Stress Path Effects on the Shear Behaviour of Sand under Triaxial Compression and Extension

by

M. Krishna Murthy*
T.S. Nagaraj**
A. Sridharan**

Introduction

There are many practical situations in which the deformation conditions correspond to either compression or extension mode of shear with increasing or decreasing effective mean normal stresses. Stress-deformation of a soil element below a circular footing or the mobilization of active earth pressure in the soil adjacent to a retaining wall are the examples of compression mode of shear with increasing or decreasing mean normal stresses respectively. On the other hand, the passive resistance offered by the soil adjacent to a retaining wall or the bottom heave of an excavation corresponds to the extension mode of shear with increasing or decreasing mean normal stresses. For realistic assessment of mechanical properties of soils, it is necessary to simulate the appropriate mode of deformation and stress paths in the laboratory tests.

Based on the Mohr-Coulomb criterion, the shear test data obtained from the standard triaxial compression test is widely used even in the problems where the extension conditions prevail. So far as the cohesionless soils are concerned, various investigators have, however, reported the conflicting results with respect to the angle of shearing resistance in compression and extension tests (Bishop and Eldin, 1953; Kirkpatric, 1957; Peltier, 1957; Cornforth, 1964; Bishop, 1966; Barden and Proctor, 1971; Green, 1971; Thurairajah and Sithamparapillai, 1971; Reades and Green, 1976; Shankariah and Ramamurthy, 1980; Krishnamurthy, 1981). Further, many of these findings are based on tests employing conventional stress paths. In a conventional compression test, the axial stress ($\sigma_a = \sigma'_1$) is increased while holding the radial stress ($\sigma_r = \sigma'_2 = \sigma'_3$) constant so that the effective mean principal stress increases whereas in an extension test, the axial stress ($\sigma_a = \sigma'_3$) is decreased while holding the radial stress ($\sigma_r = \sigma'_1 = \sigma'_2$) constant so that the effective mean stress decreases. In many of these investigations, the peak strengths obtained from the compression and extension tests have been frequently compared although the effective mean principal stress varied differently in the two tests.

---

* Professor, Department of Civil Engineering, Sri Venkateswara University, Tirupati, INDIA.

** Professor, Department of Civil Engineering, Indian Institute of Science, Bangalore, INDIA.

(*This paper was received in November 1984 and is open for discussion till the end of August 1985*).

Further the available information on the effect of various other stress paths on the strength and stress-strain response of sand is limited. In view of these considerations, this paper presents the analysis of results obtained from several series of triaxial compression and extension tests carried out on a sand over a wide range of stress paths.

Experimental Work

A medium coarse Ennore sand was used in this study to prepare the saturated triaxial samples. It is a uniform sand passing a 1.18 mm sieve (ASTM sieve No. 16) and retained on a 600—μm sieve (ASTM sieve No. 30) with a uniformity Coefficient of 1.35 and a specific gravity of particles of 2.65. The maximum and minimum void ratios for the sand are found to be 0.92 and 0.62 respectively as per the method suggested by Kolbuszewski (1948). Triaxial tests were carried out on specimens 38 mm in diameter and 76 mm in height using enlarged lubricated end platens. Loose specimens with a relative density of 23 per cent were prepared by gently spooning freshly boiled sand into a sand former that was filled with deaired water while dense specimens having a relative density of 90 per cent were prepared by tamping in layers.

Incremental loading device

All tests have been carried out under stress control using incremental loading procedure which conveniently permitted to induce failure in the sample under any desired stress path appropriate to the field stress and strain conditions. An incremental loading device has been designed and fabricated for this purpose. Figure 1 shows the schematic diagram for the loading device arranged for compression test. It can be also used for the extension test with slight modifications. The incremental loading device consists of two pressure cylinders in which a loading piston operates hydraulically. The loading piston is guided in its vertical motion through a metal bushing and is sealed to the head by means of a rolling diaphragm called a Bellofram. The lower ends of the Bellofram pistons are connected (through a metal bar) to a proving ring that registers axial loads transferred to the cell plunger. To develop pressure inside the Bellofram, both pistons have been connected to a self-compensating mercury control through a manifold. Further details of design, assembly and testing techniques of the loading device are presented in a separate paper (Nagaraj et. al. 1981).

Stress Paths

Seven types of effective stress paths have been investigated in compression and extension tests with the incremental loading device and the procedures for stress path testing are as follows :

1. Increasing Mean Stress Path (IMS)

In a compression IMS test, the sample is isotropically consolidated and axial stress ($\sigma_a = \sigma'_1$ is increased while holding the radial stress ($\sigma_r = \sigma'_2 = \sigma'_3$) constant so that the effective mean principal stress increases. In an extension IMS test the radial stress ($\sigma_r = \sigma'_1 = \sigma'_2$) is increased

