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Prediction of Amplitude of Vibration of Machine Foundations

by

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Introduction

For the design of machine foundations the primary design criterion is the limiting displacement amplitude at the operating frequency. However, it is generally given to understand that the operating frequency should be away from the resonance frequency. The design can be carried on either by using the elastic half-space theory or mass-spring-dashpot model or empirical methods. (Richart, *et al.*, 1970; Sridharan and Nagendra, 1981) Out of these, elastic half-space theory is widely used and most popular. Using this theory many investigators Sung (1953), Bycroft (1956), Hsiesh (1962), Chase, *et al.*, (1965), Richart and Whitman (1967), Sridharan and Nagendra (1981), Nagendra and Sridharan (1981), (1982) and (1984) to name a few) have carried out research on vertical and horizontal modes of vibration taking into consideration the three different pressure distributions (viz., rigid base, uniform and parabolic) and three different displacement conditions (viz., central, average and weighted average). They brought out simplified charts and tables for the above cases to determine the frequency and amplitude of vibration at resonance. Figures 1 and 2 show the effect of displacement and contact pressure distribution conditions on the amplitude of the vibration for the frequency dependent excitation for vertical and horizontal modes of vibration. Analog solutions have been attempted in order to obtain simplified solutions (Lysmer and Richart 1966, Hall 1967, Sridharan and Nagendra 1981, Nagendra and Sridharan 1984). In many cases of machine foundation design, it is necessary to limit the displacement amplitude of the vibration at operating frequency at a desired level. Satisfying the displacement amplitude criterion at resonance without considering the limiting displacement amplitude criterion at operating frequency leads to uneconomical designs. Using mass-spring-dashpot model, one can determine the displacement amplitude at any frequency knowing the appropriate

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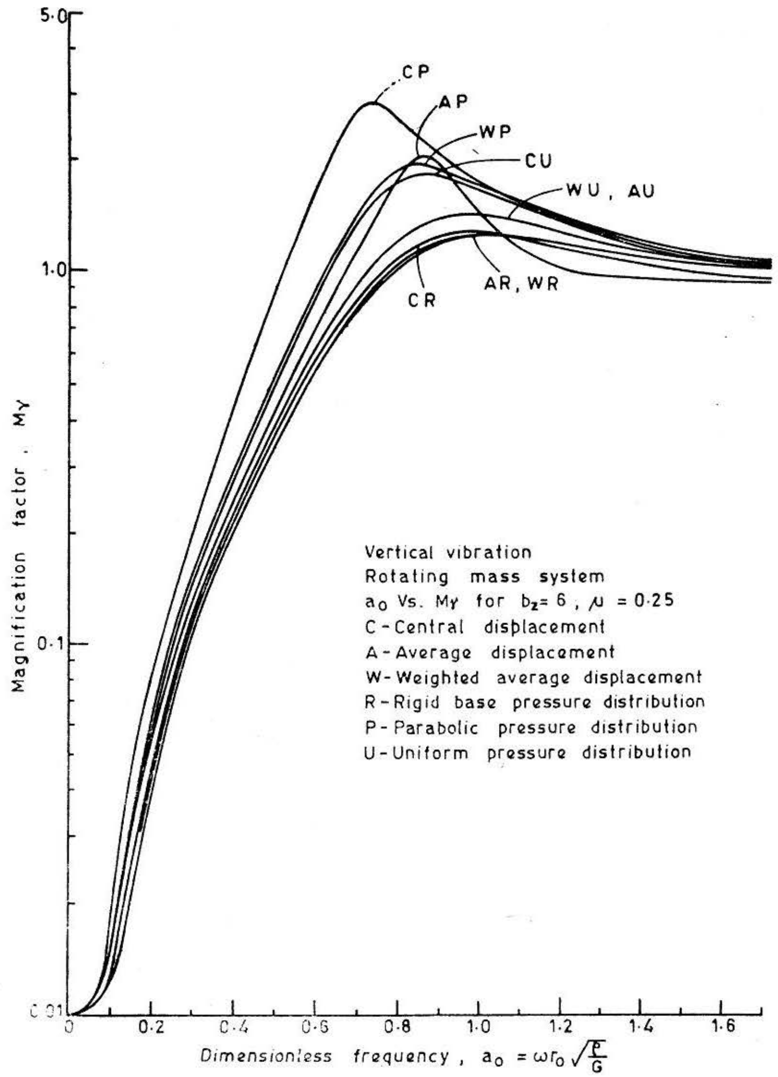

FIGURE 1. Nondimensional Frequency Factor vs Magnification Factor for Vertical Vibration

values of spring constant and damping factor. However, nowhere in the literature there is any simplified procedure for the estimation of the amplitude of the vibration at any frequency other than resonance using elastic half-space theory. Many investigators (Housner and Castellani 1969, Richart and Whitmann 1970) have shown that the weighted average displacement condition is more appropriate than other displacement conditions. In this paper solutions for the determination of the displacement amplitude at any frequency, for the vertical and horizontal modes of vibration are presented for the above mentioned displacement condition and for all the three types of pressure distributions (viz., rigid base, uniform and parabolic), using

