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A Theoretical Basis for the Estimation of Influence Radius

by

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Introduction and Literature Review :

The radius of influence of a well during pumping or recovery in confined and unconfined aquifers is a very important parameter. It is needed for the estimation of discharge, field permeability and for calculating the optimum well spacing in a well field. The very definition of the "Radius of Influence" or "Influence Radius" (sometimes also referred as "Circle of Influence") is very primitive and nebulous in an otherwise mathematically exact applied science of Groundwater Hydraulics. The definitions given in some of the existing literature are given below. The underlines and the subsequent questions are given by the author to stress the arbitrary nature of the definition.

Davis and Dewiest (1966) :

> "The radius corresponds to rather ill defined radius of influence. The circle of influence is nothing but the vertical projection of a cylinder at constant head, *not affected* by the pumping of the well". —— what is the criterion to determine whether it is affected or not ?

Aravin and Numerov (1963) :

> "The distance between axis of a well and that of cylindrical useful cross section and which the lowering of the piezometric line or of the phreatic curve becomes *unappreciable* while the well is drawing is called the influence radius of the well", —— what is the measure of appreciability ?

With such confused definitions of the concerned parameters it's not surprising that there is no scientific methodology available for estimation of magnitude of the influence radius and the values suggested are purely adhoc and abritrary in nature. Some of the available suggestions are.

D K Todd (1959) :

> "In practice, the selection of the radius of influence R is approximate

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and arbitrary, but the variation in Q (discharge) is small for a wide range of R. Suggested values of R fall in the range of 500 to 1000 ft. These distances do not indicate the limits within which drawdown can be observed, but rather, they serve as an approximation for practical application of the euqation (of steady state discharge)".

Brown, Konoplyantsen, Ineson and Kovalevsky (1972)

The above authors give the probable values (basis not known) of influence radius in Table 1.

TABLE 1

Suggested Values of R for Confined and Unconfined Flow

Type of Sediment	Groundwater Condition	R in meters
Fine and medium grained sand	Confined	250 to 500
	Unconfined	100 to 200
Coarse grained sands and gravel beds	Confined	750 to 1500
	Unconfined	300 to 500
Fissured rocks	Confined	1000 to 1500
	Unconfined	500 to 1000

Aravin and Numerov (1963) mention that as there is no available theoretical basis for calculation of influence radius, this should be determined from hydrogeological tests and for computation purpose what they say is, "It is always possible to assign it a value which is clearly adequate i.e. take R equal to say a few kilometers". For unconfined flow condition, suggested values as given by the authors are given in Table 2.

TABLE 2

Suggested Values of R for Unconfined Flow

Type of Soil	Suggested Value of R
Fine grained soils	100 to 200 meters
Medium grained soils	250 to 500 meters
Coarse granted soils	700 to 1000 meters

Aravin and Numerov (1963) also recommend an *empirical* formula for calculation of R for unconfined flow condition (equation 1).

$$R = 3000\ D\ K \tag{1}$$

in which R is the influence radius in meters, K is the permeability coefficient in m/sec and D is the pumping depth in meters.

Existing literature contains another empirical formula (Aravin and Numerov, 1963) for influence radius as a function of the pumping time (eqn. 2).

$$R = 1.9\sqrt{\frac{KH_t}{n_e}} \quad \dots (2)$$

in which t is the pumping time in days, K is the permeability in m/day, H is the thickness of aquifer in meters and N is the effective porosity or specific yield of the medium.

A critical look into the concept of influence radius :

Influence radius of any closed water body in contact with groundwater (e.g. wells, tanks, pits or canals) may be viewed from two distinctly different operations viz.

1. Pumping (or withdrawing) of water from the closed systems,
2. Recuperation or recovery into the system once the pumping has stopped.

