# Axial Uplift Capacity of Inclined Piles 

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## Introduction

Large inclined loads act on the foundation of retaining walls, anchors for bulkheads, bridge abutments, piers and offshore structures. Inclined or batter piles can be economically used in these foundations, wherever they are required to carry large incline pulling loads. No significant information on the axial uplift capacity of inclined piles is reported in the available literature.

Meyerhof (1973) presented an analysis to determine the axial pull out resistance of batter piles. Extending the earlier theory of vertical uplift capacity of foundations (Meyerhof and Adams, 1968) to inclined piles under axial uplift load, Meyerhof (1973) presented an approximate expression for pull out resistance of piles in sand. For a pile of inclination, $i$, with vertical, and vertical depth of embedment, $D$, the pull out resistance $\boldsymbol{P}_{\boldsymbol{i}}$, is given as,

$$
\begin{equation*}
P_{i}=\left(\zeta_{0}^{\prime} K_{u i} \tan \delta\right) A_{s} \tag{1}
\end{equation*}
$$

where

$$
\begin{aligned}
A_{s} & =\text { embedded pile surface area } \\
& =\pi d, L \\
d & =\text { diameter of pile } \\
L & =\text { embedded length of pile } \\
\delta & =\text { pile friction angle } \\
K_{u i} & =\text { uplift coefficient for pile of inclination } i \\
\zeta_{0}^{\prime} & =\text { average effective overburden pressure } \\
& =\frac{\gamma \cdot D}{2}
\end{aligned}
$$

and

$$
\gamma=\text { effective unit weight of soil. }
$$

[^0]For different values of angle of shearing resistance, $\phi$, of soil and for piles with various inclinations, the estimated uplift coefficients were presented by Meyerhof (1973) as shown in Fig. 1. For a given value of angle of shearing resistance $\phi$, the uplift coefficients do not differ much for moderate pile inclinations $0-45^{\circ}$. The ratio of the uplift capacity of an inclined pile of inclination $i$, to the uplift capacity of a vertical pile having equal vertical depth of embedment, neglecting the value of $K_{u i}$ may be written from Eq. (1) as :

$$
\begin{equation*}
\frac{P_{i}}{P_{o}}=\sec i \tag{2}
\end{equation*}
$$

where

$$
P_{i}=\text { axial uplift capacity of an inclined pile }
$$

and

$$
P_{o}=\text { axial uplift capacity of a vertical pile. }
$$

While analysing the model experimental test results of axial uplift capacity of an inclined pile $i \leqslant 40^{\circ} ; \phi=25^{\circ}$ and $D=40 \mathrm{~cm}$, Tran-VoNhiem (1971) arrived at the relationship given by Eq. (2).

Adams and Klym (1972) reported an axial pull out test on an aluminium pile of diameter 10 cm and length 65.5 cm embedded in densely packed silica sand, with batter angle of $35^{\circ}$. Experimental results indicated no appreciable change in the value of $K_{u}$ for vertical and inclined pile.


FIGURE 1 Theoretical Uplift Coefficients for Bored Circular Piles (Meyerhof, 1973).

In the absence of sufficient data on uplift capacity of inclined piles of different surface characteristics and lengths, in a soil of known properties, it is difficult to ascertain properly the validity of any theory. In case of vertical piles, it had been shown that uplift coefficient depends on angle of shearing resistance of soil, pile friction angle and slenderness ratio of pile (Chaudhuri and Symons 1983, Chattopadhyay and Pise 1985 a).

## Scope of The Work

In this paper, a generalized theory to evaluate the axial uplift capacity of a rircular inclined pile embedded in sand is proposed. The ultimate axial uplift capacity of inclined pile has been related to that of a vertical pile by dimensionless ratios which have been presented through figures.

To substantiate the theoretical results, elaborate testing programme was undertaken. Results of axial uplift capacity of circular piles of inclination varying between $2-45^{\circ}$, in dense sand, with 3 different pile surface characteristics and 3 different pile lengths having slenderness ratio, $\lambda=L / d$, ranging from 11.44 to 39.18 are reported.

