A Strength Criterion For Anisotropic Rocks

by

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Introduction

For a realistic analysis and rational design of engineering structures in rocks, it is essential to have a clear understanding of its strength behaviour. The strength of rock mass is significantly influenced by a number of factors, most important among them being bedding planes, joints, faults, and other weak planes. For example, planes of weakness which facilitate drilling and blasting, may later hinder effective roof and rib control. Thus, importance of the study of anisotropic behaviour of rocks need hardly be stressed.

In general, most rocks, especially sedimentary and metamorphic rocks exhibit some degree of anisotropy. Depending upon the nature of anisotropy, they are classified as :

(*i*) stratified materials, for instance, some sandstones and shales consist of layers with elastic moduli, where the rock mass exhibits different proporties along and perpendicular to the bedding planes

(*ii*) regularly jointed rocks, where the development of fissures and joints have a significant effect on the gross mechanical response, and

(*iii*) foliated rocks, where the schistocity planes define the most common types of anisotrophy, e.g. gneisses and schists.

In the present study, use is made of published data on all the three types of anisotropy.

Figure 1 (*a*) shows an idealized cylindrical specimen of an anisotropic rock with an oblique weak plane making an angle β with the axis of major principal stress (σ_1). This angle β is designated as the orientation angle. It is common knowledge that strength varies continuously with specimen orientation as depicted in Fig. 1 (*b*) showing a minima when bedding plane is oblique to the principal stress axis.

Large amount of experimental data reported by numerous investigators e.g. Donath (1961, 1964) on Martinsburg slate, Chenvert and Gatlin (1965)

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(*The revised paper was received in July,* 1986 *and is open for discussion till the end of December, 1986.*

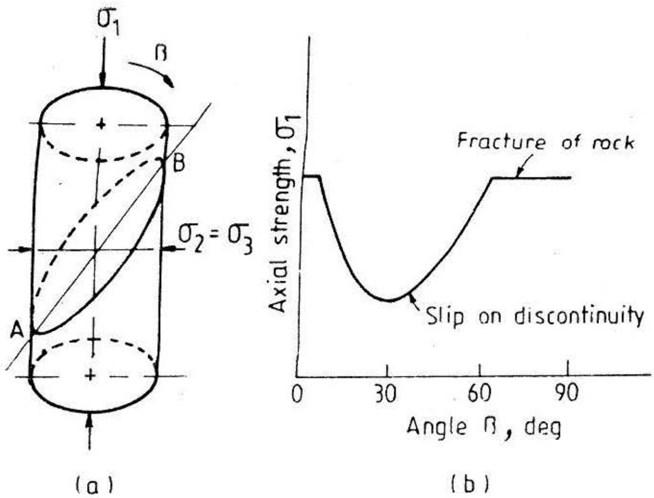

FIGURE 1 (*a*) **View of Typical Anisotropic Sample Showing Parameters Varied During Testing and** (*b*) σ_1—β **Failure Pattern for Anisotropic Rock**

on Arkansas sandstone, Hornio and Ellickson (1970) on fractured sandstone Attewell and Sandford (1974) on Penrhyn slate and Hoek and Brown (1980) on slates, clearly show that the strength (deviatoric stress at failure) for all rocks is maximum for $\beta = 0$ or 90 deg. and is minimum for $\beta = 20$ to 30 deg. Typical results demonstrating such a behaviour are presented in Fig. 2 for sandstone, slate and shales. From this figure it is also clear that the degree of anisotropy considerably diminishes with increasing confining pressure.

A systematic variation of modulus of elasticity, E, and Poisson's ratio, ν, with β was obtained by Chenvert and Gatlin (1965) for Arkansas sandstone, Permean shale and Green river shale. It was noticed that the E was highest for $\beta = 90$ deg. and lowest for $\beta = 30$ to 45 deg.

Strength Criteria for Anisotropic Rocks

Unlike isotropic rocks, the strength criteria for anisotropic rocks is more complicated, because of the variation in the orientation angle, β. A number of empirical strength criteria have been proposed in the recent past, based on the classical Navier-Coulomb and the Griffith's criteria. Some of the widely used failure theories for anisotropic rocks are tabulated in Table 1 along with their basic assumptions and limitations. Among the theories tabulated, Walsh and Brace (1964) and Jaeger (1960) variable cohesive strength theory and McLamore and Gray (1967) criterion predict the non-linear behaviour of anisotropic rocks. McLamore and Gray (1967) assume that the material fails in shear and has a variable cohesive strength, c, but constant values of internal friction, tan ϕ, whereas Walsh and Brace (1965) assume that the failure is tensile in nature and that the body is composed of long, non-randomly oriented cracks whichare superposed on an isotropic array of randomly distributed smaller cracks or Griffith's cracks. Walsh and Brace (1964) further assume that the fracture may occur through the growth of either long or small cracks depending upon the orientation of the long crack system to the applied stress, σ_1.

