Sensitivity Study of Pile Head Impedance Functions For Offshore Structures

by

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Introduction

The dynamic response to random loading of a structure supported on flexible foundations may be significantly different from that of the structure supported on rigid base. The behaviour of the pile supported structure, is influenced by the interaction between the structure, the piles and the surrounding soils. For the dynamic analysis of the structure supported on piles it is, generally, convenient to replace the soil pile system by frequency dependent impedance functions at the structure pile interface. The real and imaginary part of these impedances represent in turn the stiffness and damping of the soil-pile system. These impedance functions are largely affected by the soil properties such as the dynamic shear modulus, the material damping and the Poisson's ratio. There are inherent uncertainties in the evaluation of these soil properties because of the heterogenity, the limited sampling where samples are often disturbed and error in testing procedure. In addition the effect of the spatial variation of shear modulus with depth and the soil-pile separation near the mudline could also be significant. The effect of these parameters on the pile-head impedance functions is investigated in the present study.

Soil-Pile Model

The idealization of soil-pile system is based on the lumped parameter model (Basu and Gupta, 1983). The steel pile is descretized into a number of finite elements, each segment is assumed to have constant properties of soil and pile. The cylinderical pile is taken as linearly elastic. The material damping of pile is neglegibly small in comparison to that of the surrounding soil medium and, therefore, neglected. The discreet model can account for : (*i*) The longitudinal variation of soil-pile properties, (*ii*) the material damping of soil, (*iii*) a variety of support conditions and (*iv*) soil-pile separation near mudline. The soil separation is treated by assuming zero soil resistance in the top segment of pile.

The soil-pile model is shown in Fig. 1 (*a*) for vertical vibration and Fig. 1 (*c*) for horizontal vibration. The dynamic stiffness (k) of the soil per unit length for the different vibratory motions of pile is written as

$$\text{Horizontal: } k_x = G\,[S_{x1}(a_o, \nu, D) + iS_{x2}(a_o, \nu, D)] \qquad \ldots(1)$$

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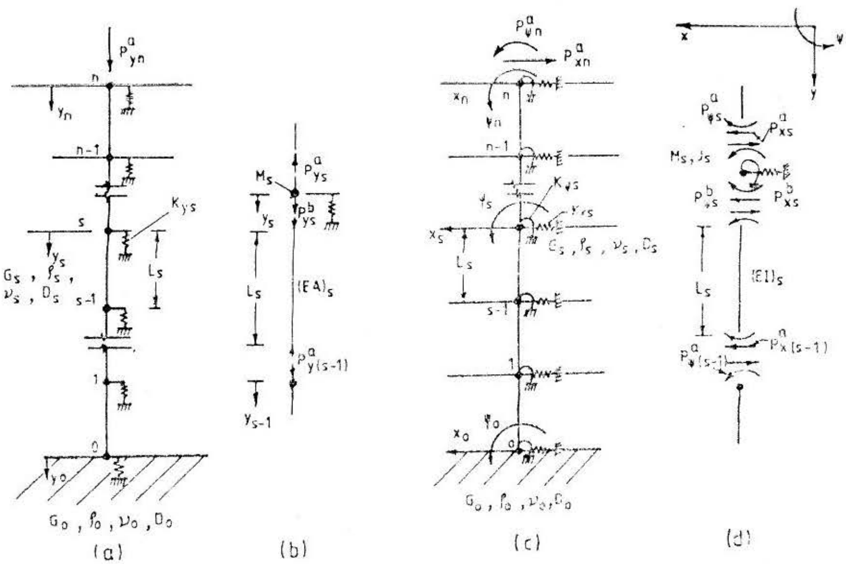

FIGURE : 1 Model of a Pile Subjected to Vertical and Horizontal Vibration

$$\text{Vertical} \quad : k_y = G\,[S_{y1}(a_o, D) + iS_{y2}(a_o, D)] \qquad \ldots(2)$$

$$\text{Rocking} \quad : k_\psi = G\,r^2\,[S_{\psi_1}(a_o, D) + iS_{\psi_2}(a_o, D)] \qquad \ldots(3)$$

where G is the dynamic shear modulus and D is the damping ratio of the soil; $i = \sqrt{-1}$; S_{j1}, S_{j2} ($j = x, y, \psi$) are respectively the non-dimensional stiffness and damping parameters in the direction of j and r is the outer radius of the pile. The non-dimensional frequency parameter, $a_o = \omega r \sqrt{\rho_s/G}$ where ρ_s is the submerged density of soil and ω is the frequency of vibration in radians per second. The parameters S_{j1} and S_{j2} obtained by Novak et al. (1978) based on plane strain assumption have been used in this study. The draw back of the plane strain assumption is that the stiffness in the horizontal and vertical direction diminish to zero. To take care of this the soil stiffness for low frequency range ($a_o \leqslant 0.3$) has been taken as constant and equal to dynamic stiffness calculated for $a_o = 0.3$ (Novak and Sheta, 1982).

The inertia property of pile and the dynamic stiffness property of soil associated with a segment are lumped at its both ends. The lumped mass. M_s and moment of inertia, J_s of pile at sth node (Fig. 1) can be written as:

$$M_s = (m_s L_s + m_{s+1} L_{s+1})/2 \qquad \ldots(4)$$

$$J_s = (m_s L^3 + m_{s+1} L^3_{s+1})/24 \qquad \ldots(5)$$

where m_s is the mass per unit length and L_s is the length of the pile segment. Similarly the lumped dynamic stiffness, K_{js} of soil at sth node is written as :

$$K_{js} = (k_{js} L_s + k_{j(s+1)} L_{s+1})/2 \qquad \ldots(6)$$

where j stands for x, y, ψ in turn for horizontal, vertical and rocking mode of vibration. K_{js} is the dynamic stiffness of the soil per unit length of pile in the *sth* layer and is obtained from Eqn. 1 to 3. The lumped mass and stiffness at the end nodes ($s = 0$ and $s=n$) of the pile are found by taking the non existing segments of lengths L_o and L_{n+1} as equal to zero.

Pile-Head Impedance

The lumped parameter soil-pile model is analysed by transfer matrix approach to obtain the pile-head forces (impedances) needed to produce a unit harmonic displacement of pile-head. The analysis is effectively carried out by means of matrix operation. The transfer matrix relates the forces and displacements, at a node to the corresponding forces and displacements at the adjacent node of the pile. The column vector representing forces and dislacements at a node is called the state vector which provide information regarding the force displacement state of the pile at each node.