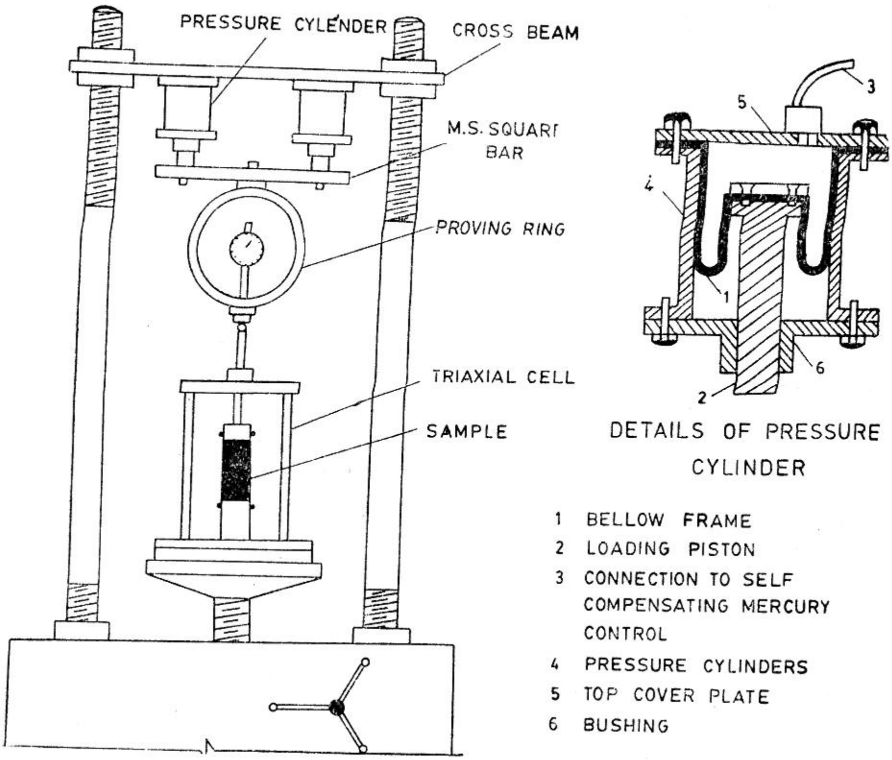


FIGURE 1 Incremental Loading Device

while holding the axial stress ($\sigma_a = \sigma_3'$) constant.

2. Constant Mean Principal Stress Path (CMS)

In the CMS test, the axial and radial stresses are varied so that the effective mean principal stress remains constant. In a compression CMS test the axial stress ($\sigma_a = \sigma_1'$) increases while the radial stress ($\sigma_r = \sigma_2' = \sigma_3'$) decreases. In an extension CMS test, the axial stress ($\sigma_a = \sigma_3'$) decreases and the radial stress ($\sigma_r = \sigma_1' = \sigma_2'$) increases. The sample is sheared under pure deviatoric stresses since the effective mean principal stress during shearing remains constant.

3. Decreasing Mean Stress Path (DMS)

In a compression DMS test, the radial stress ($\sigma_r = \sigma_2' = \sigma_3'$) is decreased while holding the axial stress ($\sigma_a = \sigma_1'$) constant. However, in an extension DMS test, the axial stress ($\sigma_a = \sigma_3'$) is decreased while holding the radial stress ($\sigma_r = \sigma_2' = \sigma_1'$) constant.

4. Increasing-Decreasing Mean Stress Path (IDMS)

An IDMS test is a combination of stress paths 1 and 3. During first phase of shearing the procedure is the same as the one followed in Stress Path SP 1. In a compression test, the axial stress σ_a is increased while holding the radial stress constant. In an extension test, the radial stress σ_r is increased while holding the axial stress constant. In the second phase the procedure is similar to that followed in SP 3. In a compression test, the radial stress σ_r is decreased while holding the axial stress constant. In an extension test the axial stress σ_a is decreased while holding the radial stress σ_r constant.

5. Decreasing-Increasing Mean Stress Path (DIMS)

A DIMS test is the reverse of SP 4. Initially the procedure is similar to SP 3 and later it is similar to that in SP 1.

6. Increasing-Decreasing-Constant Mean Stress Path (IDCMS)

During the first two stages, the procedure for IDCMS test is the same as in SP 4 (IDMS). In the final stage, the axial and radial stresses are both changed as in SP 2 (CMS) so that the sum of the principal stresses (attained at the end of second stage) remains constant, with the sample shearing to failure under pure deviatoric stresses.

7. Decreasing-Increasing-Constant Mean Stress Path (DICMS)

In the first two stages, the stress path follows as in SP 5 (DIMS). In the final stage, the sample is sheared to failure under pure deviatoric stress as in SP 2 (CMS).

Stress-Strain-Volume Change Behaviour

Figs. 2 and 3 show the typical stress-strain-volume change characteristics of specimens consolidated isotropically to 4 kg/cm^2 and later sheared under six different stress paths. The stress paths to which the specimens have been sheared are shown in the inset. Although the specimens have been sheared from the same preshear consolidation stress, stress-strain-volume change characteristics including strains to failure, rate of dilatancy are significantly affected by the variation of octahedral stress during the test. Referring to the compression tests, the specimens either loose or dense, exhibited the net volume expansion, the magnitude dependent on the stress path. In extension tests, however, the dense specimens exhibited initially the volume contraction and later expansion while the loose specimens experienced only volume contraction. The amount of net volume expansion or contraction depended on the stress path.