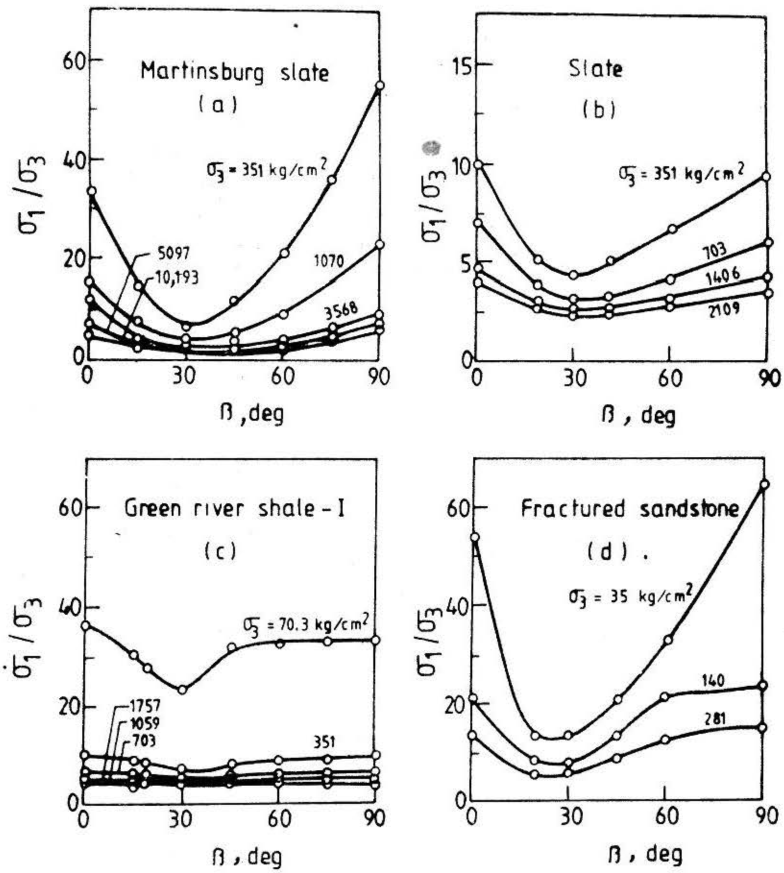

FIGURE 2 **Dependence of Stress Ratio with Orientation of Weak Plane for** (*a*) **Martinsburg Slate (Donath, 1972),** (*b*) **Texas Slate,** (*c*) **Shale (McLamore and Gray, 1967),** (*d*) **Fractured Sandstone (Hornio and Ellickson, 1970)**

To evaluate all these failure criteria, it is necessary to conduct triaxial tests at a minimum of three different confining pressures on specimens of at least three different orientations i.e., $\beta = 0$, 90 and 30 deg. Due to theoretical restrictions, Walsh-Brace, and Jaeger criteria cannot predict the extreme regions for β (near 0 and 90 deg.) and form 'horizontol shoulders', whereas the McLamore and Gray modification which introduces anisotropic factor (n) predicts the end shoulder portion also.

Based on an analogy with the non-linear failure envelope predicted by classical Griffith's crack theory for plane compression and by using a process of trial and error, Hoek and Brown (1980) have developed the following empirical failure criterion for isotropic and anisotropic rocks :

$$\sigma_1 = \sigma_3 + (m\,\sigma_c\,\sigma_3 + s\sigma_c^2)^{1/2} \qquad ...(7)$$

where, σ_1 and σ_3 = major and minor principal stresses respectively.

σ_c = uniaxial compressive strength of intact rock, and

TABLE 1

Anisotropic Strength Criteria

Proposed by	Strength criteria	Assumptions	Limitations
	Single plane of weakness theory	(*i*) This is a generalization of Mohr-Coulomb linear theory	(*i*) Predicts linear behaviour only
Jaeger (1960)	(*i*) Failure within the matrix, $\beta = 0$ or $90°$ $(\sigma_1 - \sigma_3) = \dfrac{2\,(c \cos \phi + \sigma_3 \sin \phi)}{(1 - \sin \phi)}$ ...(1) c = cohesive strength ϕ = friction angle	(*ii*) Isotropic body that contains a single or a set of parallel planes of weakness within the material that has different values of c and $\tan \phi$ than the surrounding matrix.	(*ii*) Both end portions ($\beta = 0$ and $90°$) cannot be predicted. (*iii*) Tests needed at $\beta = 0$, 30 and 90° and at different σ_3.
	(*ii*) Failure through weak plane, $\beta = 30°$ $(\sigma_1 - \sigma_3) = \dfrac{(c \cos \phi + \sigma_3 \sin \phi)}{(\cos (\theta + \beta) \sin \beta)}$ ...(2) $\theta = (45 - \phi/2)$	(*iii*) The body fails in shear and has variable c, and constant $\tan \phi$.	(*iv*) Solves for values of β within 25° of ϕ.
Jaeger (1960)	Variable cohesive strength theory $(\sigma_1 - \sigma_3) = \dfrac{2c - 2\,\sigma_3 \tan \phi}{(\tan \phi - (\tan^2 \phi + 1)^{1/2})}$ ...(3) where $c = A - B\,[\cos 2\,(\xi - \beta)]$ $\tan \phi \approx$ constant ξ = the orientation that has minimum c, A and B are constants	(*i*) The body fails in shear and has variable c and constant value of $\tan \phi$	(*i*) Tests should be carried out at $\beta = 0$, 30, 90° and at several σ_3. (*ii*) Least efficient method, since a wider range of tests are needed (*iii*) Cannot predict the end regions.

Walsh and Brace (1964)	(*i*) When $\beta = 0$ or $90°$ $(\sigma_1 = \sigma_3) = \sigma_c + \dfrac{2\mu\ \sigma_3}{(1+\mu^2)^{1/2}-\mu}$ ...(4) (*ii*) when $\beta = 30°$ $(\sigma_1-\sigma_3) = \dfrac{\sigma_{cj}[(1+\mu^2)^{1/2}-\mu)+2\mu\ \sigma_3}{2\sin\beta\cos\beta\,(1-\mu\tan\beta)}$ ...(5) μ = slope of σ_3 vs. $(\sigma_1-\sigma_3)$ curve σ_c = compressive strength when $\beta = 0$ or $90°$ σ_{cj} = compressive strength other than $\beta = 0$ or $90°$	(*i*) This is an extension of McClintock and Walsh (1963) and modification of Griffith's tensile failure. (*ii*) Body composed of long random cracks, superposed on an isotropic array of randomly distributed smaller cracks. (*iii*) Fracture occurs through the growth of either the long or the small cracks depending on the orientation of the long crack system	(*i*) 'Horizontal shoulders' in the ends cannot be predicted. (*ii*) Tests should be done at least at $\beta = 0$, 30 and $90°$ at several σ_3.
McLamore and Gray (1967)	$(\sigma_1-\sigma_3) = \dfrac{2c-2\ \sigma_3\tan\phi}{\tan\phi-(\tan^2\phi+1)^{1/2}}$ ...(6) where $c = A_{1,2} - B_{1,2}\ [\cos 2(\xi-\beta)]^n$ A_1, B_1 constants, variance of the range of $0° \leqslant \xi \leqslant \beta$ A_2, B_2 variance of the range of $\beta \leqslant \xi < 90°$ n = anisotropy factor that has the value of 1 or 3 for planar type of anisotropy (cleavage, schistocity) 5 or greater for the linear type as with bedding planes.	(*i*) Material fails in shear and has a variable c and constant $\tan\phi$	(*i*) Tests at $\beta = 0$, 30, $90°$ and at several σ_3 are necessary. (*ii*) 'Horizontal shoulder' portion can be eliminated by introducing proper constant n

