Indian Geotechnical Journal, 18 (4), 1988

Semiconfined Flow Through an Aquifer

by

K. Sridharan*

A. Siva Reddy**

V.G. Kulkarni***

Introduction

This study is concerned with the computation of seepage flow through a semiconfined aquifer. Figure 1*a* shows the physical flow domain in z plane, $z = x + iy$, in which CG is the interface between two strata of finite thickness with k_1 and k_2 as coefficients of permeability. On the downstream side, a level water surface is assumed such that a phreatic line, ED, occurs.

This paper presents an analytical solution to the problem of finding the shape and location of the phreatic line when steady flow takes place through the aquifer.

In the present investigation it is assumed that the water level on the downstream side of CG remains unchanged and on the upstream side of CG, a constant head of water is assumed to act across AF. The flow is assumed to be two dimensional. Analysis has been done using the hodograph and method of inversion. Numerical results in non-dimensional form have been presented for the seepage quantity and for location of phreatic line for various inclinations of CG.

Derivation of Equations

The complex potential plane w, where $w = \phi + i\Psi$, for the flow region is shown in Fig. 1*b*. Here, ϕ is the velocity potential function defined as

$$\phi = -k_1 \left(\frac{p}{\gamma_\omega} - Y \right) + c$$

Associate Professor, Department of Civil Engineering, Indian Institute of Science, Bangalore 560 012, India.

**Professor, Department of Civil Engineering, Indian Institute of Science, Bangalore 560 012, India.

***Assistant Professor, B V B College of Engineering, Hubli, Karnataka, India.

(*The modified manuscript of this paper was received in August* 1988 *and is open for discussion till end of June,* 1989)

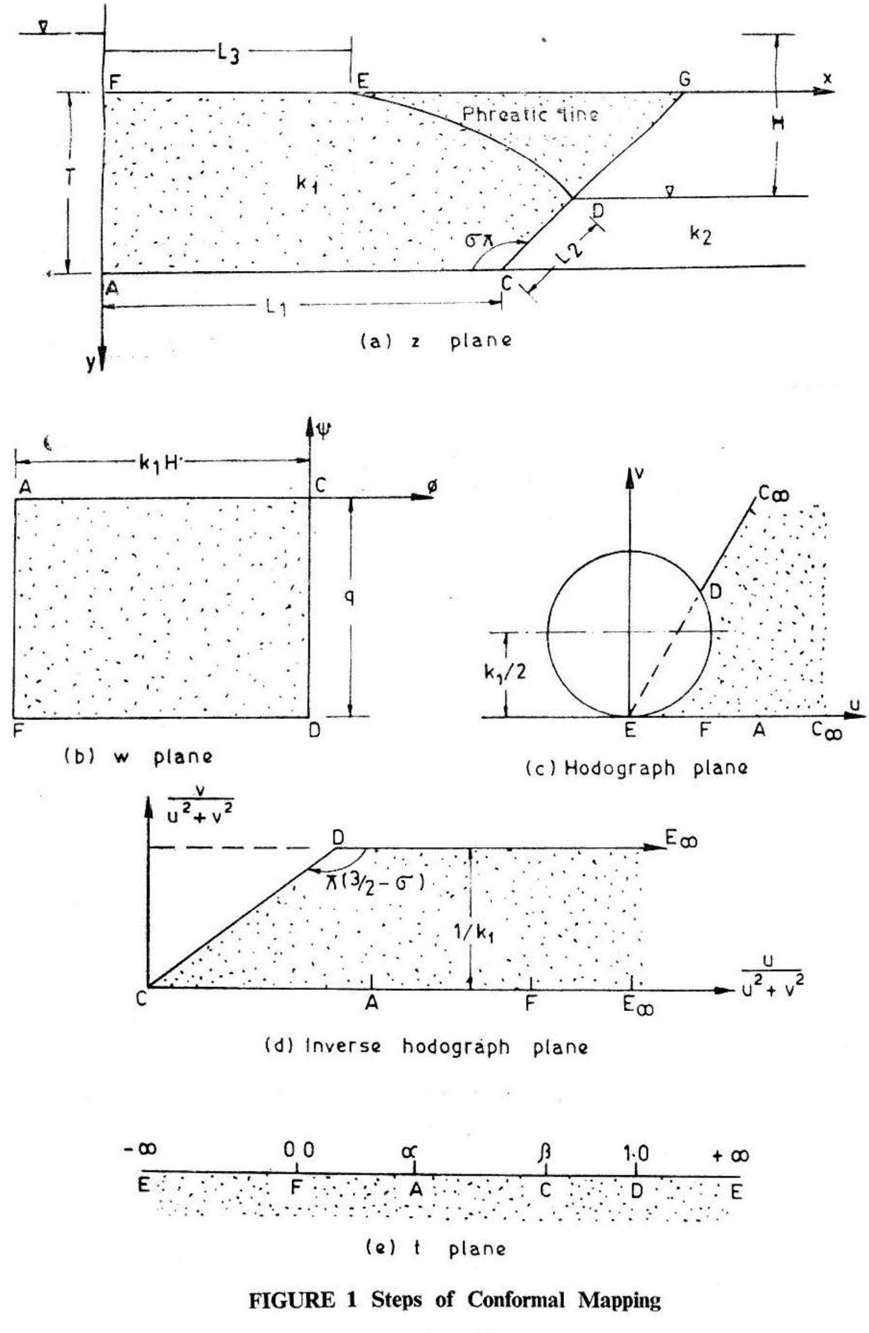

FIGURE 1 Steps of Conformal Mapping

where p is the pressure, γ_ω is the unit weight of water, k_1 is the co-efficient of permeability, Y is the vertical coordinate, c is an arbitrary constant and Ψ is the stream function. The value of Ψ for the streamline AC is assumed to be zero and for the streamline FED, it is assumed to be $-q$. Since the pressure along the phreatic line is atmospheric, ϕ will vary linearly with the elevation along the phreatic line ED.

