Differences between Cantilever and Gravity Retaining Walls under static Conditions

by

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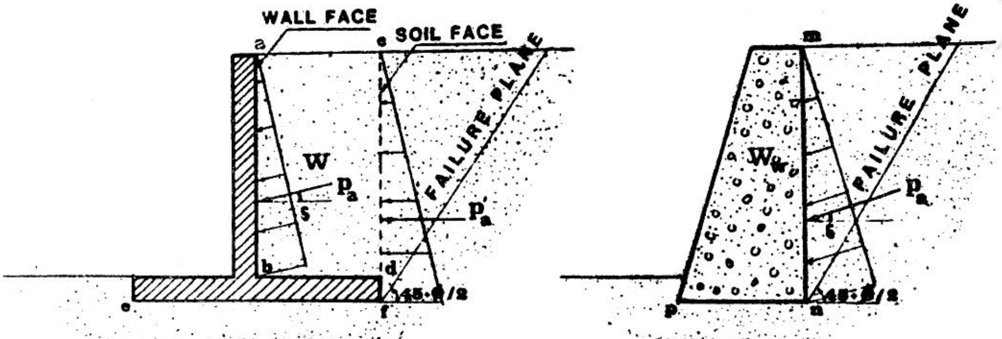
Introduction

In practice, the design codes used for cantilever and gravity walls do not differentiate between these two types of walls. The difference between them is that in a gravity retaining wall the massive volume of the wall provides its stability against sliding and overturning. In the cantilever wall, on the other hand, the weight of the soil mass above the wall base is a main contributing factor in the wall-backfill stability. Gravity walls are most likely to fail as a result of sliding and hence may be called displacement-governed walls. Cantilever walls, which fail because of fracturing or yielding of the wall stem, may be considered force-governed walls. In both types of walls, the lateral earth pressure behind the walls (see Figure 1a, section a-b, for the cantilever wall and Figure 1b, section m-n, for the gravity wall), is calculated by the use of Coulomb's theory, taking into account the effect of the wall-soil interaction. For the cantilever wall, the lateral earth pressure computed at the wall-face (Figure la, section a-b) is used in the structural design of the wall elements, whereas the stability of the cantilever wall against sliding and overturning is governed by the earth pressure calculated at the soil face (Figure la, section c-f). Since no wall-soil interaction takes place along section c-f, the lateral earth pressure can be calculated with the use of Rankine's theory. Table 1 summarizes the differences between the two types of retaining walls.



(a) CANTILEVER

(b) GRAVITY

FIGURE 1 Cantilever and Gravity Retaining Structure with Earthpressure Behind Them.

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TABLE 1

	Cantilever Wall	Gravity Wall
Material	Reinforced Concrete	Masonry, Plain Concrete
Design Forces	Coulomb's Theory at Section a-b, Rankine's Theory at Section c-f for K_0 or K_a condtions.	Coulomb's Theory at Section n-m for the Active Condition K_a.
Deformation Due to Earth Pressure	Rotation in the Stem About Its Connection With the Base. Smaller Translation of the Entire Wall.	Three Possible Wall Displacements : (1) Translation (2) Rotation About Wall Top (3) Rotation About Wall Base Translation is most likely due to the massive weight of the wall.
Stability Against Sliding	Weight of Soil Block abdc(W) over a Large Base Friction Area (ef). The Contribution of the Wall Height is Minimal.	Weight of the Wall over a Small Base Friction Area (pn).
Stability Against Bearing Capacity And Overturning	Weight of the Soil Block abdc(W) Forms a Resisting Force. Large Base Area Reduces Stresses at Base.	Weight of the Wall Forms the Resisting Force. Stresses are High at Base Due to the Small Base Area.
Practical Usage	Soft Foundation. Overturning Moments Govern the Design. Limited Construction Space.	Stiff Foundation. Sliding Governs the Design. Rural Areas.
Economy	Expensive : More Workmanship. Special Construction Procedure.	Less Expensive : Less Workmaship.
	Figure 1-a	Figure 1-b

This paper is an investigation of the differences between cantilever and gravity walls, specifically a nondisplacing wall (N.D.W.) or a wall displacing away from the backfill (D.W.). The finite element method is used in modelling the wall-backfill systems in both cases. The results of several other studies on earth pressure problems using the finite element method show the suitability of this method in modelling such problems. The studies reported by Finn (1963), Clough and Duncan (1971), and Nadim and Whitman (1982) are examples of such work.