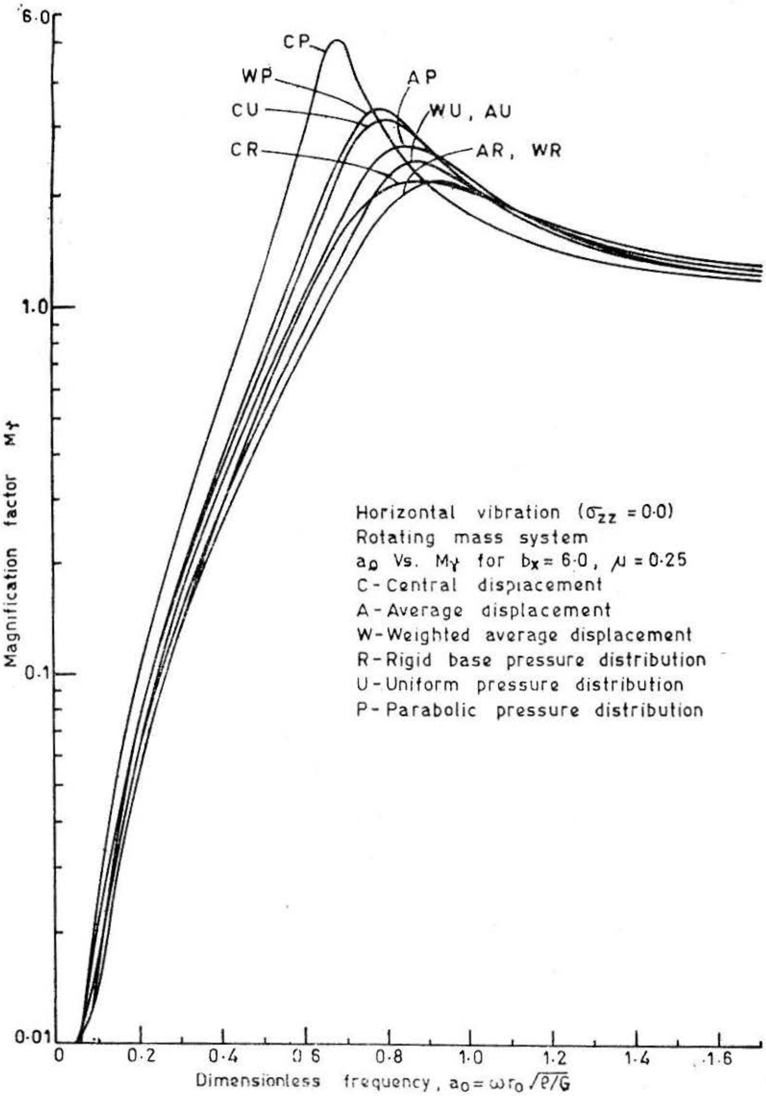

FIGURE 2. Nondimensional Frequency Factor vs Magnification Factor for Horizontal Vibration (σ_{zz}=0.0)

elastic half-space theory. Results are given for the frequency independent and frequency dependent excitations in the form of charts using appropriate non-dimensional parameters.

Analysis

The design of machine foundations can be done either by using elastic half-space theory or mass-spring-dashpot model. From the mass-spring-dashpot model the amplitude of the vibration at any frequency can be determined by the Eqs. (1*a*) and (1*b*) (Richart, *et al.* 1970).

For frequency independent excitation

$$A = \frac{Q_o}{K} \cdot \frac{1}{\left[\left(1 - \left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2D\frac{\omega}{\omega_n}\right)^2\right]^{0.5}} \quad \text{.. (1}a\text{)}$$

For frequency dependent excitation.

$$A = \frac{m_e e}{m} \frac{\left(\frac{\omega}{\omega_n}\right)^2}{\left[\left(1 - \left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2D\frac{\omega}{\omega_n}\right)^2\right]^{0.5}} \quad \text{...(1}b\text{)}$$

where, A = Amplitude of the vibration,

D = Damping factor of the system,

e = Eccentricity of the eccentric mass,

K = Spring constant of the soil system,

m = Total mass of the system,

m_e = Eccentric mass of the system,

ω = Circular frequency at which the amplitude of the vibration is required,

ω_n = Circular natural frequency of the system,

Q_o = Constant dynamic force acting on the system.

The above method of estimation can give very erronious results if the value of damping factor, D, is not properly estimated. Sridharan and Raman (1977) discuss at length the various parameters influencing the damping factor of foundation soil system. They have brought out that the geometrical damping factor (Eq. 2) obtained from the elastic half-space theory at resonance condition can also result in unsafe side.

$$D = \frac{D_k}{\sqrt{B}} \quad \text{.. (2)}$$

where, D = Damping factor,

D_k = Constant depending on the type of contact pressure distribution and displacement conditions (Given in Table 1).

B = Modified mass ratio.

TABLE 1

Values of D_k for Weighted Average Displacement Condition for Vertical and Horizontal Vibrations

Pressure Distribution	Values of D_k	
	Vertical vibration	Horizontal vibration
Rigid Base	0.416	0.252
Uniform	0.380	0.231
Parabolic	0.311	0.188

The values of D_k have been taken from Nagendra (1984). These values of D_k are applicable only at the resonance condition.

Using the elastic-half space theory, the values of nondimensional amplitude factor, $\overline{A}$ or magnification factor, M_r are evaluated at different frequency ratios viz., ω/ω_m or $\frac{\omega}{\omega_{mr}}$ (where ω is operating frequency, ω_m and ω_{mr} the resonance frequency for frequency independent and frequency dependent excitations respectively) from Eq. 3 and 4, (Sung (1953) and Richart, *et al.* (1970)), with the help of a computer.

For the frequency independent excitation (constant force system)

$$\overline{A} = \frac{Gr_o}{Q_o} A = \left[\frac{f_1^2 + f_2^2}{(1+ ba_0^2\, f_1)^2 + (ba_0^2\, f_2)^2} \right]^{0.5} \qquad ..(3)$$

For the frequency dependent excitation (Rotating mass system)

$$M_k = \frac{m}{m_e e} A = ba_0^2 \left[\frac{f_1^2 + f_2^2}{(1 + ba_0^2 f_1)^2 + (ba_0^2\, f_2)^2} \right]^{0.5} \qquad ...(4)$$

where,

A = Amplitude of the vibration at any given frequency,

$\overline{A}$ = Nondimensional amplitude factor,

a_o = Nondimensional frequency factor = $\omega r_o \sqrt{\rho/G}$,

b = Nondimensional mass ratio = $m/\rho r_0^3$,

e = Eccentricity of the eccentric mass,

f_1 & f_2 = Functions of a_o, depending on the displacement and contact pressure distribution conditions and Poisson's ratio,

G = Shear Modulus of the soil,

M_r = Nondimensional magnification factor,

Q_o = Constant dynamic force acting on the system,

ρ = Mass density of the half-space (soil),

r_o = Radius of the contact area of foundation,

ω = Operating frequency of the system.