m and s = dimensionless parameters which characterize the degree of interlocking between particles in a jointed rock mass.

For intact rock, $s = 1$ and for completely broken, $s = 0$. The range of variation of m is very wide and is believed to be a function of rock type and rock quality.

The above failure theories, due to their limitations, cannot be used for evaluating the strength behaviour of all rocks. The practical utility is very less as a large number of tests are to be performed at different σ_3 and β to evaluate the anisotropic rock strength. In this paper a failure criterion to predict anisotropic rock behaviour has been attempted.

Proposed Strength Criterion for Anisotropic Rocks

Due to limitations in the applicability of the existing failure criteria in one way or the other in the prediction of non-linear behaviour of anisotropic rocks, an attempt has been made to propose an empirical strength criterion based on the second order parabolic equation as follows :

$$\frac{\sigma_1}{\sigma_3} = A_2 \left(\frac{\sigma_c}{\sigma_3} \times \beta\right)^2 + A_1\left(\frac{\sigma_c}{\sigma_3} \times \beta\right) + A_0 \qquad \text{...(8)}$$

where σ_1 and σ_3 = the major and the minor principal stresses respectively

σ_c = uniaxial compressive strength (when $\beta = 0^0$),

β = orientation of weak plane/bedding, with reference to σ_1 (in radians), and

A_0, A_1, and A_2 = constants.

For which the normal equations are

$$NA_0 + A_1\,\Sigma\chi + A_2\,\Sigma\chi^2 = \Sigma y \qquad \text{...(9)}$$

$$A_0\,\Sigma\chi + A_1\Sigma\chi^2 + A^2\,\Sigma\chi^3 = \Sigma\chi y \qquad \text{...(10)}$$

$$A_0\Sigma\chi^2 + A_1\,\Sigma\chi^3 + A_2\,\Sigma\chi^4 = \Sigma\chi^2 y \qquad \text{...(11)}$$

where $y = \dfrac{\sigma_1}{\sigma_3}$, $\chi = \left(\dfrac{\sigma_c}{\sigma_3} \times \beta\right)$, N = number of data.

A_0, A_1 and A_2 can be obtained as follows :

$$A_2 = \frac{\Sigma\chi^2 y\,[N\Sigma\chi^2 - (\Sigma\chi)^2] - \Sigma\chi y\,(N\Sigma\chi^3 - \Sigma\chi^2\Sigma\chi)}{\Sigma\chi^4\,[(N\Sigma\chi^2 - (\Sigma\chi)^2] - \Sigma\chi^3\,(N\Sigma\chi^3 - \Sigma\chi^2\,\Sigma\chi) + \Sigma\chi^2}$$

$$\frac{+\Sigma y\,[\Sigma\chi^3\,\Sigma\chi - (\Sigma\chi^2)^2]}{[\Sigma\chi^3\,\Sigma\chi - (\Sigma\chi^2)^2\,]} \qquad \text{...(12)}$$

$$A_1 = \frac{(N\Sigma\,\chi y - \Sigma\chi\,\Sigma y) - A_2\,(N\Sigma\,\chi^3 - \Sigma\chi^2\,\Sigma\chi)}{N\Sigma\,\chi^2 - (\Sigma\chi)^2} \qquad \text{...(13)}$$

$$A_0 = \frac{\Sigma y - A_1\,\Sigma\chi - A_2\,\Sigma\chi^2}{N} \qquad \text{...(14)}$$

The constants A_0, A_1 and A_2 in Eq. (8) can be evaluated from the triaxial tests using the Eqs. (12) to (14). This criterion is valid for any orientation angle, β.

The validity of this criterion to predict the strength behaviour of anisotropic rocks is tested by conducting the experimental work described below. Analysis of the published experimental data on several anisotropic rocks is also carried out.

Experimental Work

Rock Tested

The anisotropic specimens of sandstone used in this study were collected from Kota, Rajasthan belonging to the Bhander series of upper Vindhyans. This type of rock is a common foundation rock for several river valley projects e.g. Ranapratap Sagar and Jawahar Sagar in Rajasthan. The colour is varigated shades of red, buff or grey, mottled or speckled, owing to variable dissimination of the colouring matter or its removal by its deoxidation. Thin and perfect bedding planes are clearly discernible. Scanning Electron Micrographs (SEM) and X-ray diffraction patterns show that the rock mainly consists of moderately sorted, well cemented medium quartz grains (95 per cent) and amorphous ferruginous material (5 per cent) (Rao, 1984).

Specimen Preparation

Specimens were cored using diamond bit drills of 38 mm diameter and prepared as per relevant ISRM (1981) Code. The base of the conventional laboratory drilling machine was fitted with special frames to obtain specimens at different orientations (β) 0, 30, 65 and 90 degrees. The uniaxial compressive strength and triaxial tests were carried out on specimens of $L/D = 2$ to eliminate any effect of the specimen length on the strength and to decrease the possibilities of buckling. The tolerance limits suggested by ISRM were met for all specimens. The polished specimens were first oven dried at $105 \pm 1°C$ for 24 h and kept in desiccators for cooling.

