Indian Geotechnical Journal, 17 (1), 1987

Short Communication

Dynamic Active Earth Pressure of Cohesionless Soil

by

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Introduction

Earth pressure on structures have been determined scientifically as early as in the 1776 when Coulomb's classical theory was propounded. But for dynamic earth pressure Mononobe (1929) and Okabe (1929) assumed a plane ruptured surface and gave the analytical solution of dynamic earth pressure in terms of seismic coefficients. Investigators like Kapila (1962), Prakash and Saran (1968), Soni and Murthy (1983), Satyanarayan and Murthy (1983) suggested different approaches for determining dynamic earth pressure of soils.

In the present paper, a scientific approach has been suggested for determining the dynamic active earth pressure of cohesionless soils. In this approach, dynamic load has been considered in terms of sinusoidal wave which produces maximum effect when acts on the line of action of the weight of failure wedge.

Analysis of the Approach

Present analysis is based on the following assumptions,

(*i*) The method is based on Coulomb's theory and deformation takes place so that a plane ruptures from the rest of soil mass.

(*ii*) The soil is cohesionless.

(*iii*) The reaction on plane ruptured surface acts at an angle 'ϕ' to the normal of the wall. 'ϕ' is the angle of shearing resistance of back-fill material.

(*iv*) The moving dynamic load is sinusoidal in nature and depends on magnitude of dynamic load and position of load from the face of the wall.

Moving dynamic load, $W_d = Q \text{ Sin } \omega t$ ... (1)

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(The revised manuscript of this paper was received in August, 1986 and is open for discussion till the end of March, 1987).

Where Q = moving load per unit length.

L = length of the failure wedge along the backfill line (Fig.) 1)

$\omega = \frac{3\pi}{2L}$

t = distance at which the load acts from the face of the wall

W_d will be maximum when $\omega t \frac{\pi}{2}$

$$\text{or } t = \frac{\pi}{2\omega}$$

$$\text{or } t = \frac{\pi}{2}\frac{2L}{3\pi} = \frac{L}{3}$$

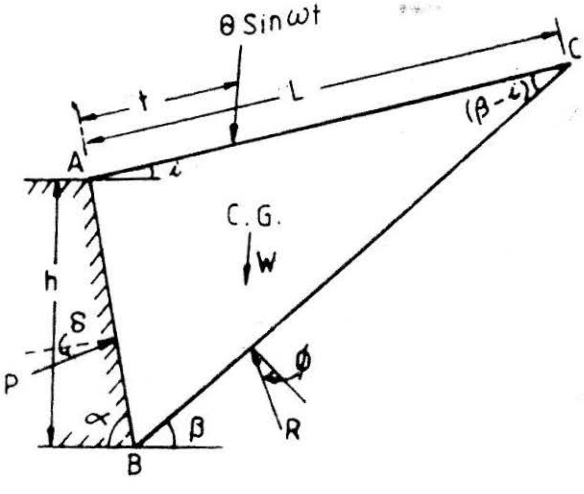

FIGURE 1 Failure Wedge

It shows that dynamic load will be maximum when it acts at $L/3$ distance from face of wall which will coincide through centre of gravity of the failure wedge.

The resultant equivalent weight of the failure wedge will be

$$W_e = W + W_s \qquad \dots (2)$$

Where W_e = equivalent weight of the failure wedge

W = static weight of wedge.

W_s = equivalent static load of moving dynamic load.

W_s may be given by

$$W_s = \int_0^t Q \text{ Sin } \omega\, t \qquad \dots (3)$$

$$\text{or } W_s = \frac{1}{\omega}\left[-Q \operatorname{Cos} \omega t\right]_0^t$$

$$\text{at } t = L/3,\ W_s \text{ will be maximum,}$$

$$(W_s) \text{ max.} = \frac{Q}{\omega}(1 - \operatorname{Cos} \omega L/3)$$

$$\text{or } (Ws) \text{ max.} = \frac{2LQ}{3\pi}\left(1 - \operatorname{Cos} \frac{3\pi}{2L}\frac{L}{3}\right)$$

$$\text{or } (W_s) \text{ max.} = \frac{2LQ}{3\pi}(1-0)$$

$$\text{or } (W_s) \text{ max.} = \frac{2LQ}{3\pi} \qquad \dots (4)$$

If the force polygon is considered for the forces acting on the failure wedge, this may be seen from the triangle rule (Fig. 2),

