Stability of Slopes in Desiccated Clays

by

P.K. Basudhar*

A.J. Valsangkar**

M.R. Madhav***

Introduction

Analysis of stability of slopes is a widely studied field of soil mechanics. In most of the references available on this subject soil has been treated as a homogeneous isotropic material with constant strength throughout the slope under consideration or into layers into which the soil has been divided arbitrarily. Assuming the validity of $\phi = 0$ analysis, Gibson and Morgenstern (1962) and Kenney (1963) have presented rigorous solutions for the case of linearly increasing strength with depth. But in nature most of the soils exhibit inherant or induced anisotropy. The importance of studying the effect of anisotropy on the conventional factor of safety in the design of earth slopes and cuts, needs no emphasis. Lo (1965),Chen et al. (1975), Ranganatham and Srinivasulu (1979), Koppula (1984) have presented very useful contribution in this direction. In these works also a linear strength distribution with depth is assumed. Although this is an entirely reasonable asssumption in many cases, there are other situations where it is decidedly in error. Throughout the world there exist areas of surface clays in climates in which there are alternating dry and wet periods. In such climates, the surface clays are subjected to cycles of desiccation and saturation. The capillary forces in the drying clay have subjected the clay to preconsolidation pressure which produce overconsolidation to considerable depths. This results in a higher insitu undrained shear strength over this region than the strength predicted by linear strength depth distribution assumption. James et al. [1969), based on published evidence, has postulated an empirical relationship to account for increased undrained shear strength in the desiccated zone. In the studies pertaining to the stability of slopes this fact has been ignored.

In the present investigation considering nonhomogeneous strength distribution as suggested by James et al. (1979) in both the horizontal and vertical directions and assuming strength anisotropy as suggested by Casagrande and Carrilo (1944) the factor of safety is evaluated by using the ordinary method of slices. With these assumptions the degree of anisotropy turns out to be a function of depth. Determination of the critical failure surface and the corresponding minimum factor of safety has

* Assistant Professor, Civil Engineering Department, Indian Institute of Technology, Kanpur-208016, India.

** Professor, Civil Engineering Department, University of New Brunswick, Fredericton, New Brunswick, Canada.

*** Professor, Civil Engineering Department, Indian Institute of Technology, Kanpur-208016, India.

(The paper was received in April, 1986 and is open for discussion till the end of December, 1986)

been formulated as a nonlinear programming problem. The influence of various parameters on the factor of safety has been studied.

Analysis

Basic Assumptions

1. A $\phi = 0$ Analysis is valid (undrained case)
2. The potential failure surfaces are cylindrical i.e. circular in cross section (Fig. 1).
3. The soil is nonhomogeneous and anisotropic with respect to strength (Figs. 2 and 3).
4. The degree of anisotropy is a function of depth (Fig. 2).