Influence Radius during Pumping:

Let the time be measured from the commencement of pumping operation. Hence at t=O, the piezometric line is as shown in Fig. 1. At some other time t, a different cone of depression will be formed and the piezometric line will have to hit the original piezometric line at a finite distance (say at A), because, a finite amount of water had been released and it will have to come out, from a finite volume of physical space. As time progresses,

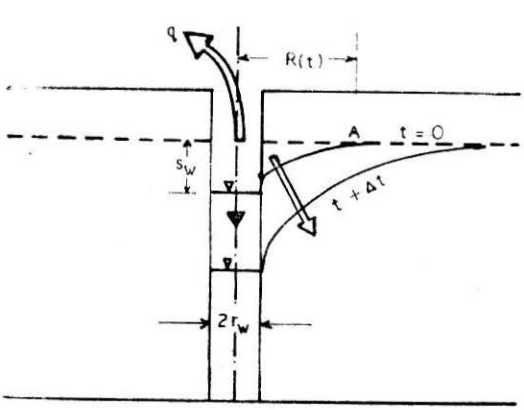

FIGURE 1 Moving Influence Radius for Pumping Well

the piezometric line around well will continue to lower and the point will continue to recede further and further from the centre of the well. The influence radius (R) being the distance between the centre of the well and the point A, it is now obvious, that during pumping operation, R will have to be an increasing function of time. A close inspection of the system (Fig 1) also will reveal that for pumping well,

$$R=f\ (q,\ n_e,\ s_w,\ r_w,\ t) \qquad \ldots(3)$$

It may further be intuitively felt that $\frac{R}{r_w}$ will have to be an increasing function of time and after a prolonged pumping, R may be insensitive or may even be independent of q, s_w and t. The theoretical derivation of this functional relationship (eqn. 3) forms a part of the present investigation.

Influence Radius during Recovery:

After a prolonged pumping as mentioned earlier, when the value of R has more or less stablilised, let us assume the pumping has stopped and the well is allowed to recover. Time in this case is counted from the instant of stoppage of pumping. At t=O, the piezometric line, at a distance R, will be very close to the original line as shown in Fig. 2. From time $t = o^+$ onwards, the line pp' (Fig 2) will act as a recharge boundary and the piezometric line will move upwards, starting from the well face and ending at the recharge boundary at P. Without any external sources of water, the movment of the recharge boundary itself (pp'), does not seem to be plausible. Hence, prima facie the influence radius during recovery is likely to be independent of time and the well drawdown (s_w). Consequently, the parameters which might affect the magnitude of R during recovery seems to be well radius (r_w) and effective porosity or specific yield (storage coefficient in case of confined flow), i.e. during recovery,

$$R=\phi\ (r_w,\ n_e \text{ or } S) \qquad \ldots(4)$$

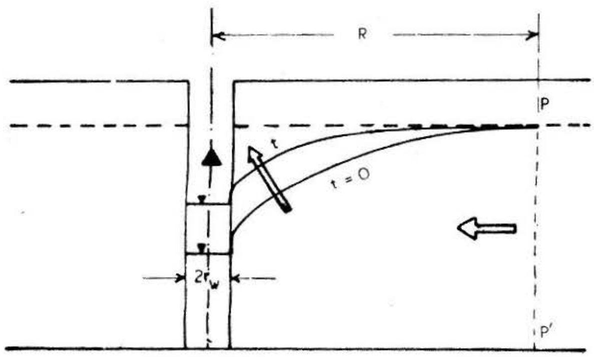

FIGURE 2 Fixed Influence Radius for Recovery

Taking the clue from this intuitive mechanistic logic, the actual functional relationship has been theoretically derived later in this note. Before taking up the derivation proper, an attempt to re-define the influence radius on more specific and scientific line is perhaps due.

Suggested Definition of Influence Radius

From the earlier paragraphs, it can be contemplated that the concept of influence radius hinges on two points viz

1. the distance from the centre of well at which draw-down is nil or *negligible* and
2. the distance from the centre of the well at which the slope of the piezometric line *almost* coincides with the original piezometric line.

It is obvious that a definition of the influence radius which is based on both the points will be more specific and rational provided the terms "negligible" and "almost" can be given a quantitative expression.

In any flow field involving a single pumping or recuperating well (Fig 1 and 2), the inflow rate decreases as one goes away from the centre of the well and the inflow rate at any distance from the well is function of both the drawdown and the slope of the piezometric line at the section in question. Hence an alternative or more rational definition of influence radius will be the distance from the centre of the well at which the inflow rate (i.e. discharge per unit time) is very close to the original inflow rate and very much small compared to the inflow at the well face. To make it more precise, the final definition recommended is the distance from the centre of the well at which the inflow rate q_R is given by $q_R = pq_w$, in which q_w is the inflow rate at the well face and p is a small fraction of the order of 0.01 to 0.001, the exact value of which may be fixed on the desired accuracy with which the influence radius R is to be obtained".