Figure 1c shows the hodograph plane, *i.e.*, $\frac{dw}{dz}$ plane, for the flow domain. At point C, the velocity is infinite and point E is a stagnation point (velocity is zero). The phreatic line ED maps into a circle of diameter k_1 passing through the origin and it is completely defined in the hodograph plane (Polubarinova—Kochina, 1962). According to inversion rule, circles and straight lines passing through the origin will be straight lines. The inverse of the hodograph, *i.e.*, $\frac{dz}{dw}$ plane, has been constructed and it is shown in Fig. 1d. In the inverse hodograph, point C is at the origin and point E lies at infinity.

$\frac{dz}{dw}$ and t relations for various ranges are obtained as follows.

According to the Schwarz—Christoffel transformation, the conformal mapping of the polygon ECDE in the $\frac{dz}{dw}$ plane to the lower half of the auxiliary t plane (Fig. 1e) is given by

$$\frac{dz}{dw} = M \int \frac{dt}{(t-\beta)^{3/2-\sigma}\,(t-1)^{\sigma-1/2}} + N \qquad \ldots(1)$$

the vertices E, C, D and E of the polygon in $\frac{dz}{dw}$ plane being mapped on $-\infty, \beta, 1, \infty$, respectively, of the t plane; M and N being constants.

For $\beta < t < 1.0$, Eq. (1) may be written as

$$\frac{dz}{dw} = M \int_{\beta}^{t} (t-\beta)^{\sigma-3/2}\,(t-1)^{1/2-\sigma}\,dt + N_3 \qquad \ldots(2)$$

Substituting $t = (1-\beta)\,r + \beta$ and performing the integrations (Gradshteyn and Ryzhik, 1965), Eq. (2) reduces to

$$\frac{dz}{dw} = (-1)^{1/2-\sigma}\, M.B_r\,(\sigma-\tfrac{1}{2}, \tfrac{3}{2}-\sigma) + N_3 \qquad \ldots(3)$$

where $B_r\,(\sigma-\frac{1}{2}, \frac{3}{2}-\sigma)$ is incomplete Beta function and N_3 is a constant.

Since at $t = \beta, \frac{dz}{dw} = 0$, the constant $N_3 = 0$

At point D ($t = 1$), Eq. (3) becomes

$$\frac{dz}{dw} = (-1)^{1/2-\sigma} M B (\sigma-\tfrac{1}{2}, \tfrac{3}{2}-\sigma) \quad ...(4)$$

where $B (\sigma-\frac{1}{2}, \frac{3}{2}-\sigma)$ is complete Beta function.

From Fig. 1d, at point D ($t = 1$),

$$\frac{dz}{dw} \mid D = \frac{i}{k_1} - \frac{\tan \sigma\pi}{k_1} \quad ...(5)$$

where $i = \sqrt{-1}$.

Hence, from Eq. (4), the constant M can be obtained as

$$M = \frac{\frac{1}{k_1} (i-\tan \sigma\pi)}{(-1)^{1/2-\sigma} B(\sigma-\frac{1}{2}; \frac{3}{2}-\sigma)} \quad (6)$$

For $\alpha \leqslant t \leqslant \beta$, Eq. (1) may be written as

$$\frac{dz}{dw} = M \int_{\beta}^{t} (t-\beta)^{\sigma-3/2}(t-1)^{1/2-\sigma} dt + N_2$$

$$= - M \int_{\alpha}^{t} (\beta-t)^{\sigma-3/2} (1-t^{1/2-\sigma} dt + N_2 \quad ...(7)$$

where, N_2 is a constant.

Since $\frac{dz}{dw} = 0$ at $t = \beta$, Eq. (7) becomes

$$0 = - M \int_{\alpha}^{\beta} (\beta-t)^{\sigma-3/2} (1-t)^{1/2-\sigma} dt + N_2 \text{ and}$$

$$N_2 = M. I_2 \quad ...(8)$$

$$\text{where } I_2 = \int_{\alpha}^{\beta} (\beta-t)^{\sigma-3/2}(1-t)^{1/2-\sigma} dt \quad ..(9)$$

At point A ($t = \alpha$),

$$\frac{dz}{dw} \mid_A = N_2 = M. I_2 \quad ...(10)$$

For $0 \leqslant t \leqslant \alpha$, Eq. (1) may be written as

$$\frac{dz}{dw} = -M \int_{0}^{t} (\beta-t)^{\sigma-3/2} (1-t^{1/2-\sigma} dt + N_1 \quad ...(11)$$

where N_1 is a constant.

Applying the conditions at point A

$$\left(\frac{dz}{dw} = M.\,I_2 \text{ at } t = \alpha,\ \text{Eq. } 10\right),$$

$$N_1 = M\,(I_1 + I_2) \qquad \ldots(12)$$

$$\text{where } I_1 = \int_0^{\alpha} (\beta - t)^{\sigma - 3/2}\,(1-t)^{1/2-\sigma}\,dt \qquad \ldots(13)$$

At point F $(t = 0)$,

$$\frac{dz}{dw}\,\Big|_F = M\,(I_1 + I_2) \qquad \ldots(14)$$

For $1 < t < \infty$, Eq. (1) may be written as

$$\frac{dz}{dw} = M \int_1^t (t-\beta)^{\sigma-3/2}\,(1-t)^{1/2-\sigma}\,dt + N_4 \qquad \ldots(15)$$

where N_4 is a constant.