Using the proposed theory, the experimental results of Meyerhof (1973) as well as thosé of authors have been analysed. Comparison between Meyerhof's and authors' analysis has also been made.

## Theoretical Analysis

Figure 2 shows an inclined pile of diameter, $d$, embedded length, $L$, with vertical depth of embedment, $D$. The pile axis is inclined at an angle of $i$, to the vertical. The ultimate axial uplift resistance to uplift load is assumed to be mobilised when total frictional resistance due to normal pressure by the surrounding soil on the pile surface is overcome. For evaluating the resistance, an elemental length of pile, $\Delta 1$, at a height, $Z$, above the pile tip is considered.

On face $A A^{\prime} / C C^{\prime}$, vertical pressure $\triangle q$ due to soil is given by :

$$
\begin{equation*}
\Delta q=\gamma(L \cos i-Z)=\gamma \cos i(L-1) \tag{3}
\end{equation*}
$$

The lateral pressure acting on $A A^{\prime} / C C^{\prime}$ is

$$
K \triangle q=K_{\gamma} \cos i(L-1)
$$

where $K=$ coefficient of lateral earth pressure.
The normal component of the pressure acting on $A A^{\prime} / C C^{\prime}$

$$
\begin{aligned}
& =K \triangle q \cos i+\Delta q \sin i \\
& =\triangle q(K \cos i+\sin i)
\end{aligned}
$$

On face $B B^{\prime} / D D^{\prime}$, the normal component of the pressure acting

$$
=K \Delta q
$$

Hence unit uplift resistance on face $A A^{\prime} / C C^{\prime}$

$$
\begin{equation*}
=\Delta q \tan \delta(K \cos i+\sin i) \tag{4}
\end{equation*}
$$

and unit uplift resistance on face $B B^{\prime} / D D^{\prime}=K \triangle q \tan \delta$


FIGURE 2 (a) Schematic Diagram of an Inclined Pile
(b) Horizontal Elliptical Section of a Pile

Along the elliptical periphery of the horizontal section of the pile, unit resistance to pulling is symmetrical as seen from Eqs. (4) and (5) and the distribution is assumed to be elliptical as shown in Fig. 2 (c).


FIGURE 2 (c) Distribution of Pulling Resistance on the Elliptical Section of a Pile

## Evaluation of Total Resistance

From Fig. 2 (c), for the chosen axis system, the co-ordinates of any point $P$, which lie on the elliptical section of the pile, may be described from the properties of the ellipse as :

$$
\begin{align*}
& r=\frac{d}{2}\left[\frac{1+\cos ^{2} i \tan ^{2} \phi_{1}}{\cos ^{2} i+\cos ^{2} i \tan ^{2} \phi_{1}}\right]^{1 / 2}  \tag{6a}\\
& \phi=\tan ^{-1}\left[\cos i \tan \phi_{1}\right] \tag{6b}
\end{align*}
$$

where $\phi_{1}$ is the eccentric angle for the point.
From Eqs. ( $6 a$ ) and ( $6 b$ )

$$
\begin{equation*}
r d \theta=\frac{d}{2} \frac{\sec \phi_{1} d \phi_{1}}{\left[1+\cos ^{2} i \tan ^{2} \phi_{1}\right]^{1 / 2}} \tag{7}
\end{equation*}
$$

On the elemental perimetric length $r d \theta$, in Fig. $2(c)$, on the elliptical section of the pile, unit resistance to pulling is $p$.