In this research, two finite element meshes are designed to model two identical backfills retained, in one case, by a cantilever wall and in the other case by a gravity wall. The behaviour of both walls and the effect of wall displacement on the magnitude and distribution of the earth pressure are studied. Before presenting this comparison, however, the results of a study on a gravity wall will be summarized to highlight a few important points regarding the calculation of lateral earth pressure by the finite element method and the design of the finite element mesh.

Lateral Earth Pressure behind Gravity Walls

This section reports a comparison of the field measurements of lateral earth pressure hehind a 10-meter-high gravity wall shown in Figure 2a (Matsuo et. al., 1978) with the results obtained by use of the finite element mesh shown in Figure 2b. The finite element mesh consisted of 247 two-dimensional isoparametric quadratic quadrilateral eight-noded elements. The interaction of the soil with the back and the base of the wall was modelled by two sets of interface friction elements. The backfill elements were assumed to represent a dry-cohesionless soil with the properties given in Table 2. The values of the initial tangent modulus (E), the unit weight (γ) and Poisson's ratio (ν) were all taken from the original paper by Matsuo et. al. (1978). The properties of the wall elements were chosen to represent typical values for concrete (see Table 2).

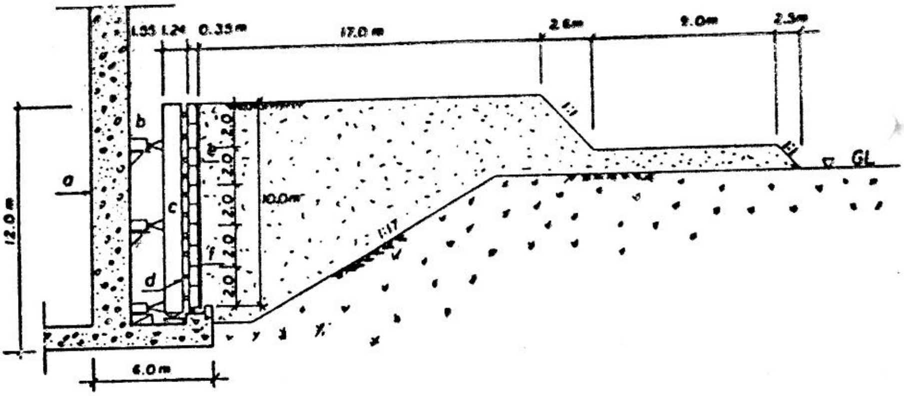


FIGURE 2a Gravity Retaining Wall Set-up used by Matsuo et al. (1978).

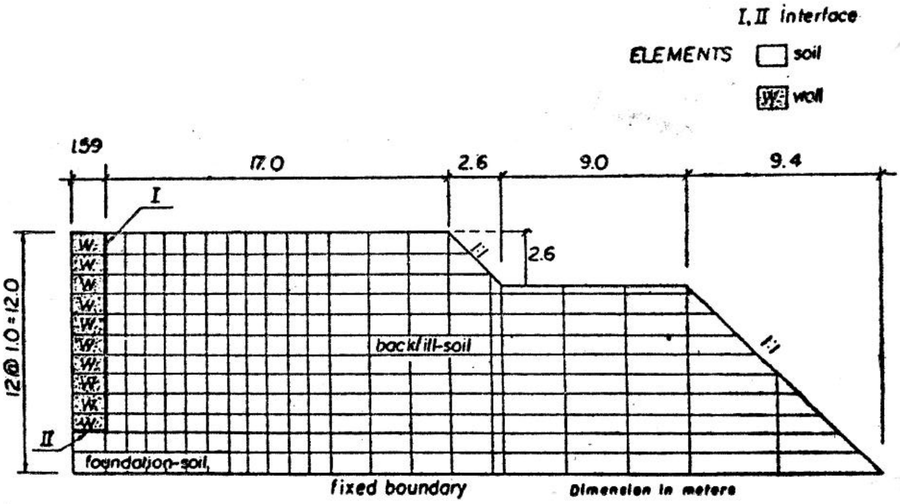


FIGURE 2b Finite Element Mesh Used for the Analysis to Represent the Test Set-up Conditions by Matsuo et. al. (1978).