Since, $t = 1$ at point D, from Eq. (15),

$$N_4 = \frac{1}{k_1}\,(i - \tan\sigma\pi) \qquad \ldots(16)$$

The w-t relations for various ranges of t are obtained as follows.

The conformal mapping of the complex potential plane, w, onto the t plane is given by

$$w = M' \int \frac{dt}{\sqrt{(t-1)\,(t-\beta)\,(t-\alpha)t}} + N' \qquad \ldots(17)$$

where M' and N' are constants.

For $\beta < t < 1$, Eq. (17) can be written as

$$w = \pm\,(i)\,M' \int_\beta^t \frac{dt}{\sqrt{(1-t)(t-\beta)(t-\alpha)t}} + N_3' \qquad \ldots(18)$$

where N'_3 is a constant.

Taking negative sign which is pertinent with the given physical problem, after integration, (Byrd and Friedman, 1954)

$$w = -\,iM'\,\frac{2}{\sqrt{\beta(1-\alpha)}}\,F\left(\text{si}^{-1}\sqrt{\frac{(t-\beta)(1-\alpha)}{(t-\alpha)(1-\beta)}}\,\sqrt{\frac{\alpha(1-\beta)}{\beta(1-\alpha)}}\right) + N_2' \qquad \ldots(19)$$

where $F(\theta, m)$ is incomplete elliptic integral of the first kind having modulus m and amplitude θ.

Applying the conditions at point $C\,(t=\beta, w=0)$ and at point $D\,(t=1$ $w=-iq)$, $N'_3=0$ and an expression for the discharge q in terms of M' is obtained as

$$q=-M'\frac{2}{\sqrt{\beta(1-\alpha)}}\,K\left(\sqrt{\frac{\alpha(1-\beta)}{\beta\,(1-\alpha)}}\right) \qquad \ldots(20)$$

where, $K(m)$ is the complete elliptic integral of first kind with modulus m.

For $\alpha<t<\beta$, Eq. (17) can be written as

$$w=\pm M'\int_{\alpha}^{t}\frac{dt}{\sqrt{(1-t)(\beta-t)(t-\alpha)t}}+N_2' \qquad \ldots(21)$$

where N'_2 is a constant.

Taking negative sign which is pertinent with the given physical problem, after integration (Byrd and Friedman, 1954),

$$w=M'\frac{2}{\sqrt{(1-\alpha)\beta}}\,F\left(\sin^{-1}\sqrt{\frac{\beta(t-\alpha)}{t(\beta-\alpha)}}\sqrt{\frac{\beta-\alpha}{\beta(1-\alpha)}}\right)+N_2' \qquad \ldots(22)$$

Applying the conditions at point $A\,(t=\alpha, w=-k_1 H)$ and at point $C\,(t=\beta, w=0)$

$$N_2'=-k_1 H \qquad \ldots(23)$$

$$M'=-\frac{k_1.H.\sqrt{(1-\alpha)}}{2.k\left(\sqrt{\dfrac{\beta-\alpha}{\beta(1-\alpha)}}\right)} \qquad \ldots(24)$$

For $0<t<\alpha$, Eq. (17) can be written as

$$w=iM'\int_{0}^{t}\frac{dt}{\sqrt{(1-t)(\beta-t)(\alpha-t)t}}+N_1' \qquad \ldots(25)$$

where N_1' is a constant.

Performing the integration (Byrd and Friedman, 1954),

$$w=iM'\frac{2}{\sqrt{\beta(1-\alpha)}}\,F\left(\sin^{-1}\sqrt{\frac{t(1-\alpha)}{\alpha(1-t)}}\sqrt{\frac{\alpha(1-\beta)}{\beta(1-\alpha)}}\right)+N_1' \qquad \ldots(26)$$

At point F, $t = 0$ and $w = -k_1 H - iq$ and hence,

$$N_1' = -k_1 H - iq \qquad \text{...(27)}$$

For $1 < t < \infty$, Eq. (17) is written as

$$w = M' \int_1^t \frac{dt}{\sqrt{(t-1)(t-\beta)(t-\alpha)t}} + N_4' \qquad \text{...(28)}$$

where N'_4 is a constant.

Performing the integration (Byrd and Friedman, 1954),

$$w = M' \frac{2}{\sqrt{\beta(1-\alpha)}} F\left(\sin^{-1}\sqrt{\frac{\beta(t-1)}{t-\beta}}, \sqrt{\frac{\beta-\alpha}{\beta(1-\alpha)}}\right) + N_4' \qquad \text{...(29)}$$

At point D, $t = 1$ and $w = -iq$ and hence,

$$N_4' = -iq \qquad \text{...(30)}$$

z—t relations for various ranges of t are obtained as follows. Multiplying Eq. (1) by the derivative of Eq. (17) results in

$$\frac{dz}{dw} \cdot \frac{dw}{dt} = \frac{dz}{dt} = [M \int \frac{dt}{(t-\beta)^{3/2-\sigma}(t-1)^{1/2-\sigma}} + N]. \left[\frac{M'}{\sqrt{(t-1)(t-\beta)(t-\alpha)t}}\right] \qquad \text{...(31)}$$

Integrating Eq. (31) by parts and rearranging the terms,

$$z = [M \int (t-\beta)^{\sigma-3/2} (t-1)^{1/2-\sigma} dt + N] [M' \int \frac{dt}{\sqrt{(t-1)(t-\beta)(t-\alpha)t}} + N')$$
$$- \int [(M' \int \frac{dt}{\sqrt{(t-1)(t-\beta)(t-\alpha)t}} + N') M(t-\beta)^{\sigma-3/2} (t-1)^{1/2-\sigma}] dt + C' \qquad \text{...(32)}$$

where C' is a constant.