Unique Stress-Strain Behaviour

Figs. 4 and 5 show the results of compression and extension tests for the dense specimens having different consolidation pressures and stress paths. Fig. 4 shows the effective stress paths followed during shear in terms of $(\sigma_1-\sigma_3)$ and σ'_{oct} (i.e. the effective mean principal stress). Fig. 5 shows the stress-strain behaviour in terms of the normalised octahedral shear stress, τ_{oct}/σ_{oct}' and the the octahedral shear strain, γ_{oct}. The stress ratio τ_{oct}/σ_{oct}' at every point has been corrected for the energy spent

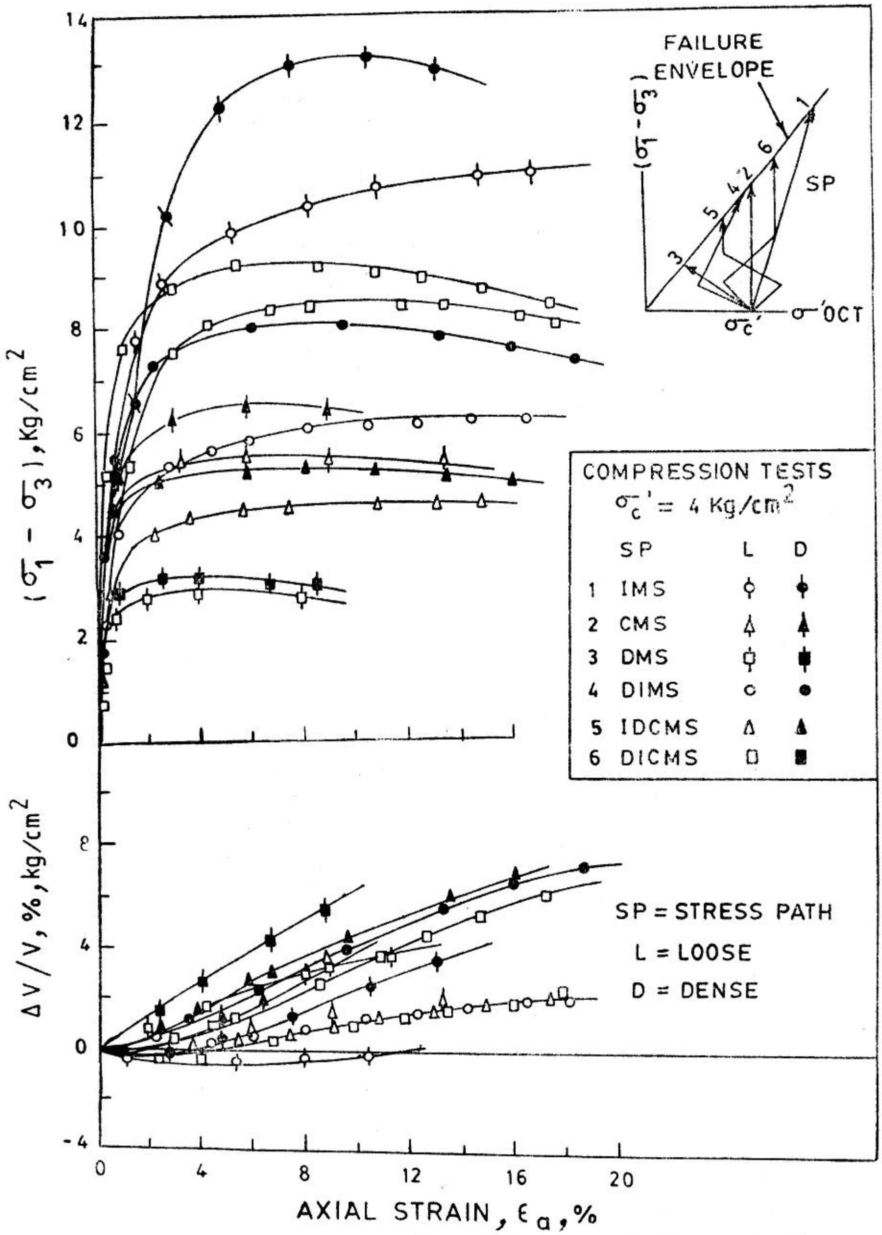


FIGURE 2 Stress-Strain-Volume Change Relationships for Compression Tests ($\sigma_c = 4$ kg/cm^2)

