Dynamic Characteristics of Tapered Piles

by

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Introduction

Dynamics of piles has been receiving attention mainly due to its applications in machinery foundations and structures exposed to dynamic forces such as those produced by wind, earthquakes or operation of machines. The response of the structures to these loads depends to a high degree, on the stiffness and damping generated by the piles. These characteristics of the piles are products of the interaction between the pile and the surrounding soil. This interaction being very complex, the dynamic behaviour of piles is far from being completely understood.

In recent years, significant progress has been made in the dynamic analysis of pile foundations. The earlier methods of analysis were most often based on the concept of the equivalent cantilever (Hayashi et al, 1965; Prakash and Sharma, 1968; Singh et al, 1977), that does not allow to take into account of energy dissipation or a number of other factors (Novak, 1982). Penzien (1970), Prakash and Chandrasekharan (1973), and Kuhlemeyer (1981) presented lumped mass solutions for piles under dynamic loads. The more recent approaches (Novak, 1974, 1977; Nogami and Novak, 1976; Novak and Grigg, 1976; Novak and Aboul-Ella, 1978) consider the soil as a continuum and allow for dynamic soil-pile interaction, energy dissipation through propagation of elastic waves and also group effects (Nogami, 1983; Novak and Sheta, 1982). Equally realistic but more complicated finite element solutions (Blaney et al, 1976; Kuhlemeyer, 1976; 1979, Wolf and Von Arx, 1978) are also available. Brief descriptions and evaluation of the available methods are presented by Tajimi (1977) and Roesset (1980). These analytical and numerical studies on piles indicate that dynamic pile-soil interaction modifies pile stiffness, which is frequency-dependent and generates geometric damping through energy radiation.

All the above contributions in the field of pile dynamics relate to the study of the dynamic characteristics of cylindrical piles only. To the best of authors' knowledge, no work has been initiated so far on the dynamic response of foundations supported by tapered piles, although tapered piles are quite often used to support foundations and structures subjected to dynamic loads. In this paper, a simple approximate method is presented for dynamic analysis of foundations supported by tapered piles, that accounts for soil-pile interaction in relatively simple manner and predicts dynamic response of tapered piles with a fair degree of accuracy. A computer-based numerical approach is presented, as the dynamic soil reactions become complicated with the pile diameter reducing with depth.

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(This paper was received in September 1984 and is open for discussion till the end of July, 1985).

The analysis is restricted to the vertical vibration of tapered piles driven into a soil stratum that can be approximated as a homogeneous elastic half-space.

Analysis

Nogami and Novak (1976) and Novak (1974) assumed fixed tip conditions for cylindrical piles subjected to vertical vibration. Subsequently Novak (1977) considered the motion of the pile tips. Experiments (Novak and Grigg, 1976) have also shown that the vertical response is highly affected by the motion of the tip, as the axial stiffness of the pile is very high. Since the inclusion of the motion of the pile tip in a rigorous way is extremely difficult, an approximate approach is chosen in this paper.

The main assumptions are : (1) The pile is vertical, elastic and of circular cross-section but uniformly tapered down with depth and has a perfect contact with the soil; (2) the soil is assumed to be elastic, isotropic, homogeneous semi-infinite medium; (3) The soil reactions acting on the tip and also on the projected (Fig. 1) annular horizontal surfaces of the pile

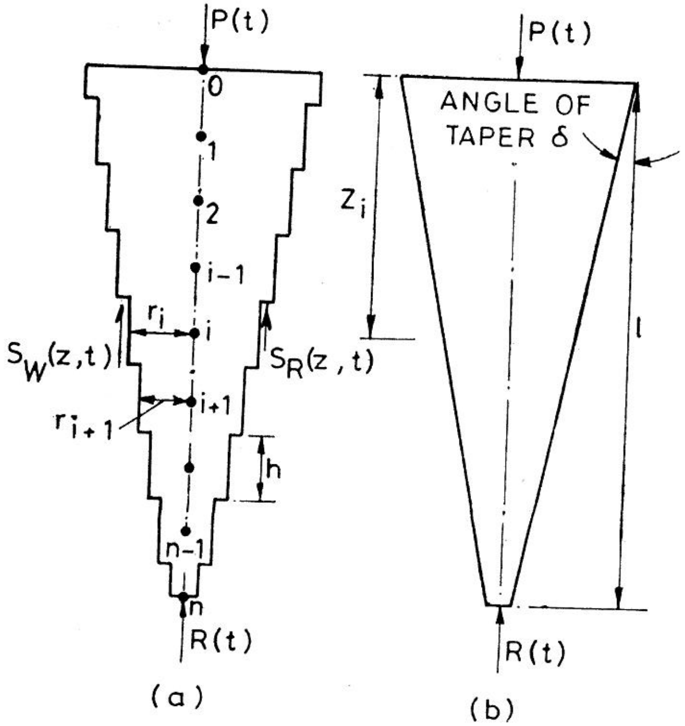

FIGURE 1 Tapered Pile (*a*) Idealised and (*b*) Real

elements are equal to that of an elastic half-space; (4) The motion of the pile is small and the excitation is harmonic, i.e.,