Using Eqs. 3 and 4 and a numerical method, the maximum values of the nondimensional amplitude factor, $\overline{A}$ and Magnification factor, M_r were obtained with corresponding nondimensional frequency factor, $a_{om} = \omega_{mr} r_o \sqrt{\rho/G}$, for frequency independent excitation and $a_{omr} = \omega_{mr}\ r_o \sqrt{\rho/G}$, for frequency dependent excitation. Knowing ω_m or ω_{mr} one can express the operating frequency in terms of resonance frequency as a ratio, $\frac{\omega}{\omega_m}$ or $\frac{\omega}{\omega_{mr}}$ and corresponding $\overline{A}$ or M could be obtained using Eqs. 3 and 4

Tables 2 and 3 give the values for functions f_1 and f_2 for vertical and horizontal vibrations respectively. For vertical vibration the functions f_1 and f_2 were calculated using coefficients given by Bycroft (1956), Nagendra and Sridharan (1982). In the case of horizontal vibration, solutions are available in the literature for two boundary conditions. They are (*i*) the vertical stress, σ_{zz} at any point due to the horizontal dynamic force is equal to zero. (*ii*) the vertical displacement, w, at any point due to horizontal dynamic force is equal to zero (Nagendra, et al. 1982, Nagendra and Sridharan, 1984). Of the two boundary conditions, the first one is more realistic and has been recommended (Richart and Whitmann 1967, Housner and Castellani, 1969, Nagendra and Sridharan, 1981). Figure 3 shows the comparison of the magnification factor, M for the above two boundary conditions at different frequency ratios for the frequency independent excitation obtained by the authors. It can be seen that the difference in magnification factor is marginal for all the frequency ratios except for the case of $\frac{\omega}{\omega_m} = 0.8$. However, the boundary condition $\sigma_{zz} = 0.0$ is more rational and hence further results are given only for this boundary condition. For the calculation of the displacement amplitude at any frequency, the co-efficients for displacement functions f_1 and f_2 for the weighted average displacement condition, horizontal vibration ($\sigma_{zz} = 0.0$) given by Nagendra and Sridharan (1984) have been used (Table 3).

TABLE 2

Values of Functions f_1 and f_2 for Vertical Vibration—Weighted Average Displacement

$$-f_1 = P - Q\,a_0^2 + R a_0^4; \qquad f_2 = S a_0 - T a_0^3 - U a_0^5$$

Type of Pressure Distribution	Poisson Ratio, μ	P	Q	R	S	T	U
Rigid Base	0.00	0.250000	0.145833	0.034896	0.214474	0.078832	0.013035
	0.25	0.187500	0.093750	0.019618	0.148594	0.047354	0.006901
	0.50	0.125000	0.062500	0.011458	0.104547	0.029434	0.003824
Uniform	0.00	0.271090	0.126089	0.022988	0.214474	0.059124	0.007638
	0.25	0.202642	0.018057	0.012923	0.148594	0.035516	0.004044
	0.50	0.135095	0.054038	0.007548	0.104547	0.022076	0.002241
Parabolic	0.00	0.329374	0.102472	0.012738	0.214474	0.038416	0.003564
	0.25	0.247031	0.065875	0.007161	0.148594	0.023677	0.001887
	0.50	0.164687	0.043917	0.004183	0.104547	0.014717	0.001046

TABLE 3

Values of Functions f_1 and f_2 for Horizontal Vibration ($\sigma_{zz} = 0.0$), Weighted Average Displacement

$$-f_1 = P - Q\,a_0^2 + R a_0^4;\quad f_2 = S a_0 - T a_0^3 + U a_0^5$$

Type of Pressure Distribution	Possion Ratio, μ	P	Q	R	S	T	U
Rigid Base	0.00	0.250000	0.093750	0.017448	0.113688	0.021399	0.002082
	0.25	0.218750	0.067708	0.010851	0.016076	0.019742	0.001956
	0.50	0.187500	0.052083	0.007813	0.094542	0.018311	0.001882
Uniform	0.00	0.270189	0.081057	0.011494	0.113688	0.016049	0.001220
	0.25	0.236416	0.058541	0.007148	0.106076	0.014807	0.001146
	0.50	0.202642	0.045032	0.055146	0.094542	0.013733	0.001103
Parabolic	0.00	0.329374	0.065875	0.006369	0.113688	0.010700	0.000569
	0.25	0.288202	0.047576	0.003961	0.016076	0.009871	0.000535
	0.50	0.247031	0.036597	0.002852	0.094542	0.009156	0.000515

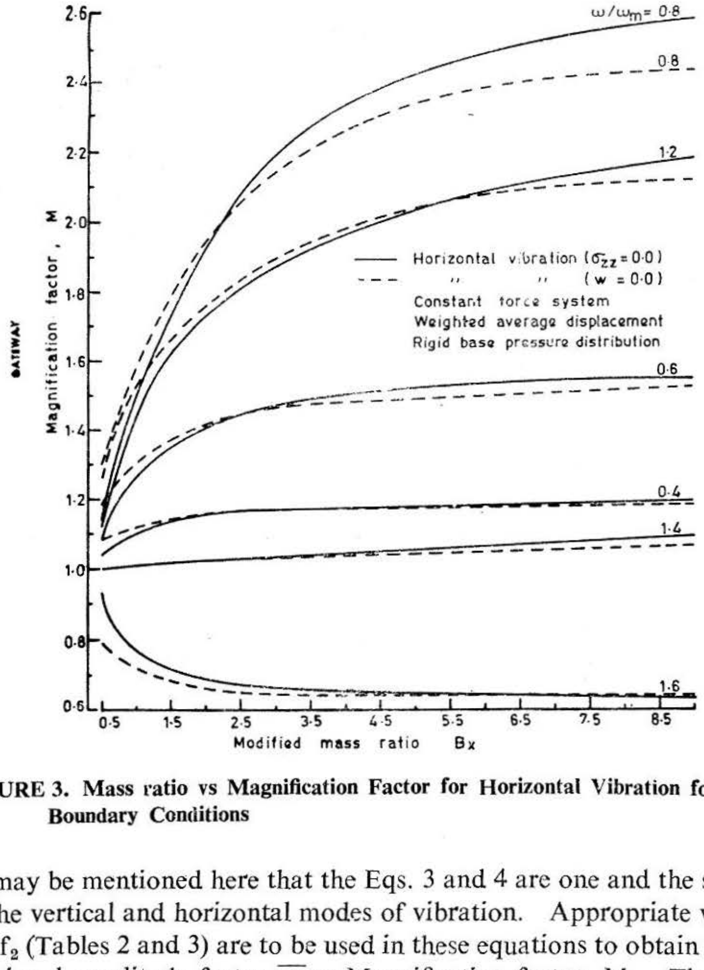

FIGURE 3. Mass ratio vs Magnification Factor for Horizontal Vibration for Two Boundary Conditions