Tests Conducted

Apart from the mineralogical and physical properties, strength index tests e.g. uniaxial compression, Brazilian and point load tests were carried out to estimate the general characteristics of the sandstone. A modified triaxial cell (Ramamurthy, 1975) was used for the high pressure triaxial tests. Tests were conducted at confining pressures of 25, 50, 75, 100 and 125 kg/cm^2. Both axial (ϵ_a) and diametral (ϵ_d) strains were measured using electrical resistance strain gauges fixed to the specimen.

Results and Discussion

General Characteristics of Kota Sandstone

The general characteristics of Kota sandstone are presented in Table 2. The uniaxial compression, Brazilian and point load strengths at different orientations are depicted in Fig. 3 (*a*) and (*b*). It is clear from the figure that the highest σ_c was obtained when $\beta = 0$ and the lowest when

TABLE 2

General Characteristics of Kota Sandstone ($\beta = 0$)

Water Absorption:	3.26 %
Specific Gravity:	2.68
Density:	2.31g/cm³
Effective Porosity:	7.31 %
Sonic Wave Velocity:	3.09 km/sec
σ_c	801.28 kg/cm²
σ_t (Brazilian)	77.78 kg/cm²
E_t	1.4×10^5 kg/cm²
ν	0.21
c	212.38 kg/cm²
ϕ	43.42°

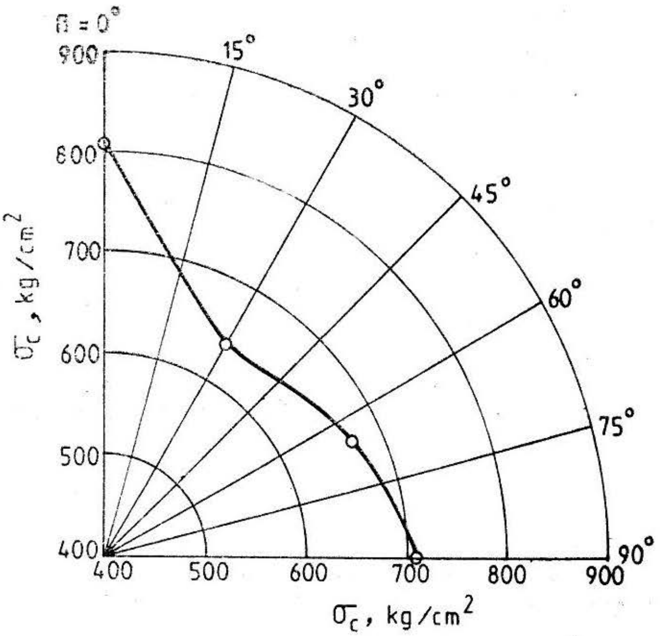

FIGURE 3 (*a*) Variation of Uniaxial Compressive Strength with Orientation for Kota Sandstone

$\beta = 30$ degrees. At $\beta = 90$ degrees it is only slightly lower than that at $\beta = 0$ degree. Similar behaviour was observed by other researchers. It is also observed from the figure that the Brazilian and point load strengths increase gradually when β changes from 0 to 90 degrees. The anisotropy ratio ($= \sigma_{cmax}/\sigma_{cmin.}$) is only 1.248. Unlike slates and shales, Kota sandstone with its high percent of fine grained quartz bonded strongly with ferruginous cement exhibits low strength anisotrophy.

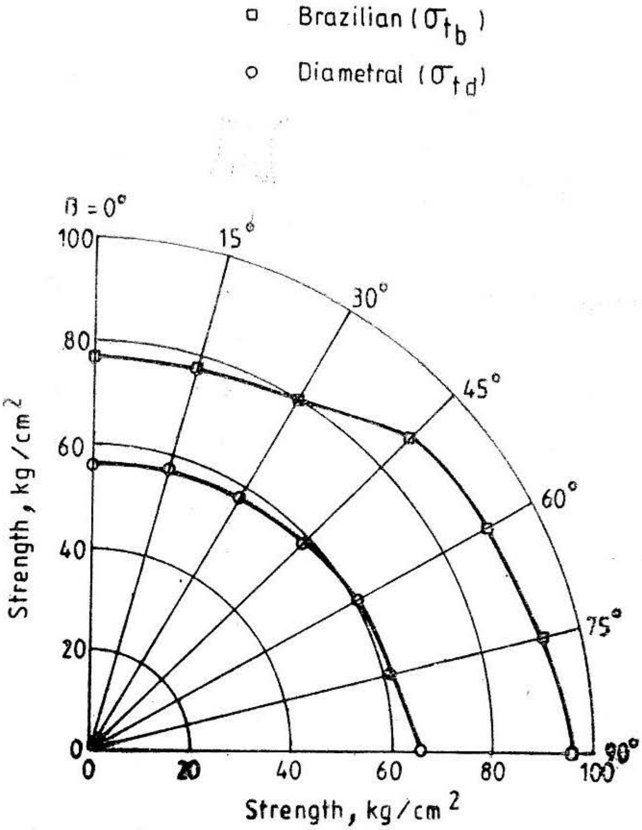

FIGURE 3 **(*b*) Variation of Brazilian and Point Load Strengths with Orientation for Kota Sandstone**

Evaluation of Parameters in the Proposed Criterion

From the experimental results, the parameters involved in the proposed criterion (Eq. 8) have been evaluated and presented in Table 3. This analysis has been carried out on an ICL 2960 computer at IIT, Delhi. The table shows the values of A_0, A_1 and A_2 at different confining pressures. The constants A_0 and A_1 decrease with increase of σ_3 whereas A_2 values increase with increase of σ_3 with negative sign. Using these values the strength at failure at σ_3 are calculated and plotted in Fig. 4, with experimental results for comparison. The strength predictions from Walsh and Brace (1964), Jaeger (1960), Hoek and Brown (1980) criteria for Kota sandstone are also presented in this figure. The results at only two values of σ_3 i.e. 25 and 125 kg/cm^2 are presented for clarity. The experimental data and the predicated values at all the confining pressures used in testing are presented separately in Fig. 5. From these two figures, it may be clearly inferred that the proposed criterion predicts strength more accurately than the other theories. It is also clear that both Walsh and Brace and Jaeger criteria yield poor prediction.