For $0 < t < \alpha$, Eq. (32) becomes

$$z = [M \int_0^t (t-\beta)^{\sigma-3/2} (t-1)^{1/2-\sigma} dt + N_1] [M' \int_0^t \frac{dt}{\sqrt{(t-1)(t-\beta)(t-\alpha)t}} + N_1']$$
$$- \int_0^t [(M' \int_0^t \frac{dt}{\sqrt{(t-1)(t-\beta)(\alpha t-\alpha)t}} + N_1') M(t-\beta)^{\sigma-3/2}(t-1)^{1/2-\sigma} dt] + C_1 \qquad \text{...(33)}$$

where C_1 is a constant.

At point F, $t = 0$ and $z = 0$. Hence,

$$C_1 = -N_1 N'_1 \qquad \ldots(34)$$

At point A, $t = \alpha$ and $z = iT$.

Introducing the limits and values of constants in Eq. (33) and simplifying, an expression for T is obtained as

$$T = | M(q.\ I_2 - M' \frac{2}{\sqrt{\beta(1-\alpha(}} I_3) | \qquad \ldots(35)$$

$$\text{where } I_3 = \int_0^\alpha F\left(\sin^{-1}\sqrt{\frac{t(1-\alpha}{\alpha(1-t)}}, \sqrt{\frac{\alpha(1-\beta}{\beta(1-\alpha)}}\right)(\beta-t)^{\sigma-3/2}(1-t)^{1/2-\sigma}dt \qquad \ldots(36)$$

For $\alpha < t < \beta$, Eq. (32) becomes

$$z = [M \int_\alpha^t (t-\beta)^{\sigma-3/2}(t-1)^{1/2-\sigma}dt + N_2]\,[M' \int_\alpha^t \frac{dt}{\sqrt{(t-1)(t-\beta)(t-\alpha)t}} + N'_2]$$
$$- \int_\alpha^t [(M' \int_\alpha^t \frac{dt}{\sqrt{(t-1)(t-\beta)(t-\alpha)t}} + N'_2)M(t-\beta)^{\sigma-3/2}(t-1)^{1/2-\sigma}]dt + C \qquad \ldots(37)$$

where C_2 is a constant.

At point A, $t = \alpha$ and $z = iT$ and hence C_2 is obtained as

$$C_2 = iT + M\ .\ I_2\ .\ k_1\ H \qquad \ldots(38)$$

At point C, $t = \beta$ and $z = iT + L_1$.

Introducing the limits and values of constants in Eq. (37), and samplifying, an expression for L_1 is obtained as

$$L_1 = -M\ M'.\frac{2}{\sqrt{\beta(1-\alpha)}} I_4 \qquad \ldots(39)$$

$$\text{where, } I_4 = \int_\alpha^\beta F\left(\sin^{-1}\sqrt{\frac{\beta(t-\alpha)}{t(\beta-\alpha)}}, \sqrt{\frac{\beta-\alpha}{\beta(1-\alpha)}}\right)(\beta-t)^{\sigma-3/2}(1-t)^{1/2-\sigma}dt$$

From Eqs. (35) and (39)

$$\left|\frac{L_1}{T}\right| = \left|\frac{M'\frac{2}{\sqrt{\beta(1-\alpha)}}I_4}{q.I_2 - M'\frac{2}{\sqrt{\beta(1-\alpha)}}.I_3}\right| \qquad \ldots(40)$$

For $\beta < t < 1.0$, Eq. (32) becomes

$$z = [M \int_{\beta}^{t} (t-\beta)^{\sigma-3/2}(t-1)^{1/2-\sigma}\mathrm{d}t + N_3]\,[M' \int_{\beta}^{t} \frac{\mathrm{d}t}{\sqrt{(t-1)(t-\beta)(t-\alpha)t}} + N_3']$$

$$- \int_{\beta}^{t} [\,(M' \int_{\beta}^{t} \frac{\mathrm{d}t}{\sqrt{(t-1)(t-\beta)(t-\alpha)t}} + N_3').M(t-\beta)^{\sigma-3/2}(t-1)^{1/2-\sigma}]\mathrm{d}t + C_3$$

...(41)

At point C, $t = \beta$, $z = z_c = iT + L_1$ ...(42)

Hence, $C_3 = iT + L_1$

At point D, $t = 1.0$, $z = z_D = iT + L_1 + L_{2H} - iL_{2V}$

where $L_{2H} = L_2 \cos[\pi(1-\sigma)]$

and $L_{2V} = L_2 \sin[\pi(1-\sigma)]$

Introducing the limits and values of constants in Eq. (41) and simplifying, an expression for L_2 is obtained as:

$$L_2 = (-1)^{-\sigma} M.M' \frac{2}{\sqrt{\beta(1-\alpha)}}\left[I_6 - I_5.\,K\left(\sqrt{\frac{\alpha(1-\beta)}{\beta(1-\alpha)}}\right)\right] \quad ...(43)$$

$$\text{where } I_5 = \int_{\beta}^{1.0} (t-\beta)^{\sigma-3/2}(1-t)^{1/2-\sigma}\,\mathrm{d}t \quad ...(44)$$

$$\text{and } I_6 = \int_{\beta}^{1.0} F\left(\sin^{-1} \frac{\sqrt{(t-\beta)(1-\alpha)}}{(t-\alpha)(1-\beta)}, \sqrt{\frac{\alpha(1-\beta)}{\beta(1-\alpha)}}\right)(t-\beta)^{\sigma-3/2}(1-t)^{1/2-\sigma}dt$$

.. (45)