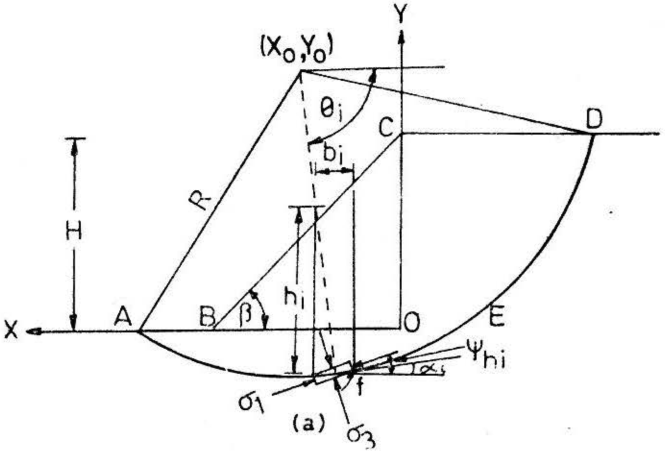

FIGURE 1 Typical Slope with a Trial Arc

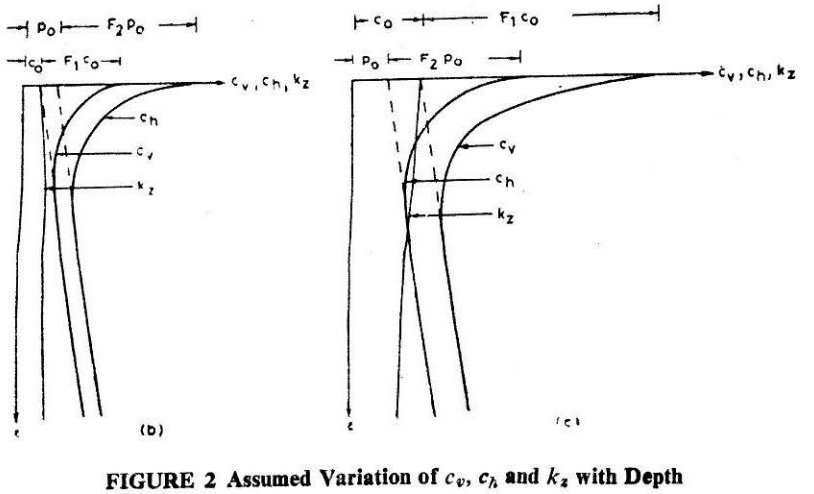

FIGURE 2 Assumed Variation of c_v, c_h and k_z with Depth

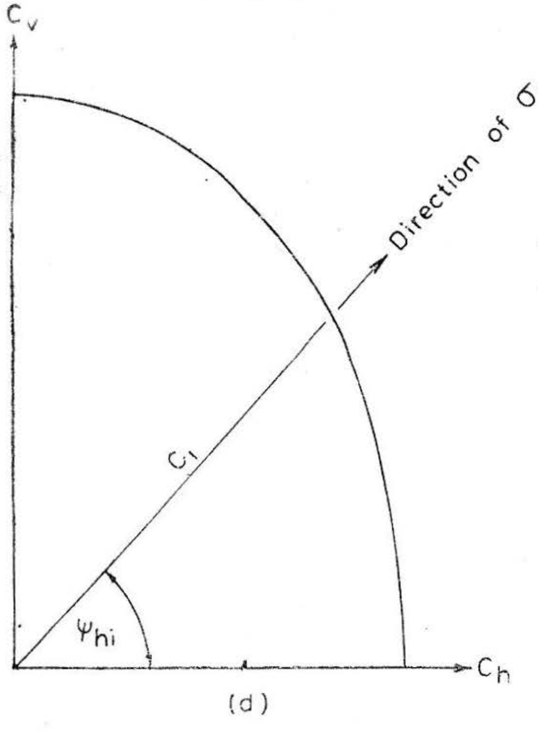

FIGURE 3 Assumed Anisotropic Strength Variation.

Variation of Strength with Depth and Direction

The insitu undrained shear strength is often a function of depth and the orientation of the principal stresss. Typical variations of undrained shear strength with depth are shown in Fig. 2. An empirical rule as suggested by James et al. (1969) is considered in the analysis. Thus the undrained shear strength in the horizontal and vertical direction c_v and c_h (at any depth z below the surface) are as follows:

$$c_v = c_o + \Psi_1 \nu_z + F_1 C_o \exp\left\{-\left(\frac{z}{\beta_1 H}\right)^{n_1}\right\} \quad \ldots (1)$$

$$c_h = P_o + \Psi_2 \nu_z + F_2 P_o \exp\left\{-\left(\frac{z}{\beta_2 H}\right)^{n_1}\right\} \quad \ldots (2)$$

in which

ν is the unit weight of the soil, C_o and P_o are the zero depth intercepts on the strength axis of the linear position of the composite strength curve.

F_1, F_2, β_1, β_2, n_1 and n_2 are empirical coefficients.

Ψ_1 and Ψ_2 are soil paramaters to be determined empirically.

The considered strength distribution rules are general enough to include cases where undrained shear strength either is constant, increases or decreases with depth.

From the findings of many investigators, over the years, James et al. (1969) have suggested the most probable range of values for various parameters for normally consolidated and slightly overconsolidated soils.

C_o and P_o are assumed to be related as follows:

$$C_o = k_s P_o \qquad \text{... (3)}$$

where k_s is the ratio of the zero depth strength intarcepts of the linear portions of the composite strength curves.

The degree of anisotropy as a function of depth is defined as follows:

$$k_z = c_v / c_h \qquad \text{... (4)}$$

For the degree of anisotropy to be constant with depth, the following relations must be valid

$$F_1 = F_2 \text{ (say } F), \; n_1 = n_2 \text{ (say n)}, \; \beta_1 = \beta_2 \text{ (say } \beta_a), \; \Psi_1 = \Psi_2 \, k_s \qquad \text{... (5)}$$

In the present study anisotropy due to the rotation of principal stress direction is considered. The anisotropic strength distribution as suggested by Casagrande and Carrilo (1944) is assumed to be valid.

The shear strength of the soil when the major principal stress direction is inclined at an angle Ψ_{hi} with the horizontal is expressed as follows:

$$c_i = c_h + (c_v - c_h) \sin^2 \Psi_{hi} \qquad \text{... (6a)}$$

where

$$\Psi_{hi} = \Pi - (f + \theta_i) \qquad \text{... (6b)}$$

in which f is the angle between the failure plane and the major principal plane, and θ_1 is a geometric parameter as shown in Fig. 1.

Using the relations 1 to 4, Eq. (6a) can be expressed in nondimensional form for any depth z_i as follows:

$$C_i^* = \frac{C_o^*}{k_z}\left[1 + \Psi_1 \frac{z_i^*}{C_o^*} + F_1 \exp\left\{-\left(\frac{z_i^*}{\beta_1}\right)^{n_1}\right\}\right] \cdot [1 + (k_z - 1)\sin^2 \Psi_{hi}] \qquad \text{... (7)}$$

where $z_i^* = \frac{z_i}{H}$, $C_i^* = \frac{C_i}{\nu H}$, $C_o^* = \frac{C_o}{\nu H}$

H : Height of the slope.

Objective Function and its Minimization

The most commonly used ordinary method of slices is applied to analyse the problem. Considering only the moment equilibrium, the factor of safety of a slope in undrained condition, in terms of material properties and geometric parameters can be obtain as follows:

$$F_s = \frac{\sum_{i=1}^{N} C_i^* \; b_i^* \; \sec\alpha_i}{\sum_{i=1}^{N} h_i^* \; b_i^* \; \sin\alpha_i} \qquad \ldots(8)$$

in which

$b_i^* = \frac{b_i}{H}$: *nondimensional width of i^{th} slice.*

$h_i^* = \frac{h_i}{H}$: nondimensional height of the i^{th} slice.

$z_i^* = \frac{z_i}{H}$: nondimensional depth co-ordinate of the i^{th} slice.

α_i : angle of inclination of the slip surface with the horizontal at the base of the i^{th} slice.

N : total number of slices.

From Eq. (7), the value of C_i^* to be used in Eq. (8), is evaluated. As there are three possible types of failure viz. base failure, toe failure and slope failure, all these possibilities are taken care of in the analysis. The nondimensional height (h_i^*) of the slices will depend on the location of the slices; for the sake of brevity the exclusive expressions are not reported here in.