It may be mentioned here that the Eqs. 3 and 4 are one and the same for both the vertical and horizontal modes of vibration. Appropriate values of f_1 and f_2 (Tables 2 and 3) are to be used in these equations to obtain the non-dimensional amplitude factor, $\overline{A}$ or Magnification factor, M_r. The coefficients to be used to obtain f_1 and f_2 are dependent on Poisson's ratio, hence they are listed for three values of Poisson's ratios, namely 0.0, 0.25 and 0.50.

Results and Discussion

Solutions have been presented in the form of charts for the frequency independent (constant force system) and frequency dependent (rotating mass system) excitations, for both, vertical and horizontal modes of vibration. It has been found that in the case of frequency dependent excitation the effect of poisson's ratio on the magnification factor, M_r is insignificant for $\frac{\omega}{\omega_{rm}} \leqslant 0.8$ and $\frac{\omega}{\omega_{rm}} \geqslant 1.2$. Hence the solutions are given in terms of mass ratio, b, itself. In the case of frequency independent excitation the effect of poisson's ratio on the amplitude factor, $\bar{A}$, is significant throughout the range under consideration $0.4 \leqslant \frac{\omega}{\omega_m} \leqslant 1.6$). Hence in this case the solutions are given in terms of modified mass ratio, B (which takes care of the effect of Poisson's ratio (Lysmer, *et al*, 1966). The value of magnification factor, M for the frequency independent excitation is got by multiplying the amplitude factor, $\bar{A}$ with the proper Poisson ratio factor. The Modified mass ratios are given as

$$B = P.F \times b$$

and Magnification factor, M, for the frequency dependent excitation is given as

$$M = \bar{A}/P.F$$

where, $P.F$ = Poisson ratio factor, given in Table 4,

b = Nondimensional mass ratio,

$\bar{A}$ = Nondimensional amplitude factor.

TABLE 4

Values of Poisson Ratio Factors for Vertical and Horizontal Vibration, Weighted Average Displacement Condition

Pressure Distribution	Poisson Ratio Factors	
	Vertical vibration	Horizontal vibration
Rigid base	$\frac{(1-\mu)}{4}$	$\frac{(2-\mu)}{8}$
Uniform	$\frac{8(1-\mu)}{3\pi^2}$	$\frac{4(2-\mu)}{3\pi^2}$
Parabolic	$\frac{1024(1-\mu)}{315\pi^2}$	$\frac{512(2-\mu)}{315\pi^2}$

Figures 4, 5 and 6 give the relationship between Modified mass ratio and Magnification factor for the frequency independent excitation, vertical vibration, weighted average displacement and for the rigid base, uniform and parabolic pressure distributions respectively. Figures 7, 8 and 9 give the relationship between mass ratio and magnification factor for the frequency dependent excitation, vertical vibration, weighted average displacement and for different pressure distribution conditions.

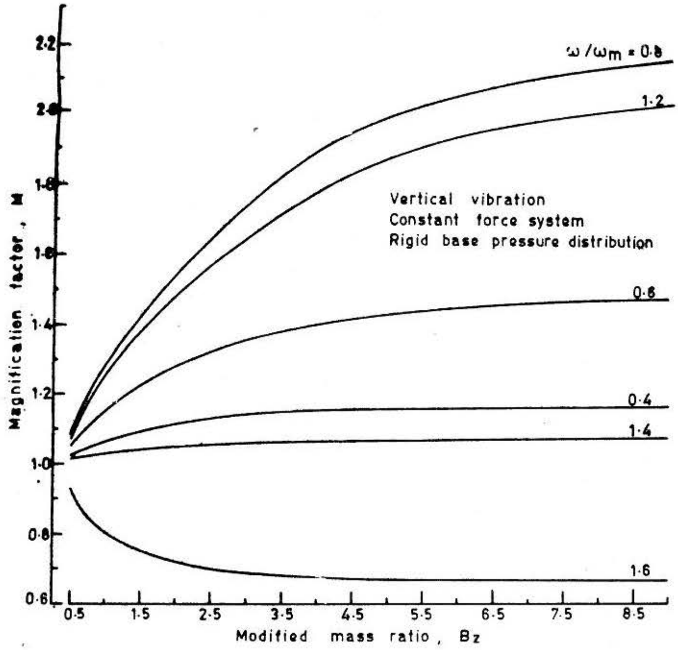

FIGURE 4. Modified Mass Ratio vs Magnification Factor for Vertical Vibration—Rigid base Pressure Distribution.

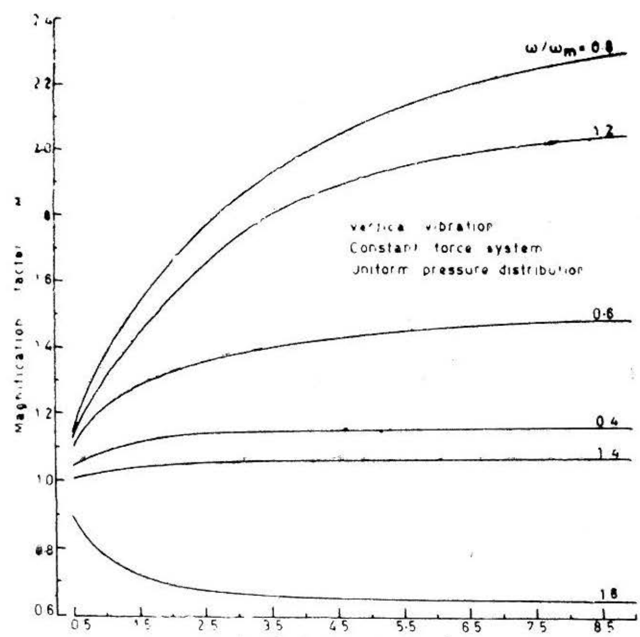

FIGURE 5. Modified Mass Ratio vs Magnification Factor ~~or~~ Vertical Vibration—Uniform Pressure Distribution

