Indian Geotechnical Journal, 18 (1), 1988

Pile Displacements Due to Tensile Loads

M.R. Madhav*

Hormoz B. Poorooshasb**

Introduction

If a structure is tall or high and is subjected to large lateral forces such as wind or wave loads, the resulting moments could induce tension in some of the piles supporting the structure. Piles driven into soil or rock may be used as anchors to specifically withstand pull out forces. During some pile load tests, the reaction from the pile under test is resisted by tensile loads in adjacent piles. In order to evaluate unit shaft resistance and the deformation modulus of the soil, a tensile load test is simpler and economical than a compression test, as the loads required to cause failure are much smaller. In this note, a method of analysis of a single pile subjected to a tensile load and the application of the results to a practical case are presented.

Analysis

The approach used in this work follows closely the method developed by Poulos and Davis 1968). The pile is assumed to be rigid and the soil as a linear, elastic continuum. A vertical pile of length, L, and diameter, d, subjected to an axial pull, P, is considered (Fig. 1a). The soil offers resistance along the shaft. The loads on the pile and the soil are shown in Fig. 1. The pile is divided into n elements each of length L/n. Due to the stresses at the soil - pile interface, the vertical displacement, ${}_s\rho_{ij}$, at node i, due to the stress, p_j, on element j, based on Mindlin (1936), can be written as

$$ {}_s\rho_{ij} = \frac{-d}{E_s} I_{ij} p_j \qquad \ldots(1) $$

*Professor, Dept. of Civil Engg., Indian Institute of Technology, Kanpur, 208016, India.

**Professor, Dept. of Civil Engg., Concordia University, 1455 de maisonneuve Blvd. West, Montreal, Que., Canada 44G IM8.

(*The modified manuscript of this paper was accepted in December, 1987 and is open for discussion till the end of June 1988*).

