

# Earth pressures Due To Inclined Surcharge Loads

by

N. Babu Shanker\*

## Introduction

Lateral earth pressures exerted on retaining structures due to self weight of backfill soils are calculated from the classical earth pressure theories of Rankine and Coulomb. Magnitudes of such pressures depend upon the relative movement of the wall with respect to the backfill. Earth pressures are also caused due to additional (surcharge) loads acting on the surface of the backfill. This paper discusses the lateral earth pressures on retaining walls due to inclined surcharge loads giving rise to plane strain conditions. Solutions are presented for the intensities of earth pressures, resultant lateral forces and moments.

## Analysis

Fig. 1 shows an inclined point load ( $Q$ ) acting at a point ( $O$ ) on the surface of the backfill soil. On the left of the load is a long retaining wall with a vertical back. Consider the influence of the point surcharge load at a point ( $O_1$ ) on the back of the wall. The spatial coordinates of point ( $O_1$ ) with respect to ( $O$ ) are  $x, y, z$ .  $x$  is the lateral distance of load away from the wall,  $y$  is the longitudinal distance of the point ( $O_1$ ) away from the load and  $z$  is the depth below the surface of backfill. The three orthogonal components of the load  $Q$  in the three spatial directions are  $Q_x$ ,  $Q_y$  and  $Q_z$ . These components give rise respectively to three horizontal earth

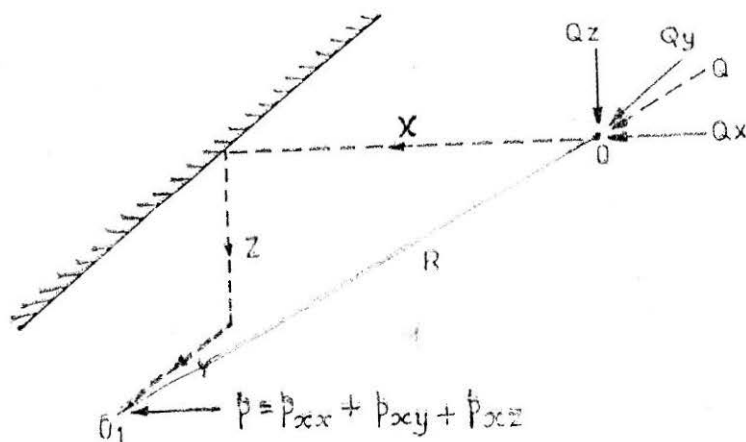


FIGURE. 1. General Inclined Point Surcharge Load.

\*Professor of Civil Engineering, Regional Engineering College, Warangal-506 004 (AP), India.

(This paper was received in November, 1985 and is open for discussion till the end of June, 1986)

pressure components perpendicular to the wall  $p_{xx}$ ,  $p_{xy}$ ,  $p_{xz}$ . Ignoring the presence of the wall and treating the problem as that of lateral stress distribution in a semi-infinite elastic medium, the following will be the expression for  $p_x$  from Boussinesq's theory (Scott, 1963).

$$\begin{aligned} p_{xx} &= \frac{Q_x \cdot x}{2\pi R^3} \left[ \frac{3x^2}{R^2} - (1-2\nu) + \frac{(1-2\nu) R^2}{(R+z)^2} \left\{ 3 - \frac{x^2 (3R+z)}{R^2 (R+z)} \right\} \right] \\ p_{xy} &= \frac{Q_y \cdot y}{2\pi R^3} \left[ \frac{3x^2}{R^2} - (1-2\nu) + \frac{(1-2\nu) R^2}{(R+z)^2} \left\{ 1 - \frac{x^2 (3R+z)}{R^2 (R+z)} \right\} \right] \\ p_{xz} &= \frac{Q_z}{2\pi R^2} \left[ \frac{3x^2 z}{R^3} - (1-2\nu) \left\{ \frac{z}{R} - \frac{R}{R+z} + \frac{x^2 (2R+z)}{R (R+z)^2} \right\} \right] \end{aligned} \quad \dots(1)$$

where  $R^2 = x^2 + y^2 + z^2$  and  $\nu$  is the Poisson's ratio of the soil.