It is shown later that the above definition not only unifies all the earlier definitions (minus the earlier arbitrariness) but also provides a ratio basis for a theoretical expression connecting the influence radius with the relevant measurable parameters of the system in question.

Theoretical Derivation for Influence Radius during Pumping

At any time t, after the pumping has started, the piezometric line is given by say CDB (Fig 3). Assuming

$$\text{Area ABC} \approx \text{Area ABDC}$$

and following law of conservation mass, one gets

$$1/2 \left[s_w (R - r_w) \times 2\pi \left(r_w + \left(\frac{R - r_w}{2}\right) \times n_e \right] = qt$$

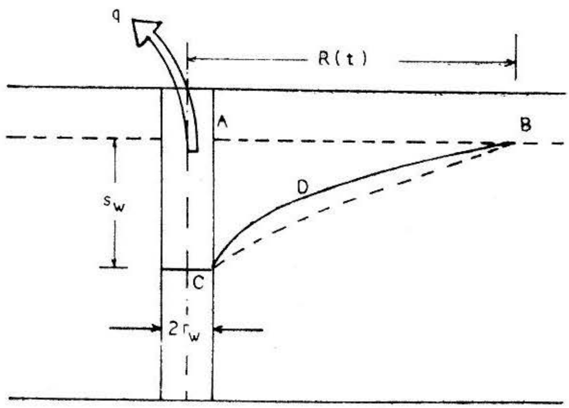

FIGURE 3 Definition Sketch for Pumping Well

For confined flow, n_e will be replaced by the storage coefficient S.

$$\text{or} \quad R^2 - r_w^2 = \frac{2\,qt}{\pi\, s_w\, n_e}$$

$$\text{or} \quad \frac{R}{r_w} = \sqrt{1 + \frac{2}{n_e}\left(\frac{qt}{as_w}\right)} \qquad \dots (5)$$

in which

q = Pumping rate

a = Well cross section ($=\pi r_w^2$)

s_w = Well drawdown

For confined flow, the above expression simply becomes

$$\frac{R}{r_w} = \sqrt{1 + \frac{2}{S}\left(\frac{qt}{as_w}\right)} \qquad \dots (6)$$

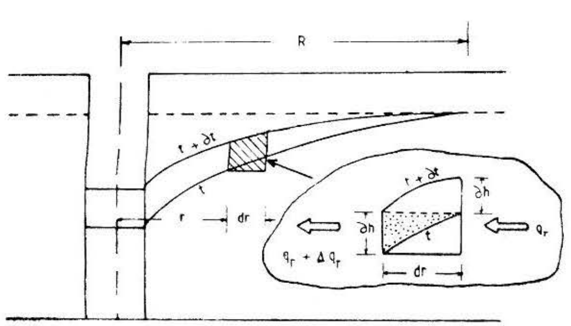

FIGURE 4 Flow in An Elemental Volume During Recovery

Eqs 5 & 6 confirm the earlier hypothesised functional relationship (eq. 3). Variation of the non-dimensional influence radius (R/r_w) with respect to another non-dimensional parameter $\left(\frac{qt}{a s_w}\right)$ as given by equations 5 and 6 for various values of n_e or S is shown in Fig 5.

Theoretical Derivation of Influence Radius during Recovery

The analysis which follows is valid for both confined and unconfined flow like the earlier case. The parameter n_e or S has to be used depending on whether the flow is unconfined or confined.