There are many possible circular arcs through a cross-section. The location of the critical arc is determined by minimizing the value of the factor of safety with respect to the co-ordinate of the centre and the radius of the slip circle ensuring that a portion of the slip circle must pass through the slope. The problem is one of nonlinear programming with strict inequality constraints (Basudhar et al. 1979). A computer program for automated slope stability analysis was developed; results had been obtained for typical sets of data and compared with the values obtained by Lo (1965) and Chen (1975) to check the correctness of the developed program (Basudhar, 1976).

The reported analysis is general enough to include the cases reported by Gibson and Morgenstern (1962) and Lo (1965).

Results and Discussion

The factor of safety (F_s) for a given slope angle (β) is computed using a value of $f = 55°$ which is the average experimentally observed value (Lo, 1965). The safety factors are evaluated for different combination of n (1.0), C_0^* (0.25, 0.5), Ψ (0, 0.15, 0.3), F (0, 1, 4), β_a (0.025, 0.1, 0.4) and k_s

(0.5, 1.0, 2.0). However, only representative results are reported herein for the sake of brevity. Qualitative and quantitative idea about the influence of various parameters on the factor of safety can be obtained from Figs. 4 through 12.

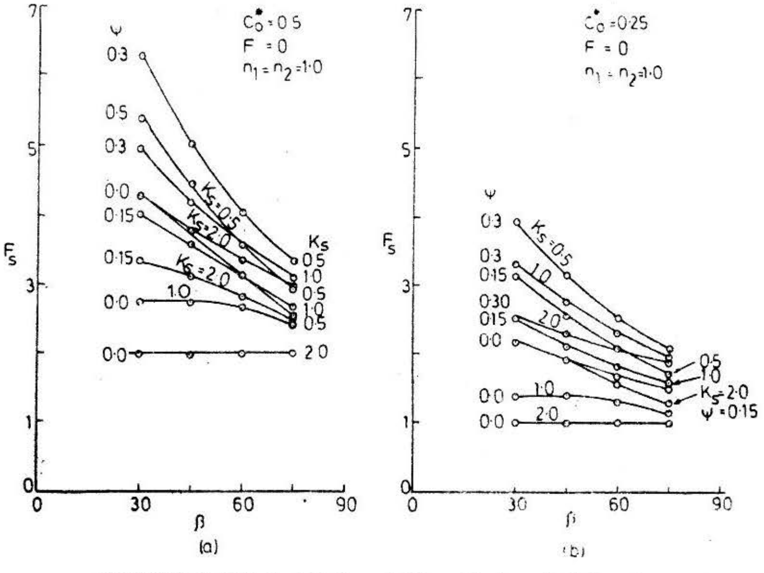

FIGURE 4 Effect of β, k_s and Ψ on Factor of Safety F_s

In Fig. 4 (*a*) the influence of β, k_s and Ψ for constant values of C_0^* ($= 0.5$), F ($= 0$) and n ($=1.0$) has been shown qualitatively. It may be observed that for $\Psi = 0$ and $K_s = 2.0$, the slope angle (β) does not have any influance on factor of safety (F_s), over the range of values it has been studied. This is identical with the observation made by Lo (1965). For $\Psi = 0.0$ and $k = 1.0$ representing a homogenous and isotropic case it is observed that β does not have any influence on factor of safety for any slope angle less than the critical angle 53° ; it is also an established well known conclusion. For any other particular value of Ψ and k_s it may be observed that with the increase in β value, the factor of safety decreases sharply. It may also be observed that for any particular slope angle (β) and k_s, factor of safety increase as Ψ increases. It is due to the fact that at higher value of Ψ, greater undrained shear strength is mobilized along the failure surface. The figure also reveals that when $k_s = 0.5$, there is an increase in factor of safety from the isotropic case and when k_s is 2.0, there is decrease in the factor of safety. Similar behaviour is observed for $C_0^* = 0.25$ and is presented in Fig. 4 (*b*).

Figure 5 shows the influence of β, 8_s Ψ and C_0^* on F_s for a given set of F (4.0), β_a (0.1) and n (1.0). The following general observations are made from the presented set of curves. Other parameters remaining constant as β increases F_s decreases, as k_s increases F_s decreases; as Ψ increases, F_s increases, as C_0^* increases F_s also increases. The safety factors