If the load  $Q$  acts in the  $x$ - $z$  plane,  $Q_y = 0$ . For such a case, if the angle of obliquity from vertical is  $\theta$ , then

$$\begin{aligned} Q_x &= Q \sin \theta \\ Q_z &= Q \cos \theta \end{aligned} \quad \dots(2)$$

For the case of  $\nu = 0.5$  (incompressible soil), one obtains the total lateral earth pressure ( $p$ ) from Eqs. (1) and (2) as  $p = p_{xz} + p_{xx}$

$$p = \frac{3Qx^2z}{2\pi(x^2+y^2+z^2)^{5/2}} \left[ \cos \theta + \frac{x}{z} \sin \theta \right] \quad \dots(3)$$

The structure of this formula is similar to that given for a vertical component of stress for a similar problem by Ramaiah and Chickanagappa (1981) and hence their tabular solutions could be used to evaluate  $p$ . The second term of Eq. (3) is the contribution of horizontal load and first term that of vertical loads, which have been discussed in literature (Spangler, 1960; Babushanker and Ali, 1979). The basic equation (3) valid for point loads may be extended to the cases of line and area loads by suitable integration.

### Infinite Line Loads

For this case of plane strain loading (Fig. 2), one obtains from Eq. (3), the intensity of lateral earth pressure as

$$p = \frac{2q'x^2z}{\pi(x^2+z^2)^2} \left[ \cos \theta + \frac{x}{z} \sin \theta \right] \quad \dots(4)$$

The total lateral earth pressure (force) for the entire height ( $h$ ) of the retaining wall per unit length of wall is given by

$$\begin{aligned} p &= \int_0^h p \cdot dz \\ &= \frac{q'}{\pi} \left[ \cos \theta \left( \frac{h^2}{x^2+h^2} \right) + \sin \theta \left\{ \frac{xh}{x^2+h^2} + \tan^{-1} \left( \frac{h}{x} \right) \right\} \right] \end{aligned} \quad \dots(5)$$

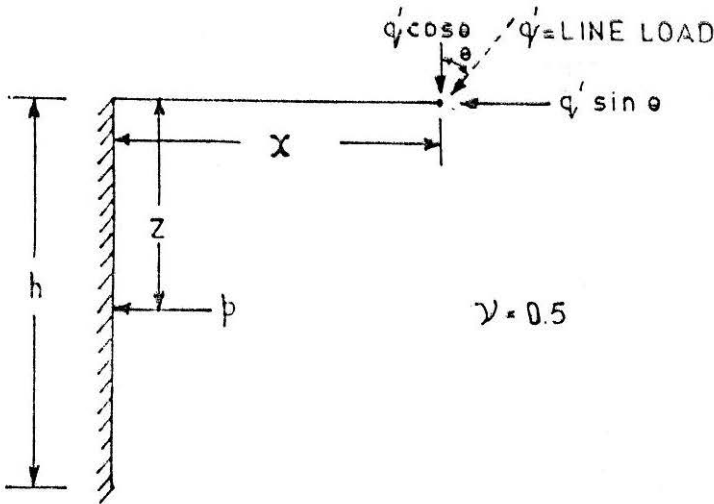


FIGURE 2 Infinite Line Load Inclined to Vertical

Moment of the force  $p$  about the base of the retaining wall is given by

$$\begin{aligned}
 M &= \int_0^h p (h-z) dz \\
 &= \frac{q'}{\pi} \left[ \cos \theta \left\{ h - x \tan^{-1} (h/x) \right\} + \sin \theta \left\{ h \cdot \tan^{-1} (h/x) \right\} \right] \dots(6)
 \end{aligned}$$

### Infinite Strip Loads

This type of plane strain loading (Fig. 3) is of greater practical utility. The intensity of lateral stress is obtained from Eq. (3) as :

$$\begin{aligned}
 p &= \frac{q}{\pi} \left[ \cos \theta \left\{ \tan^{-1} \left( \frac{x+b}{z} \right) - \tan^{-1} (x/z) - \frac{(x+b)z}{(x+b)^2 + z^2} \right. \right. \\
 &\quad \left. \left. + \frac{xz}{x^2 + z^2} \right\} + \sin \theta \left\{ \ln \frac{(x+b)^2 + z^2}{x^2 + z^2} \right. \right. \\
 &\quad \left. \left. + \frac{z^2}{(x+b)^2 + z^2} - \frac{z^2}{x^2 + z^2} \right\} \right] \dots(7)
 \end{aligned}$$

The stress distribution diagrams obtained from Eq. (7) are shown in Fig. 4 for various inclinations. As expected the pressures in general decrease with depth.