Since the unit resistance to pulling is assumed to be elliptical and is equal to $(K \cos i+\sin i) \Delta q \tan \delta$ at $C$ wherein $\phi_{1}=0$ from Eq. (4) and equal to $K \triangle q \tan \delta$ at $D$ wherein $\phi_{1}=90^{\circ}$ from Eq. (5), for point $P$, unit resistance can be expressed as :

$$
\begin{align*}
P & =\left[(K \cos i+\sin i)^{2} \cos ^{2} \phi_{1}+K^{2} \sin ^{2} \phi_{1}\right]^{1 / 2} \Delta q \tan \delta \\
& =\frac{K \Delta q \tan \delta}{\sec \phi_{1}}\left[\propto^{2}+\tan ^{2} \phi_{1}\right]^{\frac{1}{2}} \tag{8}
\end{align*}
$$

where

$$
\begin{equation*}
\propto=\frac{K \cos i+\sin i}{K}=\left(\cos i+\frac{\sin i}{K}\right) \tag{9}
\end{equation*}
$$

Thus resistance to pulling $\triangle P_{i}$, on the elemental surface area of the pile $r d \theta d l$, can be expressed from Eqs. (7) and (8),

$$
\begin{align*}
\Delta P_{i} & =p \cdot r d \theta d l \\
& =\frac{K d}{2 \cos i}\left[\frac{\propto^{2}+\tan ^{2} \phi_{1}}{\sec ^{2} i+\tan ^{2} \phi_{1}}\right]^{\frac{1}{2}} \triangle q \tan \delta d \phi_{1} d l \tag{10}
\end{align*}
$$

Substituting the value of $\Delta q$ from Eq. (3) in Eq. (10) and rearranging,

$$
\begin{equation*}
\triangle P_{i}=\frac{K d}{2}\left[\frac{\propto^{2}+\tan ^{2} \phi_{1}}{\sec ^{2} i+\tan ^{2} \phi_{1}}\right]^{\frac{1}{2}} \gamma(L-1) \tan \delta d \phi_{1} d l \tag{11}
\end{equation*}
$$

Integrating $\Delta P_{i}$ on the embedded surface of the pile, ultimate uplift resistance,

$$
P_{i}=\int_{0}^{L} \int_{0}^{2 \pi} \frac{K d}{2}\left[\frac{\propto^{2}+\tan ^{2} \phi_{1}}{\sec ^{2} i+\tan ^{2} \phi_{1}}\right]^{\frac{1}{2}} \gamma(L-1) \tan \delta d \phi_{1} d l
$$

$$
\begin{align*}
& =\gamma d K \tan \delta L^{2} \int_{0}^{\pi / 2}\left[\frac{\propto^{2}+\tan ^{2} \phi_{1}}{\sec ^{2} i+\tan ^{2} \phi_{1}}\right]^{2} d \phi_{1} \\
& =K \tan \delta \cdot \gamma d L^{2} I_{i} \tag{12}
\end{align*}
$$

where

$$
\begin{equation*}
I_{i}=\int_{0}^{\pi / 2}\left[\frac{\alpha^{2}+\tan ^{2} \phi_{1}}{\sec ^{2} i+\tan ^{2} \phi_{1}}\right]^{\frac{1}{2}} d \phi_{1} \tag{13}
\end{equation*}
$$

The above integral is evaluated numerically for different values of $K$ and $i$ and are plotted in Fig. 3.

It is of practical interest to know the variation of ultimate axial uplift capacity of inclined piles with change of inclination. Two cases have been analysed; one with constant length and variable inclination and the other with vertical depth of embedment, $D$, constant and variable inclination.

## Case : 1 Inclined Piles of Equal Vertical Depth of Embedment

For such piles, $L=D \sec i$
Eq. (12) gives $P_{i}=K . \gamma . d . D^{2} \sec ^{2} i \cdot \tan \delta \cdot I_{i}$
For a vertical pile, $i=0$ and $I_{o}=\frac{\pi}{2}$ and

$$
\begin{align*}
P_{o} & =K \cdot \gamma \cdot d . D^{2} \tan \delta \cdot I_{o} \\
& =\frac{\pi \gamma d D^{2}}{2} K \tan \delta \tag{15}
\end{align*}
$$



FIGURE 3 Values of $I_{i}$

From Eqs. (14) and (15)

$$
\begin{equation*}
\frac{P_{i}}{P_{o}}=\sec ^{2} i \frac{2 I_{i}}{\pi}=\propto_{D} \tag{16}
\end{equation*}
$$

where
$\propto_{D}=$ ratio of the ultimate uplift capacity of the inclined pile to that of a vertical pile for identical value of $D$.

