Analysis of Axially Loaded Long Piles

by

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Introduction

The piles used for offshore construction are generally very long. The piles are frequently of 80-200 m length. Under the combined effects of vertical and horizontal loads on the structures, offshore piles are subjected to very high compressive loads, requiring ultimate capacities upto 3,000 tons. The very high magnitude of the loads makes the design and construction of these piles, a very challenging task.

The design of piles is generally based on the knowledge of stress resultant in the pile section and allowable settlement of the pile. The stress resultants are required in determining the dimension of pile; and settlement is important to the functional aspect of the supported structure.

Methods of analysis of axially loaded piles have been developed to predict the load-settlement and load distribution of pile. Such methods are (1) load-transfer methods, which use measured relationship between pile resistance and pile movement at various points along the pile, (2) methods based on elastic theory approach, and (3) numerical methods such as finite element method. In the load-transfer method, the soil data required is skin friction, τ and corresponding pile movement, z.

Seed and Reese (1957) developed such curves for different depths along the pile using an instrumented pile load test. They proposed the following differential equation to calculate the strain in an element of pile at depth x.

$$\frac{dz}{dx} = \frac{P}{EA}$$

where,

P = axial load at depth x,

z = axial displacement of pile at depth x, and

EA = pile stiffness.

Coyle and Reese (1966) obtained τ vs z curves using laboratory experiments on model piles and load tests on instrumented piles. In similar way Coyle and Suliman (1967) presented load transfer curves for sand. Meyer et. al. (1975) solved τ vs z as boundary value problem using finite difference method assuming elasto-plastic interface response.

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Poulos and Davis (1968), Poulos and Mattes (1969), Randolph and Wroth (1978) analysed the axially loaded piles using elastic theory approach. In this method, displacements of piles are obtained by considering the compressibility of pile and the soil displacements are obtained by Mindlin's equation for displacement within soil mass caused by loading within the mass.

Data presented by Meyerhof (1976) and Vijayvergiya and Focht (1972) show that the skin friction factor for driven piles in clay decreases with increasing pile penetration. Randolph and Wroth (1982) and Kraft et. al. (1981) suggest that the apparent decrease in shaft capacity is due to strain softening effect of soil. They also suggest that the magnitude of this reduction in capacity depends on pile stiffness and the ratio of residual skin friction to peak skin friction. Aurora et. al. (1981) and Kraft et. al. (1981) suggest that this reduction in shaft capacity is due to degradation of pile-soil contact caused by the lateral vibration induced during pile driving. Poulos (1982) using elastic theory approach showed that the major cause of length effect on pile load capacity could be attributed to the relative compressibility of pile.

In the analysis presented in this paper, the basic governing differential equation for axial load on a pile has been devised by treating the problem as a one dimensional problem and taking into account the skin friction effect. The finite difference method is used to solve the differential equation and the results are obtained in non-dimensional form. Soil pile interface syetem bas been modelled as (*i*) linearly varying soil modulus and (*ii*) nonlinear load deflection relationship. The effect of various parameters on the behaviour of pile is studied.

Definition and Formulation

Figure 1 (*a*) shows a pile subjected to an axial load. P_t is the applied load at pile top and P_{tip} is the induced load at pile tip. The pile top undergoes a displacement of Z_t and the tip displaces by Z_{tip}. P is the axial load at depth x and z is the displacement of pile at that depth.

The governing differential equation can b^ written as:

$$\frac{d^2z}{dx^2} = \frac{\tau}{AE} \qquad ...(1)$$

In terms of nondimensional parameters:

$$Z = z.\frac{1}{R}$$

$$X = x.\frac{1}{R}$$

$$L = l.\ \frac{1}{R}$$

$$U = P.\ \frac{R^2}{EI}$$

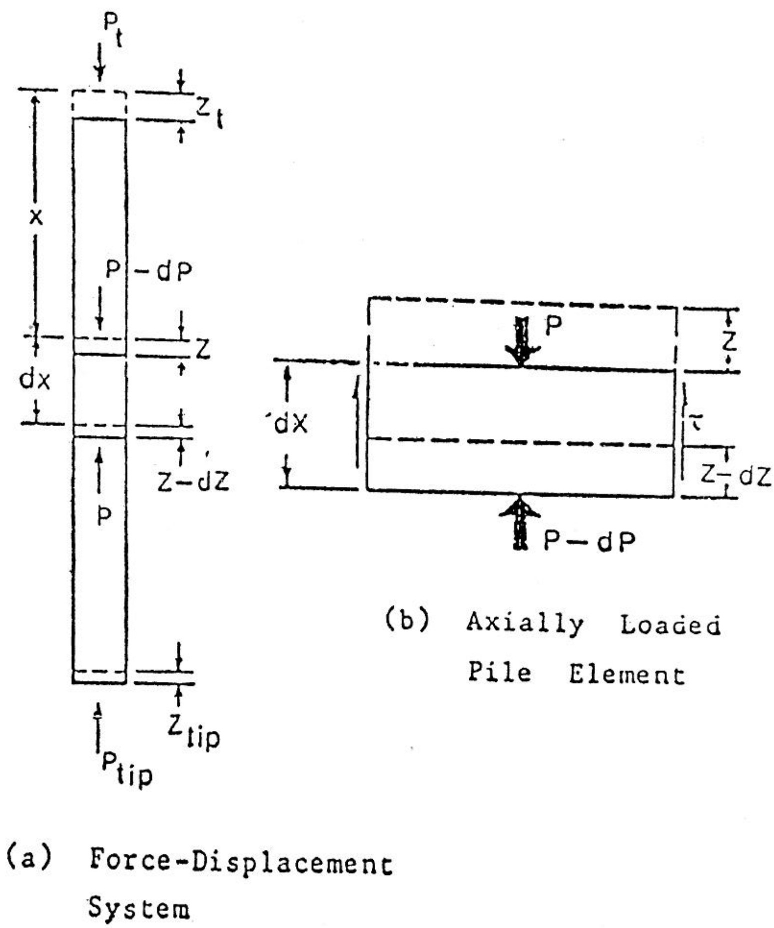

FIGURE 1 Axially Loaded Pile System

Eq. 1 can be written as:

$$\frac{d^2z}{dx^2} = \frac{R.c}{A.E} \qquad \ldots(2)$$

and

$$U = \frac{AR^2}{I} \cdot \frac{dz}{dx} \qquad \ldots(3)$$

For realistic problems, τ cannot be expressed mathematically. Further, in the problem under consideration only one boundary condition is known at the tip such as pile tip movement. At top of the pile, the applied load is known whereas the pile top movement is not known as no constraint is imposed for the movement of pile top. Therefore a numerical type solution has to be obtained and a closed form solution is not possible. Finite difference method is used herein for the analysis. The pile is divided into N discrete elements of length L/N as shown in Fig. 2.

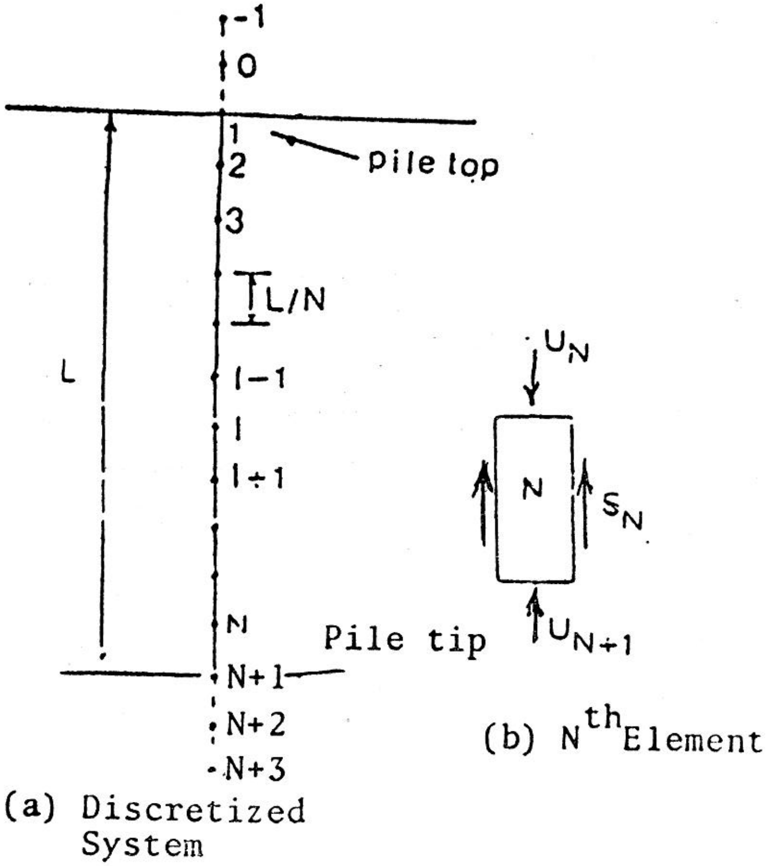

FIGURE 2 Finite Difference Analysis of Axially Loaded Piles

Referring to Fig 2 (*a*), the finite difference form of the Eqs. 2 and 3 for general node *I* is

$$U(I) = g\,(Z\,(I-1) - Z\,(I)) \qquad \ldots(4a)$$

or,

$$U(I) = g\,(Z\,(I) - Z(I+1)) \qquad \ldots(4b)$$

$$Z(I) = h\,(Z\,(I+1) - 0.5\,Z\,(I+2)) \qquad \ldots(5)$$

where.

$$g = \frac{AR^2}{I(L/N)}$$

$$h = \frac{2}{\left(1 - \dfrac{R^2(L/N)^2}{AE}\right)}$$

Eqs. 4 and 5 are the recurssion relationship for U and Z. The known boundary condition is pile tip movement and the corresponding pile tip force.

When $I = N+1$,

$$U(N+1) = U_{tip} \qquad \text{...(6a)}$$

$$Z(N+1) = Z_{tip} \qquad \text{...(6b)}$$

Substituting Eqs. 6 (*a*) and (*b*) in Eq. 4 (*a*), $Z(N)$ can be calculated as following:

$$Z(N) = Z(N+1) + \frac{U(N+1)}{g} \qquad \text{...(7)}$$

Using $Z(N)$, $Z(N+1)$ in Eqs. 4 and 5, displacement Z and axial load U at all nodes can be calculated.