The total lateral earth pressure force is given as

$$\begin{aligned}
 P &= \frac{qh}{\pi} \left[ \cos \theta \left\{ \tan^{-1} \left( \frac{x+b}{x} \right) \tan^{-1} (x/h) \right\} + \sin \theta \left\{ \ln \frac{(x+b)^2 + h^2}{x^2 + h^2} \right. \right. \\
 &\quad \left. \left. + \frac{x+b}{h} \tan^{-1} \left( \frac{h}{x+b} \right) - x/h \tan^{-1} (x/h) \right\} \right] \dots(8)
 \end{aligned}$$

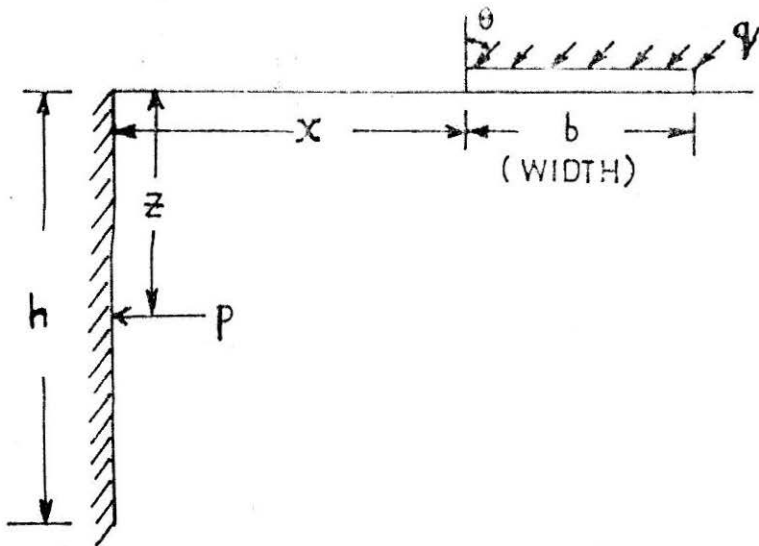


FIGURE 3. Inclined Surcharge Load Acting on an Infinite Strip

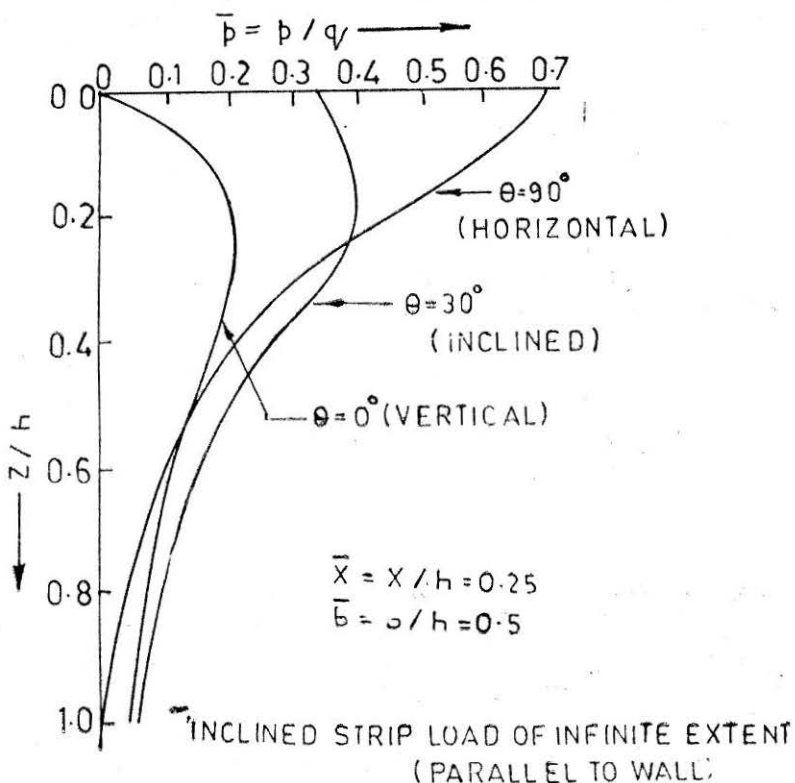


FIGURE 4. Effect of Depth and Angle of Inclination on Pressure Distribution

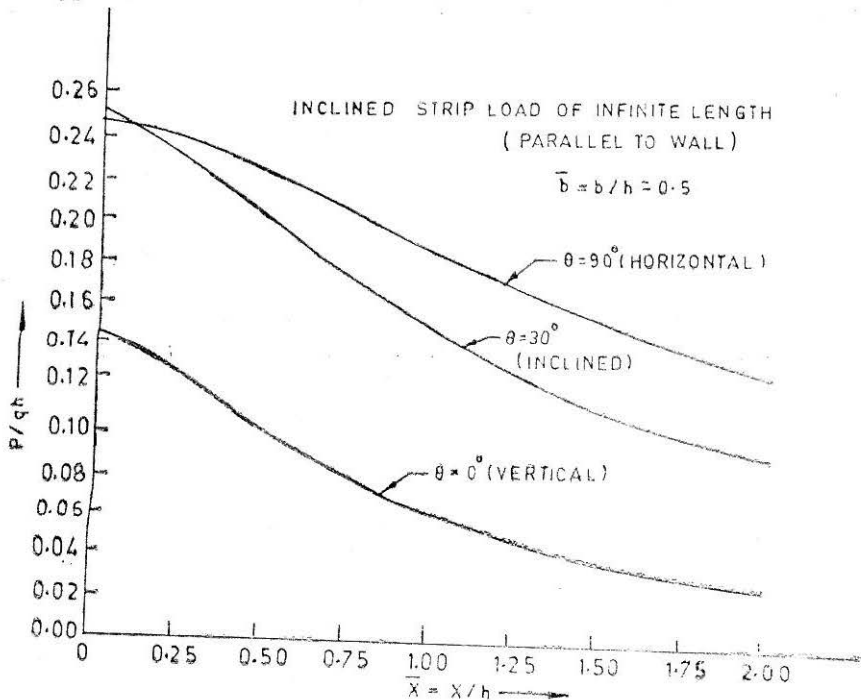
The earth pressures decrease with increasing edge distance ( $x$ ) but increase with increasing width ( $b$ ) of strip. (See Figs. 5 and 6). The lateral moment is given by :

$$M = \frac{q}{2\pi} \left[ \cos \theta \left\{ hb + (x^2 + h^2) \tan^{-1} \left( \frac{h}{x} \right) - (h^2 + (x+b)^2) \tan^{-1} \left( \frac{h}{x+b} \right) \right\} + \sin \theta \left\{ h^2 \ln \frac{(x+b)^2 + h^2}{x^2 + h^2} + 2(x+b)h \tan^{-1} \left( \frac{h}{x+b} \right) - 2xh \tan^{-1} \left( \frac{h}{x} \right) \right\} \right] \dots(9)$$

The factors  $x$  and  $b$  have similar influence on  $M$  as that of  $P$ .

**Discussion**

The earth pressures obtained from Boussinesq's theory are strictly valid for semi-infinite soil medium. But backfill may be considered as quarter-infinite medium bound by a rigid retaining wall on one side. Field experiments have proved that the actual pressures mobilised may be nearly