To verify the applicability of the proposed criterion to other anisotropic rocks, published data on Green river shale, Arkansas sandstone, and Permean shale (Chenevert and Gatlin, 1965), Martinsburg slate (Donath,

TABLE 3

Evaluation of Parameters for Kota Sandstone

Rock	σ_c kg/cm²	σ_3 kg/cm²	Values of Constants		
			A_o	$-A_1$	A_2
Kota Sandstone	801.28	25	44.02	0.63	0.01
		50	25.49	0.70	0.02
		75	18.73	0.78	0.04
		100	14.97	0.81	0.06
		125	12.66	0.86	0.08

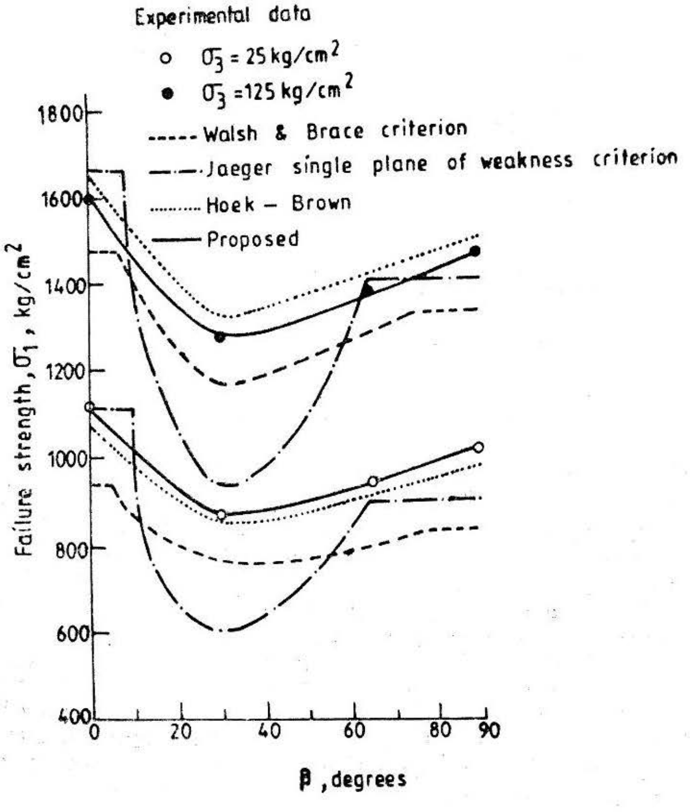

FIGURE 4 Comparsion Between Predicted and Observed Strength for Kota Sandstone

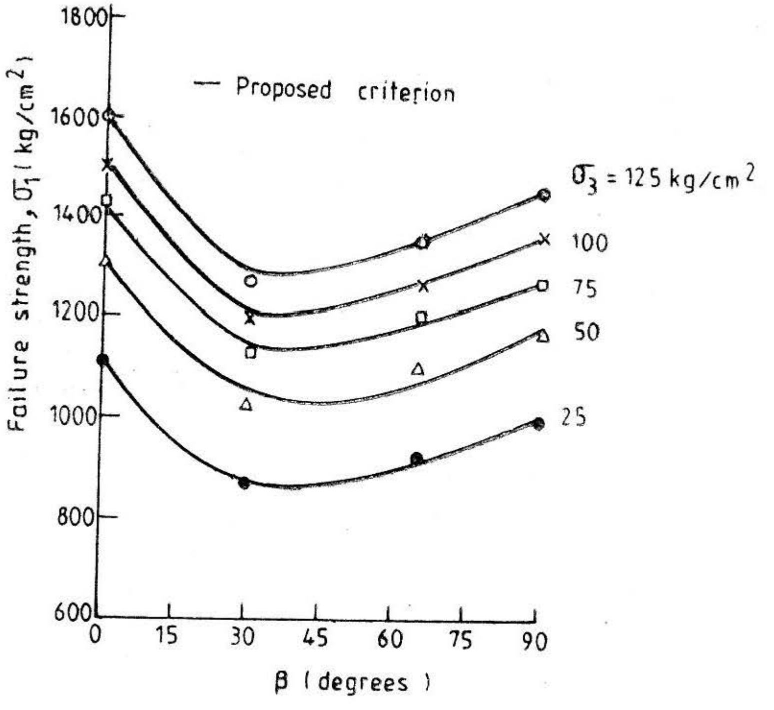

FIGURE 5 Comparison Between Predicted and Observed Strength for Kota Sandstone

TABLE 4

Evaluation of Parameters for Shale and Slates

Rock \ Source	σ_c kg/cm²	σ_3 kg/cm²	Values of Constants		
			A_0	$-A_1$	A_2
Martinsburg Slate (Donath, (1964)	*1696.17*	*38.28*	*33.19*	*1.65*	0.03
		114.83	14.49	1.93	0.10
		382.76	7.27	2.97	0.48
		546.80	6.56	3.57	0.82
		1093.60	4.92	4.68	2.15
Green River Shale (Chenevert and Gatlin, 1965)	1934.35	210.90	16.66	1.12	0.08
		421.80	9.79	1.37	0.21
		632.70	7.94	1.21	0.25
		843.60	6.53	1.16	0.37
Penrhyn Slate (Attewell and Sandford, 1974)	1690.41	140.66	14.86	1.72	0.09
		281.36	10.43	2.22	0.25
		422.04	8.16	2.64	0.43
		652.72	6.76	2.76	0.67
		703.40	7.00	3.86	1.01

1964), Texas slate, Green river shale—1 and Green river shale—II (McLamore and Gray, 1967) Barnsley hard coal (Pomeray et al. 1970), and fractured sandstone (Attewell and Sandford, 1974) have been analysed. To conserve space, only the values for Martinsburg slate, Green river shale and Penrhyn slate are given in Table 4. The predicted strength values with actual experimental results are shown in Figs. 6 to 8. All these results show the good agreement with experimental values.