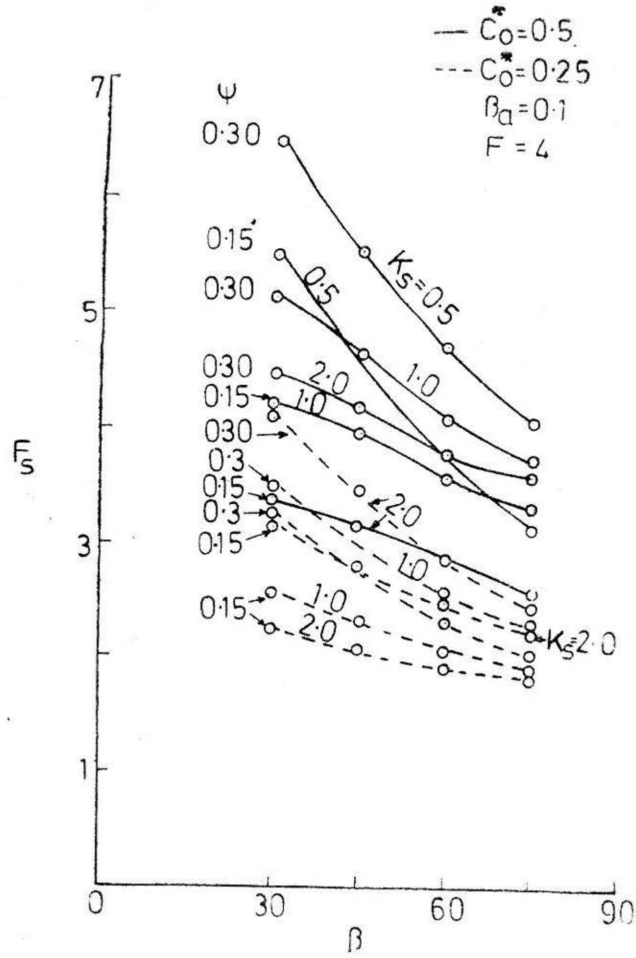

FIGURE 5 Effect of β, k_s, Ψ and C_0^* on Factor of Safety F_s

corresponding to $\Psi = 0$ and $F = 4$ are not presented in the figure as these values are very close to the obtained factor of safaty values for $\Psi = 0$ and $F = 1.0$. This signifies that for $\beta_a = 0.1$ and $\Psi = 0.0$, F does not have much of influence on the factor of safety.

To emphasise the influence of k_s on F_s values, they are plotted in Fig. 6 as abscissa and ordinate respectively. For a given slope (45°) values of factor of safety for a range of k_s, Ψ and F values are presented in the figure for paticular values of C_0^* and β_a. It can be observed that for a given set of Ψ and F values, as k_s increases factor of safety (F_s) decreases. However, when k_s is increased from 0.5 to 1.0 there is a sharp decrease in the factor of safety whereas when it is increased from 1.0 to 2.0 the increase in the factor of safety is gradual. To have a quantitative idea, percentage differences in the safety factors for a 45° slope and C_0^* = 0.25 from the isotropic value are presented in Fig. 7. The

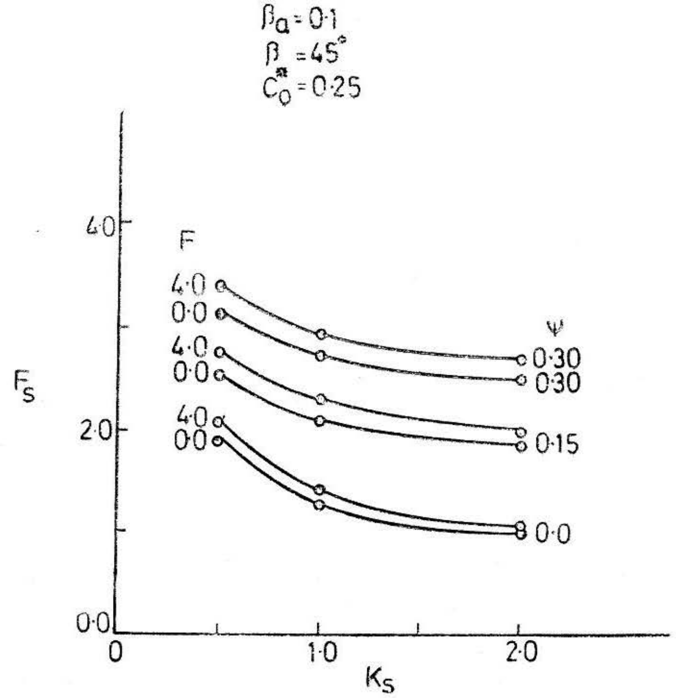

FIGURE 6 Influence of k_s on Factor of Safety F_s

maximum percentage change in factor of safety value is observed for $F = 4.0$ and $\Psi = 0.3$. It is observed that when k_s is decreased to 0.5 from 1.0, the increment in the factor of safety is about 46 per cent whereas when k_s is increased from 1.0 to 2.0 the decrease in factor of safety value is to the extent of 30 per cent. Similar trend was observed for other values. This signifies a strong dependence of the factor of safety on the ratio of zero depth cohesion intercepts k_s.

In Fig. 8 to emphasise the influence of Ψ on F_s, these are plotted as abscissa and ordinate respectively for $\beta = 45°$, $\beta_a = 0.1$, $C_0^* = 0.25$ and different combination of k_s and F values. It may be observed from the figure that as Ψ is increased from 0 to 0.3 there is a steep increase in the factor of safety values. The percentage variation in the factor of safety values taking $\Psi = 0$ as the base value, is evaluated and presented in Fig. 9. It may be observed that for $k_s = 2.0$ and $F = 4.0$ when Ψ is increased from 0 to 0.3, the increase in factor of safety may be even to the extent of about 17 per cent. From the figure it may be observed that the percentage variation ranges from about 60 to 175 percent.

The effect of F on F_s is shown in Fig. 10, where β ranges from 30° to 75°. The percentage variations in factor of safety are calculated taking $F = 0$ as the base and presented in Fig 10. The curves denoted by p_1 and p_2 correspond to the cases when F is increased from 0 to 1 and 4 respectively. The percentage variations in F_s are presented only for k_s equals to 0.5. It may be observed from the figure that the effect of F on F_s is more pronounced