$$W(z, t) = W(z).e^{i\omega t} \qquad ...(1)$$

in which $W(z)$ = complex amplitude at depth z, $i = \sqrt{-1}$, t = time and ω = circular frequency.

The differential equation of damped vibration of a tapered pile element at depth z can be written as

$$m(z)\frac{\partial^2 W(z,t)}{\partial t^2} + C\frac{\partial W(z,t)}{\partial t} - EA(z)\frac{\partial^2 W(z,t)}{\partial z^2} + \{S_W(z)+S_R(z)\} W(z,t) = 0 \quad \ldots(2)$$

in which, $m(z)$ = mass per unit length of pile

C = coefficient of pile material damping

E = Young's modulus of pile material

$A(z)$ = cross-sectional area of pile

$S_W(z)$ = soil reaction function on the projected vertical surface of the pile element

$S_R(z)$ = soil reaction function on the projected annular horizontal surface of the pile element.

On substitution of Eq. (1) into Eq. (2) and neglecting pile material damping, the Eq. (2) is reduced to

$$EA(z)\frac{d^2W(z)}{dz^2} + \{m(z)\omega^2 - S_W(z) - S_R(z)\} W(z) = 0 \quad \ldots(3)$$

in which $$S_W(z) = F_W(z) + i\omega C_{Wz}(z) \quad \ldots(4)$$

$F_W(z)$ = elastic stiffness function derived using Mindlin (1936) solution

$C_{Wz}(z)$ = equivalent viscous damping function

$$= \frac{2G\, C_{U2}(z)}{\omega} \quad \ldots(5)$$

where $C_{U2}(z)$ = half-space damping parameter (Horizontal mode) depending on Poisson's ratio of soil ν and frequency factor $a_o(z) = \omega r(z)/V_s$, V_s being shear wave velocity in soil and $r(z)$ is the radius of the pile at depth z, and G = shear modulus of soil.

$$S_R(z) = Gr(z)\{C_{W1}(z) + iC_{W2}(z)\} - Gr(z+dz)\{C_{W1}(z+dz) + iC_{W2}(z+dz)\} \ldots(6)$$

where

$C_{W1}(z)$, $C_{W2}(z)$ = half-space stiffness and damping parameters (vertical mode) at depth z and depend on ν and $a_o(z)$, and $C_{W1}(z+dz)$, $C_{W2}(z+dz)$ = half-space stiffness and damping parameters (vertical mode) at depth $(z+dz)$ and depend on ν and $a_o(z+dz)$. All the half-space stiffness and damping parameters are derived after Arnold et al (1955) and Hsieh (1962) and shown in Fig. 2.

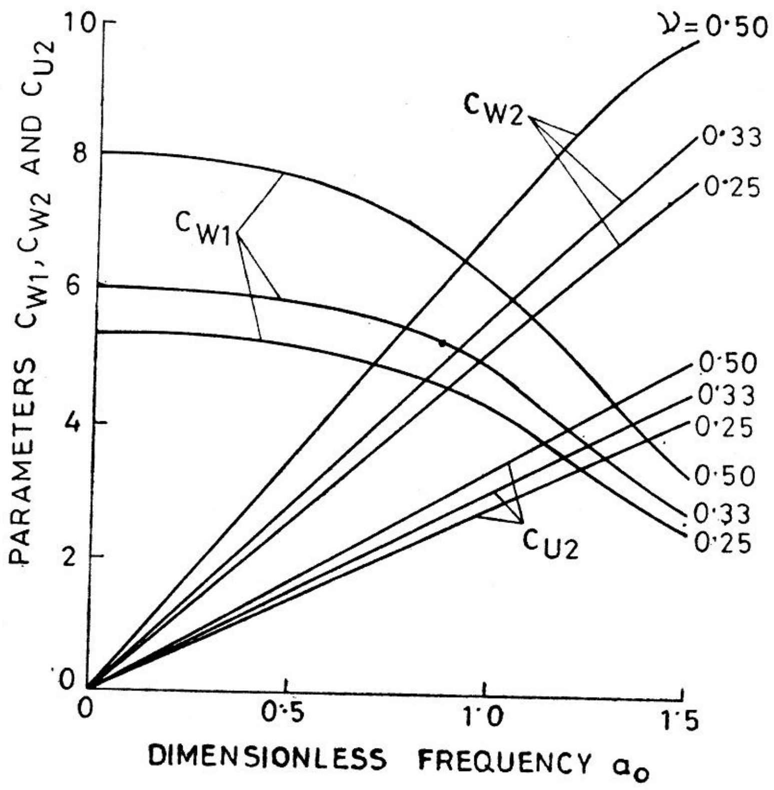

FIGURE 2 Half-Space Stiffness and Damping Parameters.