Vertical Vibration

Referring to Fig. 1 (*b*), the force and displacement in vertical direction at a pile section just above *sth* node in terms of the corresponding force and displacement at $(s-1)th$ node are given as :

$$\begin{Bmatrix} P_y \\ y \end{Bmatrix}_s^a = \begin{bmatrix} 1+L\,(K_y-\omega^2\,M)/EA & (K_y-\omega^2\,M) \\ L/EA & 1 \end{bmatrix} \begin{Bmatrix} P_y \\ y \end{Bmatrix}_{s-1}^a \quad \ldots(7)$$

in which (2×2) matrix is the transfer matrix involving properties of sth segment and sth node. (EA) is axial rigidity and superscript 'a' denotes above a node. Denoting the transfer matrix by $[A_s]$, the force and displacement at the pile head can be expressed in terms of the state vector existing above the node at pile tip.

$$\begin{Bmatrix} P_y \\ y \end{Bmatrix}_n^a = [A_n]\,[A_{n-1}]\ldots\ldots\ldots\ldots\ldots[A_2]\,[A_1]\begin{Bmatrix} P_y \\ y \end{Bmatrix}_0^n \quad \ldots(8)$$

If the pile tip is supported on a rigid stratum, $y_o = y_o^a = 0$ and p_y^a can be taken as unity. In case of floating pile y_o^a is taken as unity and p_{yo}^a as $(K_{yo}-\omega^2 M)$. Eqn. 8 then gives p_{yn}^a and y_n^a both of which are complex valued. As the soil-pile system is assumed to be linear, the pile head impedance K_{yy} can be found from

$$K_{yy} = p_{yn}^a \,/\, y_n^a = k_{yy}+c_{yy} \quad \ldots(9)$$

in which the real and imaginary part represent pile head stiffness and damping.

Horizontal Vibration

Referring to Fig. 1 (*d*) the force and displacement above the node s given in terms of corresponding force and displacement at node $(s-1)$ as :

$$\begin{Bmatrix} P_x \\ P_\psi \\ x \\ \psi \end{Bmatrix}_s^a = [B_s]_{4\times 4} \begin{Bmatrix} P_x \\ P_\psi \\ x \\ \psi \end{Bmatrix}_{s-1}^a \quad ...(10)$$

The elements of the transfer matrix $[B_s]$ are given as :

$b_{11} = 1-L^3 (K_x-\omega^2 M)/6\,\mathrm{EI}$, $b_{12} = -L^2 (K_x-\omega^2 M)/2\mathrm{EI}$

$b_{13} = -(K_x-\omega^2 M)$, $b_{14} = -L(K_x-\omega^2 M)$

$b_{21} = L + L^2 (K_\psi - \omega^2 J)/2\mathrm{EI}$, $b_{22} = 1+L(K_\psi - \omega^2 J)/\mathrm{EI}$

$b_{23} = 0$, $b_{24} = (K_\psi - \omega^2 J)$, $b_{31} = L^3/6\mathrm{EI}$

$b_{32} = L^2/2\mathrm{EI}$, $b_{33} = 1$, $b_{34} = L$

$b_{41} = L^2/2\mathrm{EI}$, $b_{42} = L/\mathrm{EI}$, $b_{43} = 0$, $b_{44} = 1$

The successive operations on the transfer matrices associated with the segment $s = 1, 2, \ldots n$ provides the relation between state vectors at the pile head and the pile tip.

$$\begin{Bmatrix} P_x \\ P_\psi \\ x \\ \psi \end{Bmatrix}_n^a = [B_n]\,[B_{n-1}]\ldots\ldots\ldots\ldots[B_2]\,[B_1] \begin{Bmatrix} P_x \\ P_\psi \\ x \\ \psi \end{Bmatrix}_o^a \quad ...(11)$$

The nature of the horizontal vibration problem is such that two of the four forces/displacements at the bottom of pile tip i.e. below the node 0 are known from the specified support conditions. Each of the two remaining quantities are given unit value in turn while the other one is taken as zero, thus yielding two independent sets of forces/displacements at the pile head. If the pile tip is embedded in a rigid stratum; $x_0^a = \psi_0^a = 0$ and p_{xo}^a and p_ψ^a are to be given unit values in turn. In case of floating pile x_0^a and ψ_0^a are given unit values in turn and the corresponding values for p_{xo}^a and $p_{\psi o}^a$ are taken as $-(K_{xo}-\omega^2 M)$ and $-(K_{\psi o}-\omega^2 J)$ respectively.

Let the two sets of pile-head forces/displacement be denoted as $\left\{ p^{a}_{xn1} \; p^{a}_{\psi n1} \; x^{a}_{n1} \; \psi^{a}_{n1} \right\}$ and $\left\{ p^{a}_{xn2} \; p^{a}_{\psi n2} \; x^{a}_{n2} \; \psi^{a}_{n2} \right\}$. Then, from the definition of impedance matrix one has :

$$\begin{bmatrix} K_{xx} & K_{x\psi} \\ K_{\psi x} & K_{\psi\psi} \end{bmatrix} \begin{bmatrix} x_{n1} & x_{n2} \\ \psi_{n1} & \psi_{n2} \end{bmatrix} = \begin{bmatrix} P_{xn1} & P_{xn2} \\ P\psi_{n1} & P\psi_{n2} \end{bmatrix} \quad ...(12)$$

which yields the following relationship

$$\begin{bmatrix} K_{xx} & K_{x\psi} \\ K_{\psi x} & K_{\psi\psi} \end{bmatrix} = \begin{bmatrix} P_{xn1} & P_{xn2} \\ P_{\psi n1} & P_{\psi n2} \end{bmatrix} \begin{bmatrix} x_{n1} & x_{n2} \\ \psi_{n1} & \psi_{n2} \end{bmatrix} \quad ...(13)$$

It may be noted that all the elements in the above matrix are in general complex valued. For convenience the elements of the impedance matrix can be written in the following form as in case of vertical vibration.

$$K_{pq} = k_{pq} + ic_{pq} \; (p, q = x, \psi) \quad ...(14)$$

in which the real and imaginary part represents pile head stiffness and damping respectively. Also $K_{x\psi} = K_{\psi x}$.

Influence of Various Soil Parameters on the Pile Head Impedance Functions

The pile head impedance functions are dependent on soil properties such as the dynamic shear modulus, the material damping and the Poisson's ratio. A parametric study of the pile head impedance functions using the expected range of these properties in submarine soils is carried out to gain confidence in the pile head impedances to be used in computation of dynamic response of offshore pile supported structures. Further, the effect of spatial distribution of shear modulus with depth and the effect of soil-pile separation near the mud line on the pile head impedance are also included in the present study. The sensitivity analysis of pile-head impedance functions is carried out by obtaining results for a steel pile (Fig. 2) of length 40m, outer diameter meter 0.85m and wall thickness 0.025m.

Effect of Soil's Shear Modulus

Three types of soil characterized by their dynamic shear modulus are considered to study their influence on pile-head impedance functions. These soils represent stiffer, medium and soft stratum. The other properties of soil are kept the same in all the three cases of soil in order to carry out the comparative study with respect to shear modulus. The constant soil properties along with the three values of shear modulus considered in the present work are given below :

Unit weight (kg/m^3)		1.82×10^3
Poisson's ratio,		0.5
Material damping, tan δ		0.1
Shear modulus (kg/m^2)	Stiff G_1	6.524×10^6
	Medium G_2	3.262×10^6
	Soft G_3	1.631×10^6