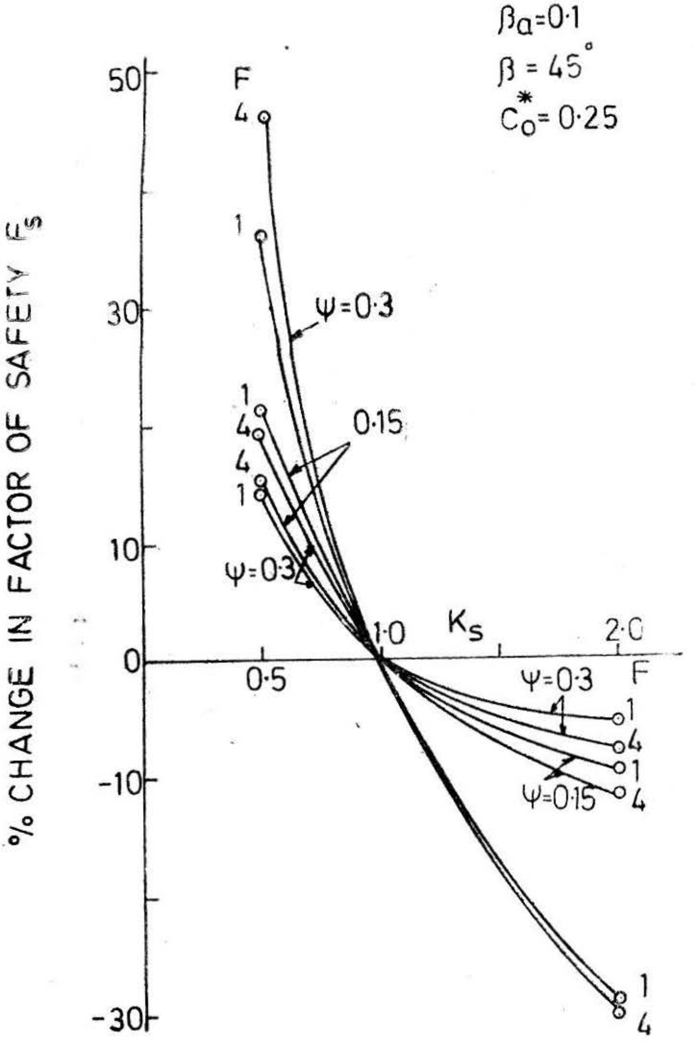

FIGURE 7 Influence of k_s on Factor of Safety F_s

in steep slopes. Maximum change is observed when F is increased from 0 to 4. At higher values of F greater shear strength is mobilized along the failure surface and as such the effect is more significant.

The influence of β_a, the attenuation rate on the factor of safety is depicted qualitatively in Fig. 11, for different slope angles and constant C_0^*, k_s, Ψ