Theoretical Results
Values of $\propto_{D}$ are plotted in Fig. 4 for piles of equal vertical depth of embedment, against inclination angle. The effect of $K$ has also been shown in the same figure. It is observed that with increase in inclination of pile $\propto_{D}$, and in turn uplift capacity increases. Theoretically when the inclination approaches $90^{\circ}$ ultimate uplift capacity approaches to infinity as given by Eq. (16).

## Case 2 : Inclined Piles of Equal Lengths

For a vertical pile, $i=0$ and $D=L$. Assuming the coefficient of earth pressure as $K$ for the pile, Eq. (15) yields

$$
\begin{equation*}
P_{o}=\frac{1}{2} K \tan \delta . \pi d \gamma L^{2} \tag{17}
\end{equation*}
$$



For a pile with inclination $i, D=L \cos i$. For a vertical pile of length $L \cos i$, uplift capacity $P_{i v}$ is given by Eq. (17) as :

$$
\begin{equation*}
P_{i v}=\frac{1}{2} K_{i \nu} \tan \delta . \pi d \gamma L^{2} \cos ^{2} i \tag{18}
\end{equation*}
$$

where $K_{i v}=$ the coefficient of earth pressure for vertical pile of length, $L \cos i$.

It has been shown by experimental test results on vertical piles over wide range of slenderness ratios that the value of $K$ is not constant but depends on slenderness ratio, pile friction angle and angle of shearing resistance (Chaudhuri and Symons 1983). A theoretical method to evaluate uplift capacity of vertical pile, incorporating the effects of above factors are presented elsewhere (Chattopadhyay and Pise $1985 a$ and $b$ ). To evaluate the uplift capacities of two vertical piles of different lengths of embedment $L$ and $L$ ©os $i$, two different values of $K$, namely $K$ and $K_{i v}$ in Eqs. (17) and (18) are used.

From Eqs. (17) and (18),

$$
\begin{equation*}
\frac{P_{o}}{P_{i v}}=\frac{K}{K_{i v} \cos ^{2} i} \tag{19}
\end{equation*}
$$

But from Eq. (16),

$$
\begin{equation*}
\frac{P_{i}}{P_{i \nu}}=\sec ^{2} i \frac{2 I_{i \nu}}{\pi} \tag{20}
\end{equation*}
$$

where $I_{i v}$ is the integral given by Eq. 13 for a value of $K_{i v}$.
From Eqs. (19) and (20)

$$
\begin{align*}
& \quad \frac{P_{i}}{P_{o}}=\frac{P_{i}}{P_{i v}} \cdot \frac{P_{i v}}{P_{o}}==\sec ^{2} i \frac{2 I_{i v}}{\pi} \frac{K_{i v} \cos ^{2} i}{K} \\
& \text { i.e. } \frac{P_{i}}{P_{o}}=\frac{2 I_{i v}}{\pi} \frac{K_{i v}}{K}=\propto_{L} \tag{21}
\end{align*}
$$

where $\propto_{L}=$ ratio of ultimate uplift capacity of an inclined pile to that of a vertical pile with same value of $L$.

Use of Eq. (21) to analyse the model test results of the authors has been demonstrated later.

## Experimental Set-up and Programme of Tests

## (a) Model Piles

Aluminium alloy tubes 19 mm outside diameter and 0.88 mm wall thickness were used as model smooth piles. Smooth surfaced pile was coated with adhesive and sand to simulate rough pile. Coarse Ennore sand was applied to make rough piles while brown fine local sand was used to simulate medium rough pile. The adhesive and sand coating increased the pile diameter to 21.4 mm and 20.5 mm respectively. For each type of piles, 3 lengths of embedments 246,496 and 744 mm were used.

