5$ | $B=9$ | Plane strain <br> Analysis |
| 1 | -31.7 | -41.6 | -49.6 | -58.2 | -58.5 |
| 2 | -48.0 | -61.7 | -85.2 | -123.8 | -133.5 |
| 3 | -46.1 | -73.7 | -88.6 | -125.8 | -137.7 |
| 4 | -48.5 | -68.3 | -90.9 | -125.2 | -142.2 |
| 5 | -46.8 | -64.6 | -84.6 | -111.2 | -116.5 |
| 6 | -45.6 | -64.8 | -79.2 | -106.4 | -107.8 |
| 7 | -38.6 | -54.4 | -72.7 | -89.1 | -85.5 |



000 . 180
FIGURE 5 Vertical Movements $w$ at Central Elevations over Central Sections
height. The value of $\sigma_{x}$ increases with valley width in about $20 \%$ height near the ground level and decrease with increasing valley width in the remaining $80 \%$ height, in both shells and transitions. In the upper $80 \%$ height of the dam, the $\sigma_{x}$ values are smaller and the effect of valley width on $\sigma_{x}$ values is thus minimal. The effect of valley width is evident only in a small height near the base of the dam and in the core trench. However it is evident that all through, the $\sigma_{x}$ values for valley width factor of 4.5 and 9 are of the same order all along the height and quite close to the plane strain values. The $\sigma_{x}$ values at the base of core trench on the upstream face are $-209-297,-382,-392$ and $-411 \mathrm{t} / \mathrm{m}^{2}$ for 3-D analyses with valley width factors $\beta=1.12,2.25,4.5$ and 9 and plane strain analysis respectively. In terms of plane strain values the 3-D $\sigma_{x}$ values are $51 \%, 72 \%, 93 \%$ and $95 \%$ respectively for valleys with valley width factors $\beta=1.12,2.25,4.5$ and 9 . At the $\mathrm{d} / \mathrm{s}$ face of core trench at the base the corresponding $\sigma_{x}$ values are $-154-182,-244,-245$ and -279


VERTICAL NO. 2



VERTICAL NO. 4



VERTICAL NO. 6


VERTICAL NO. 5

$$
\begin{array}{ll}
\longleftrightarrow & 2-0 \\
\hdashline & \beta=1.12 \\
\longmapsto & \beta=225 \\
\hdashline & \beta=45 \\
\hdashline & \beta=9
\end{array}
$$

FIGURE 6 Horizontal Normal Stress $\sigma_{x}$ along at Different Locations over Central Section
$\mathrm{t} / \mathrm{m}^{2}$ and the corresponding percentages are $55 \%, 65 \%, 87 \%, 88 \%$ respectively. The $\sigma_{x}$ values in the bottom $20 \%$ height increase with valley width but the stress values are not affected by valley shapes beyond a valley width factor of 4.5 or more.

The variation of $\sigma_{x}$ along horizontal planes is shown in Fig. 7 for different analyses. It is inferred from the figure that at a given elevation the $\sigma_{x}$ values are minimum at the dam faces and gradually increase towards the interiors. The increase in $\sigma_{x}$ is sharp near the transition/shell interface. In the core the $\sigma_{x}$ values are higher than in the adjoining shells in the lower third height and smaller than in the adjoining shells in the upper two third portion of the dam. The effect of valley width is predominant near the ground level and gradually decreases in the upper reaches. The stress gradients in shells and transitions in horizontal direction is constant all over the height, as is evidenced by the parallel nature of curves at all elevations. The effect of valley width almost vanishes for valleys with valley width factor $\beta$ equal to or more than 4.5 .


FIGURE 7 Horizontal Normal Stress, $\sigma_{x}$ at Constant Elevations over Central Section

## Horizontal Longitudinal Normal Stress $\sigma_{y}$

The distribution of $\sigma_{y}$ along height is shown in Fig. 8. It is seen from the figure that:
(i) The distribution pattern of $\sigma_{y}$ along height is almost same as that of $\sigma_{x}$ i.e. it gradually increases with depth of overburden.
(ii) The $\sigma_{y}$ values are affected with the valley width, approaching plane strain values for valleys with valley width factor equal to 4.5 or more.
(iii) In the upper two third portion of dam, the 3-D values are higher than the plane strain values. The $\sigma_{y}$ values decrease with increase in valley width. However, the values of $\sigma_{y}$ being small in this portion, the effect of valley width is not significant here.
(iv) The $\sigma_{y}$ values are almost of the same order as that of $\sigma_{x}$ for all