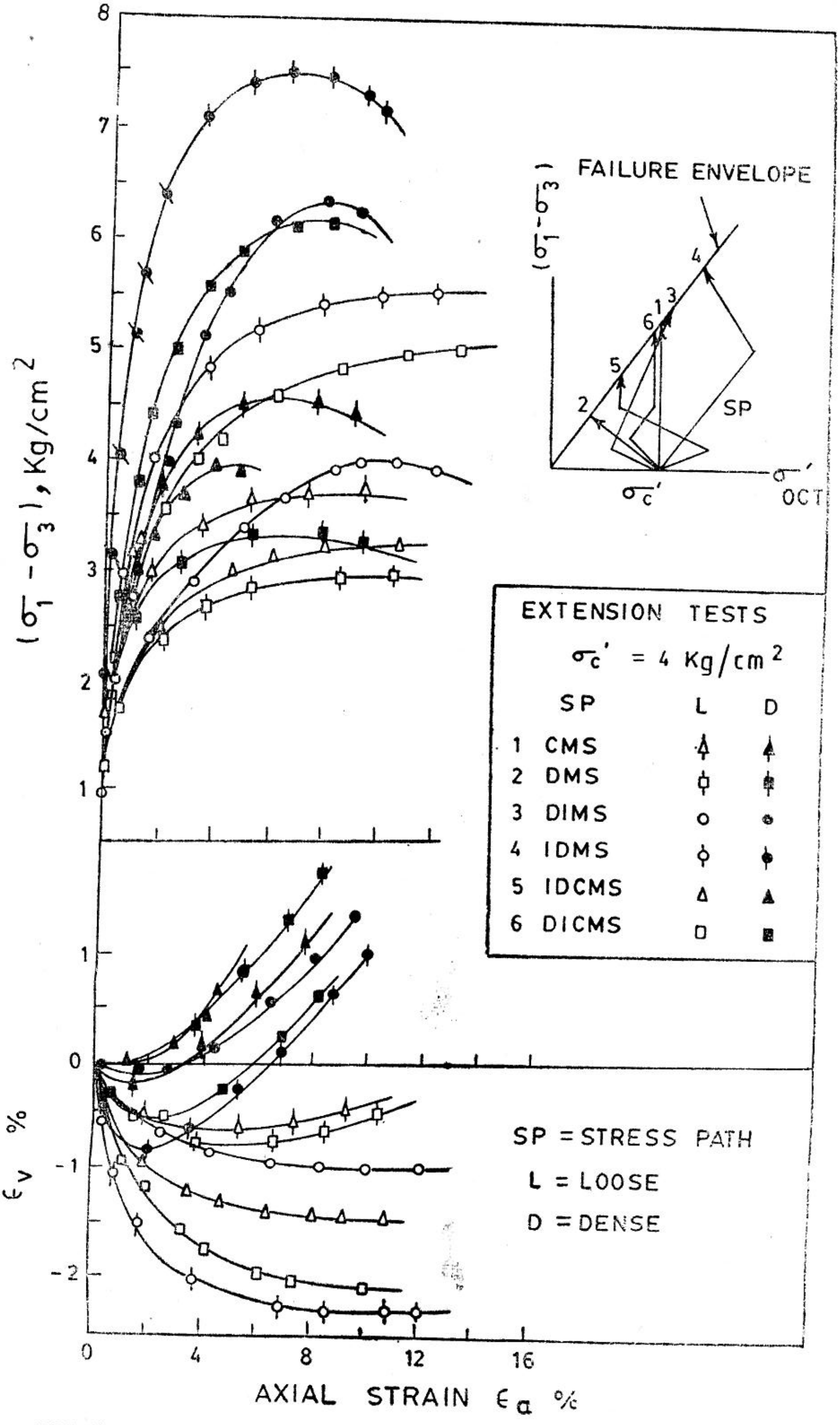


FIGURE 3 Stress-Strain-Volume Change Relationships for Extension Tests ($\sigma_c = 4$ kg/cm^2)

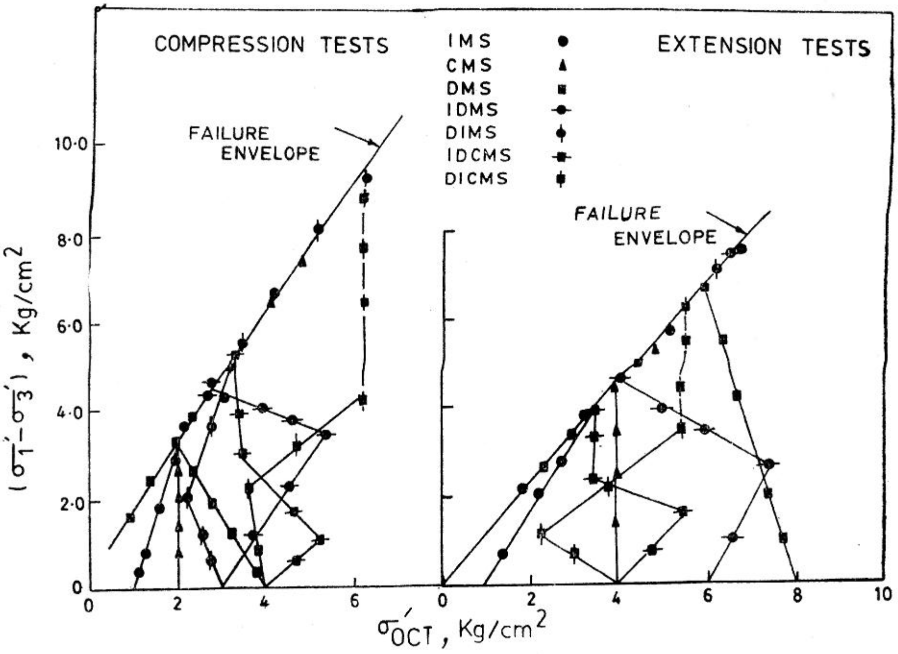


FIGURE 4 Effective Stress Paths for Compression and Extension Tests

in dilation as proposed by Bishop (1954). It can be seen that within the limits of experimental scatter, the normalised stress-strain behaviour is uniquely represented independent of both pre-shear consolidation stress as well as the stress path followed during the test. Similar behaviour was observed even for loose sand (Krishna Murthy, 1979). From the practical point of view, it is an important finding since the stress-strain behaviour of an element soil shearing under any stress path and stress level in the prototype structure can be predicted by the conventional compression/extension test.