From Eqs. (35) and (43),

$$\left|\frac{L_2}{T}\right| = \left|\frac{M' \frac{2}{\beta(1-\alpha)}\left[I_6 - I_5.K\left(\sqrt{\frac{\alpha(1-\beta)}{\beta(1-\alpha)}}\right)\right]}{q.\,I_2 - M\frac{2}{\sqrt{\beta(1-\alpha)}}\,I_3}\right| \quad ...(46)$$

Location of Phreatic Line

For $1 < t < \infty$, Eq. (32) can be written as

$$z = [M \int_{1}^{t} (t-\beta)^{\sigma-3/2}(t-1)^{1/2-\sigma}\,\mathrm{d}t + N_4]\,[M' \int_{1}^{t} \frac{\mathrm{d}t}{\sqrt{(t-1)(t-\beta)(t-\alpha)t}} + N_4']$$

$$- \int_{1}^{t} ([M' \int_{1}^{t} \frac{\mathrm{d}t}{\sqrt{(t-1)(t-\beta)(t-\alpha)\,t}} + N_4')\,M(t-\beta)^{\sigma-3/2}(t-1)^{1/2-\sigma}]\mathrm{d}t + C_4$$

...(47)

where C_4 is a constant.

or $t = 1, z = z_D = iT + L_1 + L_{2H} - iL_{2V}$

Hence, C_4 is obtained as:

$$C_4 = iT + L_1 + L_{2H} - iL_{2V} - N_4 N'_4 \qquad ...(48)$$

For a given value of t, introducing the values of constants in Eq. (47), performing the integrations and simplifying,

$$z = x + iy$$

$$= M M'G (I_7 F (\phi_u m_u) - I_8) + \frac{M'}{k_1} (i - \tan \sigma\pi) G.E (\phi_u, m_u) + iT - iL_{2V} + L_{2H} + L_1 \qquad ...(49)$$

where $I_7 = \int_1^t (t-\beta)^{\sigma-3/2}(t-1)^{1/2-\sigma} dt \qquad ...(50)$

$$I_8 = \int_1^t F(\phi_u, m_u) (t-\beta)^{\sigma-3/2}(t-1)^{1/2-\sigma} dt \qquad ...(51)$$

$$\phi_u = \sin^{-1} \sqrt{\frac{\beta(t-1)}{(t-\beta)}} \qquad ...(52)$$

$$\text{and } m_u = \sqrt{\frac{(\beta-a)}{\beta(1-a)}} \qquad ...(53)$$

$$G = \frac{2}{\sqrt{\beta(1-a)}}$$

From real and imaginary parts of Eq. (49),

$$X = \text{Re}(M).\ M'G (I_7 F (\phi_u, m_u) - I_8) - \frac{M'}{k_1} \tan \sigma\pi.\ G.F(\phi_u, m_u) + L_1 + {}_{2H} \qquad ...(54)$$

$$\text{and } Y = Im (M) M'G (I_7. F (\phi_u, m_u) - I_8) + \frac{M'}{k_1} G.F. (\phi_u, m_u) + T - L_{2V} \qquad ...(55)$$

The real and imaginary parts of M are to be determined to evaluate Eqs. (54) and (55).

Referring to Eq. (6),

$$M = \pm \frac{(-1)^{\sigma}}{k_1\, B\,(\sigma - \frac{1}{2}, \frac{3}{2} - \sigma)}\,(1 + i \tan \sigma\pi) \qquad \text{..(56)}$$

or $$M = \pm \frac{\cos \sigma\pi + i \sin \sigma\pi)}{k_1\, B(\sigma - \frac{1}{2}, \frac{3}{2} - \sigma)}\,(1 + i \tan \sigma\pi) \qquad \text{...(57)}$$

Separating and equating the real parts and imaginary parts in Eq. (57),

$$R_e(M) = \pm \frac{\cos \sigma\pi + \sin \sigma\pi \tan \sigma\pi}{k_1\, B(\sigma - \frac{1}{2}, \frac{3}{2} - \sigma)}$$

$$I_m(M) = \pm \frac{-\sin \sigma\pi \pm \cos \sigma\pi \tan \sigma\pi}{k_1\, B(\sigma - \frac{1}{2}, \frac{3}{2} - \sigma)}$$

As can be seen from Eqs. (15), (16) and Fig. 1d in the range of $1 < t < \infty$, the constant imaginary value of $\frac{dz}{dw}$ corresponds to the imaginary value of N_4 and hence M is real. M is also positive as seen from Eq. (15) and Fig. 1d (the integrand in Eq. (15) is positive for $1 < t < \infty$),

Hence, $$M = -\frac{\cos \sigma\pi + \sin \sigma\pi \tan \sigma\pi}{k_1\,(B(\sigma - \frac{1}{2}, \frac{3}{2} - \sigma)} \qquad \text{...(58)}$$

Introducing the value of M from Eq. (58) into Eqs. (54) and (55), the points on the phreatic line corresponding to the given values of t can be obtained.