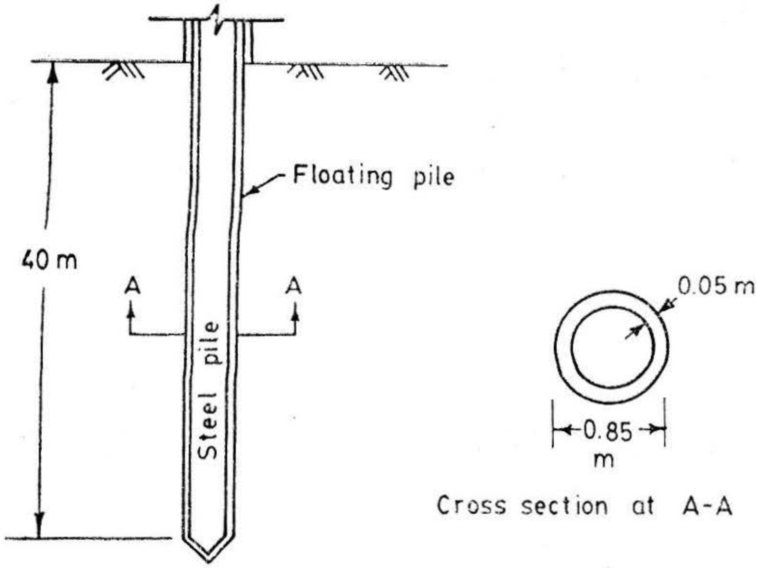

FIGURE 2 Details of the Pile Foundation

The frequency dependent pile-head stiffness elements (k_{yy}, k_{xx}, $k_{\psi\psi}$) and damping elements (c_{yy}, c_{xx}, $c_{\psi\psi}$) corresponding to vertical, horizontal and rotational directions respectively and the coupled stiffness element ($k_{x\psi}$) and damping element ($c_{x\psi}$) corresponding to all the three values of shear modulus are presented in Figs. 3 to 6. In the analysis of steel jacket subjected to wave loading, the consideration of pile head impedance functions is important in the low frequency range. As such, the stiffness and damping coefficients are calculated corresponding to the frequency range of 0.0—2.5 Hz only. It is evident from the results that the pile-head

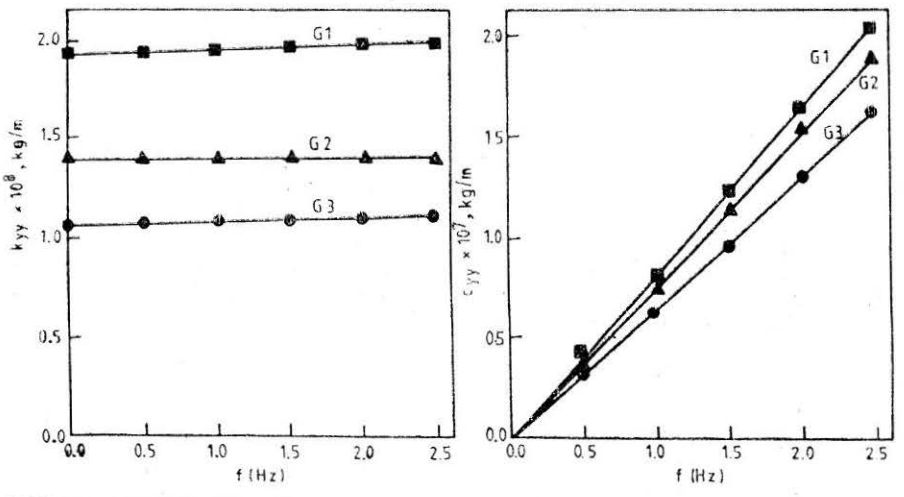

FIGURE 3 Pile Head Impedance Function in Vertical Direction Corresponding to Shear Modulus G1, G2, and G3

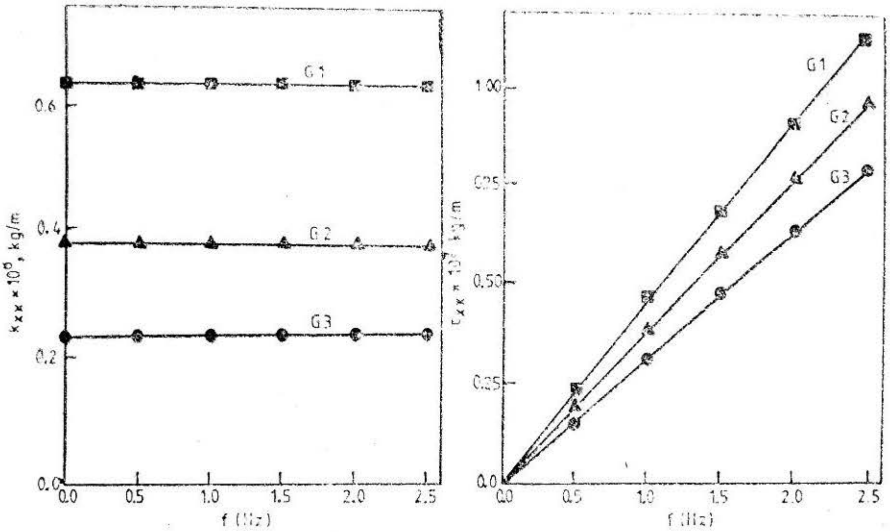

FIGURE 4 **Pile Head Impedance Function in Horizontal Direction Corresponding to Shear Modulus G1, G2, and G3**

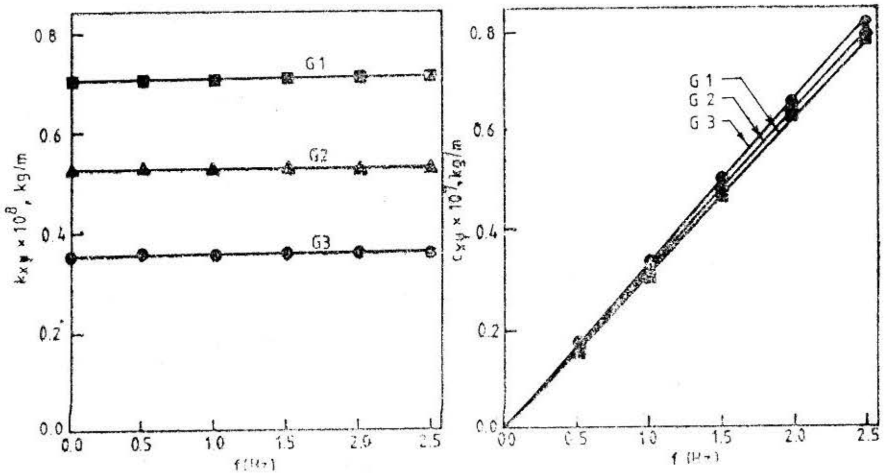

FIGURE 5 **Pile Head Coupled Impedance Function Corresponding to Shear Modulus G1, G2 and G3**

stiffness is almost frequency independent in the low frequency range. This is because the soil stiffners parameters in the vertical and horitontal directions are taken as constant for nondimensional frequency $a_o \leqslant 0.3$ and the effect of the variation in soil damping parameters on the pile-head impedance is quite small. The variation of damping element with respect to frequency is almost linear in the low frequency range. The pile-head stiffness and damping elements are also given in Fig. 7 to signify the variation in the impedance coefficients with respect to the shear modulus of soil. As the deformation behaviour of soil is largely influenced by its shear modulus, the effect of shear modulus on the impedance are found to be large. While damping element c_{yy} and c_{xx} increase with the increase in the value of shear modulus, the damping elements $c_{\psi\psi}$ and $c_{x\psi}$ decrease.