TABLE 2

Material	Modulus of Elasticity E t/m^2	Strain per cent ϵ	Poisson's Ratio ν	Unit Weight γ t/m^3	Internal Friction Angle ϕ Degrees
Silty-Sand	1600*	0-1	0.31*	1.9*	27*
	1000	1-1.2			
	250	1.2-2.0			
	Failure	>2.0			
Concrete Yield > 2.0	1.4E6	0-2.0	0.15	2.3	

* Given by Matsuo et. al. (1978).

The boundary conditions of the mesh were selected such that the nodes along the base of the mesh were restrained against vertical and horizontal movements, thereby simulating the infinite extension in the foundation soil. All other nodes were allowed to move freely in all directions. The conditions of a free wall and a free lateral boundary, which represent the actual field conditions, were found to yield solutions in good agreement with field measurements (Bhatia and Bakeer, 1984a). The analysis was performed by the use of the computer program ABAQUS (Hibbitt et. al., 1980). Several computer runs were performed on the mesh for a non-displacing wall (N.D.W.) and for a wall translating or rotating in the direction away from the soil (D.W.). Figure 3 and Table 3 present a

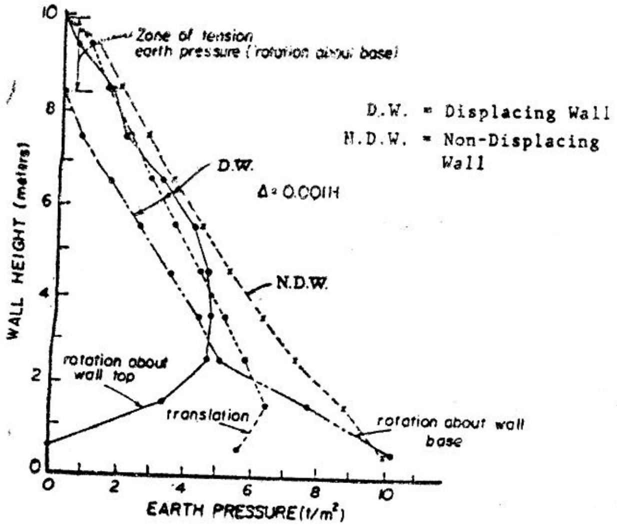
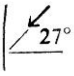
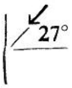
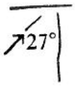
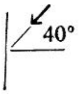
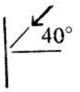
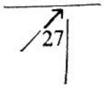


FIGURE 3 Earth Pressure Distribution Behind a Gravity Wall.

TABLE 3

Case	Reference	Initial Wall Displacement cm	Reaction Direction		Earth Pressure P ton/m^2	Coefficient of Earth Pressure K	Position of Resultant Y/H
			Wall Face	Wall Base			
N.D.W.	Matsuo et. al. (1978)	Assumed Zero		Unknown	52.3	0.55	0.34
	F.E.M.	0			54.0	0.57	0.34
	Matsuo et. al. (1978)	1.6 (0.0016H)		Unknown	38.0	0.40	0.27
	F.E.M.	1.6 (0.0016H)			30.8	0.33	0.33
D.W.	F.E.M.	0.1 (0.0010H)			43.1	0.45	0.34
	F.E.M.	1.6 (0.0016H)			39.4	0.42	0.30

sample of the results obtained by the finite element method along with those reported by Matsuo *et. al.* (1978). A complete presentation of the results is given by Bhatia and Bakeer (1984b).

From these results, the following observations can be made:

(1) The earth pressures obtained by the finite element method agree well with the field measurements. Agreement is within 5 per cent for earth pressure, its distribution, and the coefficient of earth pressure.