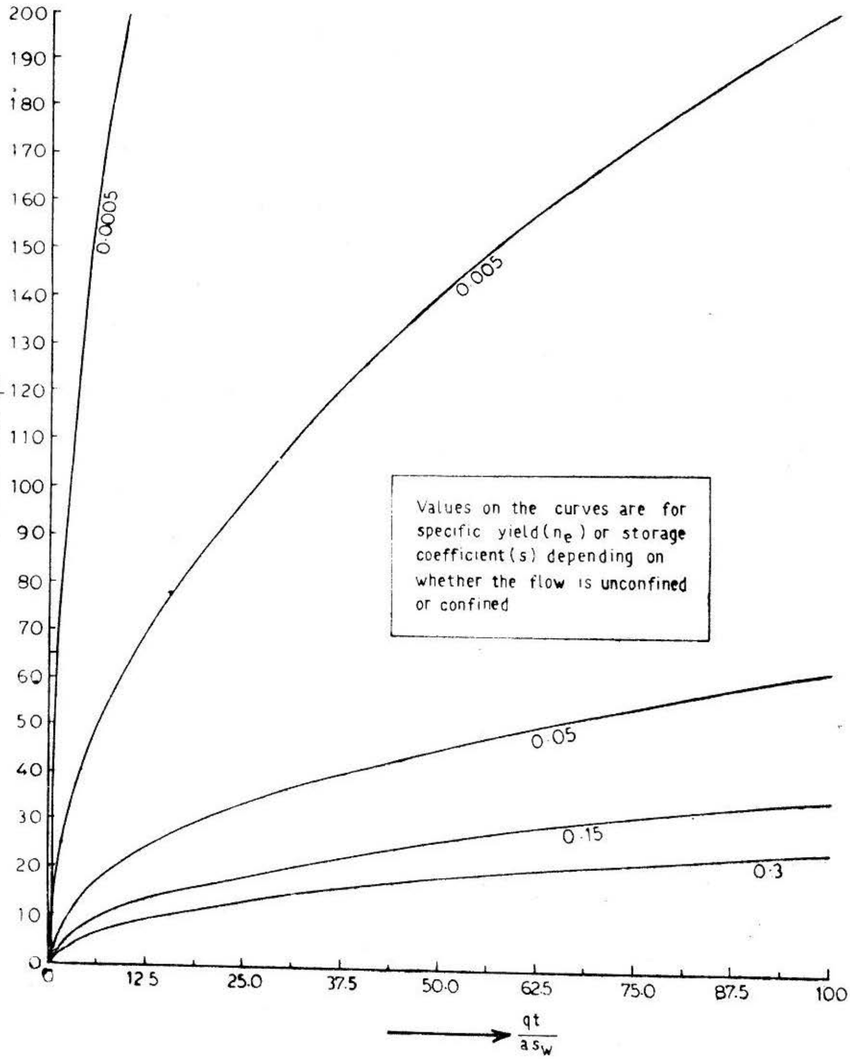

FIGURE 5 Influence Radius During Pumping

With reference to Fig 4 change of inflow rate in the elemental volume

$$=q_r+\Delta q_r-q_r=\Delta q_r=-\frac{\partial q_r}{\partial_r}\,dr \quad \ldots (7)$$

Storage (water released in this case) in the same element volume in unit time	=	Dotted triangular volume x specific yield or effective porosity

$$=1/2\left(\frac{dr\times\partial h\times 2\pi r}{\partial t}\right)\times n_e \quad \ldots (8)$$

$$=\pi n_e r\ dr\ \frac{\partial h}{\partial t}$$

By principle of conservation

$$-\frac{\partial qr}{\partial r}\,dr=\pi\ n_e\,r\ \ dr\ \frac{\partial h}{\partial t}$$

or

$$-\frac{\partial q_r}{\partial r}=\left(\pi\ n_e\ \frac{\partial h}{\partial t}\right)r \quad \ldots (9)$$

If it is assumed that the piezometric lines are parallel to each other at successive time intervals (Basak, 1979) then,

$$\frac{\partial h}{\partial t}=f(r) \quad \ldots (10)$$

By (3) and (4)

$$q_r=-\left(\pi\ n_e\,\frac{\partial h}{\partial t}\right)\frac{r^2}{2}+C_1 \quad \ldots (11)$$

In which C_1 is the constant of integration

But at

$$r=R,\ q_r=q_R \quad (12)$$

By (11) and (12)

$$q_r-q_R=1/2\ \pi\ n_e\ \frac{\partial h}{\partial t}\,(R^2-r^2) \quad \ldots (13)$$

At

$$r=r_w,\ q_r=q_{rw} \quad (14)$$

hence

$$q_{rw}-q_R=1/2\ \pi\ n_e\,(R^2-r_w^2)\ \frac{\partial h}{\partial_t} \quad \ldots (15)$$