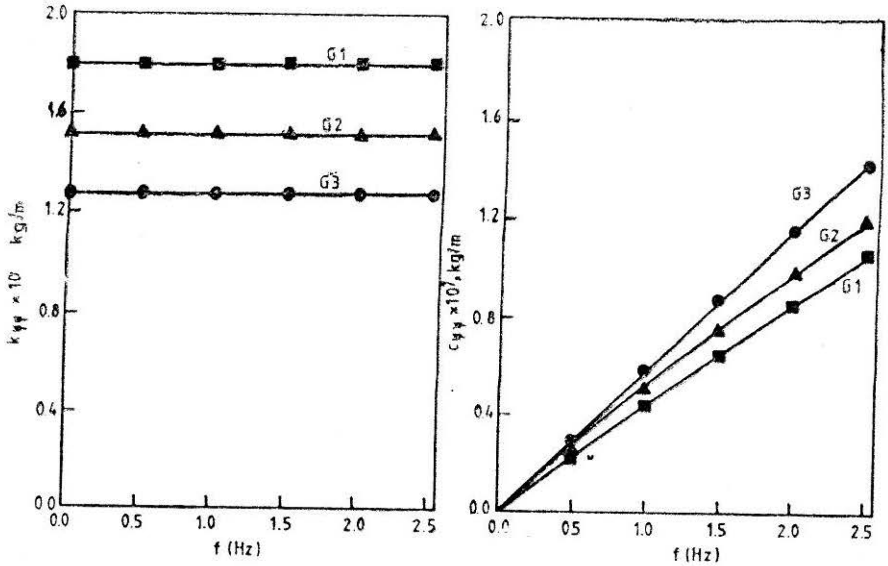

FIGURE 6 Pile Head Impedance Function in Rotation Corresponding to Shear Modulus G1, G2 and G3

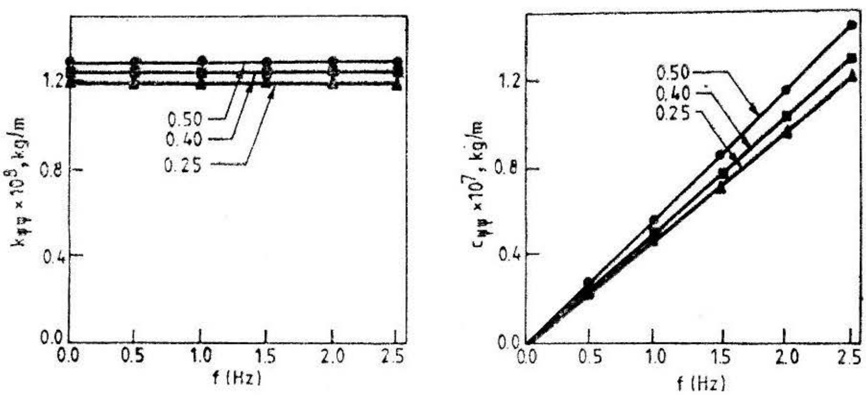

FIGURE 7 Variation of Pile Head Impedance with Shear Modulus at Frequency 2.5 Hz

For the medium soil G_2 the values of stiffness elements k_{yy}, k_{xx}, $k_{x\psi}$, $k_{\psi\psi}$ are respectively 1.31, 1.67, 1.38 and 1.19 times and the damping elements c_{yy} c_{xx}, $c_{x\psi}$ and $c_{\psi\psi}$ are respectively 1.17, 1.20, 0.99 and 0.86 times the corresponding values for the soft soil G_3. The figures for the stiff soil G_1 are 1.81, 2.82, 1.91 and 1.41 for stiffness elements and 1.27, 1.44, 0.96 and 0.73 for damping elements respectively.

This shows that the pile head impedances in the horizontal direction are affected largely by the variation in shear modulus, This is so because the pile is most flexible in the horizontal direction and, therefore, any change in the soil stiffness significantly influences the pile head impedance in the horizontal direction.

Effect of Poisson's Ratio of Soil

The soil stiffness and damping in the horizontal direction are also dependent on the Poisson's ratio of soil. The Poisson's ratio equal to 0.25, 0.40 and 0.50 are considered in turn to calculate the dynamic stiffness and damping of pile head. The other properties of soil taken into consideration are given below:

Shear modulus (kg/m²)	1.631×10^6
Unit weight (g/cm³)	1.820
Material damping, tan δ	0.1

The frequency dependent pile head stiffness elements (k_{xx}, $k_{\psi\psi}$, $k_{x\psi}$) and damping elements (c_{xx}, $c_{\psi\psi}$, $c_{x\psi}$) are presented in Figs. 8 to 10. The pile head stiffness is again observed to be nearly frequency independent and the damping almost varying linearly with frequency. The pile head stiffness and damping elements are again presented in Fig. 11 to signify the variation of pile head impedance with respect to Poisson's ratio. The pile head stiffness and damping are observed to increase with the increase

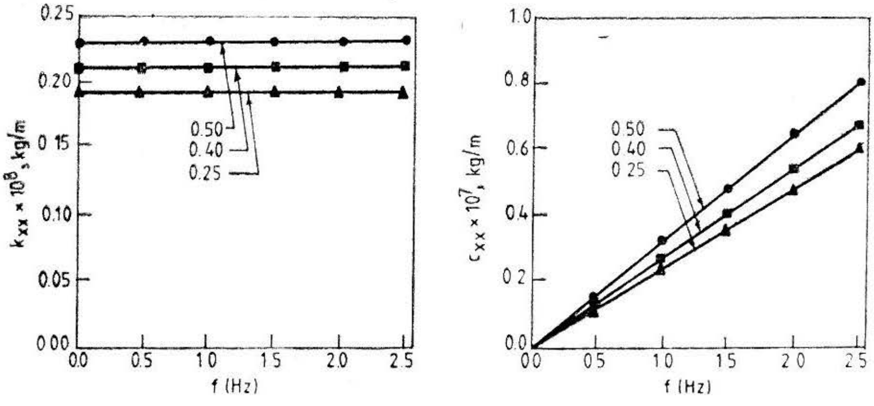

FIGURE 8 Pile Head Impedance Function in Horizontal Direction Corresponding to Poisson's Ratio.