Failure Envelopes

Separation of strength due to dilatancy

For a given material the factors which influence the volume change characteristics are the density and the level of external stresses applied. Even for the same density, the extent of dilation depends on the magnitude of the external stresses Thus, for the same density, the angle of shearing resistance ϕ' obtained from tests without considering the effect of dilatancy can not be used directly for external stress conditions other than those adopted in the test. So, many efforts have been made in the past to separate the component of strength due to dilatancy and obtain the shear strength corresponding to constant volume condition. The important analytical expressions for separating the strength component due to dilatancy applicable to triaxial compression are as follows :

$$(\sigma_1-\sigma_3)_r = \sigma_1-\sigma_3+\sigma_3' \frac{\delta\epsilon_v}{\delta\epsilon_a} \quad \text{Bishop, 1954)} \qquad \ldots (1)$$

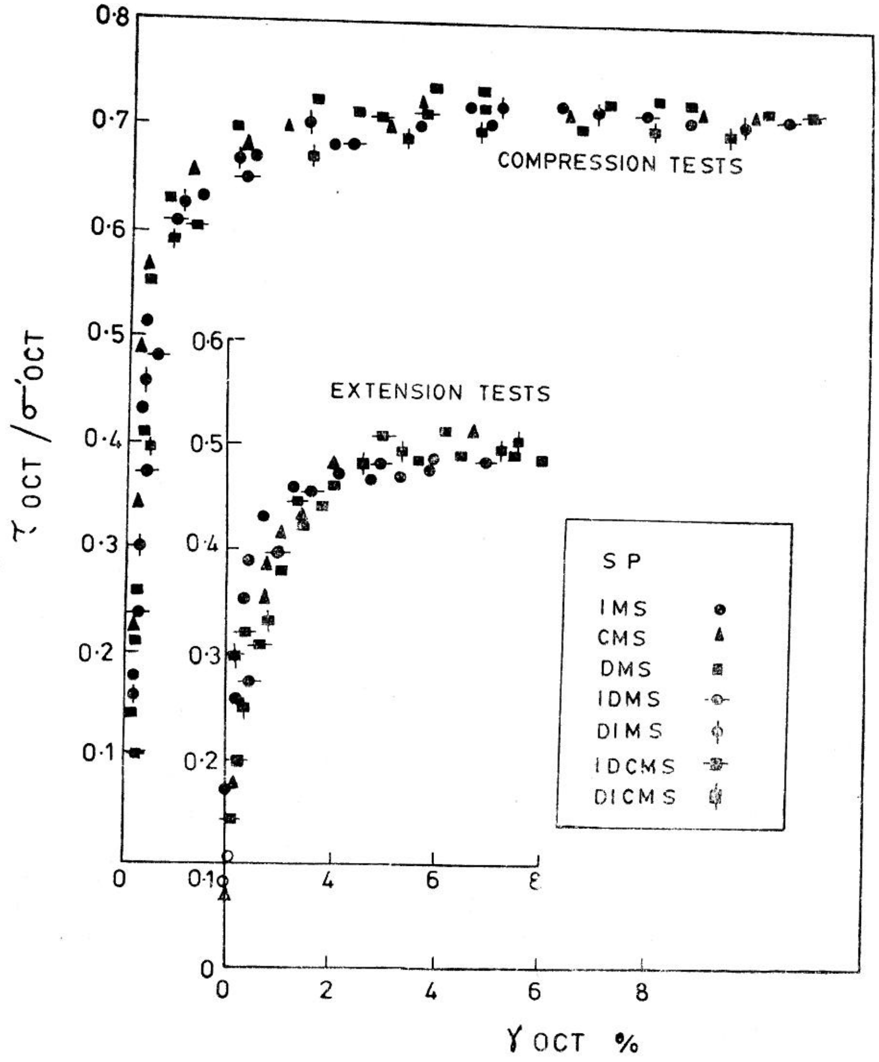


FIGURE 5 Octahedral Stress Ratio-Strain Relationships

$$(\sigma_1-\sigma_3)_R = (\sigma_1-\sigma_3) + \frac{(\sigma'_1+2\sigma'_3)}{3} \times \frac{\delta\epsilon_v/\delta\epsilon_a}{1-\frac{1}{3}(\delta\epsilon_v/\delta\epsilon_a)} \qquad \ldots (2)$$

(Poorooshasb and Roscoe, 1961)

Rowe et. al. (1964) extended the energy principle of Bishop (1954) to the triaxial extension test and gave the expression as

$$(\sigma_1-\sigma_3)_r = (\sigma_1-\sigma_3) - \sigma'_1 \frac{\delta\epsilon_v}{\delta\epsilon_a} \qquad \ldots (3)$$

where $(\sigma_1-\sigma_3)_r$ and $(\sigma_1-\sigma_3)_R$ are the corrected deviator stresses and $(\sigma_1-\sigma_3)$ is the measured deviator stress. $\frac{\delta\epsilon_v}{\delta\epsilon_a}$ is the rate of dilatancy. In all the above expressions, axial compression and volume decrease are considered as positive and axial extension and volume increase are taken as negative. In the present investigation, all the above relations have been used to separate the strength component due to dilatancy and obtain the strength corresponding to constant volume condition.