The non-dimensional skin friction and elastic compression $E(I)$ at element I are given by

$$S(I) = U(I) - U(I+1) \qquad \text{...(8)}$$

$$E(I) = Z(I) - Z(I+1) \qquad \text{...(9)}$$

Hence the shear force and the elastic compression at all elements can be calculated.

Results and Discussions

In order to study non-linear behaviour of pile, actual soil data (Ching, 1984) is used for the analysis. The shear strength profile shown in Table 1

TABLE 1

Undrained Shear Strength of Soft and Stiff Clay with Depth

Depth (*m*)	*Undrained Shear Actual* Soil Deposit (Soft Clay Condition)	Strength (kN/sq. *m*) Idealised Soil Deposit (Stiff Clay Condition)
0.0	13.4	53.6
2.0	16.4	65.6
4.0	19.4	77.6
7.0	23.9	95.6
10.0	28.4	113.6
15.0	37.9	150.8
20.0	43.4	173.6
25.2	51.2	179.2
40.0	76.8	192.0
54.8	100.0	200.0
75.0	125.0	259.0
101.5	151.0	282.0

indicates that first 25 *m* depth of the clay is soft. Below 25 m the clay becomes stiff and from 55 m downwards the clay is very stiff up to 101.5 m. Since the top layer is soft clay for distinguishing purpose, this actual soil deposit will be referred to as "soft clay condition".

To study the effect of the soil stiffness on the behaviour of piles, an ideal stiff soil profile has been assumed by increasing the shear strength of clay used in the analysis of "soft clay condition". By doing so, the layer pattern of soil profile is unaltered, only the shear strength of the layers are increased which facilitate comparison of the effect of soil stiffness on the behaviour of piles. The undrained shear strength of the soft and stiff clay conditions at vari s depths are shown in Table 1.

The τ—z curve or soft and stiff clay are shown in Fig 3.

The analysis presented earlier is used to solve the problem of axially loaded piles. Separate programs are written for linearly varing soil modulus and non-linear soil modulus. The program calculates the axial load distribution, axial displacement, skin friction and soil modulus at all nodes for a given value of pile tip movement. The program is initialised with input value of pile tip movement and the axial load is calculated back.

The characteristics length *R* for linearly varying soil moduls and non-linear soil moduls are taken as equal in magnitude. Therefore comparison can be made among the non-dimensional results for different type of soil moduli.

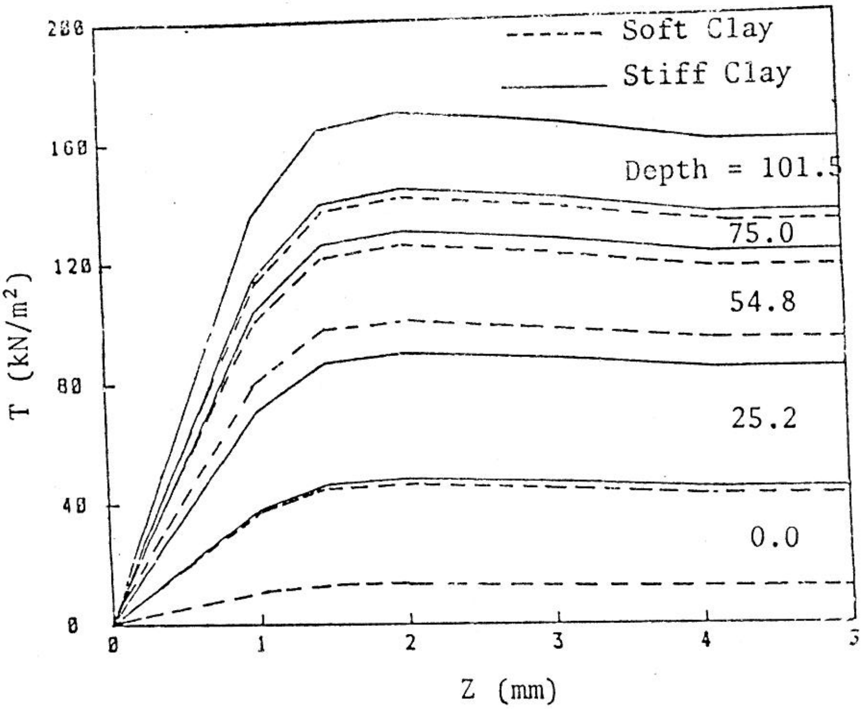

FIGURE 3 τ-z Curves for 1 m Diameter Pile in Soft and Stiff Clay Conditions

Effect of Soil Stiffness

Figure 4 shows the load-settlement behaviour of pile in soft and stiff clay condition. It can be seen that for a same axial load the settlement in stiff clay is smaller than that in soft clay. Figs. 5 to 7 show the