## (b) Test Medium

Dry Ennore sand, having $D_{60}=0.7 \mathrm{~mm}$, and $D_{10}=0.62 \mathrm{~mm}$ of uniformity coefficient 1.1 and angle of shearing resistance $\phi=41^{\circ}$, was used as a foundation medium in the tank. Maximum, minimum and test dry density of sand were $1.667,1.395$ and $1.606 \mathrm{~g} / \mathrm{cc}$. Angles of friction $\delta$ between smooth, medium, rough and rough surface of piles and foundation medium were 15,34 and 37 respectively.

## (c) Procedure

All the axial uplift tests were conducted in the laboratory on model piles embedded in sand, in a steel tank of size $91.4 \times 76.2 \mathrm{~cm}, 91.4 \mathrm{~cm}$ deep. The pile was kept in position at the desired inclination by means of a set of longitudinal split pile caps having holes of different inclination. The pile cap with pile in position was supported on flat surface running across the width of the tank. The axial inclination of the pile was ensured by checking the horizontality of the pile cap. A standardised rainfall technique of sand pouring (Pise 1969, Pal 1983) was adopted to get reproducible sand density. Sand pouring was continued till 2 cm of pile were exposed for insertion of a ball and socket arrangement with wire rope at the top of the pile. Loadings were applied on the pile top at desired angle by a wire, one end of which was attached to the pile top through ball and socket arrangement and the other to a loading pan which accepted dead weights. The wire was taken first over an inverted adjustable pulley with frictionless ball bearing and then over the second fixed pulley. Normal and axial displacement of the pile top were measured with dial gauges of sensitivity of 0.002 mm .

## Experimental Results

Axial movements of the top of the piles were measured against the applied pull for different lengths and surface characteristics of piles. The uplift capacity of the pile was taken as the load at which the pile was pulled out. Axial uplift load versus axial movement of piles for typical cases have been presented through Figs. 5 to 7. From Figs. 5 to 7 it is observed that for rough and medium rough piles, axial movement of 2-3 mm is required to mobilise the ultimate uplift resistance. Since the pile friction angles for these surfaces are $37^{\circ}$ and $34^{\circ}$, which are comparable, the amount of axial movement is also nearly same in both the cases. Further it is seen to be independent of the inclination of pile. But for smooth surfaced piles ( $\delta=15^{\circ}$ ), large axial movement 7 to 15 mm (Fig. 7) is necessary to mobilise the full pulling resistance. However, plots are shown only upto limited axial movement. Other piles also showed qualitatively similar behaviour.

The uplift capacity of piles for different lengths and surface characteristics have been plotted against the inclination of the pile in Figs. $8(a)-10(b)$. From these figures, it is observed that with increase of inclination, $i$, uplift capacity of a pile increases gradually to a peak value and then decreases. Figures $8(b)-10(b)$ show the variation of the ratio $\alpha_{L}$ with $i$ for all piles. For rough piles (Fig. 8b), it is observed that there is significant increase of $\propto_{L}$ from 1.00 at $i=0$, to 1.20 at $i$ lying between $15-22 \frac{1}{2}^{\circ}$ for $\lambda=23.07$ and 34.7. But for $\lambda=11.44$, there is practically no change in uplift capacity upto $i=22 \frac{1}{2}^{\circ}$ and thereafter it


FIGURE 5 Uplift Load Versus Axial Movement of Pile, Rough Piles ( $L=49.6 \mathrm{~cm}$, $\delta=37^{\circ}$ )


FIGURE 6 Uplift Load Versus Axial Movement of Pile, Medium Rough Piles ( $L=49.6$ $\mathrm{cm}, \delta=34^{\circ}$ )
reduces with increase of $i$. Similar conclusions are drawn for medium rough and smooth piles (Fig. $9 b$ and Fig. 10 b ). The maximum uplift capacity occurs when $i$ lies between $15^{\circ}$ to $22 \frac{1}{2}^{\circ}$ and it depends on the slenderness ratio of the pile and pile friction angle.