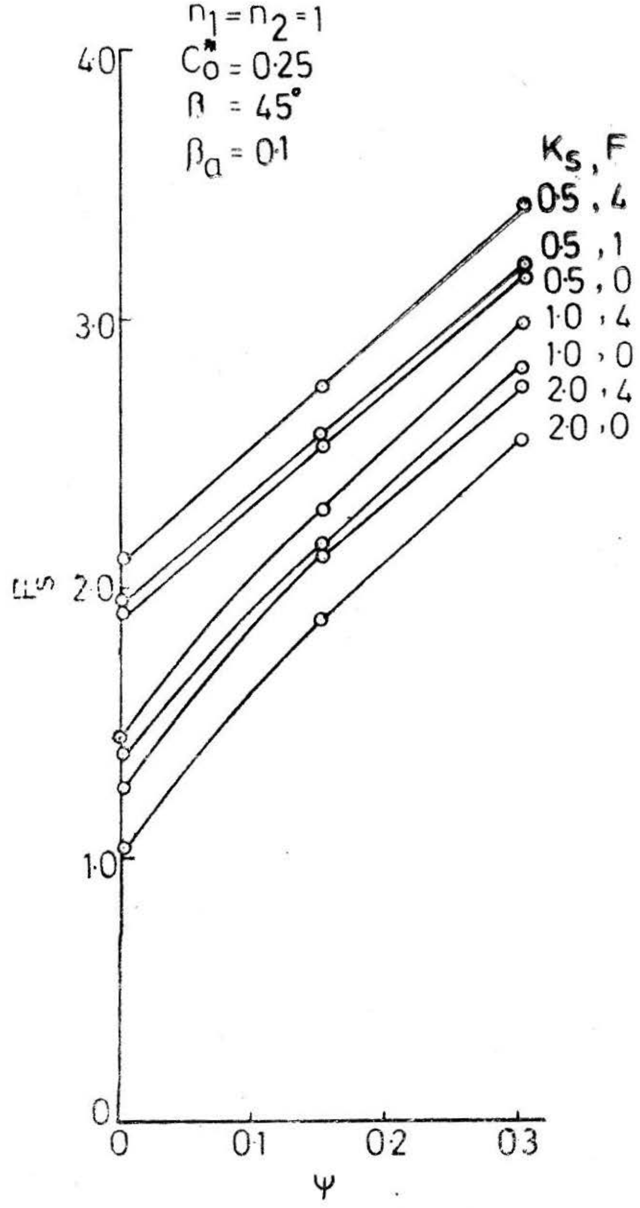

FIGURE 8 Influence of ψ on Factor of Safety F_s

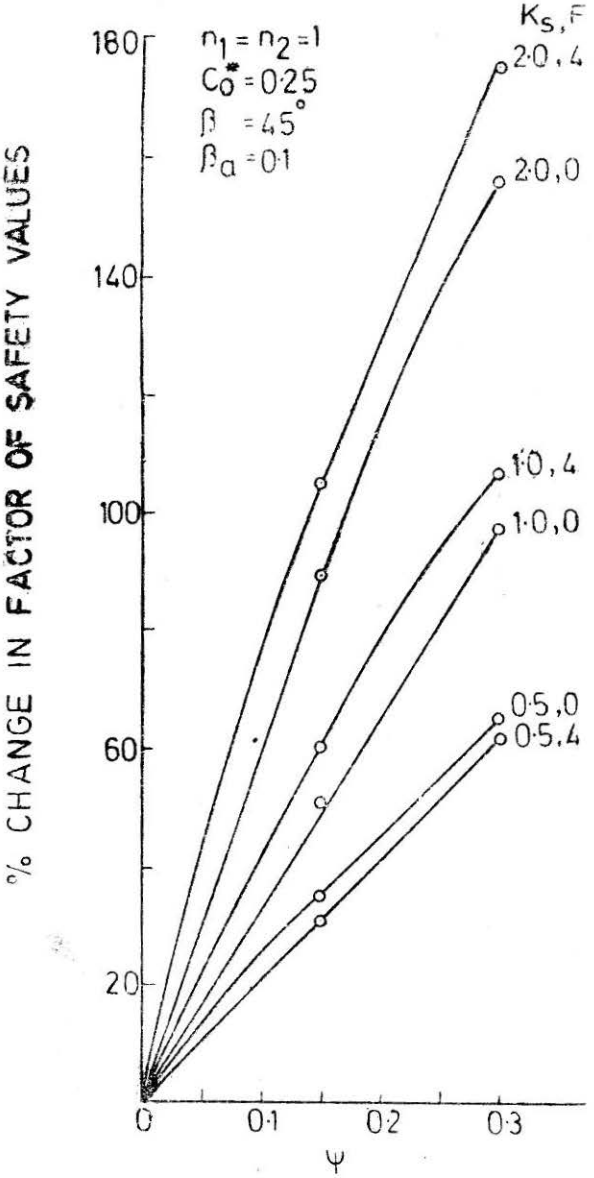

FIGURE 9 Influence of ψ on Factor of Safety F_s

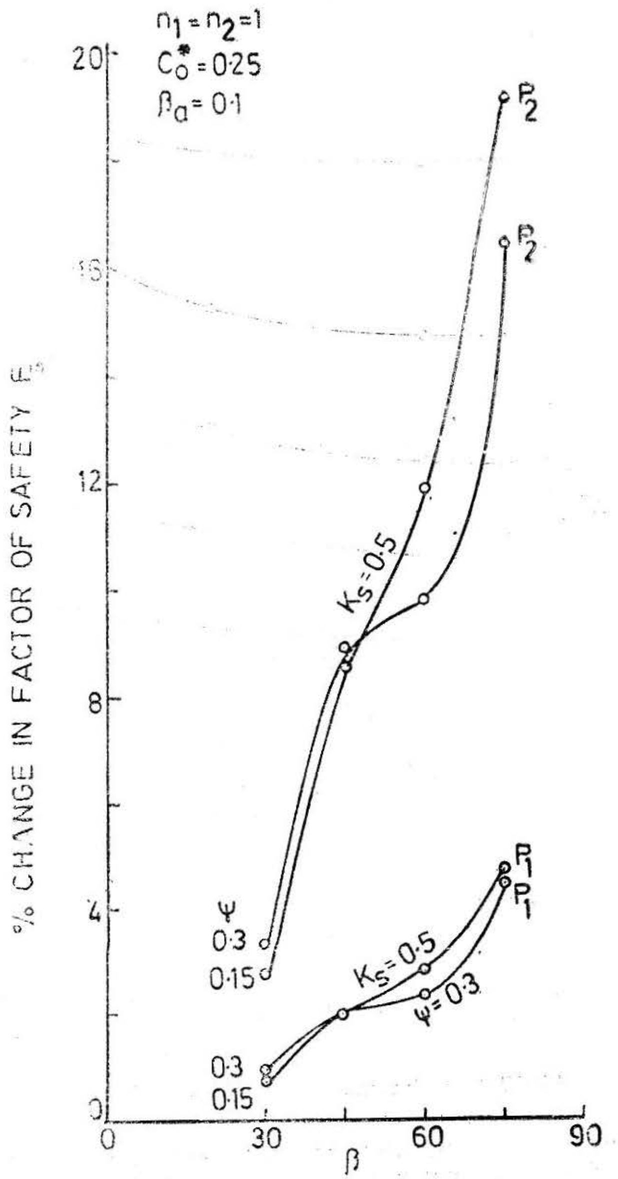

FIGURE 10 Influence of F on Factor of Safety F_s

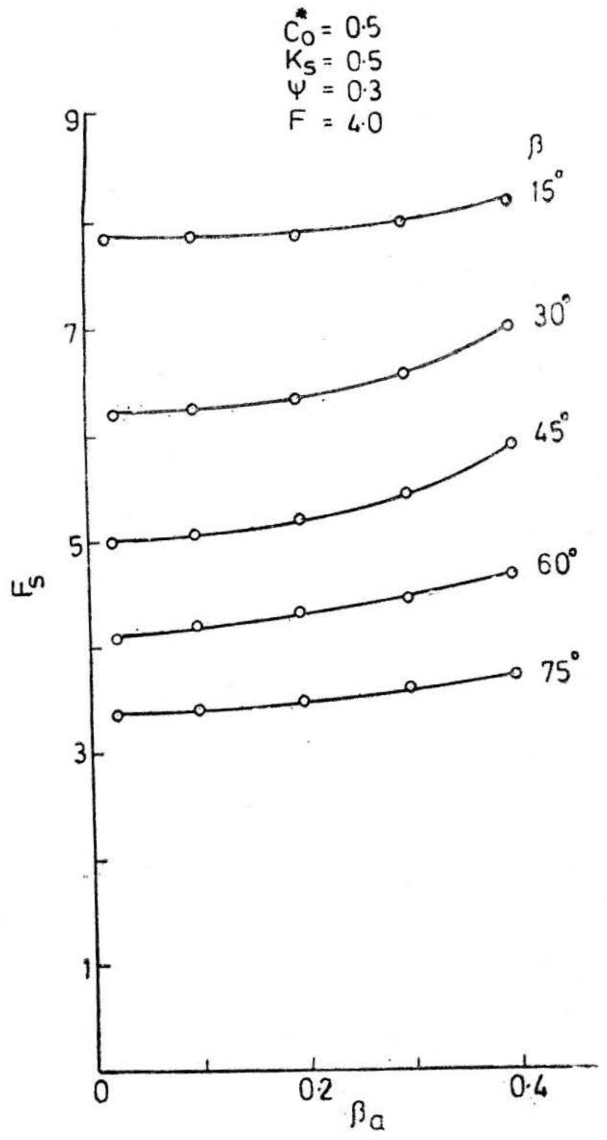

FIGURE 11 Influence of β_a on Factor of Safety F_s

and F values. It may be observed that lower values of β_a do not have appreciable effect on the safety factor F_s. Lower values of β_a signifies that the higher strength at the surface decreases very sharply within a limited depth and as such very less or no effect is felt as the major portion of the failure surface lies beyond this zone. Taking $\beta_a = 0.25$ as the base value the percentage variations in F_s value due to any increment in β_a is shown in Fig. 12. It may be observed that for $\beta_a = 0.4$ for 45° slope the change is maximum and is of the order of 16 percent whereas for 75° slope it is only 7.7 percent.

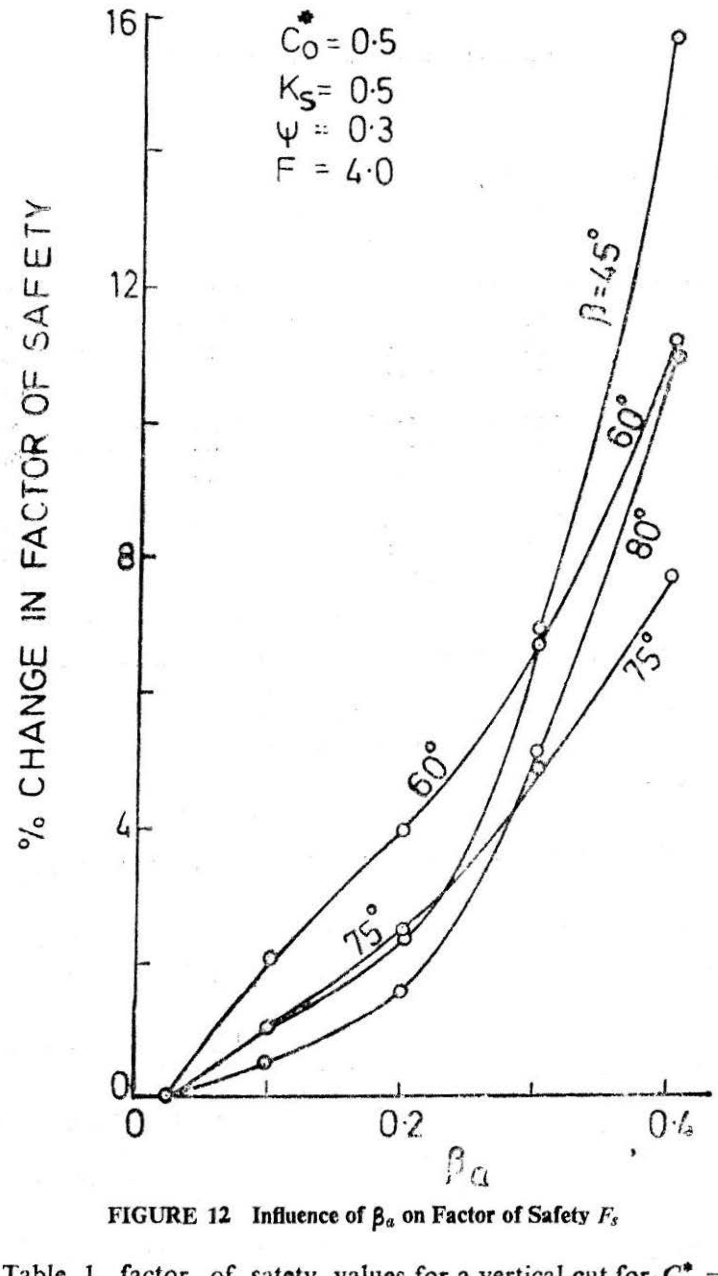

FIGURE 12 Influence of β_a on Factor of Safety F_s