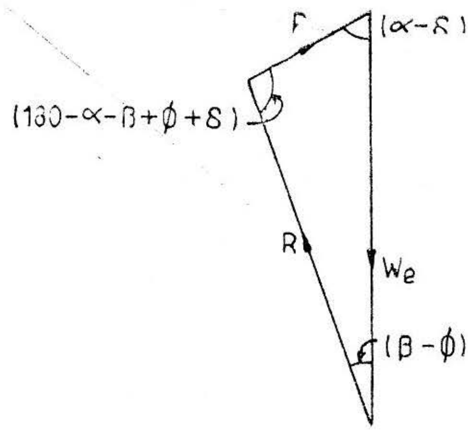

FIGURE 2 Force Polygon

$$\frac{P}{\operatorname{Sin}(\beta-\phi)} = \frac{W_e}{\operatorname{Sin}(180^\circ-\alpha-\beta+\phi+\delta)}$$

$$\text{or} \quad P = W_e \frac{\operatorname{Sin}(\beta-\phi)}{\operatorname{Sin}(\alpha+\beta-\phi-\delta)} \qquad \dots (5)$$

where

P — dynamic active earth pressure

β — angle of inclination of failure plane to the horizontal

α — angle of retaining wall to the horizontal

δ — angle of wall friction.

Weight of the wedge, W = Area of the failure wedge X density of backfill

$$W = \tfrac{1}{2}\gamma \frac{h}{\text{Sin}\,\alpha} L \,\text{Sin}\,(\alpha+i) \qquad \dots (6)$$

Where γ = density of the backfill material

i = horizontal slope of backfill.

h = height of retaining wall.

But $$L = \frac{h\,\text{Sin}\,(\alpha+\beta)}{\text{Sin}\,\alpha\,\text{Sin}\,(\beta-i)} \qquad \dots (7)$$

Therefore, from Eq. (6),

$$W = \tfrac{1}{2}\gamma \frac{h^2\,\text{Sin}\,(\alpha+\beta)\,\text{Sin}\,(\alpha+i)}{\text{Sin}^2\alpha\,\text{Sin}\,(\beta-i)} \qquad \dots (8)$$

From Eqs. (2), (4), (7) and (8)

$$W_e = \tfrac{1}{2}\,\gamma\,\frac{h^2}{\text{Sin}^2\alpha}\,\frac{\text{Sin}\,(\alpha+\beta)\,\text{Sin}\,(\alpha+i)}{\text{Sin}\,(\beta-i)} + \frac{2Q\,h}{3\pi\,\text{Sin}\,\alpha}\,\frac{\text{Sin}\,(\alpha+\beta)}{\text{Sin}\,(\beta-i)} \qquad \dots (9)$$

From Eqs. (5) and (9),

$$P = \left[\tfrac{1}{2}\,\gamma\,\frac{h^2}{\text{Sin}^2\alpha}\,\frac{\text{Sin}\,(\alpha+\beta)\,\text{Sin}\,(\alpha+i)}{\text{Sin}\,(\beta-i)} + \frac{2\,Qh}{3\pi}\,\frac{\text{Sin}\,(\alpha+\beta)}{\text{Sin}\,\alpha.\,\text{Sin}(\beta-i)}\right]\frac{\text{Sin}\,(\beta-\phi)}{\text{Sin}\,(\alpha+\beta-\phi-\delta)}$$

or $$P = \frac{\text{Sin}\,(\beta-\phi)}{\text{Sin}\,(\alpha+\beta-\phi-\delta)}\,\frac{\text{Sin}\,(\alpha+\beta)}{\text{Sin}\,(\beta-i)}\left[\tfrac{1}{2}\gamma\,h^2\frac{\text{Sin}\,(\alpha+i)}{\text{Sin}^2\alpha} + \frac{2\,Qh}{3\pi\,\text{Sin}\,\alpha}\right] \qquad \dots (10)$$

For P to be maximum,

$$\frac{dP}{d\beta} = 0$$

or $$\frac{d}{d\beta}\left[\frac{\text{Sin}\,(\beta-\phi)\,\text{Sin}\,(\alpha+\beta)}{\text{Sin}\,(\alpha+\beta-\phi-\delta)\,\text{Sin}\,(\beta-i)}\right] = 0 \qquad \dots (11)$$

Putting $\theta = (\alpha-\phi-\delta)$ Eq. (11) will be,

$$\frac{d}{d\beta}\left[\frac{\text{Cos}\,(\alpha+\phi)-\text{Cos}\,(2\beta+\alpha-\phi)}{\text{Cos}\,(\theta+i)-\text{Cos}\,(2\beta+\theta-i)}\right] = 0$$

or $[\text{Cos}\,(\theta+i)-\text{Cos}\,(2\beta+\theta-i)]\,\text{Sin}\,(2\beta+\alpha-\phi)-[\text{Cos}\,(\alpha+\phi)-$

$(2\beta+\alpha-\phi)]\,\text{Sin}\,(2\beta+\theta-i) = 0$ ...(12)