- LEGENC

| (2) | 2-0 |
| :---: | :---: |
| $\bigcirc$ | $\beta=1.12$ |
| $+$ | $\beta=225$ |
| $x$ - | $\beta=4: 5$ |
| $\cdots$ | $\beta=9$ |



VERTICAL NO. 1


analyses. The $\sigma_{y}$ values in shells are marginally higher than those of $\sigma_{x}$. At the shell/transition interfaces and also on the upstream transition/core interface, $\sigma_{y}$ values are marginally lower than those of $\sigma_{x}$. In the remaining core portion the $\sigma_{y}$ values are marginally higher than the corresponding $\sigma_{x}$ values. This indicates that near triaxial conditions prevail in the dam.
(v) The $\sigma_{x} / \sigma_{y}$ ratios vary between 0.95 and 1.05 for all valley shapes.
(vi) The $\sigma_{y}$ values at the core centre line at the base of core trench are $-272,-325,-372$ and $-394 \mathrm{t} / \mathrm{m}^{2}$ for 3-D analyses with valley width factors of $1.12,2.25,4.5$ and 9 respectively and $-365 \mathrm{t} / \mathrm{m}^{3}$ for plane strain analysis. The 3-D values are thus $75 \%, 94 \%$, $102 \%$ and $108 \%$ respectively of the plane strain values. The $\sigma_{y}$ values, thus approach plane strain values at a valley width factor of 4.5 .

The distribution of $\sigma_{y}$ along horizontal planes is shown in Fig. 9 for 3-D analyses. It is seen that the distribution is similar to that of $\sigma_{x}$. However the $\sigma_{y}$ in core is greater than those in the adjoining shells all over the height. The effect of valley width is more pronounced on $\sigma_{y}$ as compared to that on $\sigma_{x}$. The values of $\sigma_{y}$ for $\beta=4.5$ and $\beta=9$ are very close to each other all through the dam indicating that the effect of valley width is not evident beyond $\beta=4.5$.


FIGURE 9 Horizontal Normal Stress $\sigma_{y}$ at Constant Elevations over Central Section

Vertical Normal Stress $\sigma_{z}$
The plot of $\sigma_{z}$ along height for different analyses is shown in Fig. 10. It is seen from the figure that the variation along height is on the same pattern as that for $a_{x}$ and $\alpha_{y}$. Whereas in case of $\sigma_{x}$ and $\sigma_{y}$ the effect of valley width was manifest only near the base, the effect of valley width on $\sigma_{z}$ is evident in the lower $2 / 3$ rd height, except on core/transition interfaces where the effect of valley shape is appreciable only in the lower one third height. The following points emerge from a perusal of Fig. 10.


## 1. Irrespective of the valley shape

(a) The distribution of $\sigma_{z}$ along height is parabolic in shells and transitions, with a near zero value at top and non-zero value at the ground level and a maxima at about one third height above groung level.
(b) In the core the $\sigma_{z}$ values increase nearly linearly with the overburden height upto the core trench base in the interiors of the core and in about three fourths upper height at the core faces. In the lower one fourth height, the increase of $\sigma_{z}$ with overburden is sharp at the core faces upto the ground level. In the core trench, the $\alpha_{z}$ distribution is same as that for $\sigma_{x}$ and $\sigma_{y}$ i.e. a decrease first, followed by an increase at the base of trench.
(c) At the core faces in the upper two third height and in transition and shells over the upper one third height, the $\sigma_{z}$ values are almost of same magnitudes for all valley shapes. In the remaining height the effect of valley width is significant and the $\sigma_{z}$ values increase with increasing valley width. The $\sigma_{2}$ values with valley width factors of 4.5 and 9 are very near to those of plane strain values.

Table 3 gives typical maximum values of stress $\sigma_{2}$ for different analyses.

TABLE 3

$$
\text { Maximum } \sigma_{2} \text { Values for Different Analyses }\left(t / m^{2}\right)
$$

| Vertical | 3-D Analyses |  | Plane Strain Analysis |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  | $B=1.12$ | $B=2.25$ | $B=4.5$ | $B=9$ |  |
| 1 | -127 | -163 | -206 | -213 | -212 |
| 2 | -188 | -240 | -271 | -282 | -281 |
| 3 | -231 | -329 | -409 | -421 | -440 |
| 4 | -301 | -380 | -413 | -434 | -399 |
| 5 | -296 | -392 | -448 | -470 | -403 |
| 6 | -218 | -273 | -341 | -359 | -310 |
| 7 | -166 | -235 | -319 | -344 | -328 |

The maximum vertical normal stress is about 50 to $60 \%$ and 70 to $80 \%$ of the corresponding plane strain values for valley width factor of 1.12 and 2.25 and about 90 to $100 \%$ of plane strain values for valleys with valley width factors of 4.5 and 9 .