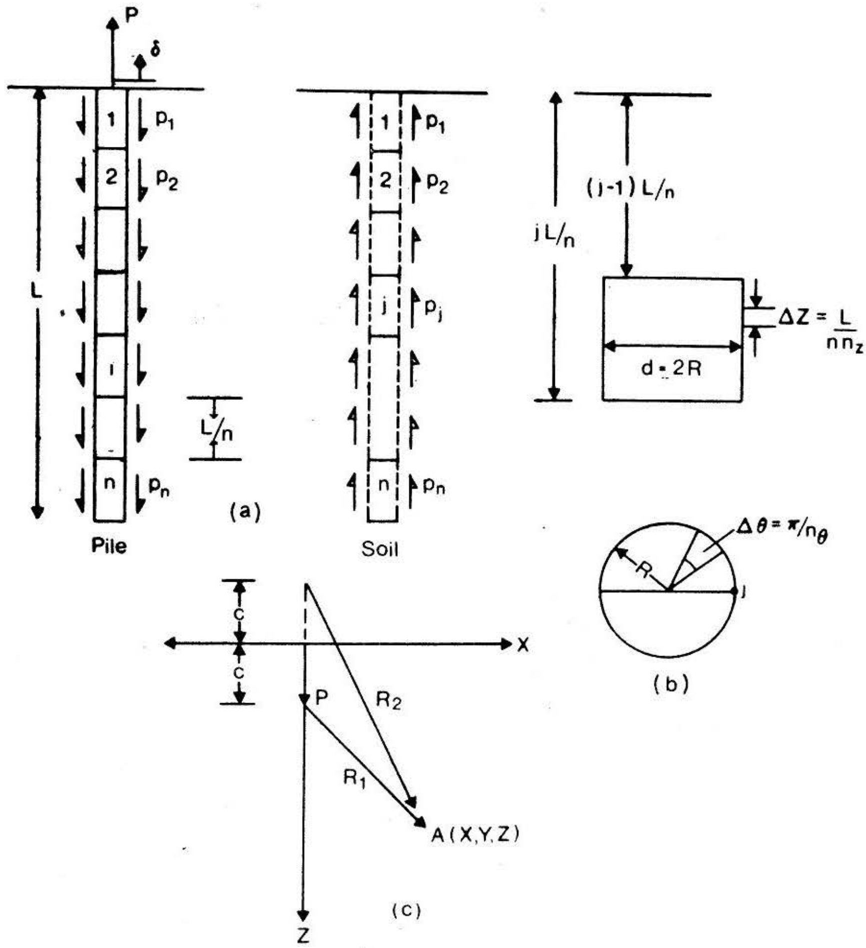

FIGURE 1. Definition Sketch

where

$$I_{ij} = \frac{(1+\nu_s)}{8\pi(1-\nu_s)} \int_0^{2\pi} \int_{(j-1)L/nd}^{jL/d} \left[\frac{(3-4\nu_s)}{\bar{R}_1} + \frac{8(1-\nu_s^2)-(3-4\nu_s)}{\bar{R}_2} + \frac{(\bar{z}-\bar{c})^2}{\bar{R}_1^3} + \frac{(3-4\nu_s)(\bar{z}+\bar{c})^2 - 2\bar{c}\,\bar{z}}{\bar{R}_2^3} + \frac{6\bar{c}\bar{z}(\bar{z}+\bar{c})^2}{\bar{R}^5} \right] d\theta d\bar{z} \qquad \ldots(2)$$

$R_{1,2} = \sqrt{x^2 + y^2 + (z \mp c)^2}$, the bar representing nondimensionalization with respect to d, E_s and ν_s — the deformation modulus and the Poisson's ratio of the soil. x, y, z, c, R_1, and R_2 are defined in Fig. 1c. The total vertical displacement, ${}_s\rho_i$, of node i, is

$$_s\rho_i = \frac{-d}{E_s} \sum_{j=1}^{n} I_{ij}\, p_j \qquad \ldots(3)$$

Collating all the soil displacements,

$$\{_s\rho\} = \frac{-d}{E_s} [I] \{p\} \qquad \ldots(4)$$

where $\{_s\rho\}$ and $\{p\}$ are vectors of soil displacement and stresses respectively, of size n, and $[I]$ — the influence coefficient matrix of size $n \times n$.

The displacements $\{_p\rho\}$, of pile assumed rigid are written as

$$\{_p\rho\} = -\{1\} \cdot \delta \qquad \ldots(5)$$

where $\{1\}$ is a unit vector, and δ — the vertical displacement of the pile. Equilibrium of forces requires

$$P = \frac{\pi d L}{n} \sum_{j=1}^{n} p_j \qquad \ldots(6)$$

The compatibility of displacements is satisfied by equating $\{_p\rho\}$ and $\{_s\rho\}$ i.e. Eqs. (4) and (5), from which one gets

$$\{p\} = \frac{-E_s}{d} [I]^{-1} \{1\} \cdot \delta \qquad \ldots(7)$$

Substituting Eq. (7) into Eq. (6), the pile displacement δ, is obtained as

$$\delta = \frac{-P}{E_s d} I_t \qquad \ldots(8)$$

where I_t is the influence factor due to tensile load.

Results

Numerical results are obtained using CYBER 835 computer at Concordia University. The analysis is carried out with $n = 20$ though for displacements alone $n = 10$ is adequate. For the intergration over each element to evaluate I_{ij} numerically, the surface of each element is subdivided with $n_z = 10$ and $n_\theta = 20$ (over half the circumference). The displacement factor I_t, is plotted against L/d — ratio for $\nu_s = 0.2$ and 0.5 (Fig. 2). As anticipated the value of I_t decreases with the length of the pile. Longer the pile the smaller will be the displacement. As L/d ratio tends to zero, values of I_t tend to infinity unlike for a pile in compression. Comparing the results with those for a pile in compression (Poulos and Davis, 1968), it may be noted that values of I_t are more by 25 to 30% over I_p . For a soil with $\nu_s =$ 0.2, the influence factor, I_t, is smaller in comparison with those for $\nu_t =$ 0.5.