Again

$$q_{rw} = \pi r_w^2 \frac{\partial h}{\partial t} \qquad \dots (16)$$

Equation (15) can be rewritten as

$$q_{rw}\left(1- \frac{q_R}{q_{rw}}\right) = 1/2\, \pi\, n_e\, (R^2 - r_w^2)\, \frac{\partial h}{\partial t}$$

or

$$\pi r_w^2 \frac{\partial h}{\partial t}\,(1-P) = 1/2\, \pi\, n_e (R^2 - r_w^2)\, \frac{\partial h}{\partial t}$$

or

$$\frac{R}{r_w} = \sqrt{1 + \frac{2}{n_e}(1-P)} \qquad \dots (17)$$

in which

$$P = \frac{q_R}{q_{rw}} << 1 \qquad \dots (18)$$

As mentioned earlier, the equivalent expression for $\frac{R}{r_w}$ for confined flow condition will be

$$\frac{R}{r_w} = \sqrt{1 + \frac{2}{S}(1-P)} \qquad \dots (19)$$

The value of P may be taken as either 0.01 or 0.001 depending on the accuracy with which the influence radius R is desired to be determined. However, it can be seen from eqns (17) and (19) that as long as $P \ll 1$, the ratio R/r_w is insensitive to the value of P. Once the value of P is fixed, the influence radius is seen to be a function of the well radius and the effective porosity (or specific yield) for unconfined flow. For confined flow condition it is a function of well radius and storage coefficient.

As long as $P \ll 1$, the value of $(1 - P) \approx 1$ and for which the R/r_w ratio is calculated for various values of n_e or S and is shown in Fig 6.

Conclusions

From the foregoing discussions and analysis, the following conclusions can be drawn.

1. Definition of influence radius based on inflow rate seems to be more rational.

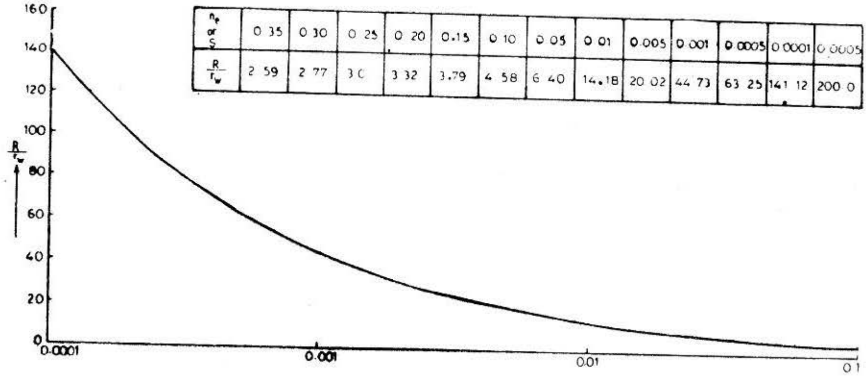

FIGURE 6 Variation of Influence Radius During Recovery

2. The mechanism of spread of influence radius during pumping and recovery are distinctly different.

3. During pumping, the ratio of the influence radius and the well radius (R/r_w) is a function of two non-dimensional parameters i.e. $\frac{qt}{as_w}$ and $\frac{2}{S}$ (or $\frac{2}{n_e}$). The approximate functional relationship and their variations are given in equations (5) and (6) and Fig. 5.

4. During recovery, the value of influence radius does not change with time but depends predominantly on the value of specific yield (n_e) or storage coefficient. The ratio R/r_w decreases with the increase of n_e or S. (Eqns 17 and 19 and Fig.6).

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Notations

D	= Pumping Depth
H	= Aquifer thickness
K	= Permeability coefficient
n_e	= Effective porosity or specific yield
q	= Inflow rate
P	= Non-dimensional parameter defined by equation (18)
q_r	= Inflow rate at any distance r
q_{rw}	= Inflow rate at the well face
q_R	= Inflow rate at the influence radius
Δq_r	= Incremental inflow rate at a distance r
r	= Distance from the centre of the well
r_w	= Radius of the well
R	= Influence radius
s_w	= Drawdown in the well
S	= Storage coefficient
	= time