In Table 1, factor of satety values for a vertical cut for $C_0^* = 0.25$, $\beta_a = 0.1$ and different combination of k_s, Ψ, F are presented. It may be observed from the table that k_s, and F do not play any significant role in the factor of safety value. For a vertical cut the failure surface may be such that the direction of the major principal stress does not rotate to a great extent and as such the effect of anisotropy is not appreciable. At $\beta_a = 0.1$, the surface excess strength decreases rapidly and the effect of

TABLE 1

Factor of Safety Values for a Vertical Cut

$\beta_a = 0.1$

C_0^*	F	Ψ	F_s		
			$k_s = 0.5$	$k_s = 1.0$	$k_s = 2.0$
0.5	0	0.00	—	1.910	1.864
		0.15	2.576	2.592	2.494
		0.30	3.184	3.140	3.110
	1	0.00	1.968	1.912	1.866
		0.15	2.577	2.529	2.494
		0.30	3.185	3.140	3.110
	4	0.00	1.970	1.917	1.868
		0.15	2.578	2.530	2.495
		0.30	3.185	3.141	3.110

increase of strength over the linear portion of the composite strength curve is not appreciably felt. It can also be observed that as Ψ increases there is an increase in the factor of safety values in a similar manner as has already been discussed for other slope angles.

Conclusions

From the limited study presented in this paper the following generalized conclusions are drawn.