(2) As reported by Matsuo and co-workers, the finite element method yields a higher value for the earth pressure under rotation conditions than the value obtained by Coulomb's theory.

(3) The earth pessure distribution, and hence the point of application of the resultant, varies according to the roughness of the wall base. With the increase in roughness, the earth pressure ordinates near

the base of the wall diverge from the linear relationship, which pulls the earth pressure resultant to a lower position. The roughness of the wall back has a negligible effect on the earth pressure distribution since it influences the earth pressure distribution uniformly along the entire wall height, thereby changing the magnitude of the earth pressure but not its distribution.

(4) The magnitude and the distribution of the earth pressure are highly dependent on the amount of wall displacement as shown by the finite element method solutions (see Figs. 3 and 4). This conclusion is supported by the experimental findings of several investigators, including Matsuo et. al. (1978), Sherif et al. (1981 and 1984), and Foukoka et. al. (1981). This conclusion, however, contradicts present practices, which assume a single value of the earth pressure magnitude and distribution. In practice. this assumption is accounted for by large factors of safety. A displacement-oriented approach for evaluating the earth pressure should be developed to minimize the construction costs by accurately estimating these safety factors (see Bhatia and Bakeer, 1984c).

(5) The magnitude and distribution of the earth pressure are also dependent on the type of wall displacement, that is, rotation or translation, as shown by the results of the finite element method and experimental tests (Sherif et. al., 1981 and 1984). This factor should be considered in the design of the wall since the mode of displacement depends on a variety of factors, such as the embedment of the wall and the backfill strength parameters (Bhatia and Bakeer, 1984b).

Static Earth Pressure Behind Cantilever and Gravity Walls

This section presents the results of the study of two wall configurations, cantilever and gravity, retaining backfills of identical dimensions and properties. A 4.5 m high cantilever wall with a base embedded at a

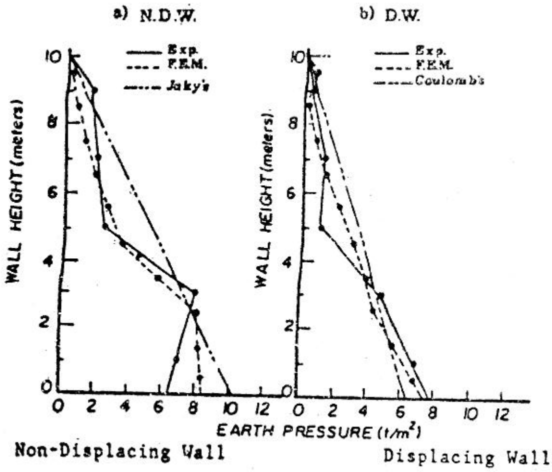


FIGURE 4 Comparison Between Finite Element and Experimental Results for a 10 m High Gravity Retaining Wall Tested by Matsuo et. al. (1978).

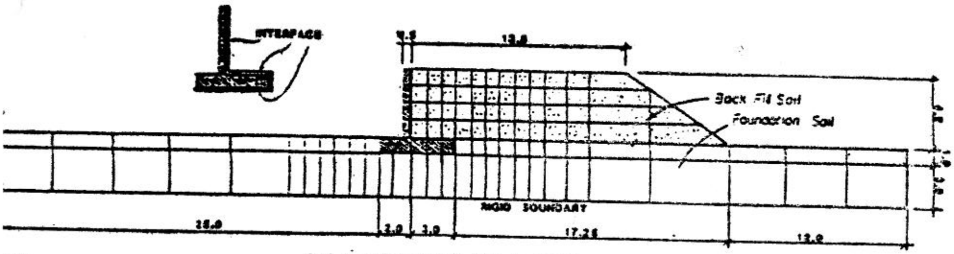
depth of 1 m is modelled by the finite element mesh shown in Figure 5a. The other two meshes shown in Figure 5b and 5c represent, respectively, a 4.5 m high gravity wall with no soil emedment and a 5.5 m high gravity wall with a soil embedment of 1 m on the wall-toe side. The purpose of using two gravity walls is to examine the effect of embedment on the behaviour of the wall and to compare this behaviour with that of the cantilever wall. The cantilever wall has two sections, a 4.5 m wall-face section, and a 5.5 m soil-face section with an embedded depth of 1 m. The soil and wall properties selected are given in Table 2. The walls are designed to have adequate factors of safety against both sliding and overturning for the active earth pressure forces calculated by Rankine's and Coulomb's theories.