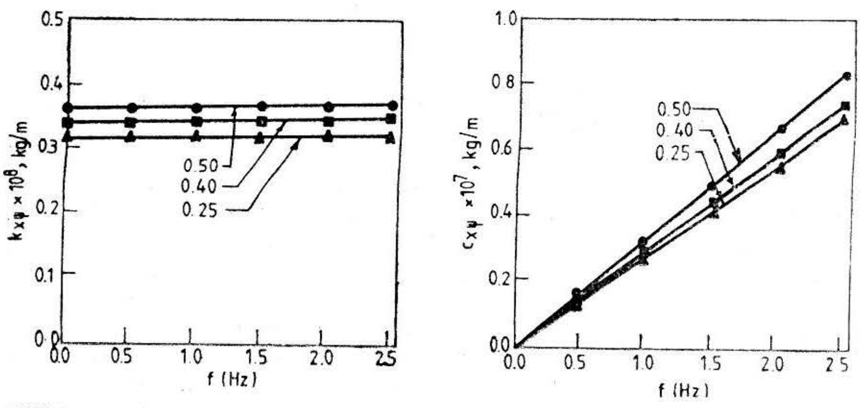

FIGURE 9 Pile Head Coupled Impedance Function Corresponding to Poisson's Ratio

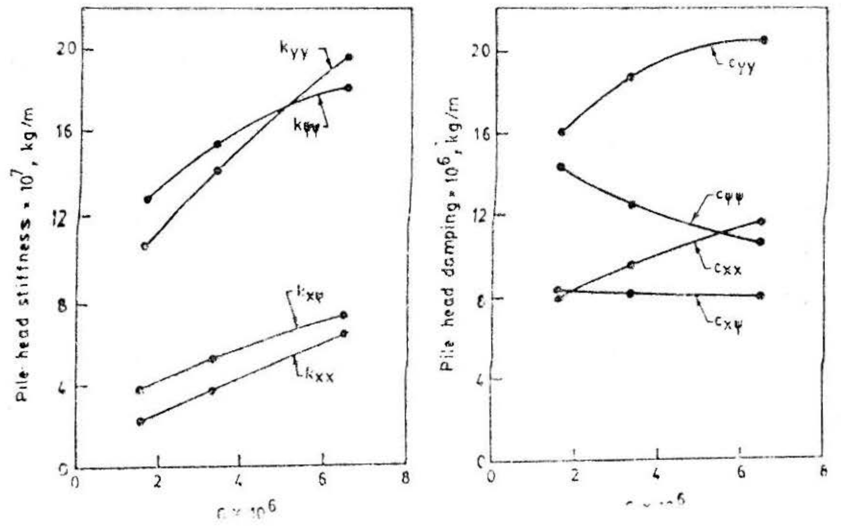

FIGURE 10 Pile Head Impedance Function in Rotation Corresponding to Poisson's Ratio

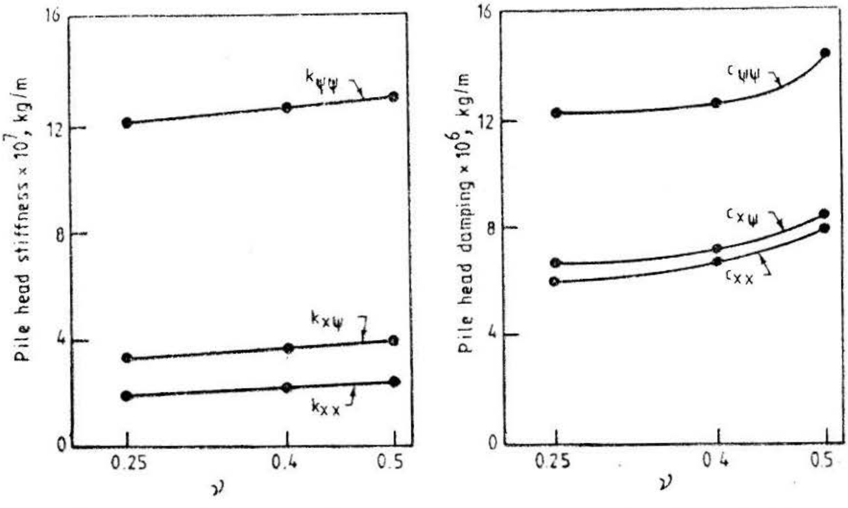

FIGURE 11 Variation of Pile Head Impedance Functions with Poisson's Ratio at Frequency 2.5 Hz

in Poisson's ratio. For soil with Poisson's ratio equal to 0.4 the stiffness elements k_{xx}, $k_{\psi\psi}$, $k_{x\psi}$ are respectively 1.11, 1.03, 1.08 times and damping elements c_{xx}, $c_{\psi\psi}$, $c_{x\psi}$ are in turn 1.10, 1.03, 1.05 times the corresponding values associated with the soil having Poisson's ratio equal to 0.25; the figure for the soil with Poisson's ratio equal to 0.5 are 1.21, 1.07, 1.14 for stiffness elements and 1.34, 1.17 and 1.24 for damping elements. The effects of Poisson's ratio on the impedance functions are smaller than those of shear modulus; influence on the dynamic response of the pile-supported structure, therefore, may not be large.

Effect of Material Damping of Soil

The plie head impedance is calculated by taking in turn the soil's material damping (tan δ) equal to 0.0, 0.1, and 0.2. The soil's properties presented below are kept same in all the three cases of damping.

Shear modulus (kg/m²)	1.631×10^6
Unit weight (g/cm³)	1.820
Poisson's Ratio, ν	0.5

The pile-head stiffness and damping corresponding to the three cases of material damping are presented in Fig. 12. The stiffness is found to be decreasing and the damping increasing with increase in material damping. For the soil with material damping equal to 0.1 the stiffness elements k_{yy}, k_{xx}, $k_{x\psi}$, $k_{\psi\psi}$ are respectively 0.97, 0.95, 0.97, 0.98 times and damping elements c_{yy}, c_{xx}, $c_{x\psi}$, $c_{\psi\psi}$ are in turn 1.16, 1.18, 1.19, 1.21 times the corresponding values associated with zero material damping of soil; the figures in case of soil with material damping equal to 0.2 are 0.95, 0.91, 0.94, 0.98 for the stiffness elements and 1.30, 1.34, 1.38, 1.42 for the damping elements. Whereas the increase in the pile-head damping is appreciable the decrease in the stiffness is small with the increase in material damping. Its effect on the structural response, however, may not be large.

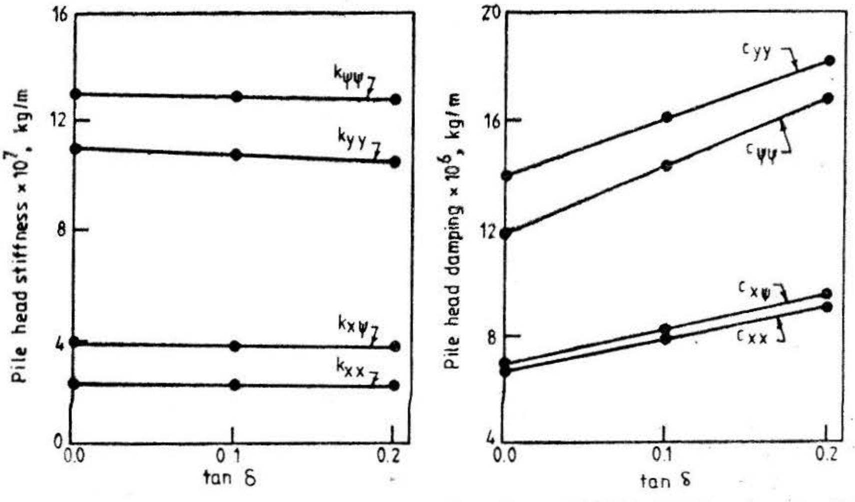

FIGURE 12 Variation of Pile Head Impedance Functions with Material Damping (Tan δ) at Frequency 2.5 Hz

Uniform versus Linear Distribution Shear Modulus of Soil

The shear modulus of soil is affected by the overburden soil pressure which increases with the increase in depth. To study its effect on the pile-head impedance functions two types of spatial variation of shear modulus are considered in the analysis of pile. In the first case the shear modulus is assumed to be uniformly distributed and its value is taken equal to the average of the shear modulus of soil existing along the pile depth. In the second case the shear modulus is assumed to vary linearly from zero value at the pile-head to the maximum shear modulus at the pile base (Fig. 13).