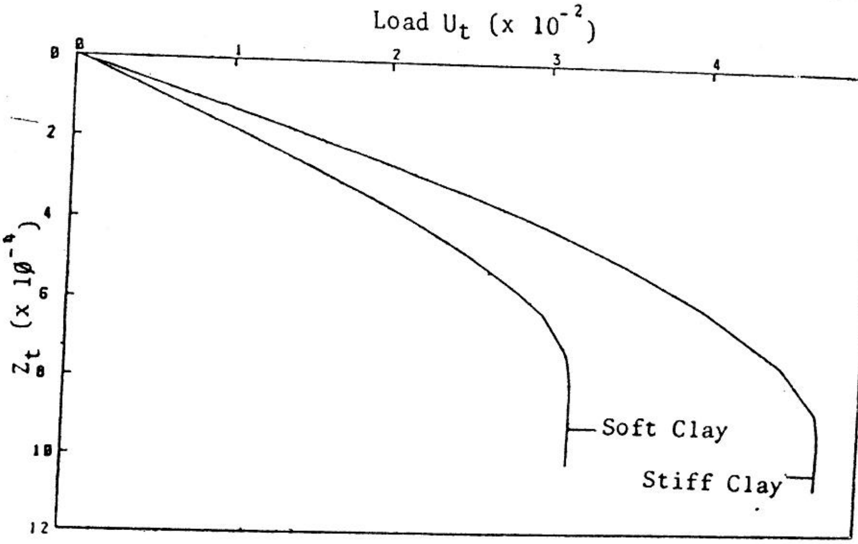

FIGURE 4 Load-Settlement Relation for Soft and Stiff Clay

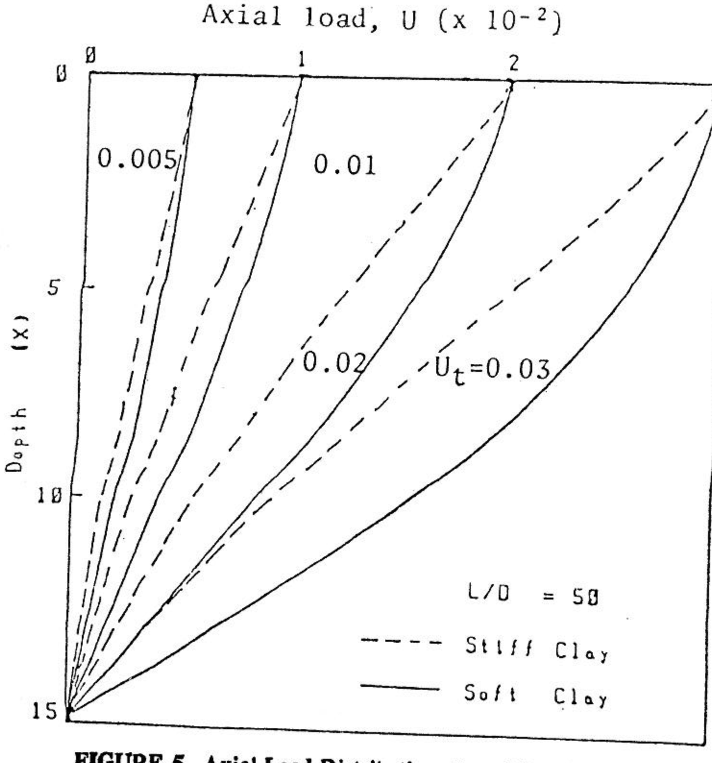

FIGURE 5 Axial Load Distribution along Pile, Soft Clay

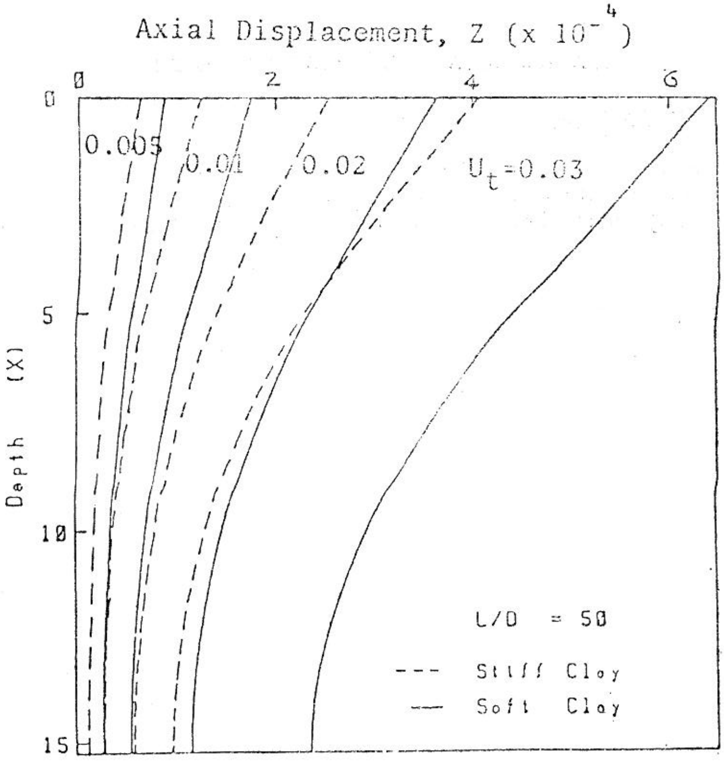

FIGURE 6 Axial Displacement along Pile, Soft Clay

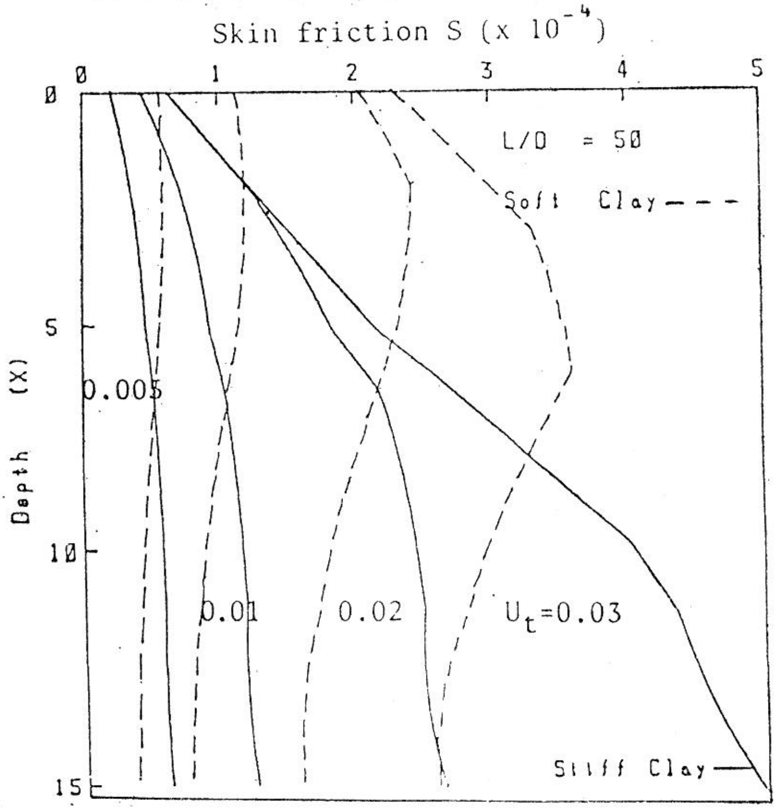

FIGURE 7 Skin Friction along Pile, Soft Clay

distribution of non-dimensional axial load, shear force and displacement respectively along the length of pile in soft clay and stiff clay condition. The curves are shown for different values of axial load on the pile.

From Figs. 6 and 7, a difference in settlement at tip and top of pile can be observed. The figures show that the elastic compression in both soft and stiff clays increase with increasing axial load. At smaller axial load, the elastic compression is almost equal in soft and stiff clays but at higher axial load, the elastic compression in soft clay is higher than that in stiff clay. In soft clay a small translation may trigger higher elastic compression. Therefore considerable head movement may take place before the pile tip experiences any change in stresses. Hence settlement behaviour of long piles is governed by the stiffness of soil. More the stiffness, smaller the settlement.

Yielding of Soil

The increase in elastic compression can be attributed to the yielding of soil. As explained above, at small axial load, elastic compression in both soft and stiff soil is approximately equal in magnitude. The difference in elastic compression increases at higher axial load. Figures 5 and 6 shows that at small axial load, the axial load distribution in both stiff and soft