In all meshes, the back and the base of the walls are connected to the soil elements by sets of friction elements with a coefficient of friction at the wall back corresponding to a friction angle of $\delta = 2/3\ \phi = 18$ degrees and a friction coefficient at the wall base corresponding to a friction angle of $\phi_b = \phi = 27$ degrees. No loads were assumed to be acting on the meshes other than the unit weight of the wall and soil materials. The nodes along the foundation-soil boundaries in the finite element mesh are restrained against horizontal and vertical movement to represent the infinite extension of the foundation-soil. The wall nodes and the nodes along the backfill lateral boundary are allowed to move freely, as are the remaining nodes in the meshes.

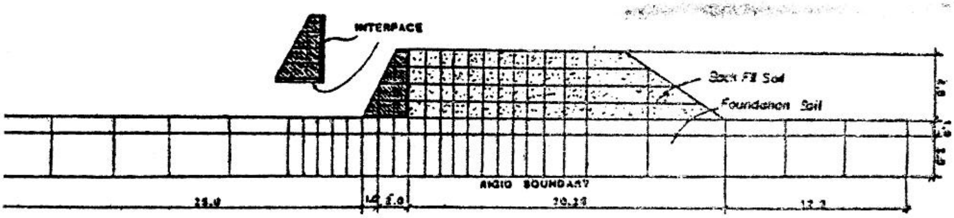
The three meshes were examined for the case of a nondisplacing wall (N.D.W.) as well as the case of an increasing wall displacement in the form of translation away from the backfill (D.W.). From the results, the following observations are made :

(1) For all walls, the strain levels are less than 0.2 per cent in the finite elements representing the soil. Higher strains are observed only at one Gaussian integration point in the finite element representing the soil next to the wall heel. High strains occur also in the soil element next to the wall toe in the case of an embedded wall. This indicates that a pure elastic analysis could be performed on retaining walls with dry-cohesionless backfills under static conditions, since the stress-strain behavior of most cohesionless soils may be considered linear up to a strain level of 0.2 per cent. Pure elastic analysis would result in a considerable savings in computer time and data storage.

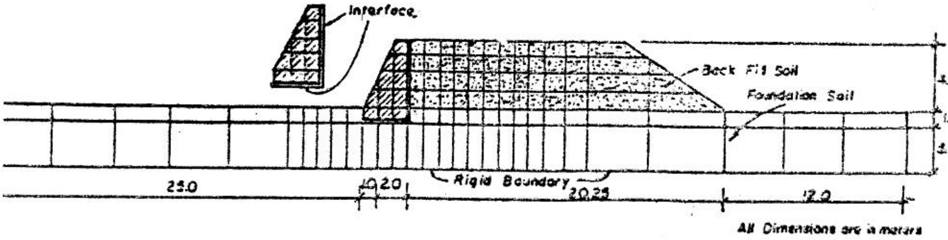
(2) The earth pressure distribution behind the non-embedded gravity wall is almost linear with the resultant acting at a level higher than the lower third point of the wall height (0.36H-0.37H) as shown in Figure 6b. For an embedded gravity wall, the earth pressure is nonhydrostatic with higher ordinates near the wall base because of the stiffness of the soil embedment, as shown in Figure 6a, and the consequent resistant to wall movements. This causes the lateral earth pressure resultant to act at a lower level (0.33H—0.34H) than for a non-embedded wall.



(a) CANTILEVER WALL MESH



(b) NON-EMBEDDED GRAVITY WALL MESH



(c) EMBEDDED GRAVITY WALL MESH

FIGURE 5 Finite Element Meshes Used to Represent a Cantilever Wall, a Non-Embedded Gravity Wall and an Embedded Gravity Wall.