Thus, Eqs. (54) and (55) become

$$X = \frac{\cos \sigma\pi + \sin \sigma\pi \tan \sigma\pi}{k_1\, B(\sigma - \frac{1}{2}, \frac{3}{2} - \sigma)}\, M'.\, G\,(I_7.$$

$$F\,(\phi_u, m_u) - I_8) - \frac{M'}{k_1} \tan \pi\sigma.$$

$$G.F\,(\phi_u, m_u) + L_1 + L_{2H} \qquad \text{...(59)}$$

$$Y = \frac{M'}{k_1}\, G.F\,(\phi_u, m_u) + T - L_{2V} \qquad \text{...(60)}$$

Evaluation of Integrals

Integral I_1 (Eq 19) is evaluated as follows. Eq. 13 can be written as

$$I_1 = \beta^{\sigma - 3/2} \int_0^{\alpha} (1 - t/\beta)^{\sigma - 3/2}\,(1 - t)^{1/2 - \sigma}\, dt \qquad \text{...(61)}$$

Since $t/\beta < 1$ in $\alpha < t < \beta$, the factor $(1 - t/\beta)^{\sigma - 3/2}$ appearing in the integrand of Eq. (61) can be expanded binominally to obtain a converging series as:

$$(1 - t/\beta)^{\sigma-3/2} = \sum_{n=0}^{n=\infty} A_n t^n \quad ..(62)$$

where $A_n = \dfrac{(-1)^n(\sigma-3/2)(\sigma-3/2-1)\,..(\sigma-3/2-\overline{n-1})}{n}\left(\dfrac{1}{\beta}\right)^n;$

$n = 1,2,3,\ldots\ldots$

and $A_o = 1$.

Inserting Eq. (62) and performing term by term integration, Eq. (61) takes the form

$$I_1 = \beta^{\sigma-3/2} \sum_{n=0}^{\infty} A_n\, B(n+1,\, 3/2-\sigma)$$

where $\beta\,(n+1,\, \frac{3}{2}-\sigma)$ is Beta function.

Integral I_2 (Eq. 9) is evaluated as follows. After substituting $r = \dfrac{t-a}{\beta-a}$ and simplifying, Eq. (9) becomes

$$I_2 = (\beta-a)^{\sigma-1/2}(1-a)^{1/2-\sigma} \int_0^1 (1-r)^{\sigma-3/2}\left((1 - \frac{\beta-a}{1-a}\right)^{1/2-\sigma} dr \quad ...(63)$$

For $a < t < \beta$, $\dfrac{\beta-a}{1-a} < 1$, and hence the factor $(1-\dfrac{\beta-a}{1-a})^{1/2-\sigma}$ appearing in the integrand of Eq. (63) can be expanded into a converging series as

$$[1-\left(\frac{\beta-a}{1-a}\right) r\,]^{1/2-\sigma} = \sum_{n=0}^{\infty} D_n r^n \quad ..(64)$$

where $D_n = (-1)^n \dfrac{(\frac{1}{2}-\sigma)\,(\frac{1}{2}-\sigma-1)\ldots\ldots(\frac{1}{2}-\sigma-\overline{n-1})}{n}\left(\dfrac{\beta-a}{1-\sigma}\right)^n;$

$n = 1,2,3,\ldots\ldots\ldots$

and $D_o = 1$.

Inserting Eq. (64) into Eq. (63) and performing term by term integration, Eq. (63) becomes

$$I_2 = (\beta-a)^{\sigma-1/2}(1-a)^{1/2-\sigma} \int_0^1 (1-r)^{\sigma-3/2}\, r^n\, dr \quad ...(65)$$

$$\text{or } I_2 = (\beta-a)^{\sigma-1/2}(1-a)^{1/2-\sigma} \sum_{n=0}^{\infty} D_n\,.\,B\,(n+1,\, \sigma-\tfrac{1}{2}) \quad ...(66)$$

The integrals appearing as I_5 are evaluated as follows:

Referring to Eq. (44), after substituting $r = \dfrac{t-\beta}{1-\beta}$ and simplifying,

$$I_5 = \int_0^{1.0} r^{\sigma-3/2}(1-r)^{1/2-\sigma}\, dr \qquad \ldots(67)$$

or $$I_5 = B(\sigma - \tfrac{1}{2}, \tfrac{3}{2} - \sigma) \qquad \ldots(68)$$

The integrals I_3, I_4, I_6, I_7 and I_8 are evaluated numerically.

Results and Discussion

Expressions for seepage quantity and location and shape of phreatic line in a semiconfined aquifer have been obtained. Numerical results are presented for the quantity of seepage, L_2/T and for the location and shape of the phreatic line for different values of parameters.

Figure 2 illustrates the relation between q/k_1H and H/T for different combinations of L_1/T and σ. It is observed that for a given σ an increase in L_1/T effects a decrease in q/k_1H and the rate of decrease of q/k_1H is found to be significant for the range of L_1/T studied. In particular, for $\sigma = 0.9$, the decrease in q/k_1H for $H/T = 2.5$ corresponding to a change in L_1/T from 0.25 to 0.5 is 30% whereas it is 36% and 41% for a change in L_1/T from 0.5 to 1.0 and 1.0 to 2.0, respectively, for the same value of H/T. The q/k_1H values are influenced only for small values of H/T. Later, as it may be observed from the curves, q/k_1H assumes a limiting value uninfluenced by H/T. Beyond $H/T = 3.5$, there is almost no influence of H/T on q/k_1H for all σ and L_1/T values within the range of study. The influence of σ on q/k_1H values is considerable for smaller values of L_1/T than for larger ones. It is evident from Fig. 2 that the curves for different σ tend to come closer as L_1/T increases.

The dependance of L_2/T on H/T for different values of σ and L_1/T is shown in Fig. 3. For small values of H/T, L_2/T steeply increases with H/T and for larger H/T, the rate of increase reduces considerably. This tendency is more pronounced for smaller value of σ.

Figure 4 shows the phreatic lines for different values of H/T for σ=0.9 and $L_1/T = 1.0$. Fig. 5 shows the phreatic lines for different values of σ and H/T for $L_1/T = 1.0$. For a given σ and L_1/T, as H/T increases, L_2/T and L_3/T increase. However, for the results presented, the shapes of the phreaitc lines are similar for different H/T values (Fig. 4). It is observed that for smaller values of σ, the phreatic line tends to become flatter (Fig. 5). As σ tends to 1.0, the phreatic line tends to become vertical at the point of intersection with the interface.