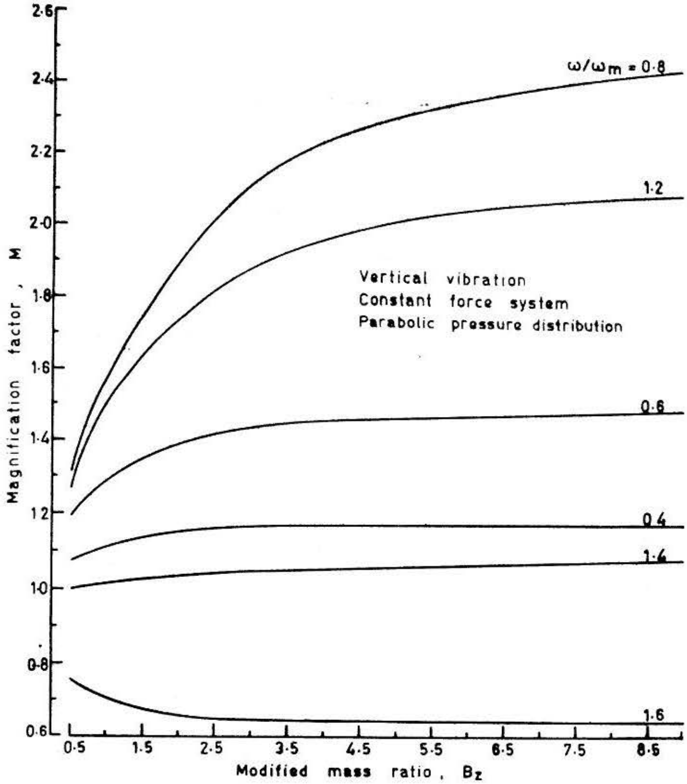

FIGURE 6. Modified Mass Ratio vs Magnification Factor for Vertical Vibration—Parabolic Pressure Distribution.

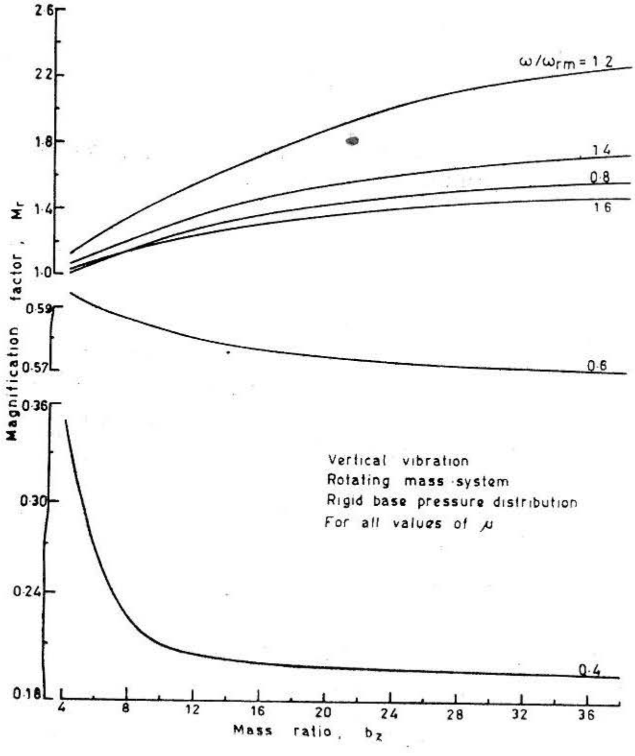

FIGURE 7. Mass Ratio vs Magnification Factor for Vertical Vibration—Rigid Base Pressure Distribution

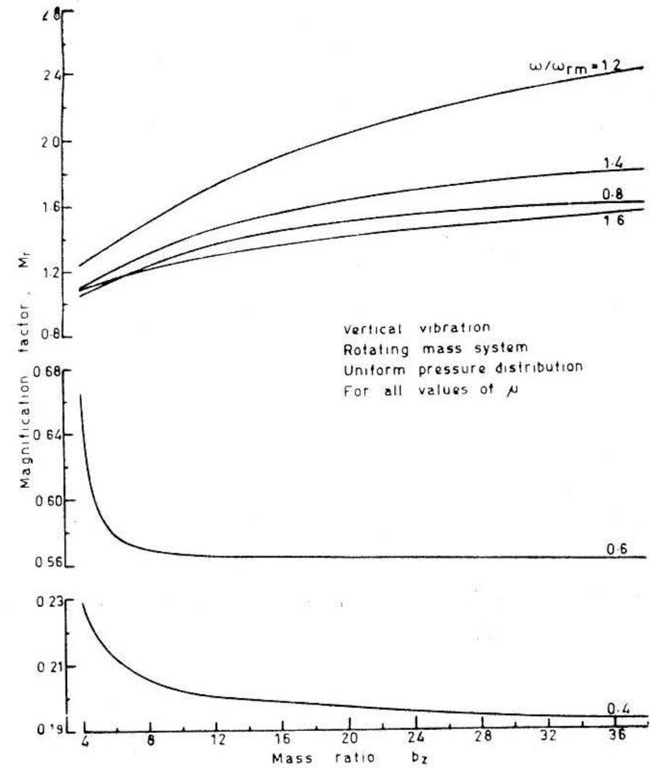

FIGURE 8. Mass Ratio vs Magnification Factor for Vertical Vibration—Uniform Pressure Distribution

Figures 10, 11 and 12 give the relationship between the modified mass ratio and magnification factor for the frequency independent excitation, horizontal vibration ($\sigma_{zz} = 0.0$), weighted average displacement and for the rigid base, uniform and parabolic pressure distributions respectively. Figures 13, 14 and 15 present the relationship between the mass ratio and Magnification factor for the frequency dependent excitation, horizontal vibration ($\sigma_{zz} = 0.0$), weighted average displacement and for different pressure distribution conditions.