(3) Compared to an embedded gravity wall, the horizontal earth pressure at the soil-face section of a cantilever wall is also nonhydrostatic but with smaller ordinates near the wall base. Its resultant acts at a level of 0.35H—0.36H from the wall base, as shown in Figure 7b. This distribution results from the effect of vertical friction at the toe side of the wall base, which reduces the horizontal component of the earth pressure near the wall base. Along the remaining portion of the wall height, the earth pressure is similar in distribution to that behind a nonembedded gravity wall. For an embedded gravity wall, on the other hand, the friction at the wall back reduces the horizontal component of the lateral earth pressure along the entire height of the wall, resulting in a smaller horizontal pressure than in the case of a cantilever wall of the same height. At the cantilever wall-face section, the earth pressure has higher ordinates than those of a non-embedded gravity wall near the stem-base connection because of the effect of fixation at that point. It also has higher ordinates near the wall top, since the soil mass above the wall base moves in perfect contact with the wall during displacement. This results in the distribution shown in Figure 7a with the resultant acting at a level of 0.4H—0.45H from the top surface of the heel

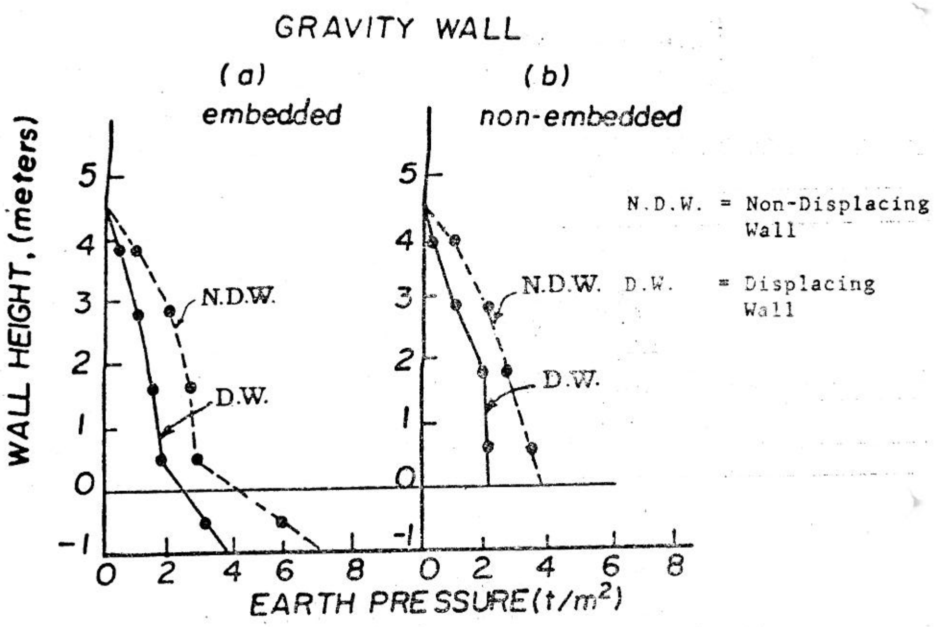


FIGURE 6 **Earthpressure Distribution Behind an Embedded and Non-Embedded Gravity Wall by Finite Element Analysis.**

of the wall. This behavior differs from that of the nonembedded gravity wall. Even if the cantilever wall displaces through translation, its stem still rotates about the connection with the base. Because of the great mass of the gravity wall, however, its expected movement is in the form of pure translation.

(4) In all cases, the lateral earth pressure decreases with the increase in the wall displacement away from the backfill. It was observed that the earth pressure reaches its constant minimum magnitude at a wall translation averaging 0.001H to 0.0015H. It should be noted that at a given wall displacement, the earth pressure at the cantilever wall face section is greater than that on the non-embedded gravity wall because of the stem fixation and the weight of the soil block above the cantilever wall base. The earth pressure at the soil-face section is greater than that behind the cantilever wall. This indicates that for walls of equal height, the design forces are in general greater on the cantilever wall sections than on the gravity wall.

(5) The results of the finite element analysis show that the non-embedded gravity wall displaces by translation away from the backfill by an average of 0.0012H, while the embedded wall also displaces by translation away from the backfill by an average of 0.001H.