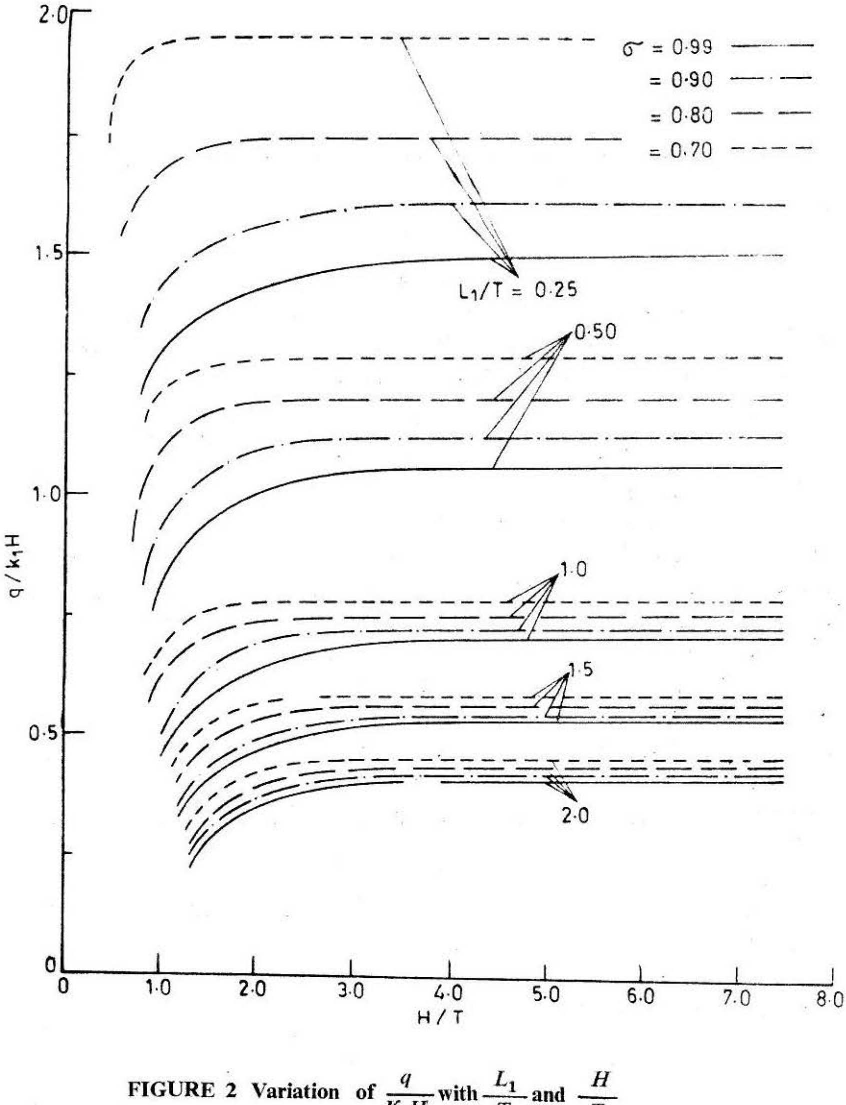

FIGURE 2 Variation of $\frac{q}{K_1 H}$ with $\frac{L_1}{T}$ and $\frac{H}{T}$

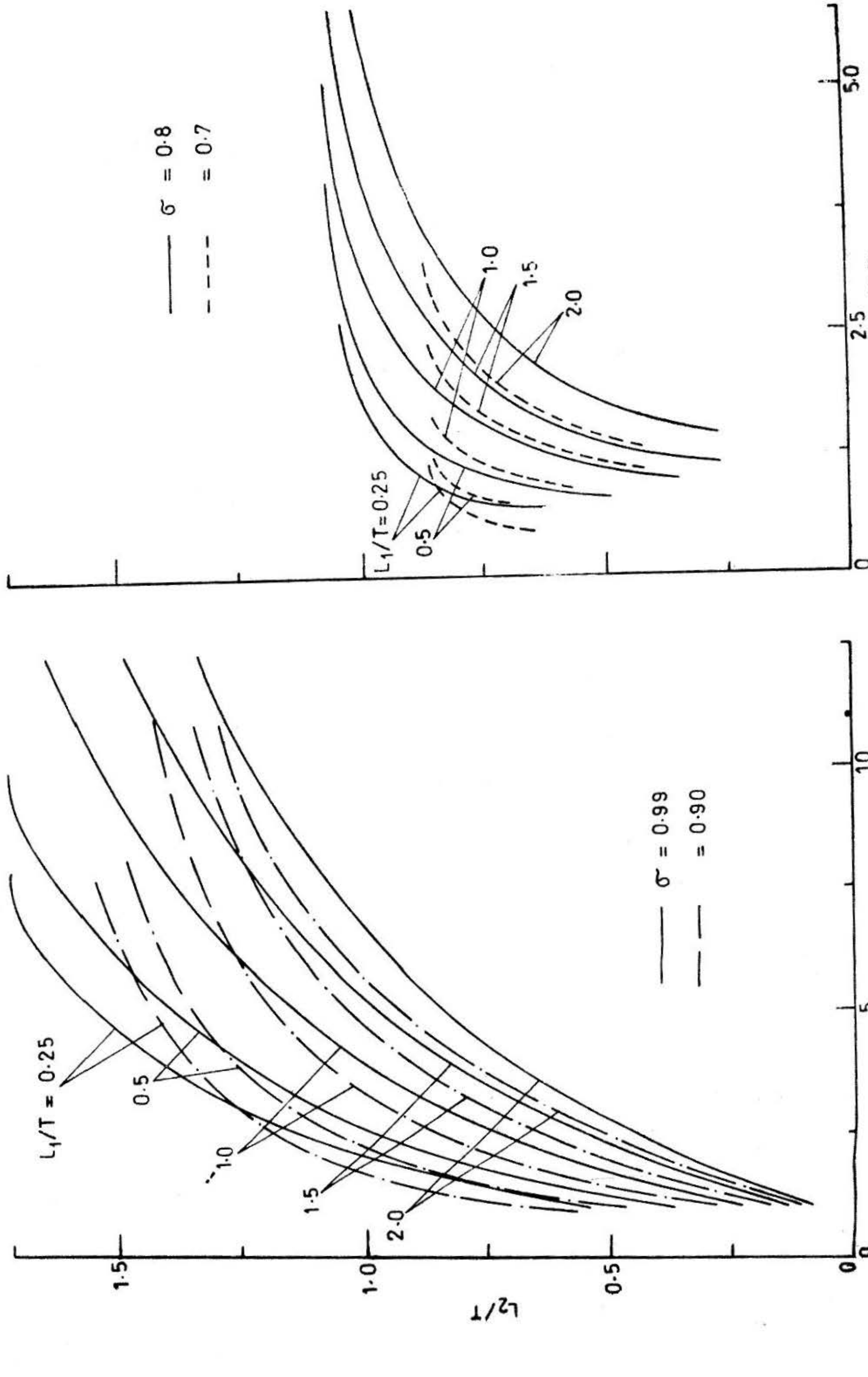

FIGURE 3 Variation of H/T on Phreatic Lines

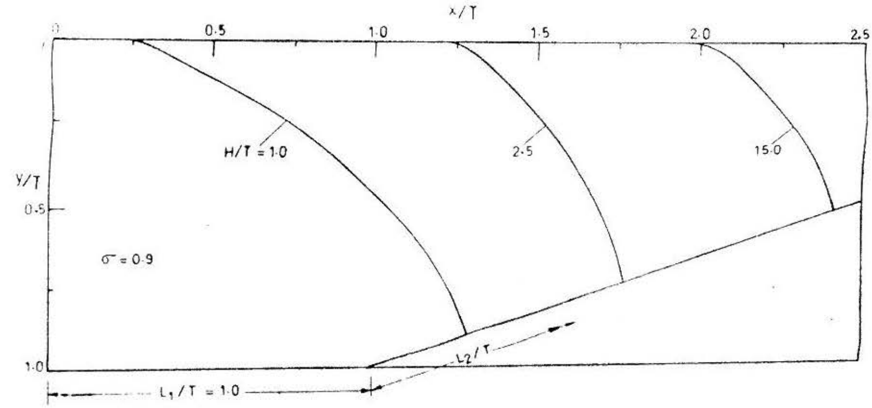

FIGURE 4 Influence of H/T on Phreatic Lines

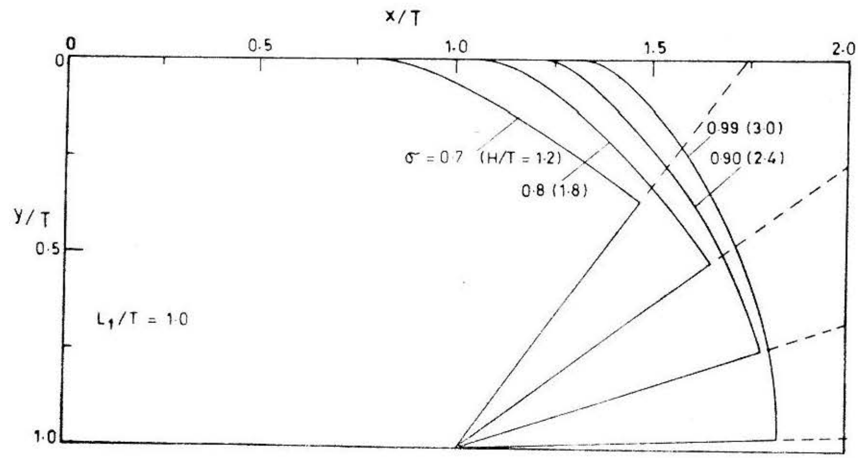

FIGURE 5 Influence of 8 and H/T on Phreatic Lines

Conclusions

Numerical results are presented for the quantity of seepage and the location and shape of the phreatic line in a horizontal semiconfined aquifer. The following conclusions are drawn from these results.

1. L_1/T has considerable influence on the quantity of seepage.

2. The nondimensional seepage quantity increases with H/T for small H/T values but for $H/T > 3.5$, it is almost constant.

3. The influence of σ on quantity of seepage decreases with increase in L_1/T.

4. For small values of H/T, L_2/T steeply increases with H/T. This tendency is more pronounced for smaller values of σ.

5. For given values of σ and L_1/T, the shapes of the phreatic lines are similar for different H/T values, with L_2/T increasing with increase of H/T.

6. For same L_1/T the phreatic line is relatively flatter for larger values of σ.

REFERENCES

BYRD, P.F. and FRIEDMAN, M.D., (1954), "Handbook of Elliptic Integrals for Engineers and Physicists," *Springer Verlag*, Berlin.

GRADSHTEYN, I.S. and RYZHIK, I.M., (1965), "Table of Integrals, Series and Products," *Academic Press*, New York, N.Y.

POLUBARINOVA-KOCHINA, P. Ya., (1962), "Theory of Groundwater Movement," *Princeton University Press*, Princeton, N.J. (Translated from Russian by J M Roger de Wiest).