twice the theoretical values (Terzaghi, 1954). Moreover the theoretical results presented in this paper may correspond to at rest conditions and hence necessary corrections may have to be applied for active and passive pressure conditions.



**FIGURE 5** Variation of Total Pressure with Edge Distance and Angle of Inclination

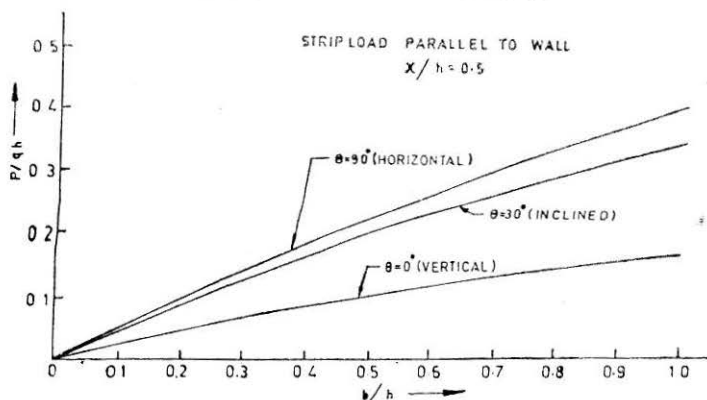


FIGURE 6. Variation of Total Earth Pressure with Width of Strip

### Conclusions

Lateral earth pressures caused due to inclined surcharge loads leading to plane strain loading conditions are discussed in this paper. Theoretical expressions are derived for intensity of earth pressure, total earth pressure force and lateral (base) moments for infinite line loads and strip loads. The previous results reported in the literature for the case of vertical loads may simply be obtained by substituting  $\theta = 0$  in the present expressions. Width of loaded strip, lateral distance of load from the retaining wall and angle of obliquity (inclination) are the important variables of the problem discussed.

### References

- BABUSHANKER, N. and MOHAMMED ALLI, (1979): "Earth Pressure due to plane strain surcharge Loads". *Indian Geotech. Jour.* 9: 4: 392-397.
- RAMAIAH, B.K. and CHICKANAGAPPA, L.S. (1981): "*Soil Mechanics and Foundation Engg.*" Oxford & IBH Pub. Co., India.
- SCOTT, R.F. (1963): "*Principles of Soil Mechanics.*" Addison Wesley Pub. Co, London.
- SPANGLER, M.G. (1960): "*Soil Engineering*". Int. Test Book Co., USA.
- TERZAGHI, K. (1954): "Anchored Bulkheads." *Trans. of ASCE.* 119: 1243.