For determining the displacement amplitude of the vibration at any frequency the above figures could be used.

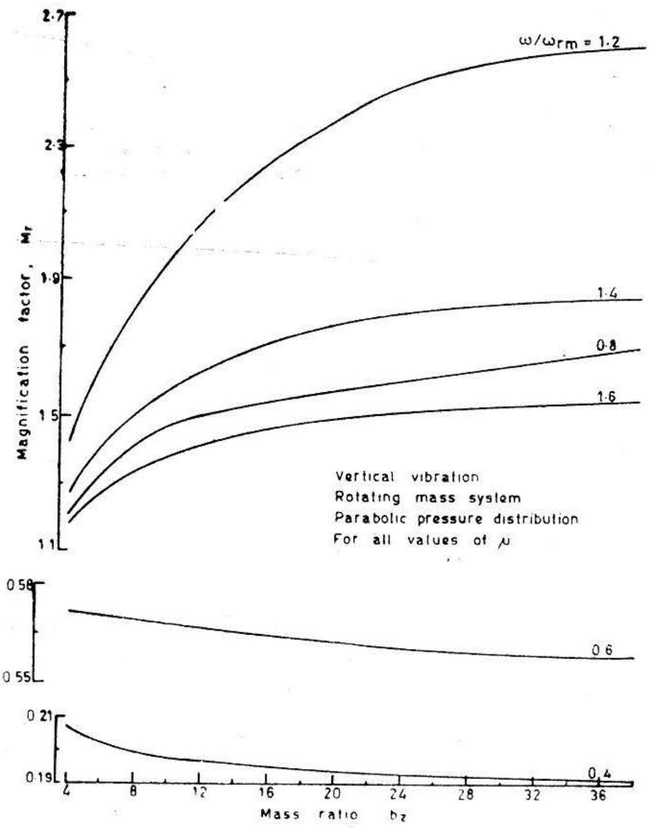

FIGURE 9. Mass Ratio vs Magnification Factor for Vertical vibration—Parabolic Pressure Distribution

Conclusions

By using the figures provided, one can easily determine the displacement amplitude of the foundation soil system at any frequency other than resonance for vertical and horizontal modes of vibration for different pressure distribution conditions and for weighted average displacement

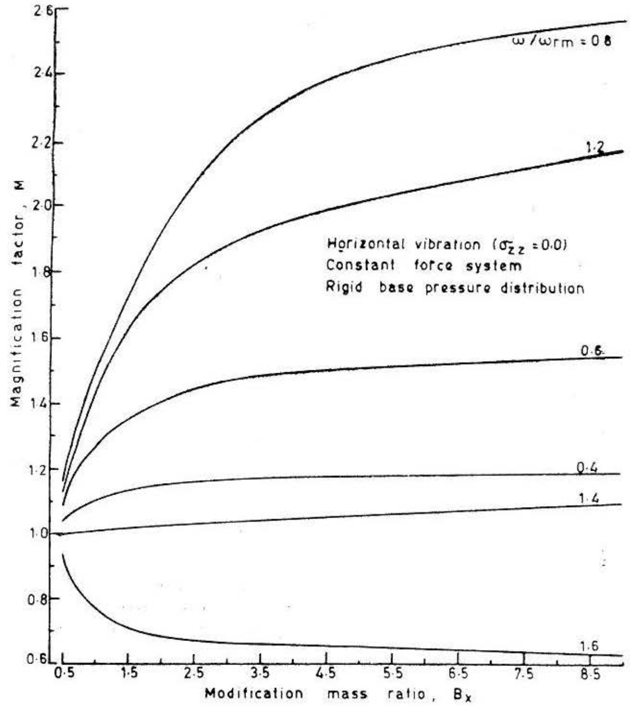

FIGURE 10. Modified Mass Ratio vs Magnification Factor for Horizontal Vibration—Rigid Base Pressure Distribuion

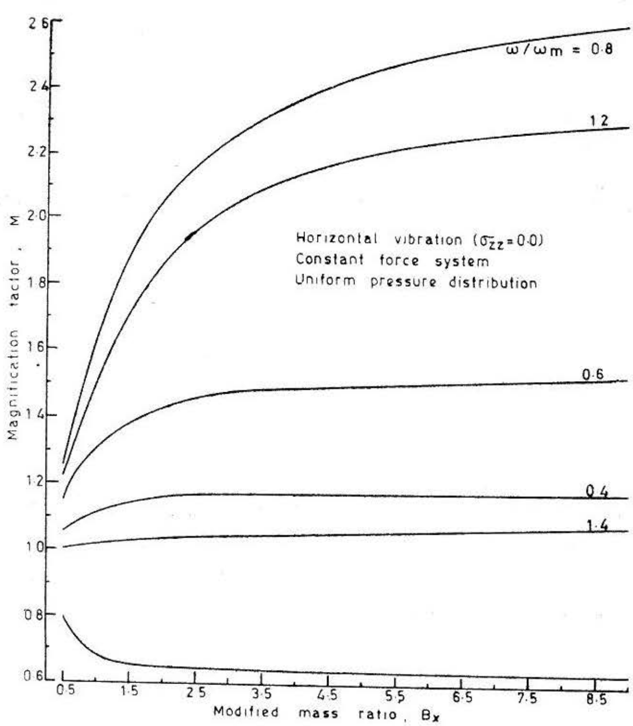

FIGURE 11. Modified Mass Ratio vs Magnification Factor for Horizontal Vibration—Uniform Pressure Distribution