FIGURE 7 Uplift Load Versus Axial Movement of Pile, Smooth Piles ( $L=49.6 \mathrm{~cm}$, $\delta=15^{\circ}$ )


FIGURE 8 Axial Uplift Capacity of Inclined Piles (Rough Piles, $\delta=37^{\circ}$ )


FIGURE 9 Axial Uplift Capacity of Inclined Piles (Medium Rough Piles, $\delta=34^{\circ}$ )

## Comparison of Experimental Results With Theoretical Predictions

Case 1: Variation of axial uplift capacity of inclined piles having equal value of $D$

Meyerhof (1973) reported uplift tests on model inclined rough piles in sand. Diameter of pile $=1.27 \mathrm{~cm}$ and $D=30.48 \mathrm{~cm}$. Angle of shearing resistance of sand used $=43^{\circ}$. Reported uplift capacity for inclined piles and the experimental values of $\propto_{D}$ are given below:

$$
\begin{aligned}
& i=0^{\circ}, P_{i}=35.452 \mathrm{~kg} \propto_{D}=1.000 \\
& i=15^{\circ}, P_{i}=49.240 \mathrm{~kg} \propto_{D}=1.389 \\
& i=30^{\circ}, P_{i}=59.080 \mathrm{~kg} \propto_{D}=1.666 \\
& i=45^{\circ}, P_{i}=70.900 \mathrm{~kg} \propto_{D}=1.999
\end{aligned}
$$

Prediction by Authors' Theory
Assuming $\gamma=1.65 \mathrm{~g} / \mathrm{cc}$, and $\delta=42^{\circ}$,
From Eq. (15), $P_{o}=\frac{1}{2} K \tan \delta \pi d \gamma D^{2}$

$$
=\frac{1}{2} K \tan 42 \cdot 3.14 \cdot 1.27 \cdot 1.65(30.48)^{2}
$$

But

$$
P_{o}=35.452 \mathrm{~kg}
$$

Therefore $\quad K=12.88$
Corresponding to $K=12.88$, values of $\propto_{D}$ for piles having $i=0^{\circ}$, $15^{\circ}, 30^{\circ}$ and $45^{\circ}$ are taken from Fig. 4, and the corresponding values are $1.000,1.048,1.185$ and 1.5000 respectively and the estimated values of uplift capacities are $35.452,37.153,42.010$ and 53.178 kg respectively.

## Prediction by Meyerhofs' Theory

According to Meyerhof's analyses, from Eq. (1),

(a) Variation of $P_{i}$ with i

(b) Variation of $\alpha_{L}$ with i

FIGURE 10 Axial Uplift Capacity of Inclined Piles (Smooth Piles, $\delta=15^{\circ}$ )

$$
\begin{aligned}
P_{i} & =\left[\frac{\gamma D}{2} K_{u_{i}} \tan \delta\right] \mathrm{A}_{t} \\
& =\frac{1}{2} K_{u_{i}} \tan \delta \cdot \pi d \gamma D^{2} \sec i
\end{aligned}
$$

From Fig. 1, values of $K_{u_{i}}$ are read for different values of $i$ and $\phi=43^{\circ}$, and are used to evaluate values of $P_{i}$ and $\propto_{\boldsymbol{D}}$. For piles having inclination 15, 30 and 45 , the corresponding values of $\propto_{D}$ are $0.985,1.0458$ and 1.212 respectively.

## Results

Values of $\propto_{D}$ obtained from experimental results and estimated from authors' and Meyerhof's analysis are shown in Fig. 11. It is observed that general trend of increase of axial uplift capacity of piles with increase in inclination as shown by experimental results, is predicted by both the theories. However, the values predicted by authors' aralysis are much closer to the experimental values than those estimated by Meyerhof's analysis.

Case 2: Variation of axial uplift capacity of inclined piles having equal lengths

The experimental values of $\propto_{L}$ for piles tested by the authors are shown in Figs. 8 to 10.