clay can be approximated to a linear variation with depth of pile. At higher axial load, the axial load distribution is still linear in stiff clay but in soft clay, the axial load distribution reaches a constant value at shallow depths ; in mathematical notation dU/dX at small X tends to infinity. From Eq. 2 it can be realised that the elastic compression will be maximum for dU/dX equal to infinity i.e. for constant axial load distribution and smaller for smaller dU/dX. The later is the case for stiff clay at all range of axial loads and for soft clay dU/dX at small X tends to infinity at higher axial loads. Therefore elastic compression is higher at higher axial load and smaller at smaller axial loads in stiff clays. From Fig. 7 it can be seen that at higher axial load the top portion of soil undergoes into yielding stage as the curves merge at shallower depths with increasing axial load. Hence value of dU/dX approaches infinity at shallow depths. Therefore settlement behaviour of axially loaded piles is not only governed by stiffness of soil but also by the yielding behaviour of soil.

Figures 8 to 10 show that the axial behaviour of pile in proportional soil is almost identical to the behaviour of pile in soft clay and the same type of load-settlement behaviour can be seen in proportional soil.

Effect of Length

The effect of length on the response of pile is analysed for soil with linearly varying modulus (proportional soil). Figures 11 to 13 show the distribution of non-dimensional axial load, shear force and displacement of pile subjected to non-dimensional axial load of 0.02. Results are presented for piles with L/D values 30, 50, 75 and 100.