Assuming, $\text{Cos}\,(\alpha+\phi) = a$,

$\text{Cos}\,(\theta-i) = b$,

$\text{Cos}\,(\alpha-\phi) = c$,

$\text{Cos}\,(\theta+i) = d$, and solving Eq. (12)

$$[d-b\,\text{Cos}\,2\beta+\sqrt{1-b^2}\,\text{Sin}\,2\beta]\,[c\,\text{Sin}\,2\beta+\sqrt{1-c^2}\,\text{Cos}\,2\beta] = (a-c\,\text{Cos}\,2\beta+\sqrt{1-c^2}\,\text{Sin}\,2\beta]\,[b\,\text{Sin}\,2\beta+\sqrt{1-b^2}\,\text{Cos}\,2\beta]$$

on solving,

$$(cd-ab)\,\text{Sin}\,2\beta+(d\sqrt{1-c^2}-a\sqrt{1-b^2})\,\text{Cos}\,2\beta+(c\sqrt{1-b^2}-b\sqrt{1-c^2}) = 0$$

$$\text{or}\quad \frac{(cd-ab)}{(c\sqrt{1-b^2}-b\sqrt{1-c^2})}\,\text{Sin}\,2\beta+\frac{(d\sqrt{1-c^2}-a\sqrt{1-b^2})}{(c\sqrt{1-b^2}-b\sqrt{1-c^2})}\,\text{Cos}\,2\beta+1 = 0 \quad \ldots(13)$$

Assuming

$$\frac{(cd-ab)}{(c\sqrt{1-b^2}-b\sqrt{1-c^2})} = x,$$

$$\frac{(d\sqrt{1-c^2}-a\sqrt{1-b^2})}{(c\sqrt{1-b^2}-b\sqrt{1-c^2})} = y, \text{ and putting the values}$$

in Eq. (13),

$$x\,\text{Sin}\,2\beta+y\,\text{Cos}\,2\beta+1 = 0$$

$$\text{or } x\,\text{Sin}\,2\beta+y\sqrt{1-\text{Sin}^2\,2\beta}+1 = 0$$

$$\text{or } x\,\text{Sin}\,2\beta+1=-y\sqrt{1-\text{Sin}^2\,2\beta}$$

$$\text{or } x^2\,\text{Sin}^2\,2\beta+1+2x\,\text{Sin}\,2\beta = y^2\,(1-\text{Sin}^2\,2\beta)$$

$$\text{or } (x^2+y^2)\,\text{Sin}^2\,2\beta+2x\,\text{Sin}\,2\beta+1-y^2 = 0$$

$$\text{or } \text{Sin}\,2\beta = \frac{-x\pm\sqrt{x^2-(x^2+y^2)(1-y^2)}}{x^2+y^2}$$

$$\text{or } \text{Sin}\,2\beta = \frac{-x\pm y\sqrt{x^2+y^2-1}}{x^2+y^2}$$

For dynamic active earth pressure,

$$\text{Sin}\,2\beta = \frac{-x+y\sqrt{x^2+y^2-1}}{x^2+y^2} \quad \ldots(14)$$

The value of 'β' can be found out from Eq. (14) by knowing the values of α, ϕ, i, δ, θ, x and y. The dynamic active earth pressure can be found out by Eq. (10) by knowing the value of 'β'.

Discussion

The method considers the dynamic load in terms of sinusoidal wave and the effect of the load becomes maximum when the line of action of dynamic load coincides with the static load of the failure wedge. The static pressure can also be found out by putting the value of 'Q' as zero. This has been observed that the results tally with Coulomb's active earth pressure of soils.

Conclusions

The method suggested here considers the effect of dynamic load in terms of sinusoidal wave which varies with the position on backfill surface. The other methods consider the maximum effect only but by this method, the effect of dynamic load can be found out at different positions also. This method will also enable one to find out critical failure plane and the magnitude of dynamic active earth pressure of soils.

References

INDIAN STANDARDS (1975) : "Criteria for Earthquake Resistant Design of Structures" *IS : 1893–1975. Indian standards Institution*, New Delhi.

KAPILA I.P., (1962) : "Earthquake Resistant Design of Retaining Walls" *Proceedings second symposium on Earthquake Engineering*, University of Roorkee, pp. 97–108.

MONONOBE, N, (1929) : "Earthquake Proof Construction of Masonary Dam" *Proc. World Engineering Congress*, Vol. 9, p. 275.

OKABE, S, (1929) : "General Theory of Earth Pressure and Seismic Stability of Retaining Walls and Dams." *Journal Japan Society of Civil Engg.* Vol. 1, No. 6.

SARAN, S. and PRAKASH S. (1968) : "Dimensionless Parameters for Static and Dynamic Earth Pressure for Retaining Walls." *Indian Geotechnical Journal*, 7 : 3.

SONI, K.M. and MURTHY, D.N. (1983) : "Graphical Method for Dynamic Earth Pressure of Cohesionless Soils", *Indian Geotechnical Journal*, 13 : 3.

SATYANARAYAN, B, and MURTHY, N.S. (1983) : "Active Earth Pressure under Dynamic Conditions", *Indian Geotechnical Conference*, I : iv 5-9.