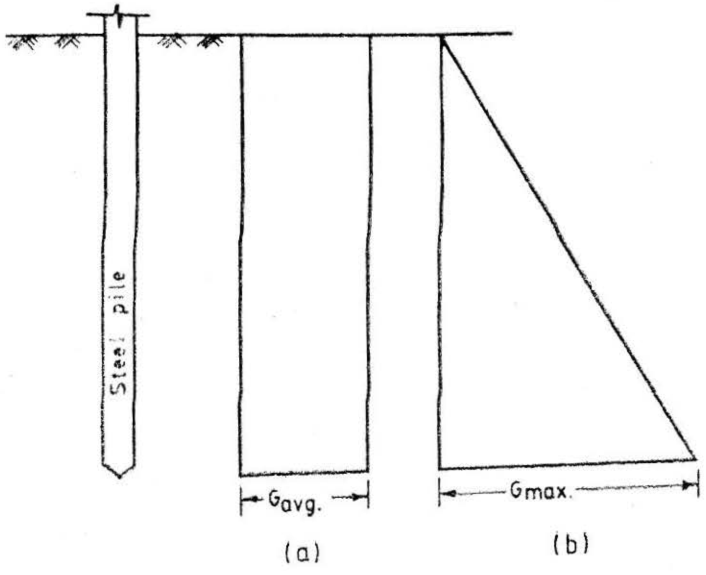

FIGURE 13 Distribution of Shear Modulus with Depth,
(*a*) Uniform Distribution,
(*b*) Linear Distribution

The shear modulus at the base is taken equal to two times the average shear modulus. The soil is assumed to have the following properties :

Average shear modulus G_{avg} (kg/m^2)	1.631×10^6
Maximum shear modulus, G_{max} (kg/m^2)	3.262×10^6
Unit weight (g/cm^3)	1.820
Poisson's ratio, ν	0.5
Material damping, tan δ	0.1

The pile-head stiffness and damping are presented in Figs. 14 to 17 for the frequency range equal to 0.0—2.5 Hz. The stiffness is found again to be frequency independent in both the soil distribution cases. Whereas, the damping in case of uniformly distributed shear modulus varies linearly with frequency, in other case of soil distribution the variation of damping is nonlinear. The spatial distribution of shear modulus is found to have large effects on the pile-head impedance. The values of stiffness and damping elements at frequency equal to 2.5Hz are given in Table 1. For the linear distribution of shear modulus case, the values of stiffness element k_{yy}, k_{xx}, $k_{x\psi}$, and $k_{\psi\psi}$ are respectively 0.81, 0.26, 0.48 and 0.72 times and the values of damping elements c_{yy}, c_{xx}, $c_{x\psi}$ and $c_{\psi\psi}$ are respectively 0.63, 0.40, 0.68, 0.94 times the corresponding values for the uniformly distributed shear modulus case. The decrease in impedance is maximum in case of impedance functions (k_{xx}, c_{xx}) in the horizontal direction. This happens because the soil's resistance against the pile's movement in the horizontal direction is reduced drastically near the mud line in case of linearly varying shear modulus.

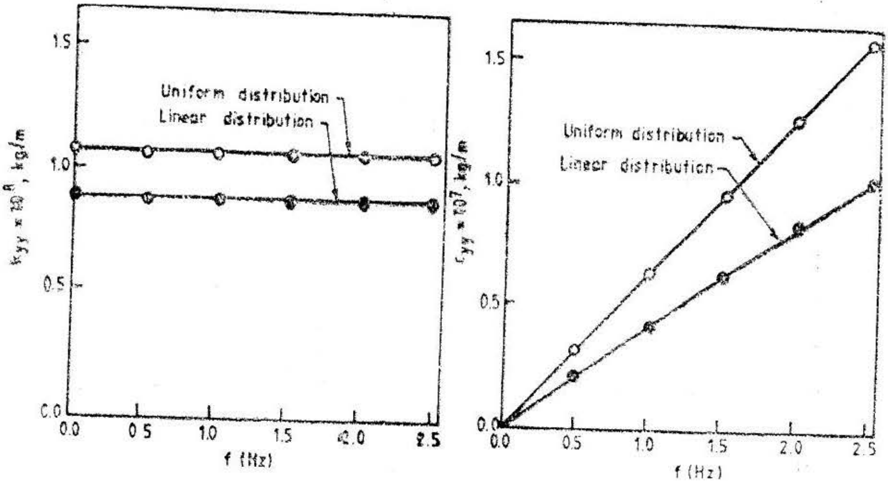

FIGURE 14 Pile Head Impedance Function in Vertical Direction Corresponding to Uniform and Linear Distribution of Shear Modulus with Depth

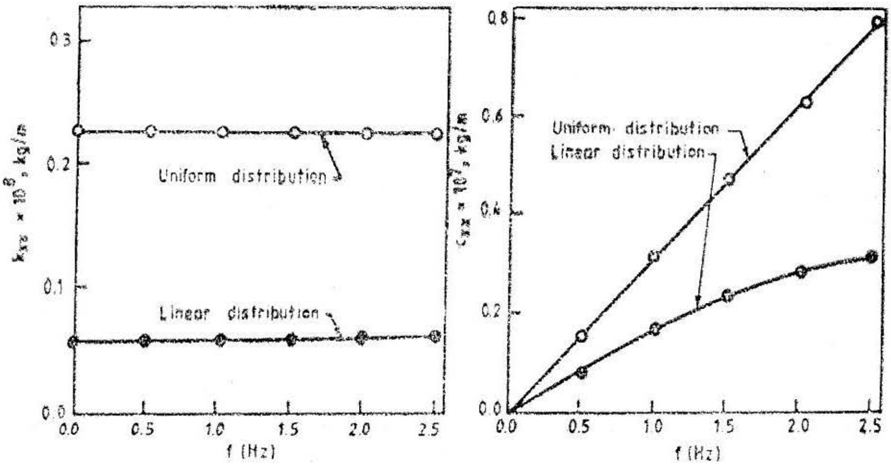

FIGURE 15 Pile Head Impedance Function in Horizontal Direction Corresponding to Uniform and Linear Distribution of Shear Modulus with Depth

Effects of Soil-Pile Separation near Mud Line

The soil-pile separation near the sea bed is caused by the cyclic movement of the jacket and scouring due to currents. To take the soil-pile separation into account the soil upto a depth equal to four times the outer radius of pile is assumed to provide zero resistance to the pile movement and as such the shear modulus in this zone is taken as zero. The shear modulus is assumed to vary linearly with depth as shown in Fig. 18. The pile head impedance functions obtained for the cases with and without soil-pile separation are compared to study the separation effect. In both the cases the soil is assumed to have the following