Compression Tests

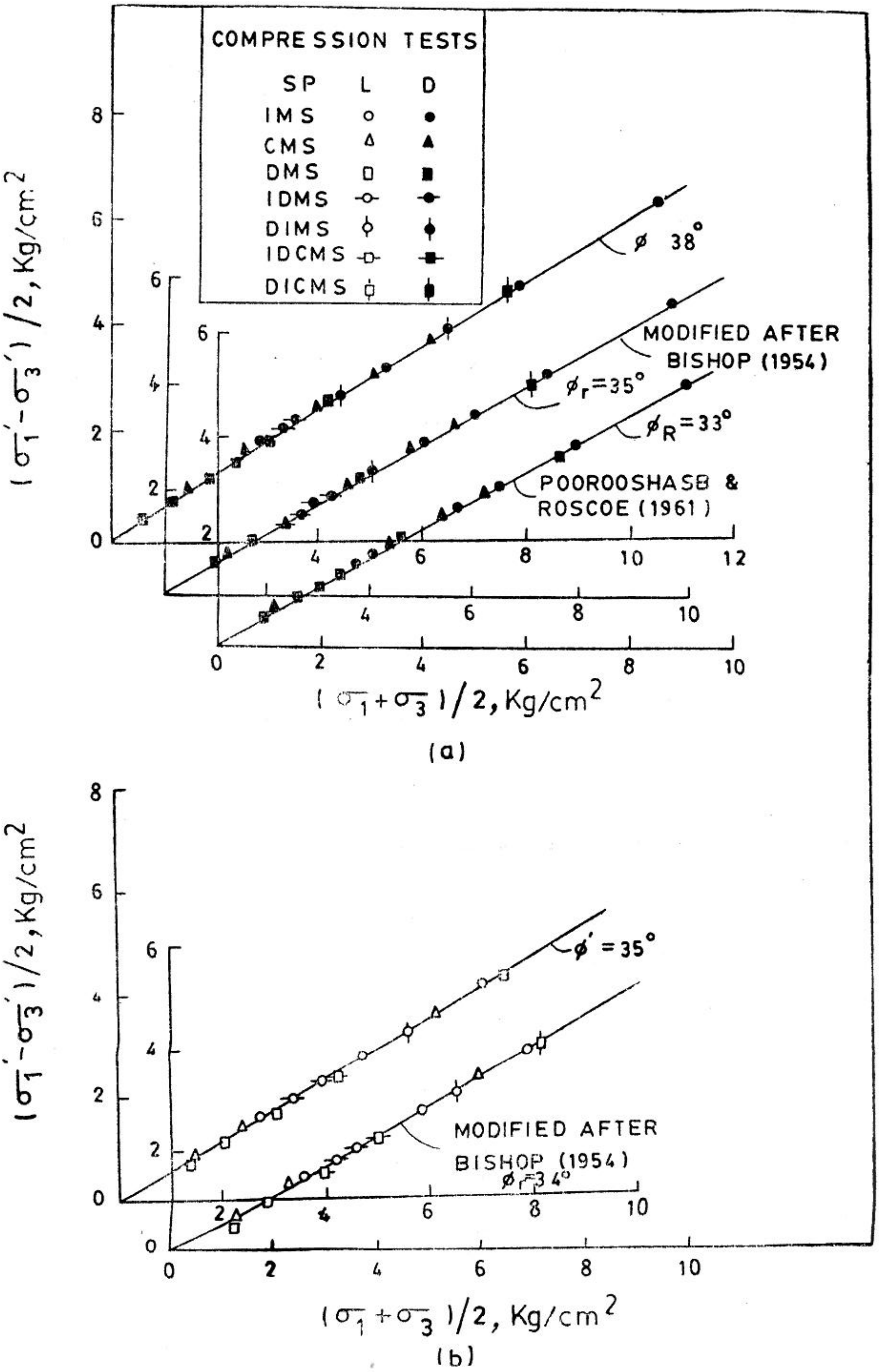
Fig 6 shows the modified Mohr-Coulomb strength envelopes for compression tests. The data points represent the failure stress conditions, defined at maximum principal stress difference, for the specimens tested over a wide range of consolidation pressures and stress paths. The test data demonstrate the remarkable uniqueness of the strength envelopes independent of both stress path and stress level. The difference in the angle of shearing resistance, ϕ', between the dense sand and loose sand is only 3°, probably due to its low uniformity coefficient and subrounded to rounded grains.

The test results also indicate that even after separating the dilatancy component using the expressions of Bishop (1954) and Poorooshasb and Roscoe (1961), the strength envelopes are uniquely defined independent of the stress path and stress level. For the dense sand, the angle of shearing resistance, ϕ_r, corrected for the volume dilatancy according to Bishop (1954) is slightly greater than ϕ_R which is corrected for dilation according to Poorooshasb and Roscoe (1961).