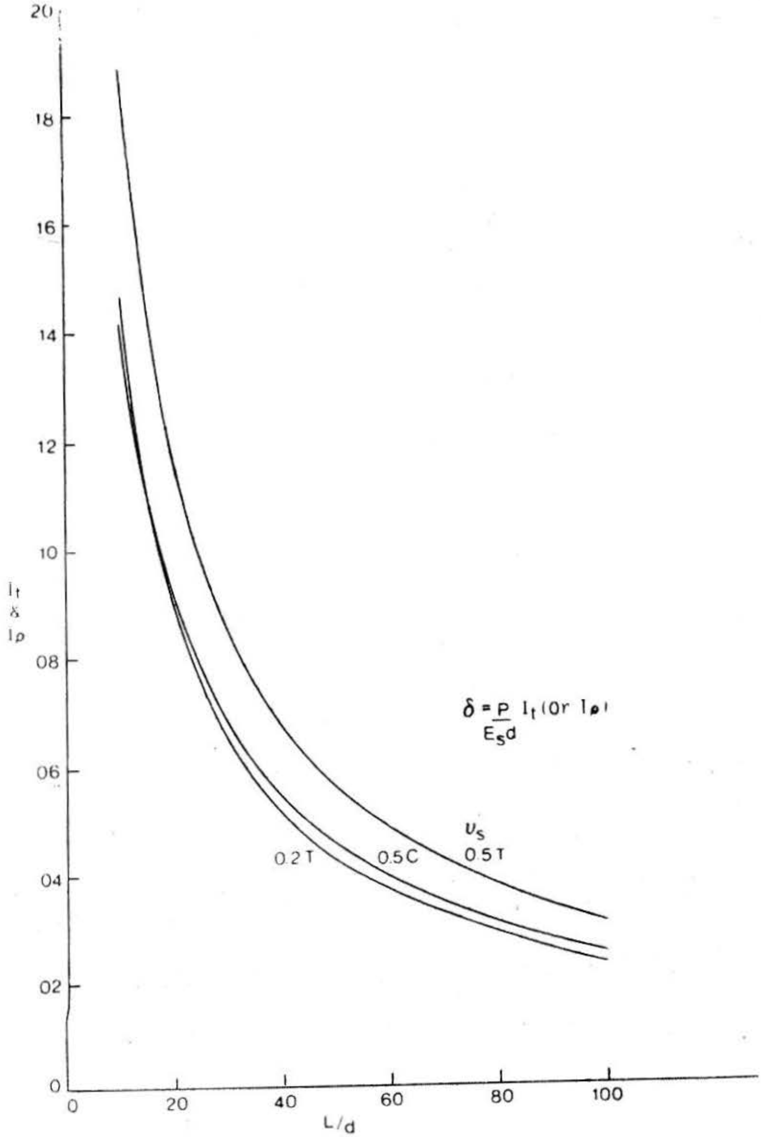

FIGURE 2 Influence Coefficient

The shear stresses on the shaft increase with depth along the pile (Fig. 3). Near the tip ($z/L \rightarrow 1.0$), the values of shear stresses are very high being more than 3 to 4 times the average value. With increasing L/D ratio, the shear stresses tend to become uniform over a large part of the pile.