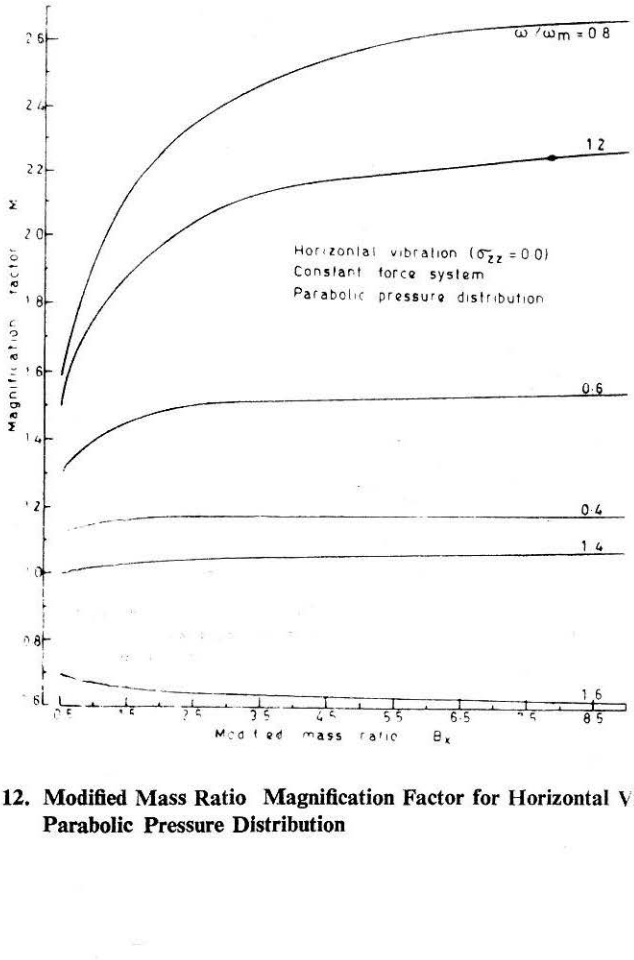

FIGURE 12. Modified Mass Ratio Magnification Factor for Horizontal Vibration—Parabolic Pressure Distribution

condition. These figures are accurate and are based on the elastic half-space theory, which is more popular, and rational. Appendix 1 presents an worked out typical example.

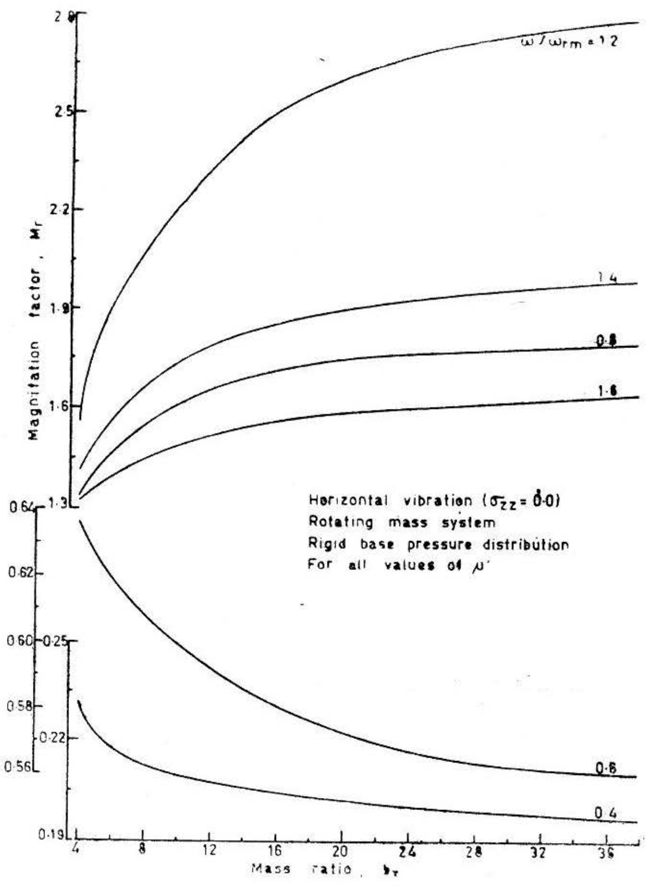

FIGURE 13 Mass Ratio vs Magnification Factor for Horizontal Vibration—Rigid Base Pressure Distribution

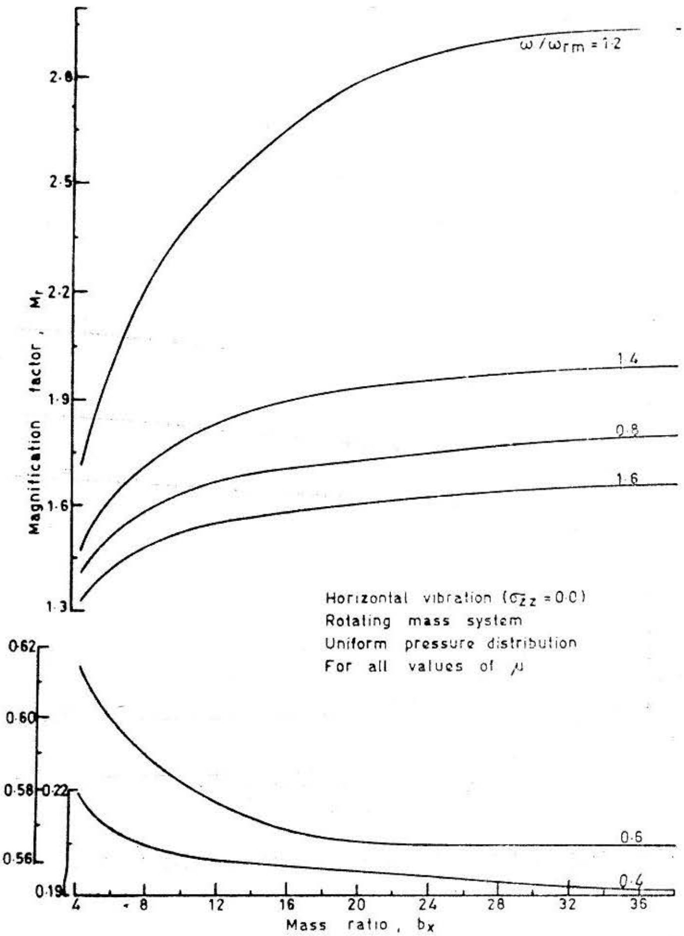

FIGURE 14 Mass Ratio vs Magnification Factor for Horizontal Vibration—Uniform Pressure Distribution