The displacement at the top of the embedded wall is slightly higher than at its base, while the opposite pattern takes place in the non-embedded wall because of the resistance of the embedment.

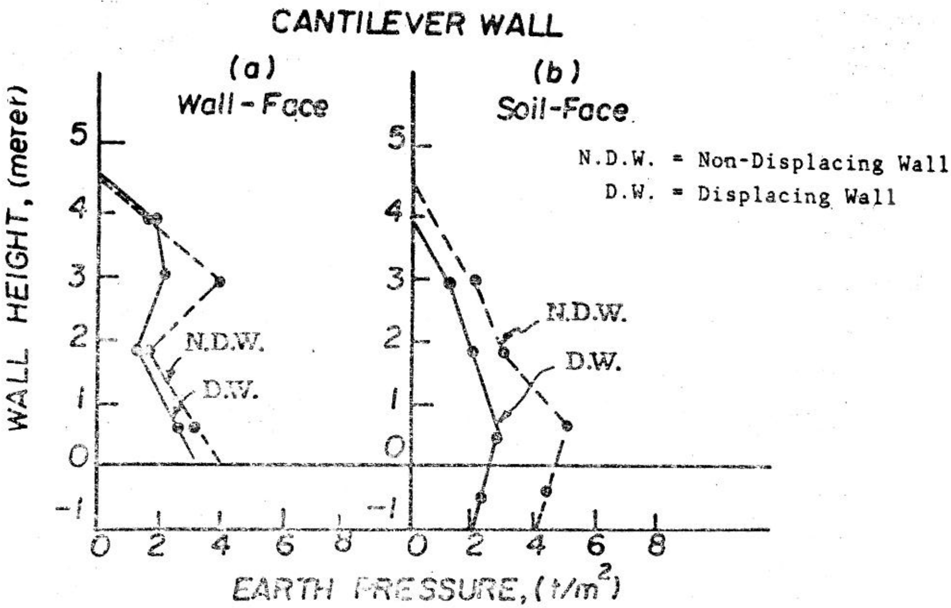
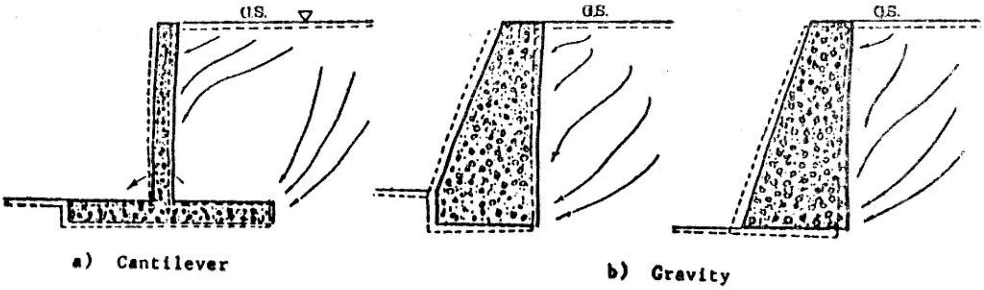


FIGURE 7 Earthpressure Distribution Behind a Cantilever Wall by Finite Element Analysis.

On the other hand, the cantilever wall displaces by overall rotation away from the soil and by local rotation of the stem about its connection with the wall base. Thus, soil embedment increases the earth pressure at the base of the wall and affects the point of application of the earth pressure resultant. Figure 8 shows the different displacement patterns described above.

Summary and Conclusion

Cantilever and gravity walls behave differently under static conditions. Gravity walls displace by translation, whereas cantilever walls displace by rotation. The earth pressure behind a gravity wall is less than that behind a cantilever wall of the same height. Both the earth pressure coefficient and the point of application of the earth pressure resultant should be obtained by a displacement-oriented approach since both are



a) Cantilever

b) Gravity

FIGURE 8 Displacement Patterns Obtained by Finite Element Analysis.

displacement dependent. Soil embedment increases the earth pressure near the wall base and affects the point of application of the resultant. This effect should be considered in designing wall sections and assessing their stability.

Acknowledgement

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