Extension Tests

In Fig 7 are shown the modified Mohr-Coulomb diagrams for the loose and dense specimens tested over a wide range of extension stress paths. The strength envelopes are uniquely defined independent of the stress path even in the extension tests. The test results also show the uniqueness of the envelopes even after separating the dilatancy component as suggested by Rowe et al (1964).

For the loose specimens, the difference in ϕ' between the compression and extension tests is negligible (Figs. 6 and 7) while for the dense specimens, the extension ϕ' is 7° greater than the compression ϕ'. These findings are in conflict with some workers and in agreement with others. Peltier (1957) and Shankariah and Ramamurthy (1980), among others, reported that the extension ϕ' was less than the compression ϕ', where as, Bishop and Eldin (1953); Kirkpatric (1957); Cornforth (1964); Bishop (1966) and Barden and Proctor (1971) have reported the same values of ϕ' in compression and extension tests. However, Green (1971) and Thurairajah and Sithampara Pillai (1971) have reported higher values of ϕ' in extension tests. Reades and Green (1976) reported that for the dense specimens the extension ϕ' was 7° greater than the compression ϕ' while for the loose specimens the difference was small or nil. Krishna Murthy (1981) also reported the similar results. A careful examination of the results reported in the literature indicates that the difference in ϕ' between compression and extension tests appears to be a function of the relative density. The difference is small or nil in loose state but it might increase with the relative density so that the maximum difference is observed in the densest state. The findings reported in this investigation are, in general, agree with this concept.



(a)

(b)

FIGURE 6 Mohr-Coulomb Envelopes for Compression Tests

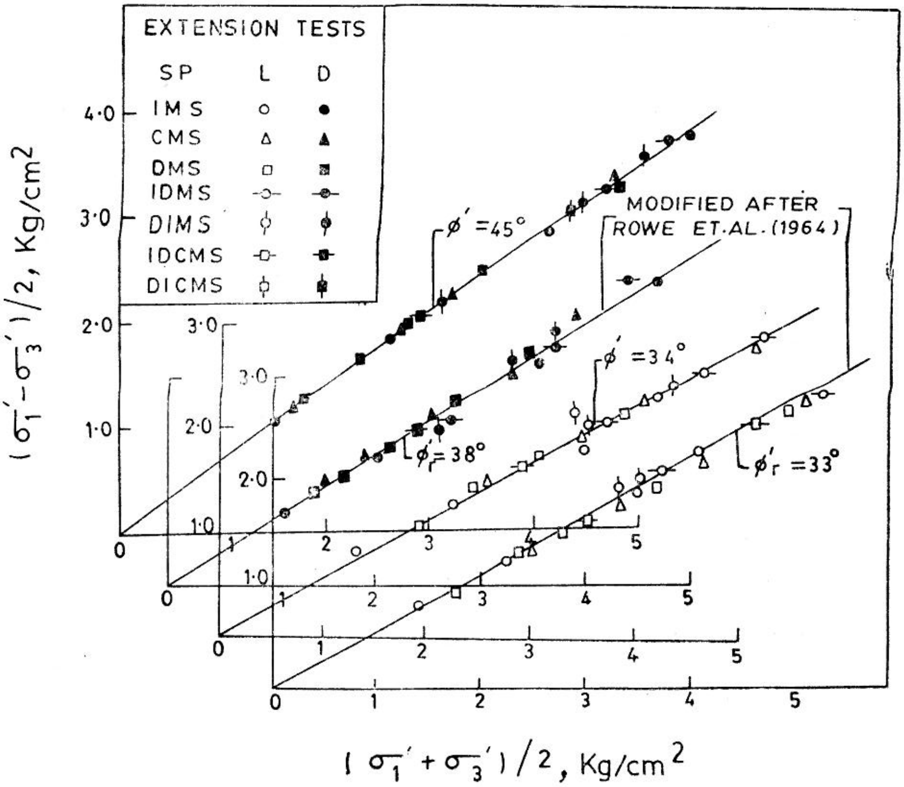


FIGURE 7 Mohr-Coulomb Envelopes for Extension Tests

Loose specimens formed by the raining technique can be generally expected to be in an isotropic state and the effect of the intermediate principal stress associated with the mode of shear (compression/extension) on such specimens is obviously negligible. Hence the difference in the angle ϕ' betwcen compression and extension tests is found to be small or nil. However, the dense specimens have been formed by tamping in layers and therefore some inherent cross anisotropy can be expected in such specimens. The difference in the angle ϕ' for the dense sand between compression and extension tests may perhaps be attributed to the coupled effect of the intermediate principal stress associated with the mode of shear and the inherent anisotropy of sand in the dense state. Perhaps, it is also due to the fact that, with the application of deviatoric stresses, the structural changes that develop in the dense specimen in the compression test do not occur or occur less intensively in the extension test.

Conclusion

Stress-strain-volume change characteristics of loose and dense specimens are greatly influenced by the stress path, stress level and mode of deformation (compression/extension). Nevertheless, by normalisation of stresses, it is possible to show unique stress-strain behaviour independent of stress path and stress level, thereby enabling the formulation of an unique constitutive stress-strain relationship. Thus, for a particular mode of shear (compression/extension), the stress-strain behaviour of an element of soil shearing under any stress path and stress level in the prototype structure can be predicted by the corresponding conventional compression/extension test.