Application

Lord and Davies (1980) report results from a tension test on an 800 mm

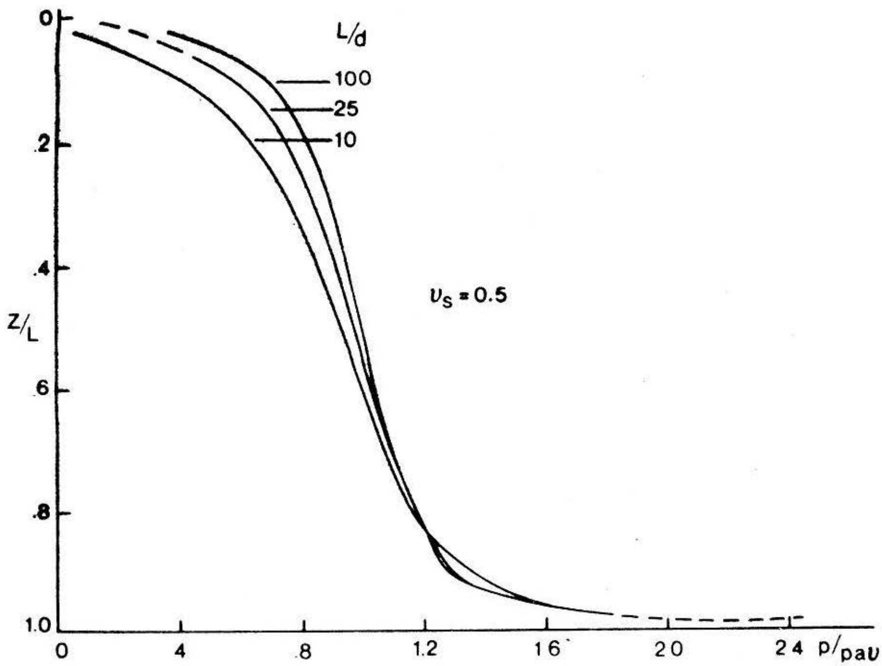

FIGURE 3 Shear Stresses Versus Depth

dia. and 4 *m* long pipe pile driven into chalk. The deformation of the pile ($L/d = 5$, $I_t = 0.207$) was 0.525 *mm* and 2.85 mm for loads of 500 and 1000 *kN* respectively. The modulus of deformation of chalk is calculated from Eq. 8 and compared in Table 1 with the predictions from plate load test (Burland and Lord, 1969), and lateral load test on a pile (Lord and Davies, 1980). From the plate load test results of Burland and Lord (1969), the values of the deformation modulus have been estimated to range between 500-100 kN/m^2 for average stress intensities of 500-800 kN/m^2 on the plate. The corresponding values of the modulus from a pull-out test on a pile range between 365-135 kN/m^2 at load levels of 500-1000 kN. Bozozuk

TABLE 1

Modulus of deformation from Load Tests

	Plate Bearing[1]		Pile Load[2]	Lateral Load[3]	Tension[4]	
E_s MN/m^2	500	100		350	365	135
E_s MN/m^2			7.36		4.98	

[1]Burland and Lord (1969); [2]Bozozuk et al. (1979); [3]Lord and Davies (1980); [4]Pre ent Analysis.

et al. (1979) report results of compression and tension load tests on 324 mm dia. 21.7 m long friction piles driven through compressible silts into dense gravel. The backfigured deformation moduli are also featured in Table 1. It can be noted that E_s—values from tension test on a pile are comparable with those from other types of tests. In case of the latter case study, the smaller E_s value from the tension test is understandable from the fact pointed by Bozozuk *et al.* that as the piles were subjected to negative skin friction prior to load tests the pile under pull out had undergone larger displacements. If corrected for the effects of negative skin friction, the results could possibly agree very closely.

Conclusions

A method of analysis of deformation of a pile subjected to a tensile load is presented. The pile is treated as rigid and the soil as an elastic half space. The displacement influence factor, I_t, is evaluated as a function of the length to diameter ratio of the pile, and ν_s of the soil. It is known to differ from that corresponding to compressive load. The modulus of deformation computed from the results of tensile load test compares well with those from other types of tests (plate load or axial compression, and lateral load on a pile).

References

BOZOZUK, M., KEENAN, C.H., and PHEENEY, P.E. (1979): "Analysis of Load Tests on Instrumented Steel Piles in Compressible Silty Soil", *Behaviour of Deep Foundations,* ASTM, STP. 670 : 133-180.

BURLAND, G.B. and LORD, J.A. (1969): "The Load-Deformation Behaviour of Middle Chalk at Mundford, Norfolk", *Proc. Conf. on In Situ Investigations in Soils and Rocks,* London.

LORD, J.A. and DAVIES, J.A.G. (1980) : "Lateral Loading and Tension Tests on a Driven Cased Pile in Chalk", *Recent Developments in the Design and Construction of Piles,* ICE, London: 113-120.

MINDLIN, R.D. (1936): "Force at a Point in the Interior of a Semi-Infinite Solid", *J. Appd. Phys.,* 7 : 195.

POULOS, H.G., and DAVIS, E.H. (1968): "The settlement Behaviour of Single Axially-Loaded Incompressible Piles and Piers", *Geotechnique,* 18 : 351-371.