In the upper portion of the dam (two third for core and one third else where) the 3-D analyses give generally higher values of $\sigma_{z}$ than those from plane strain analysis. The values however decrease with increasing valley width factor.

The variation of $\sigma_{z}$ on horizontal planes is shown in Fig. 11 for different analyses.


FIGURE 11 Vertical Normal Stress $\sigma_{z}$ at Constant Elevations over Central Section
It is seen from the figure that:
(i) In the lower one sixth height, the $\sigma_{z}$ stresses in the core are more than the corresponding values in the adjoining shells, whereas in the upper portion, the case is just opposite.
(ii) The $\sigma_{z}$ stress increases from the face towards the interior. The stress gradient in horizontal direction is practically same at all elevations, as can be inferred from the almost parallel nature of plots at all elevations.
(iii) The effect of valley width, i.e. $\sigma_{z}$ increasing with valley width factor, is maximum at the dam base, decreasing gradually upward and being practically absent in the upper third portion of the dam.
(iv) All over the dam, the values of $\sigma_{z}$ for valleys with valley width factor of 4.5 and 9 are almost same, and nearly equal to those of plane strain analysis.
(v) The vertical normal stress $\sigma_{z}$ is less than the overburden pressure for all analyses on core faces for a major part of the height. The difference between the two curves indicates the load transfer to the adjoining regions. The load transfer is maximum on the downstreame faces of the core and the downstream transition. For narrow valleys with $\beta=1.12$ and 2.25 , the stresses in shell are also less than the corresponding overburden pressure in the lower portion of dam. This indicates that for these valleys the load is transferred from the central section to the sections towards the abutments.


FIGURE 12 Ratio $\sigma_{3} / \sigma_{1}$ at Different Verticals for 3-D Analysis

Ratio of Principal Stresses $\sigma_{3} / \sigma_{1}$ ):
The ratio of minor principal stress $\sigma_{3}$ to the major principal stress $\sigma_{1}$ along height at different verticals is shown in Fig. 12. It is seen from the figure that the values of $\sigma_{3} / \sigma_{1}$ vary between 0.38 to 0.48 in shells and transitions, with higher values for narrower valley. Except near the top of the verticals where the stress values are low and the ratio may even be negative, the ratios $\sigma_{3} / \sigma_{1}$ remain in this range all over the height, with almost a uniform distribution. In the shells however, in the upper reaches wider valleys exhibit higer $\sigma_{3} / \sigma_{1}$ values.

At the core trench base, high $\sigma_{3} / \sigma_{1}$ ratio's ( 0.92 ) are obtained for all valley shapes. On the upstream face of core the ratio $\sigma_{3} / \sigma_{1}$ reduces sharply to very low values at EL 80 (higher ratio for narrow valleys), increasing again upto elevation 220 m (higher values for wider valleys). In the middle of the core, the ratio $\sigma_{3} / \sigma_{1}$ gradually reduces to $0.35-0.45$ at EL 160 (higher values for narrow valleys) whereafter the ratio again increases. On the downstream face of core the ratio $\sigma_{3} / \sigma_{1}$ is almost uniform all over the height ( $0.25-0.45$ ) with higher values for narrow valleys.

The typical values of $\sigma_{3} / \sigma_{1}$ ratio along all the verticals are given in table 4 .
TABLE 4
Typical $\sigma_{3} / \sigma_{1}$ Ratio along Verticles

| Vertical <br> No. |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
|  | 1.12 | $\sigma_{\mathbf{3}} / \sigma_{\mathbf{1}}$ | Values for Valley Width Factor |  |
| 1 | 0.48 | $\mathbf{2 . 2 5}$ | 4.5 | 9 |
| 2 | 0.47 | 0.44 | 0.40 | 0.37 |
| 3 | 0.63 | 0.37 | 0.35 | 0.35 |
| 4 | 0.57 | 0.43 | 0.32 | 0.28 |
| 5 | 0.44 | 0.35 | 0.46 | 0.41 |
| 6 | 0.45 | 0.36 | 0.31 | 0.28 |
| 7 | 0.47 | 0.43 | 0.31 | 0.26 |

It is also seen from the figure that the $\sigma_{3} / \sigma_{1}$ ratios are slightly higher in the upstream portion as compared to those in the downstream portion of the dam. The $\sigma_{3} / \sigma_{1}$ ratios change sharply for a narrow valley with $\beta=1.12$ to 2.25 whereafter the change is gradual.