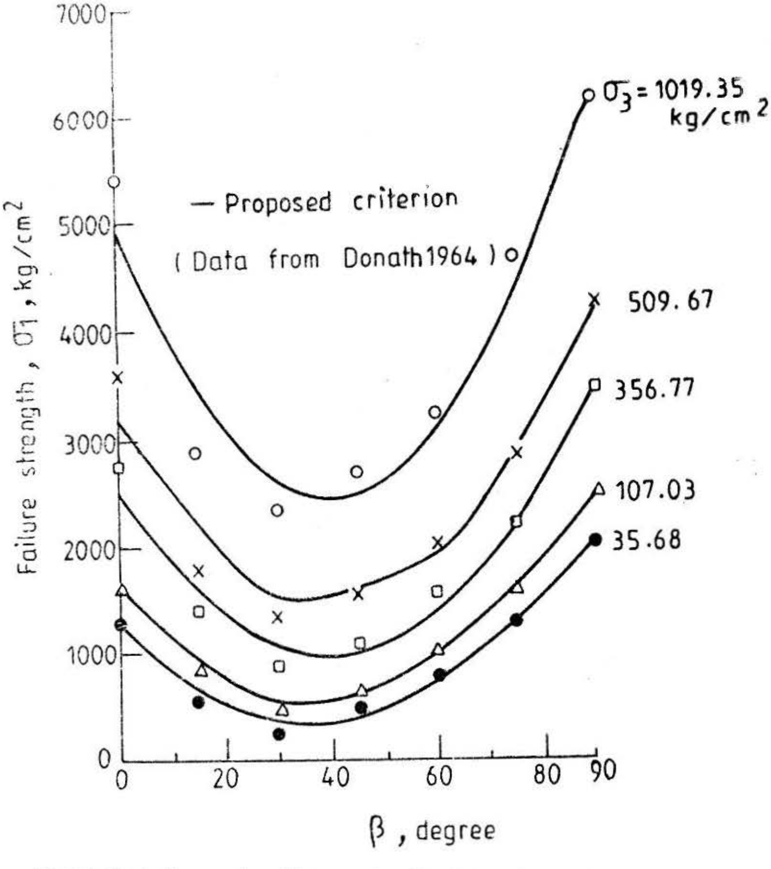

FIGURE 6 Comparison Between Predicted and Observed Strength for Martinsburg Slate

The values for the three constants are valid for all the orientations of weak plane which vary with σ_3. Further analysis has been carried out to establish relationship between the constants and the ratio of σ_c/σ_3. The variation of A_0, A_1 and A_2 are plottted against σ_c/σ_3, on log-log scale in Figs. 9, 10 and 11 respectively. Interestingly all the three constants show linear variation with σ_c/σ_3 ratio for all the rocks. Thus it is possible to extrapolate these curves to the required confining pressure. From this, one can predict the failure strengths of anisotropic rocks at higher confining pressures and also for different orientations by conducting triaxial tests at low confining pressures.

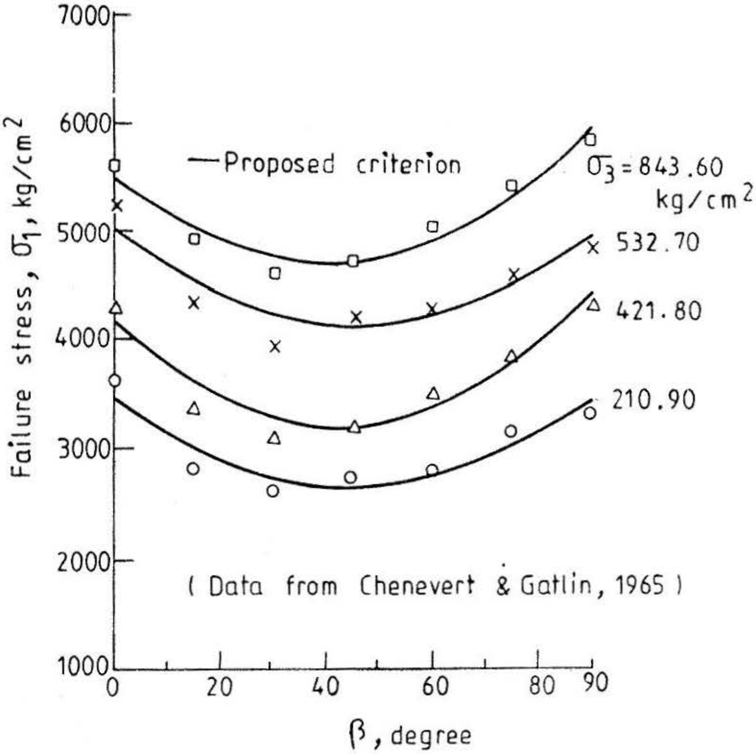

FIGURE 7. Comparison Between Predicted and Observed Strength for Green River Shale