FIGURE 11 Comparison of $\propto_{D^{-}}$-Values

Authors' Analysis
From Eq. (21) $\propto_{L}=\frac{P_{i}}{P_{o}}=2 \frac{I_{i v}}{\pi} \cdot \frac{K_{i v}}{K}$
From authors' expression (1985a), the net ultimate uplift capacity of vertical pile, $P_{u n}$ is given as

$$
\begin{equation*}
P_{u n}=A_{1} \cdot \pi d \gamma L^{2} \tag{22}
\end{equation*}
$$

where $A_{1}=$ net uplift capacity factor $=\frac{1}{2} K \tan \delta$
Chattopadhyay and Pise (1985a) have found that the coefficient $K$ for a vertical pile depends on $\phi-\delta$ combination as well as $L / d$ ratio. The results for various conditions have been presented elsewhere (Chattopadhyay and Pise, 1985 b).

The results for $\phi=41^{\circ}, \delta=15,34$ and $37^{\circ}$ and different $L / d$ ratios have been reproduced in Fig, 12.


FIGURE 12 Theoretical Net Uplift Capacity Factor $A_{3}$ Versus Slenderderness Ratio $\lambda .\left(\phi=41^{\circ}\right)$

For rough piles of length 74.6 cm , and dia 2.15 cm ;

$$
\begin{aligned}
& \lambda=L / d=34.7, \text { from Fig. } 12 \text { corresponding to } \delta=37^{\circ} \text { and } \\
& \lambda=34.7, A_{1}=1.10977 .
\end{aligned}
$$

From Eq. (23), $K=2 A_{1} / \tan \delta=2.9452$.
For $i=15^{\circ}, D=L \cos 15$. For a vertical pile of length $L \cos 15$, slenderness ratio $=L \cos 15 / d=33.51$ and corresponding value of $A_{1}=1.15$. From Eq. (23),

$$
K_{i r}=\frac{2 \times 1.1500}{\tan 37}=3.05202
$$

From Fig. 3, corresponding to $K_{i v}=3.05202, \frac{I_{i}}{\pi}=0.505$. Therefore from Eq. (21) for $i=15, \propto_{L}=1.047$. Values of $\propto_{L}$ for all piles have been evaluated as discussed above are shown in Fig. 13.

## Meyerhof's Analysis

From Eq. (1), $P_{i}=\left(\rho_{o}^{\prime} K_{u i} \tan \delta\right) A_{s}$

$$
=\frac{1}{2} K_{u_{i}} \tan \delta \pi d \gamma L^{2} \cos i
$$

and

$$
P_{o}=\frac{1}{2} K_{u o} \tan \delta \pi d \gamma L^{2}
$$

Thus

$$
\begin{equation*}
\propto_{L}=\frac{P_{i}}{P_{o}}=\frac{K_{u_{i}} \cos i}{K_{u o}} \tag{24}
\end{equation*}
$$

From Fig. 1, for $\phi=41^{\circ}$, corresponding to $i=0^{\circ}, 15^{\circ}, 30^{\circ}$ and $45^{\circ}$, the values of uplift coefficients are $2.8,2.7,2.6$ and 2.5 respectively. From Eq. (24), the values of $\propto_{L}$ for $i=0^{\circ}, 15^{\circ}, 30^{\circ}$ and $45^{\circ}$ are $1.00,0.9314$, 0.6313 respectively.

## Results

The estimated values of $\propto_{L}$ using authors' and Meyerhof's analysis, are shown in Fig. 13 along with experimental results. It is seen that the experimental uplift capacity of inclined pile increases upto a certain inclination of the pile and thereafter it decreases. Maximum value occurs when $i$ lies between $15-22 \frac{1}{2}^{\circ}$ and it depends on pile friction angle and slenderness ratio of the pile.

The estimated values of uplift capacities of piles using Meyerhof's analysis decrease with increase in inclination. Further it is independent of pile friction angle and slenderness ratio, and are very much conservative. Whereas the authors analysis predicts values, which in general, follow the experimental trend. It also predicts the optimum value of inclination at which the uplift capacity attains a maximum value. The estimated values of uplift capacities are less than the experimental values, but they are much closer to them than those predicted by Meyerhof's analysis.