The figures indicate that settlement is smaller at higher L/D values. However for higher L/D values, the top displacement reaches a limiting value. It can be seen that the piles with higher L/D values experience large elastic compression with small translation. Moreover, with increasing

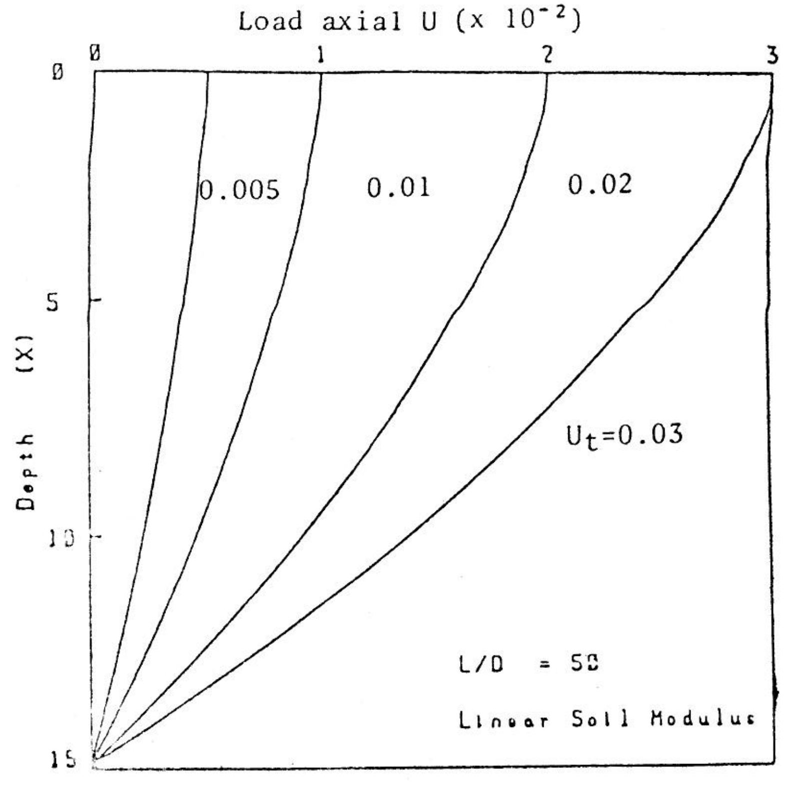

FIGURE 8 Axial Load Distribution along Pile, Proportional Soil

L/D, mobilization of skin friction takes place in the upper portion of soil and the zone where higher skin friction is mobilised moves upwards with increasing *L/D* values. The increase in *L/D* may reduce the mobilization of maximum skin friction as the top soil is weak with subgrade modulus as zero at ground surface. As can be seen in Fig. 12, the displacement of

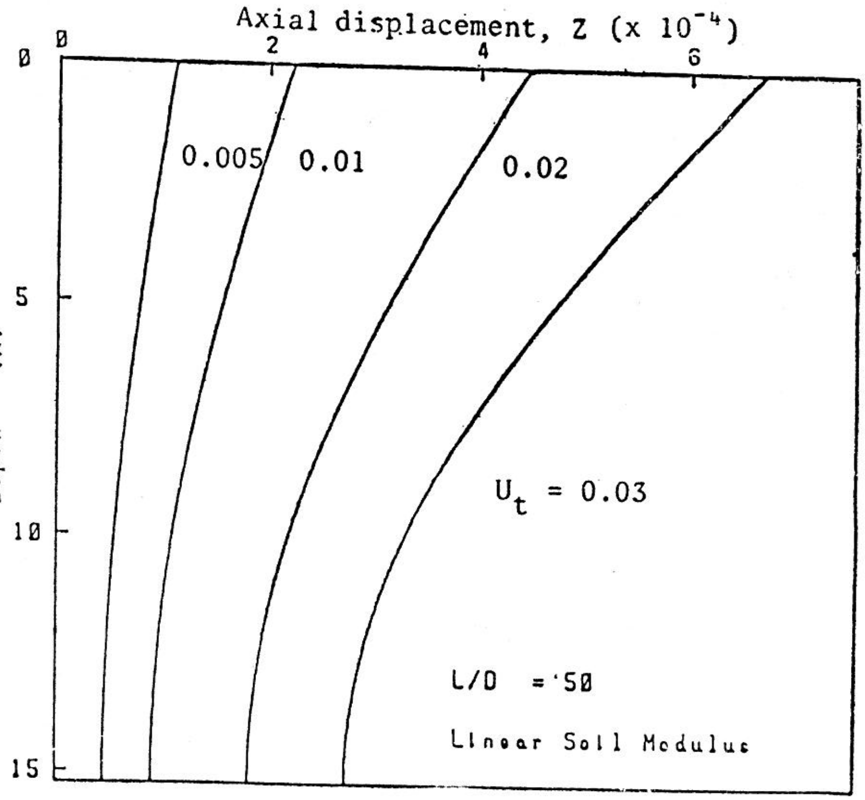

FIGURE 9 Axial Displacement along Pile, Proportional Soil

pile is small at the bottom of the pile but increases at shallow depth of soil for higher values of L/D. Thus the skin friction mobilised in piles with higher L/D is smaller at bottom portion of pile and the maximum value occurs at depths 0.2—0.4 times the total length of pile. Since the elastic compression of pile with high L/D is higher, the L/D value will lead the pile into yielding stage before the soil yields. Therefore it can be concluded that in long piles, the increase in elastic compression associated with the upward movement of maximum skin friction mobilization zone will reduce the carrying capacity of pile after a limiting value of L/D. Therefore L/D value of long pile governs the compressibility and load carrying capacity of pile.