In both compression and extension tests, loose and dense specimens have separate unique Mohr-Coulomb envelopes independent of the stress path and stress level (for the range of stresses considered).

The test results have a significant bearing on many practical problems dealing with coarse grained soils. The influence of the extension mode of shear associated with the application of stresses in the field can indeed be very important in problems like excavation, anchor piles, tensionleg offshore platforms etc., where extensive regions within the soil medium undergo unloading in a manner similar to the triaxial extension test. In deep cuttings where the extension shear conditions prevail, the factor of safety in the dense sands will be considerably underestimated if the angle ϕ' obtained from the compression tests is used in the stability analysis. Further, what affects the ultimate safety of the structure against shear failure is the mode of shear (compression/extension) rather than the stress path (i.e. the manner in which the mean effective principal stress changes during shear) as demonstrated by this investigation so far as the cohesionless sands are concerned.

References

BANDEN, L. and PROCTOR, D.C. (1971): "The Drained Strength of Granular material". *Canadian Geotechnical Journal*, 8 : 3 : 372-383.

BISHOP, A.W. (1954): "Correspondence on Shear Characterisitcs of a Saturated Silt Measured in Triaxial Compression". *Geotechnique*, 14: 43-45.

BISHOP, A.W. (1966): "The Strength of Soils as Engineering Materials (6th Rankine Lecture)", *Geotechnique*, 16: 2: 91-128

BISHOP, A.W. and ELDIN, A.K.G. (1953): "The Effect of Stress History on the Relation Between ϕ and the Porosity of Sand". *Proc. 3rd Int. Conf. Soil Mech. and Found. Engg.*, 1: 100-105

CORNFORTH, D.H. (1964): "Some Experiments on the Influence of Strain Conditions on the Strength of Sand". *Geotechnique*, 14 : 2 : 143-167.

GREEN, G.E. *(1971)* : *"Strength* and Deformation of Sand Measured in an Independent Stress Control Cell". *Proc. Roscoe Memorial Symposium, Cambridge*, pp. 285-323.

KIRKPATRIC, W.M. (1957) : "The Condition of Failure for Sands". *Proc. 4th Int. Conf. Soil Mech. and Found Engg.*, 1 : 172-178.

KOLBUSZEWSKI, J.J. (1948) : "An Experimental Study of the Maximum and Minimum Porosities of Sand". *Proc. 2nd Int. Conf. Soil Mech. and Found. Engg.*, Vol. 1.

KRISHNA MURTHY, M. (1979) : "*Studies on Stress-Deformation and Strength of Soils under Triaxial Compression and Extension*". Ph.D. thesis, Indian Institute of Science, Bangalore, India.

KRISHNA MURTHY, M. (1981) : "Stress Controlled Compression and Extension Tests on Sand". *Proc. IGS Conf. on Geotechnical Engg, Geomech-81, Hyderabad*, 1 : 285-289.

NAGARAJ, T.S., KRISHNA MURTHY, M. and SRIDHARAN, A. (1981) : "Incremental Loading Device for Stress Path and Strength Testing of Soils" *American Society for Testing and Materials, Geotechnical Testing Journal* 4 : 2 : 74-78.

PELTIER, M.R. (1957) : "Recharches Experimentals sur la courbe Intrinisique de rupture des sols Pulverulents". *Proc. 4th Int. Conf. Soil Mech. and Found. Engg.*, 1 : 179-182.

POOROOSHASB, H.B. and ROSCOE, K.H. (1961): "The Correlation of the Results of Shear Tests with Varying Degrees of Dilation". *Proc. 5th Int. Conf. Soil Mech. and Found. Engg.*, 1 : 297-304.

READES, D.W. and GREEN, G E. (1976) : "Independent Stress Control and Triaxial Extension Tests on Sand". *Geotechnique*, 26 : 4 : 551-576.

ROWE, P.W., BARDEN, L. and LEE, I.K. (1964): "Energy Components During the Triaxial Cell and Direct Shear Tests". *Geotechnique*, 14 : 247-261.

SHANKARIAH, B. and RAMAMURTHY, S. (1980) : "Axisymmetric Compression and Extension of Anisotropic Sand". *Proc. IGS Conf.* on *Geotechnical Engg., Geotech-80, Bombay*. 1 : 77-81.

THURAIRAJAH, A. and SITHAMPARA PILLAI, V. (1971) : "Strength-Deformation of Sand during Drained Triaxial Tests". *Proc. 4th Asian Conf. Soil Mech. and Found. Engg., Bangkok*, 1 : 177-182.

Notation

ϕ' = Angle of shearing resistance, effective stresses

ϕ_r = Angle of shearing resistance, corrected for volume dilatancy (Bishop, 1954)

ϕ_R = Angle of shearing resistance, corrected for volume dilatancy (Poorooshasb and Roscoe, 1961)

τ_{oct} = Octahedral shear stress

$$= 1/3\ \sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}$$

σ'_{oct} = Effective octahedral normal stress

$$= (\sigma'_1 + \sigma'_2 + \sigma'_3)/3$$

γ_{oct} = Octahedral shear strain

$$= 1/3\ \sqrt{(\epsilon_1-\epsilon_2)^2+(\epsilon_2-\epsilon_3)^2+(\epsilon_3-\epsilon_1)^2}$$