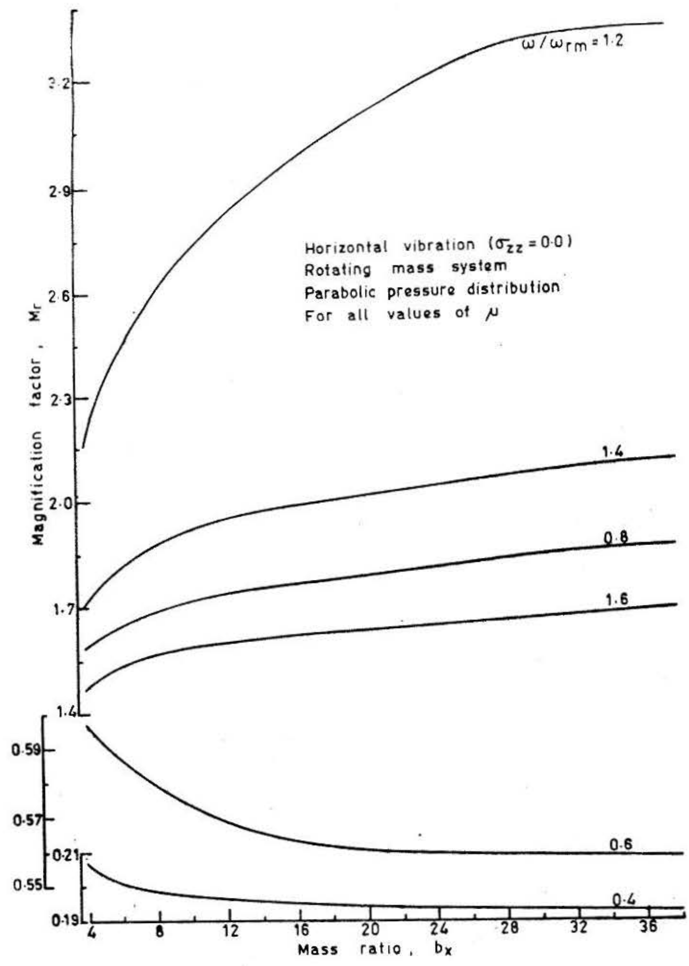

FIGURE 15 Mass Ratio vs Magnification Factor for Horizontal Vibration—Parabolic Pressure Distribution

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Notations

A = Amplitude of the vibration

$\overline{A}$ = Nondimensional amplitude factor

a_o = Nondimensional frequency factor

a_{om} = Nondimensional frequency factor at resonant frequency for frequency independent excitation

a_{omr} = Nondimensional frequency factor at resonant frequency for frequency dependent excitation

b = Mass ratio

b_z = Mass ratio in vertical vibration

b_x = Mass ratio in horizontal vibration

B = Modified mass ratio

B_z = Modified mass ratio in vertical vibration

B_x = Modified mass ratio in horizontal vibration

D = Damping factor

D_k = Constant dependent on the type of contact pressure distribution and displacement condition

e = Eccentricity

f_1, f_2 = Displacement functions

G = Shear modulus of soil

m = Mass of the machine and its foundation

m_e = Eccentric mass

M = Nondimensional magnification factor Constant force-system)

M_r = Nondimensional magnification factor Rotary mass system)

M_m = Nondimensional Magnification factor at resonant frequency for frequency independent excitation

M_{rm} = Nondimensional magnification factor at resonant frequency for frequency dependent excitation

r_o = Radius of the foundation

μ = Poisson's ratio

w = Exciting frequency

w_m = Frequency at resonance for frequency independent excitation

w_{mr} = Frequency at resonance for frequency dependent excitation

w_n = Natural frequency of the system.

Appendix-I

Sample calculation

DATA:

Unit weight of soil $\gamma = 1.87\ T/m^3$, Shear wave velocity, $V_s = 140$ m/sec, Poisson's ratio $\mu = 0.35$, Each unbalanced weight = 154 kg, Total unbalanced weight = $4 \times 0.154 = 0.616$ T, Eccentricity of unbalanced weight = 0.0025 m, Total dead weight of oscillator = 2.545 T, Total weight including footing and oscillator = 14.50 T, Diameter of footing = 150 cm, Operating frequency = 1600 RPM.

Solution to obtain the displacement amplitude at operating frequency under vertical vibration

Shear modulus calculated from shear wave velocity = $G = \rho v_s^2$

$$= \frac{1.87}{9.81} \times 140^2$$
$$= 3736.2 T/m^2$$

Radius of footing $= \frac{150}{2} = 75$ cm or 0.75 m

modified mass ratio, $B_z = \frac{(1-\mu)}{4} \frac{W}{\gamma r_0^3}$

$$= \frac{(1-0.35) \times 14.5}{4 \times 1.87 \times 0.75^3} = 2.987 \simeq 3.0$$

Dimensionless frequency factor at resonance, $a_{omr} = w_{mr}\, r_o \sqrt{\frac{\rho}{G}}$

Rich *et al.* 1970) one can get a_{omr} knowing B_z for frequency dependent excitation as $a_{omr} = 0.60$ for rigid base pressure distribution

$$\therefore w_{mr} = \frac{0.60}{0.75\sqrt{\frac{1.87}{9.81 \times 3736.2}}} = 112 \text{ rad/sec}$$

Operating frequency, $f = 1600$ RPM, $w = 167.55$ rad/sec.

$$\frac{w}{w_{mr}} = \frac{167.55}{112}$$

$$= 1.496 \simeq 1.50$$

To calculate displacement at operating frequency: Rigid base pressure distribution

$$\text{from Fig (7) for } b_z = \frac{W}{\gamma r_0^3}$$

$$= \frac{14.5}{1.87 \times 0.75^3} = 18.38$$

$$\text{for } b_z = 18.38 \text{ and } \frac{w}{w_{rm}} = 1.50$$

Magnification factor, $M_r = 1.40$.

$$M_r = \frac{m}{m_e e} \cdot A$$

$$\therefore A = \frac{m_e e}{m} \cdot M_r$$

$$= \frac{0.616 \times 0.0025 \times 9.81}{14.5 \times 9.81} \times 1.40 = 1.4869 \times 10^{-4} m$$

$$= 0.14869 \text{ mm}$$