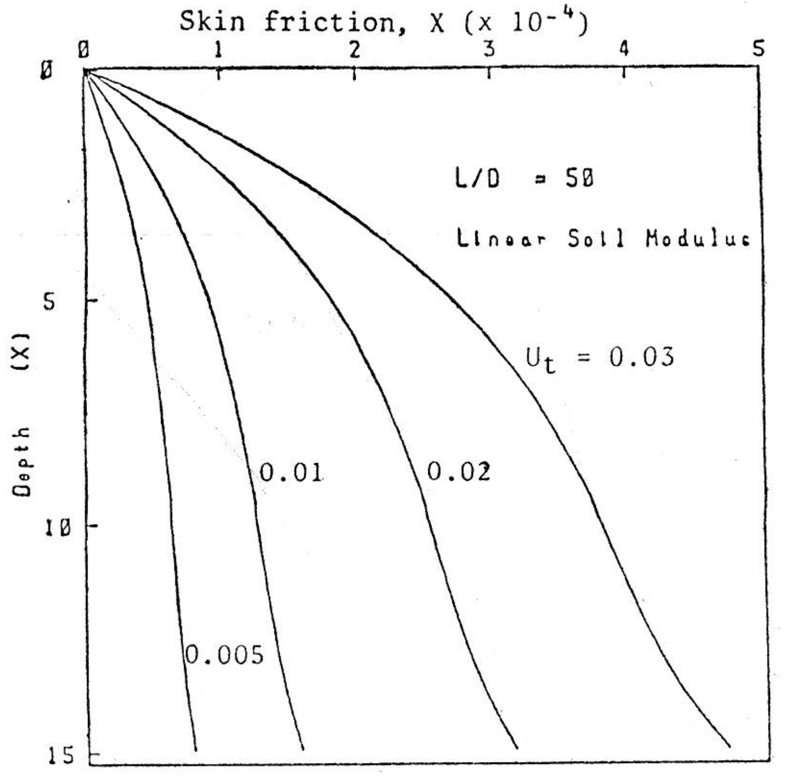

FIGURE 10 Skin Friction along Pile, Proportional Soil

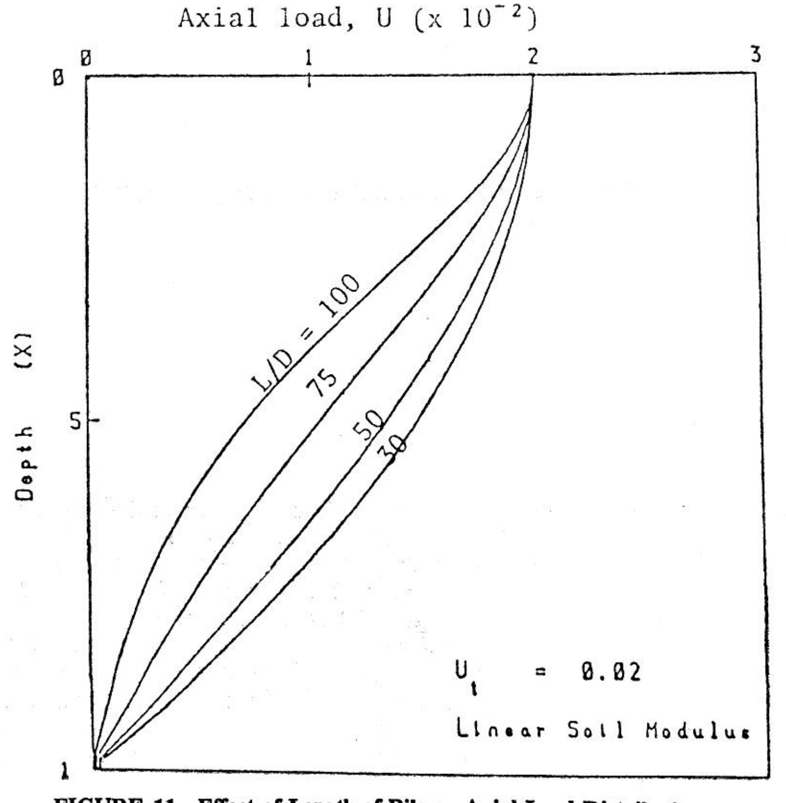

FIGURE 11 Effect of Length of Pile on Axial Load Distribution

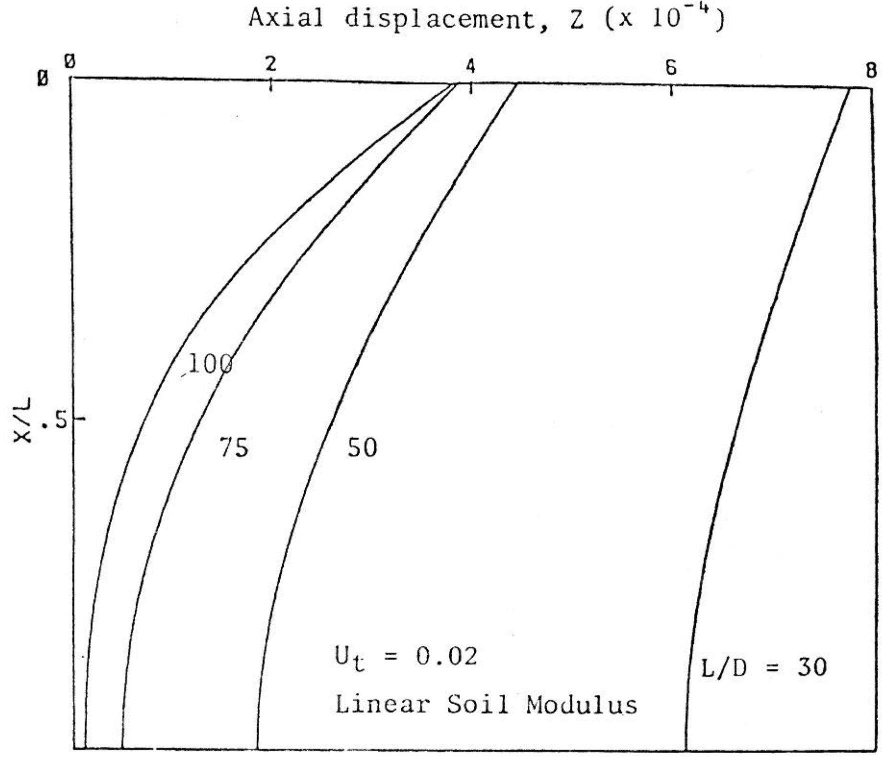

FIGURE 12 Effect of Length of Pile on Axial Displacement

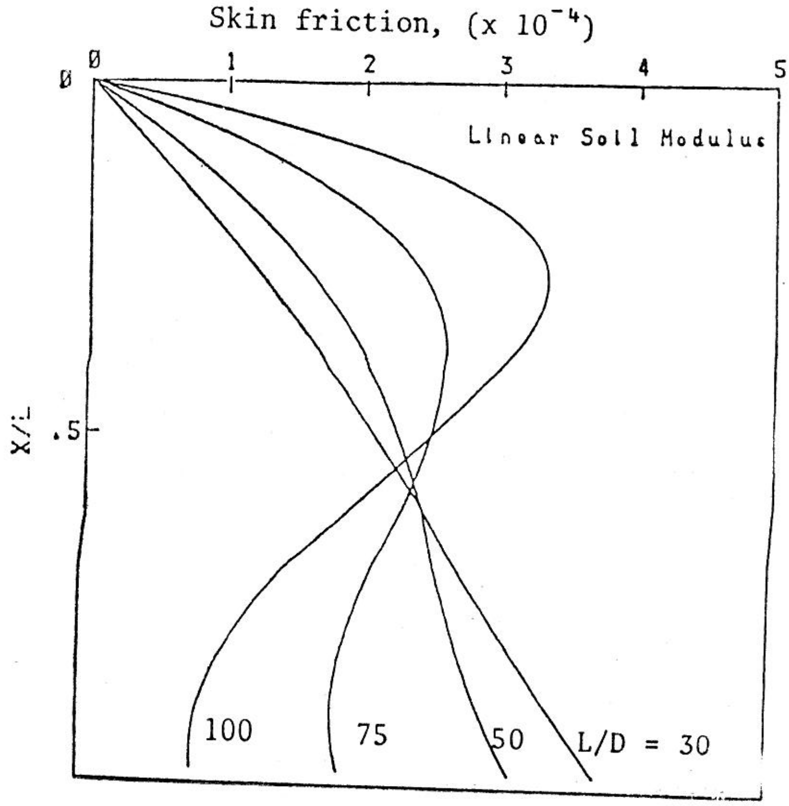

FIGURE 13 Effect of Length of Pile on Skin Friction Mobilization

Conclusions

Based on the studies carried out, the following conclusions can be drawn.

(1) Elastic compression associated with small axial load in both soft and stiff soil condition is almost equal in magnitude. But the translational displacement in soft soil is larger than that in stiff soil condition at all axial load.

(2) Yielding of soil governs the axial load distribution and settlement behaviour of pile. Because of yielding of soil in shallow depth at higher axial loads in soft soil condition, the axial load distribution changes from linear to constant variation in the yielding zone which inturn increases the settlement of long piles.

(3) The axial load distribution at small load can be approximated as a linear variation in both soft and stiff soil condition. But the variation tends to be non-linear at higher axial load in soft soil condition while the linear variation is still valid in stiff soil condition.

(4) The increase in L/D value leads to small settlement. But the settlement reaches a limiting value for further increase in L/D. With increasing L/D, the elastic compression becomes larger with less translation. The increase in elastic compression may lead to yielding of pile before soil yields, at higher L/D values.

(5) The increase in L/D reduces the mobilization of maximum skin friction since the maximum skin friction mobilization zone moves to shallower soil layers. Hence increase in L/D after a limiting value will reduce the carrying capacity of pile.

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