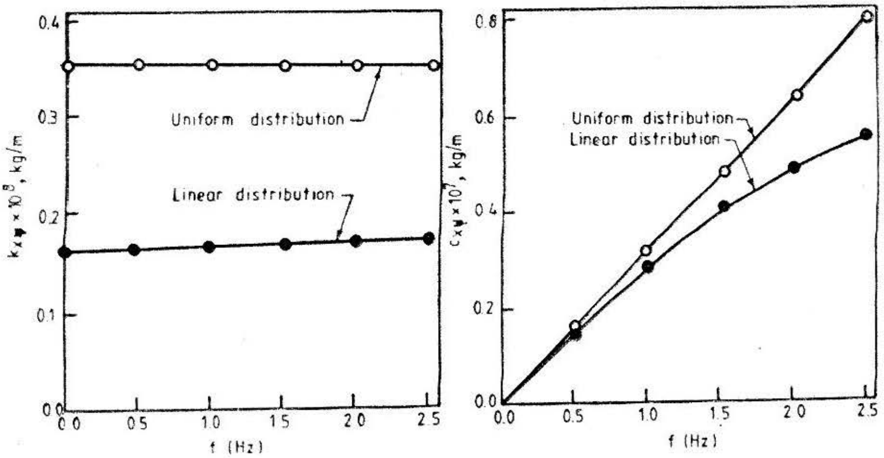

FIGURE 16 Pile Head Coupled Impedance Function Corresponding to Uniform and Linear Distribution of Shear Modulus with Depth

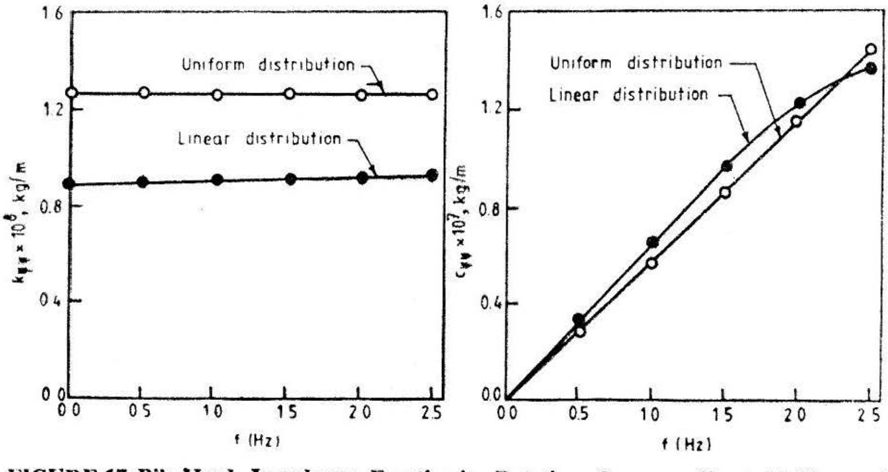

FIGURE 17 Pile-Head Impedance Function in Rotation Corresponding to Uniform and Linear Distribution of Shear Modulus with Depth

properties:

Maximum shear modulus, G_{max} (kg/m^2)	3.262×10^6
Unit weight (g/cm^3)	1.820
Poisson's ratio, ν	0.5
Material damping, tan δ	0.1

The pile head stiffness and damping corresponding to both the cases are presented in Fig. 19 to 22. The impedance functions are affected appreciably due to soil-pile separation near the mudline. The values of stiffness and damping elements at frequency equal to 2.5 Hz are given in Table 2. In cases of soil-pile separation, the pile head stiffness element

TABLE 1

Pile-Head Impedance Associated with Uniform and Linear Distribution of Shear Modulus with Depth at Frequency 2.5 Hz.

Pile Head impedance	Uniform distribution of Shear modulus (kg/m)	Linear distribution of Shear modulus (kg/m)
k_{yy}	1.08×10^8	8.70×10^7
c_{yy}	1.61×10^7	1.02×10^7
k_{xx}	2.29×10^7	5.94×10^6
c_{xx}	7.93×10^6	3.19×10^6
$k_{x\psi}$	3.76×10^7	1.82×10^7
$c_{x\psi}$	8.24×10^6	5.61×10^6
$k_{\psi\psi}$	1.29×10^8	9.33×10^7
$c_{\psi\psi}$	1.43×10^7	1.35×10^7

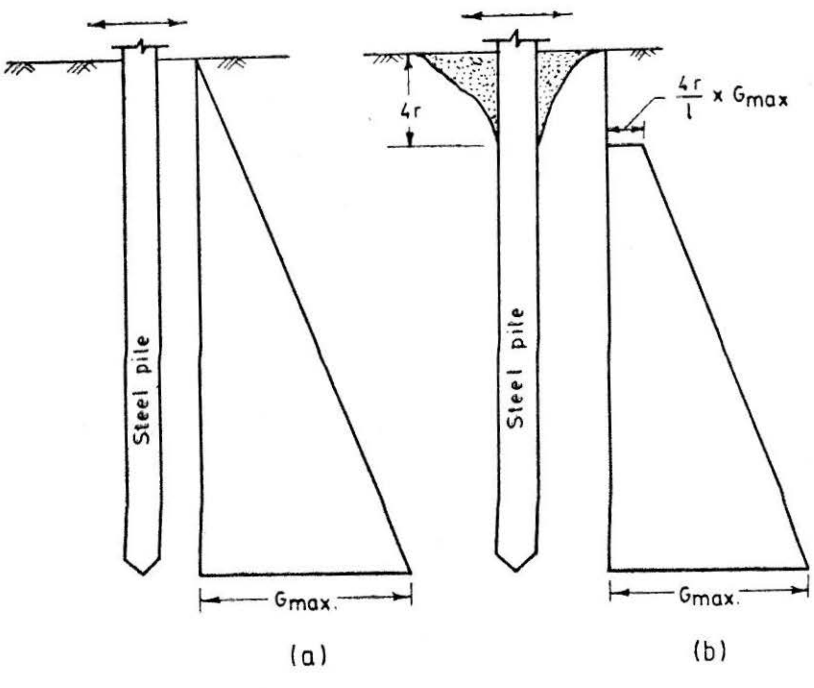

FIGURE 18 Linear Distribution of Shear Modulus with Depth,

(*a*) Without Soil-Pile Separation
(*b*) With Soil-Pile Separation

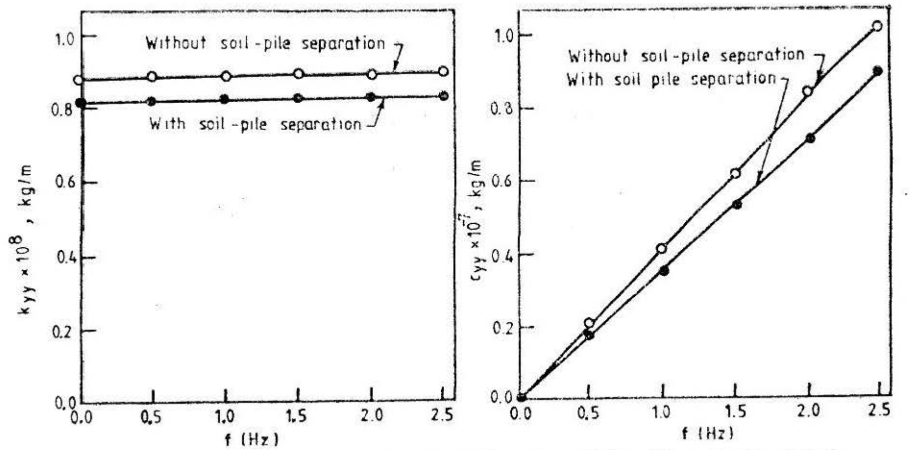

FIGURE 19 Impedance Function in Vertical Direction of Pile with and without Soil-Pile Separation