Any increment in the cohesion factor C_0^*, the nondimensional zero depth intercept of the composite strength curve showing the distribution of undrained shear strength in the vertical direction, will cause appreciable increase in the factor of safety. The usual practice of neglecting the strength at the surface leads to conservative analysis and design.

When k_s, the ratio of the zero depth intercepts of the linear portion of the composite strength curve, is less than unity there is an increase in the factor of safety value, whereas when k_s, is more than unity there is a decrease in factor of safety value.

Corresponding to any increment in the value of Ψ, there will be an appreciable increase in the factor of safety values.

The effect of F, the factor showing the excess strength at the surface over the strength predicted by the linear portion of the composite strength curve, has been observed to be more significant for steeper slope angles for $\beta_a = 0.1$, however for higher β_a values there is a possibility that the effect may even be pronounced for flater slope angles.

Higher values of β_a and the attenuation rate, signifies that the surface strength over the linear portion of the composite strength curve decreases very slowly with depth and, as such, the desiccated zone extends to a greater depth. The strength in this zone is much higher than predicted by the linear portion of the composite strength curve. It has been observed that as β_a is increased from 0.025 to 0.4 there is appreciable increase in the safety factor.

So it can be inferred that clay layers which have been subjected to alternating dry and wet periods have great importance in foundation engineering owing to the physical difference in their properties from normally consolidated noxdried clays. It has been observed that the desiccated zone have appreciable influence on the factor of safety of the slope and, as such, should always be considered for a realistic analysis and design of slopes. The effect of strength anisotropy and variation of strength with depth of the stability of slopes is well known and needs no emphasis.

References

BASUDHAR, P.K. (1976), "Some Applications of Mathematical Programming Techniques to Stability Problems in Geotechnical Engineering" Ph. D. Thesis. Indian Institute of Technology, Kanpur.

BASUDHAR, P.K., VALSANGKAR, A.J., and MADHAV, M.R. (1979), 'Non-linear Programming in Automated Slope Stability Analysis', Indian Geotechnical Journal, 9 ; 3 : 212-219.

CASAGRANDE, ARTHUR and CARRILO, NABOR (1944), "Shear Failure of Anisotropic Materials", Contributions to Soil Mechanics, 1941-1953, Boston Society of Civil Engiueers, pp. 122-135.

CHEN, W.F., SNITBHAN, N., and FANG, H.Y. (1975), 'Stability of Slopes in Anisotropic Nonhomogeneous Soils', Canadian Geotechnical Journal 12, 146-152.

GIBSON, R.E. and MORGENSTERN, N. (1962), "A Note on the Stability of Cuttings in Normally Consolidated Clays', Geotechnique, 12 : 3, 372-378.

JAMES, C.H.C., KRIZEK, R.J., and BAKER, N.H. (1969), "Bearing Capacity of Cohesive Soils with a Nonhomogenous Strength Distribution", Highway Research Record, No. 282, 48-56.

KENNEY, T.C. (1963), "Stability of Cuts in Soft Soils", Journal of the Soil Mechanics and Foundations Division, ASCE, 89 SM5 : Proc. Paper 3647.

LO, K.Y. (1965), "Stability of Slopes in Anisotropic Soils", Journal of the Soil Mechnics and Foundations Divisions, ASCE, 91 : SM4 : 85-106.

KOPPULA, S.D. (1984), 'The Stability of Slopes in Clays with Linearly Increasing Strength', Canadian Geotechnical Journal, 21 : 3 : 577-521.

RANGANATHAM, B.V. and SREENIVASULU, V. (1979), "Stability Analysis of Cuts with Variable Strength", Proceedings of the Sixth Asian Regional Conference on Soil Mechanics and Foundation Engineering, Singapore, Vol. 1, 245-248.

Notations

b_i, b^*_i = width and nondimensional width of the i^{th} slice

c_h' c_v = undrained shear strength respectively in the horizontal and vertical directions

C_o	=	zero depth undrained shear strength intercept of the linear portion of the composite strength
C_0^*	=	nondimensional undrained shear strength factor at the soil surface
C_i, C_0^*	=	undrained shear strength and nondimensional undrained shear strength respectively when the major principal stress direction is inclined at an angle Ψ_{hi} with the horizontal
F_s	=	factor of safety
F_1, F_2 F	=	empirical coefficients signifying the excess undrained shear strength over the same as predicted by linear distribution
'f'	=	angle between the failure plane and the major princial plane
H	=	height of the slope
h_i, h_0^*	=	height and nondimensional height respectively of the i^{th} slice
$K_z\left(=\frac{C_v}{C_h}\right)$	=	degree of anisotrophy as a function of depth
$K_s\left(=\frac{C_o}{F_o}\right)$	=	ratio of the zero depth undrained shear strength intercepts of the linear portion of the composite strength curves (c_v and c_h)
n_1, n_2, n	=	empirical coefficients
P_o	=	zero depth shear strength intercept of the linear portion of the composite strength curve (c_h)
P_1, P_2	=	percentage changes in the factor of safety values when k_s is changed from 0.5 to 1.0 and from 1.0 to 2.0 respectively
z	=	depth below the ground surface
z^*	=	nondimensional depth factor
α_i	=	inclination of the plane of failure to the horizontal for the i^{th} slice
β	=	slope angle
β_1, β_2	=	attenuation rate
β_a	=	attenuation rate when $\beta_1 = \beta_2$

γ = unit weight of the soil

θ_i = geometric parameter

ϕ_u = angle of shearing resistance under undrained condition

Ψ_1, Ψ_2 = rate of increase of the linear portions of the composite strength curves (c_v and c_h)

Ψ = rate of increase of the linear portion of the strength curves when $\Psi_1 = \Psi_2$

Ψ_{h_i} = inclination of the major principal stress with the horizontal