Fig. 13 shows the plot of the ratios $\sigma_{3} / \sigma_{1}$ along horizontal planes. It is seen from the plot that except near the dam faces the $\sigma_{3} / \sigma_{1}$ ratios in the shells remain equal to those on the transition/shell interface at the corresponding elevation. At the ground level, the $\sigma_{3} / \sigma_{1}$ ratios in core are higher than those in the adjoining shells. At all other elevations the ratios $\sigma_{3} / \sigma_{1}$ on core faces are generally lower than the corresponding values in shells. Inside the core, the ratios $\sigma_{3} / \sigma_{1}$ are higher than those at the faces. Higher $\sigma_{2} / \sigma_{1}$ values indicate that shear failures are less likely in narrow valleys.


FIGURE 13 Ratio $\sigma_{3} / \sigma_{1}$ at Constant Elevation over Central Section

## Maximum Values of Stresses and Movements

The maximum values of three normal stresses $\sigma_{x}, \sigma_{y}$ and $\sigma_{z}$ and the horizontal as well as vertical displacements occurring at the central section are plotted against the valley width factor $\beta$ in Fig 14. The plane strain values are also plotted as for $\beta=\infty$. It is seen that as far as the stresses are concerned, the 3-D values are more or less the same as the plane strain values for valleys with $\beta=4.5$ or more. In case of displacements, the maximum values go on increasing with valley width factor and only at $\beta=9$ the values are practically the same as those from a plane strain analysis.

## Limitations of the Study

The study suffers from the following limitations.

1. The study is only for the end of construction stage. A similar study for the reservoir full condition will be useful in generalisation of the results.
2. The foundation and abutments have been assumed rigid. Slippage condition along abutment contact and at the zonal interfaces in the dam will be more realistic.
3. The actual excavation slopes in the core trench have not been accounted for.
4. The mesh is coarse in the cross valley direction and the valley has been assumed symmetrical.


FIGURE 14 Stresses Displacemerts u/s Vally Widh Facter

## Conclusions

The following conclusions can be drawn from the study:

1. The deformations are small for narrow valleys and increase with the valley width, with the values of deformations approaching almost plane strain values for a 3-D analysis with valley width factor of 9 .
2. The horizontal movements are in the ranges of $25 \%$ to $35 \%, 30$ to $50 \%, 60$ to $75 \%$ and 70 to $100 \%$ of the corresponding values obtained from plane strain analysis with valley width factors of 1.12 , $2.25,4.5$ and 9 respectively.
3. The vertical settlements obtained from 3-D analyses with valley width factors of $1.12,2.25,4.5$ and 9 are about 35 to 50,50 to 65 , 65 to 85 and 85 to $100 \%$ of the plane strain values.
4. The effect of valley width is more on the horizontal movements than on the settlements.
5. The effect of valley width on the stresses is comparatively small in the upper two third portion and is significant in the regions near the base.
6. In the upper half of the dam, the 3-D stress values are more than the plane strain values.
7. The stresses in the upper half decrease with increase in valley width, the values with valley width factor of 4.5 and 9 being nearly the same as for plane strain analysis. The values of stresses, however, are small in this region.
8. In the lower third portion of dam the stress values increase with valley width factor.
9. The $\sigma_{x}$ values in the lower third portion for 3-D analyses are about $50 \%, 70 \%, 93 \%$ and $95 \%$ of plane strain values for valley width factors equal to $1.12,2.25,4.5$ and 9 respectively.
10. The $\sigma_{z}$ stress values in the lower third portion of dam for valley width factors of $1.12,2.25,4.5$ and 9 are about $50-60 \%$, 70 to $80 \%$, 90 to $100 \%$, of plane strain values.
11. A valley with a valley width factor of 4.5 will give the stress values comparable to those from plane strain analysis and therefore a 3-D analysis for such valley is not required. However, as far as displacements are concerned, even this valley will give smaller values as compared to those from plane strain analysis. For displacements, the limiting valley width factor is found to be 9 beyond which a plane strain analysis of the maximum section will be adequate.
12. The ratio of $\sigma_{3} / \sigma_{1}$ is higher for narrow valleys and decrease with valley width indicating that a narrow valley is stronger against shear failure.

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