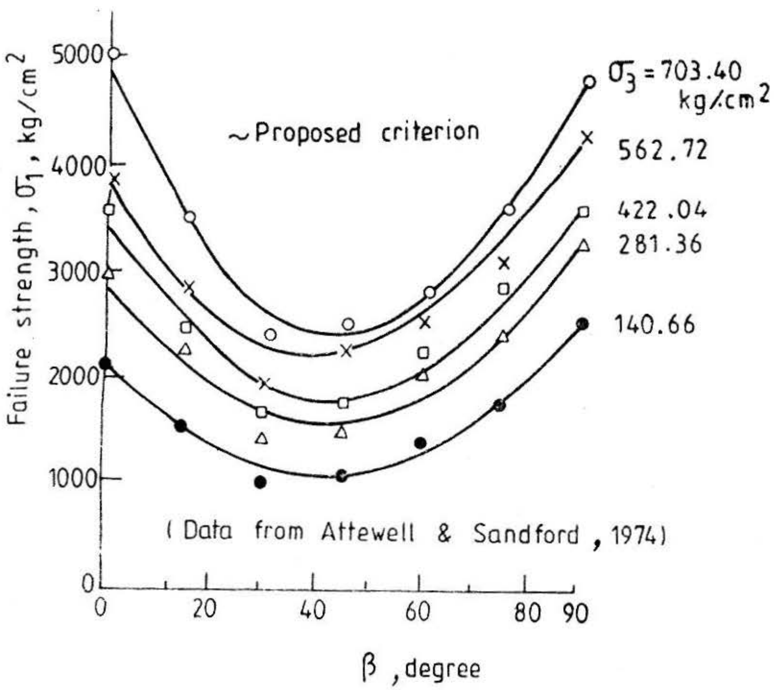

FIGURE 8 Comparison Between Predicted and Observed Strength for

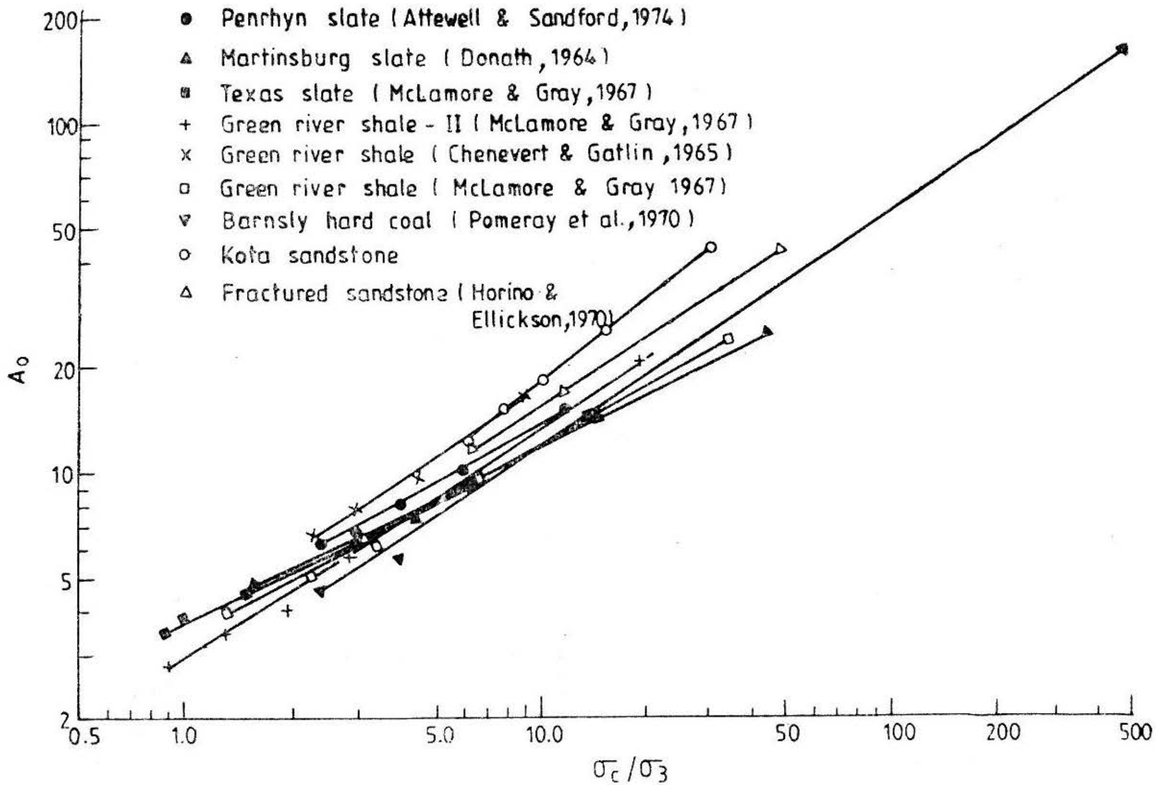

FIGURE 9 Variation of A_0 with σ_c/σ_3

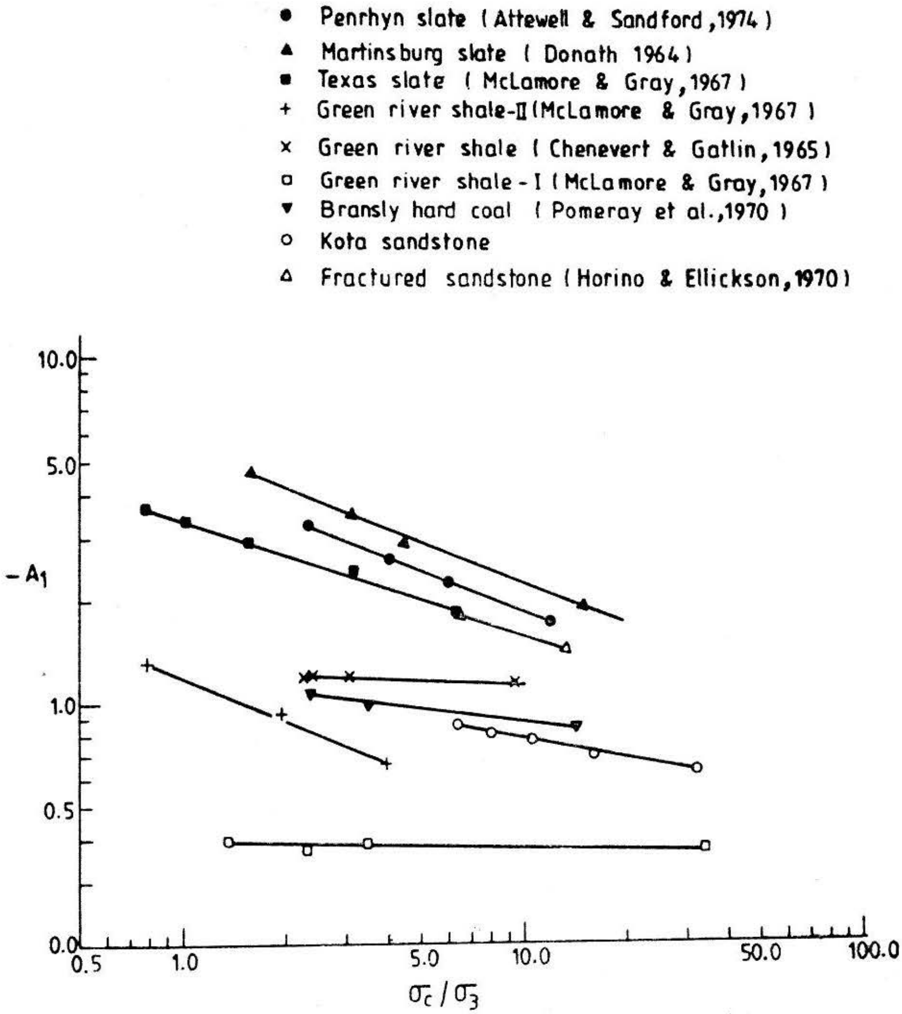

FIGURE 10 Variation of A_1 with σ_c/σ_3

Conclusions

Extensive laboratory strength tests on anisotropic Kota sandstone obtained at different orientations revealed that

(*i*) the uniaxial compressive strength and the triaxial strength upto $\sigma_3 = 125$ kg/cm^2 is least at $\beta = 30$ degrees, in a manner similar to that observed in the case of other anisotropic rocks.

(*ii*) the Brazilian strength and the point load strength index exhibit the maxima at $\beta = 90$ degrees and the minima at $\beta = 0$ degrees.

(*iii*) the strength criterion proposed takes into account σ_c and σ_3. It has three constants. From triaxial test data these three

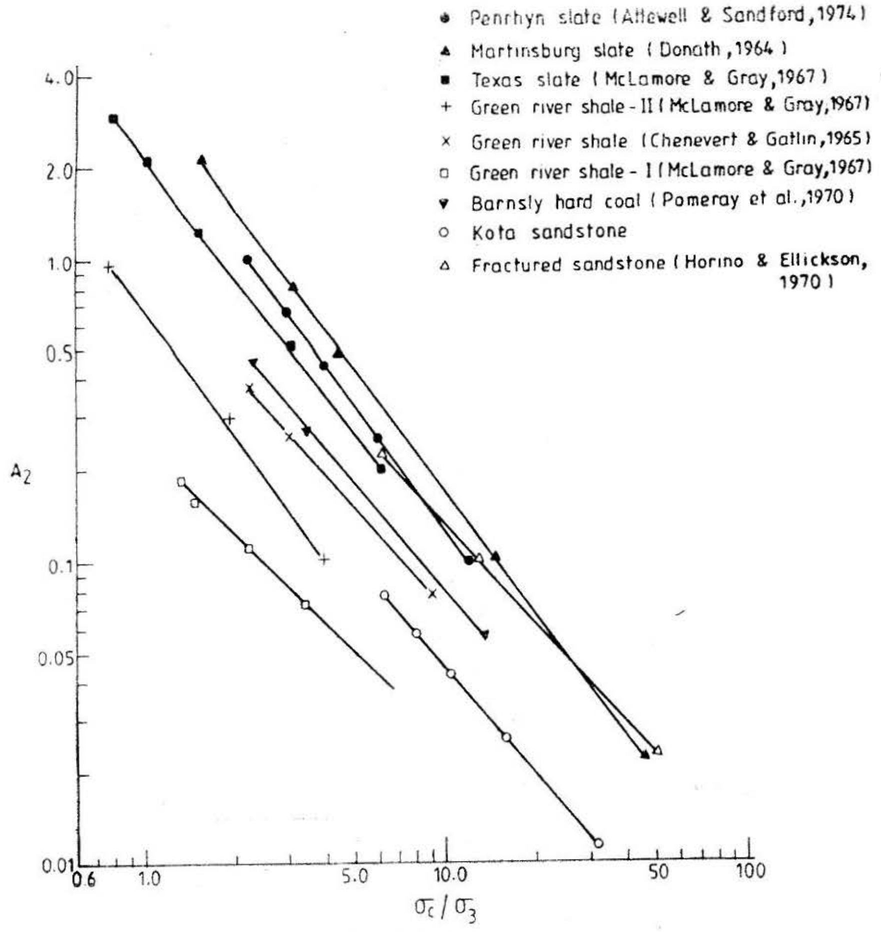

FIGURE 11 Variation of A_2 with σ_c/σ_3

constants can be obtained. The strength predictions from the proposed criterion are shown to be more accurate than any other criteria. Further, for predictions at high σ_3, a relation between the constants and σ_c/σ_3 has been presented. Thus, the criterion proposed is more reliable and accurate to predict the strength at high confining pressures and different orientations if one knows the strength variation at low σ_3.

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Notations

σ_1 = major principal stress axis

σ_3 = minor principal stress axis

β = orientation of weak/joint plane with reference to σ_1

σ_c = uniaxial compressive strength

$A_0 A_1 A_2$ = constants in the proposed strength criterion

m, s = constants in Hoek and Brown criterion

c = cohesion intercept

ϕ = coefficient of internal friction

η = anistropic factor

ϵ_t = Young's modulus

ν = Poission's ratio