## Conclusions

A theoretical analysis for predicting the axial uplift capacity of inclined piles, embedded in sand, has been proposed.

For inclined piles having equal vertical depth of embedment, theoretical values of uplift capacity showed better agreement to the reported experimental results (Meyerhof 1973) than those predicted by Meyerhof's analysis (1973).

Experimental results on uplift capacity of inclined piles of same length, conducted by authors showed that maximum value of uplift capacity occurs at inclination of pile lying between $15-22 \frac{1}{2}^{\circ}$. The proposed theory predicts values which in general follow qualitatively the experimental trend. Further, it predicts optimum value of inclination at which uplift capacity attains a maximum value. They also depend on pile friction angle and slenderness ratio. Contrary to the above observations, Meyerhof's analysis predicts values which decrease with increase in inclination of a pile. Further the values are very much conservative and independent of pile friction angle and slenderness ratio.

## Notations

$A_{1} \quad=$ net uplift capacity factor
$A_{s}=$ embedded pile surface area
$D \quad=$ vertical depth of embedment of pile
$K \quad=$ coefficient of lateral earth pressure
$K_{u i}=$ uplift coefficient
$L \quad=$ embedded length of pile
$P_{i} \quad=$ pullout resistance of an inclined pile
$d=$ diameter of pile
$i \quad=$ angle of inclination of pile axis with vertical
$\propto_{D}=$ ratio of uplift capacity of an inclined pile to that of a vertical pile for same value of $D$
$\propto_{L}=$ ratio of ultimate uplift capacity of an inclined pile to that of a vertical pile with same value of $L$
$\delta=$ pile friction angle
$\gamma \quad=$ effective unit weight of soil
$\rho_{0}^{\prime}=$ average effective overburden pressure
$\phi \quad=$ angle of shearing resistance of soil
$\phi_{1}=$ eccentric angle
$\lambda \quad=\mathrm{L} / \mathrm{d}=$ slenderness ratio

## References

ADAMS, J.I. and KLYM, T.W. (1972) : "A study of Anchorages for Transmission Tower Foundations". Canadian Geotechnical Journal, 9: 89 CHATTOPADHYAY, B.C. and PISE, P.J. (1985 a) ; "Uplift Capacity of Piles
in Sand'". Communicated for Publication in the Journal of Geotech. Engg. Div. ASCE.
CHATTOPADHYAY, B.C. and PISE, P.J. (1985b ) "Design Charts for Uplift Capacity of Piles in Sand". Proc. Indian Geotechnical Conference-1985 Roorkee. Vol. I, 243-248.
CHOUDHURI, K.P.R., and SYMONS, M.V. (1983): "Uplift Resistance of Model Single Piles". Proc. Conf. on Geoteahnical Practice in offshore Engineering ASCE, Geotechnical Engg. Div. TEXAS 335-355.
MEYERHOF, G.G. (1973) ; "Uplift Resistance of Inclined Anchors and Piles". Proc. 8th International Conference on Soil Mechanics and Foundation Engineering, MOSCOW, 2.1:167-172.
MEYERHOF, G.G. and ADAEMS, J.I. (1968): "The Ultimate Uplift Capacity of Foundations''. Canadian Geotechnical Journal, $5: 225-244$.
PAL, M. (1983): "Analysis and Behaviour of Laterally Loaded Batter Piles and Pile Bents". Ph. D. Thesis, Indian Institute of Technology, Kharagpur.
PISE, P.J. (196"): "Behaviour of Pile Groups under Vertical and Lateral Loads" Ph. D. Thesis, Indian Institute of Technology, Kharagpur.
TRAN-VO-NHIEM, (1971): "Ultimate Uplift Capacity of Anchor Piles". Proc. 4th Budapest Conference on Soil Mechanics and Foundation Engineering : 829-836.


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