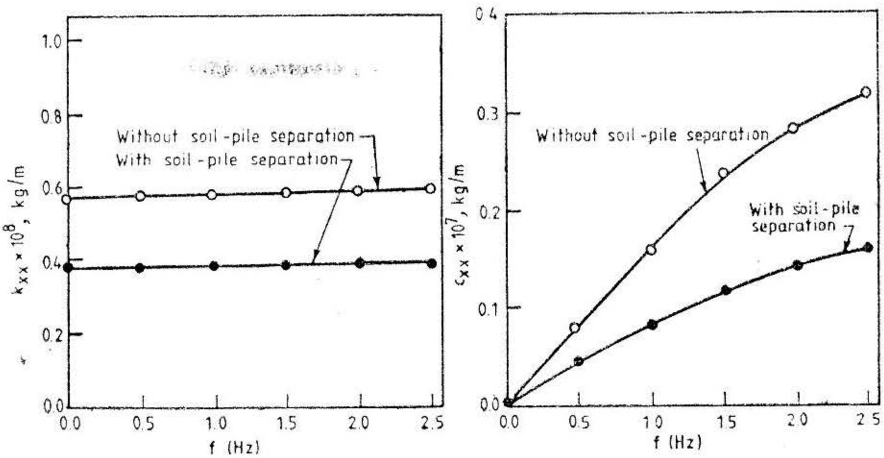

FIGURE 20 Impedance Function in Horizontal Direction of Pile with and without Soil Pile Separation

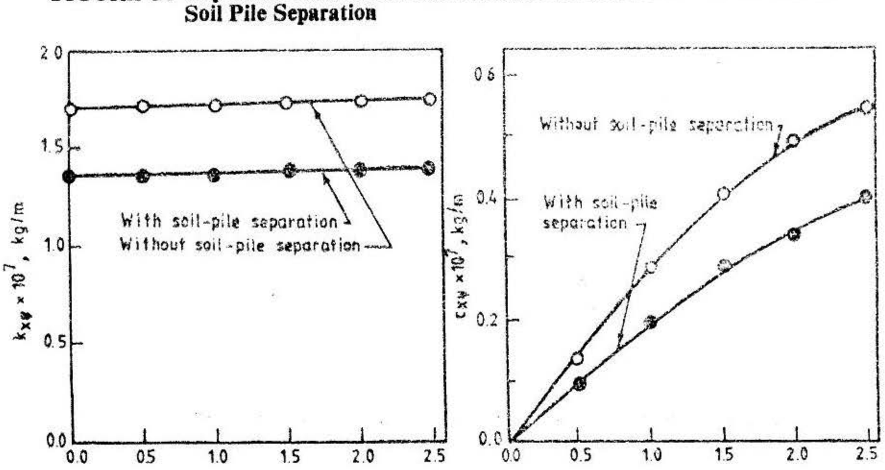

FIGURE 21 Coupled Pile-Head Impedance Functions with and without Soil-Pile Separation

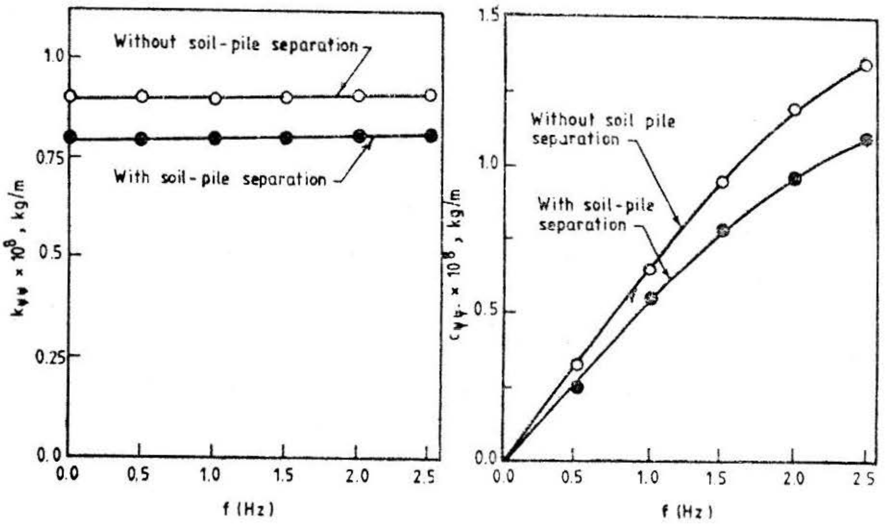

FIGURE 22 Impedance Function in Rotation of Pile with and without Soil-Pile Separation

TABLE 2

Pile Head Impedance with and without Soil-Pile Separation at Frequency 2.5 Hz

Pile Head impedance	Without separation (kg/m)	With separation (kg/m)
k_{yy}	8.70×10^7	8.54×10^7
c_{yy}	1.02×10^7	8.98×10^6
k_{xx}	5.94×10^6	3.69×10^6
c_{xx}	3.19×10^6	1.58×10^6
$k_{\psi x}$	1.82×10^7	1.44×10^7
$c_{x\psi}$	5.61×10^6	3.87×10^6
$k_{\psi\psi}$	9.33×10^7	8.45×10^7
$c_{\psi\psi}$	1.35×10^7	1.08×10^7

k_{yy}, k_{xx}, $k_{x\psi}$, $k_{\psi\psi}$ are in turn 0.97, 0.62, 0.79 and 0.91 times and damping elements c_{yy}, c_{xx}, $c_{x\psi}$ and $c_{\psi\psi}$ are respectively 0.88, 0.50, 0.69 and 0.80 times the corresponding valnes for the pile having no separation. The reduction in impedance is the largest in case of k_{xx} and c_{xx}.

Conclusions

The pile head impedance functions are largely affected by the change in dynamic shear modulus of soil. The effects of Poisson's ratio and material damping are relatively small. The influence of the distribution of shear modulus with depth and soil-pile separation near mud line on impedance functions is quite significant especially in case of pile-head stiffness and damping in horizontal direction. Thus, an accurate estimation of shear modulus of soil, an appropriate knowledge of its distribution with depth and soil-pile separation are essential in the evaluation of pile head impedance functions.

References

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Notations

A = Cross section area of pile;

a_o = Nondimensional frequency parameter;

D = Material damping ratio;

E = Young's Modulus of steel;

G = Dynamic shear modulus of soil;

I = Moment of inertia of pile;

i = $\sqrt{-1}$;

J_s = Mass moment of inertia of pile segment s,

K_{js} = Lumped dynamic stiffness of soil at sth node ($j = x, y, \psi$);

k_{pq} = Pile head impedance ($p, q = x, \psi$);

k_{yy} = Pile head impedance in vertical direction;

k_{js} = Dynamic stiffness of soil per unit length of pile segment s ($j = x, y, \psi$);

L_s = Length of pile segment s;

M_s = Lumped mass of pile at node s;

m_s = Mass per unit length of pile segment s;

P_x = Nodal force in horizontal direction;

P_y = Nodal force in vertical direction;

P_ψ = Nodal force in rotational direction;

r = Outer radius of pile;

S_{j_1} = Nondimensional stiffness parameter of soil ($j = x, y, \psi$);

S_{j_2} = Nondimensional damping parameter of soil ($j = x, y, \psi$);

x = Displacement in horizontal direction

y = Displacement in vertical direction

ω = Frequency in radians/sec;

ρ_s = Mass density of saturated soil:

ψ = Angle of rotation of pile at a node.