Boundary Conditions

The complex stiffness at the pile head is defined as the force producing unit displacement of the pile head and hence the first boundary condition is :

$$\text{At } z = 0, \quad W(z) = W(o) = 1 \qquad \ldots(7)$$

The amplitude of half-space soil reaction generated at the pile tip may be written as

$$R = -G_b r(1)\{C_{W1}(1) + iC_{W2}(1)\}\, W(1) \qquad \ldots(8)$$

where

G_b = shear modulus of soil below pile tip

$r(1)$ = pile tip radius at depth $z = 1$

$C_{W1}(1)$, $C_{W2}(1)$ = half-space stiffness and damping parameters at depth $z = 1$ and for $a_o = \omega\, r(1)/V_b$, V_b being shear wave velocity in soil below pile tip

$W(1)$ = complex displacement of pile tip.

Since the end force in the pile has to be equal to the soil reaction given by Eq. (8), the second boundary condition may be written as

$$EA(1)\ \frac{dw(1)}{dz} = G_b r(1)\{C_{W1}(1)+iC_{W2}(1)\}W(1) \qquad \ldots(9)$$

Numerical Solution

For the convenience and simplicity of analysis the tapered pile has been idealised to be a step-tapered one, dividing the pile into n number of steps. Satisfying the boundary conditions, finite difference forms of Eq. (3) are written for all the step elements of the pile. The set of n complex simultaneous equations so obtained can be written in the following form :

$$[A]\{W\} = \{B\} \qquad \ldots(10)$$

in which

$[A]$ = complex coefficient matrix in banded form,

$\{W\}$ = complex pile step (node) displacement

$\{B\}$ = complex load vector.

The solution of the Eq. (10) can be obtained on a computer with a suitable computer program for solution of complex simultaneous equations. At depth z, the complex displacement is

$$W(z) = W_R+iW_I \qquad \ldots(11)$$

where W_R and W_I are the real and imaginary parts of the complex displacement $W(z)$. The complex stiffness at the pile head can be expressed as

$$K_z^C = K_z+iC_z \qquad \ldots(12)$$

The real part of the complex stiffness is the stiffness constant of the tapered pile which can be written as

$$K_z = \frac{EA(o)}{r(o)} f_{W_1} \qquad \ldots(13)$$

and the imaginary part of the complex stiffness is the equivalent viscous damping of the tapered pile, which is written as

$$C_z = \frac{EA(o)}{V_s} f_{W2} \qquad \ldots(14)$$

where

f_{W_1}, f_{W_2} are the dimensionless stiffness and damping parameters of the tapered pile, which are presented in the following forms :

$$f_{W_1} = -\frac{r(o)}{2h}\{-3\ W_{01}+4W_{11}-W_{21}\} \qquad \ldots(15)$$

$$f_{W_2} = -\frac{V_s}{\omega}\ \{-3\ W_{02}+4W_{12}-W_{22}\} \qquad \ldots(16)$$

where

W_{01}, W_{11}, W_{21} = real parts of complex displacements at nodes 0, 1, 2 (Fig. 1) and

W_{02}, W_{12}, W_{22} = imaginary parts of complex displacements at nodes 0, 1, 2.

Then, the vertical response of a rigid footing supported by a tapered pile is

$$A_W = \frac{M}{m_e.e}.\ A_o = \frac{\omega^2}{\sqrt{(K_z/M-\omega^2)^2+(\omega\ C_z/M)^2}} \qquad \ldots(17)$$

where

A_W = dimensionless amplitude of vertical vibration at frequency ω

M = mass of the footing

$m_e.\,e$ = eccentric mass moment of excitation

A_o = real displacement amplitude at frequency ω

Results and Discussion

To examine the validity and accuracy of the proposed method of analysis of a tapered pile, an attempt has been made to compare the theoretical solutions with the experimental results reported by Novak and Grigg (1976) for piles with angle of taper $\delta = 0$. In Fig. 3 both

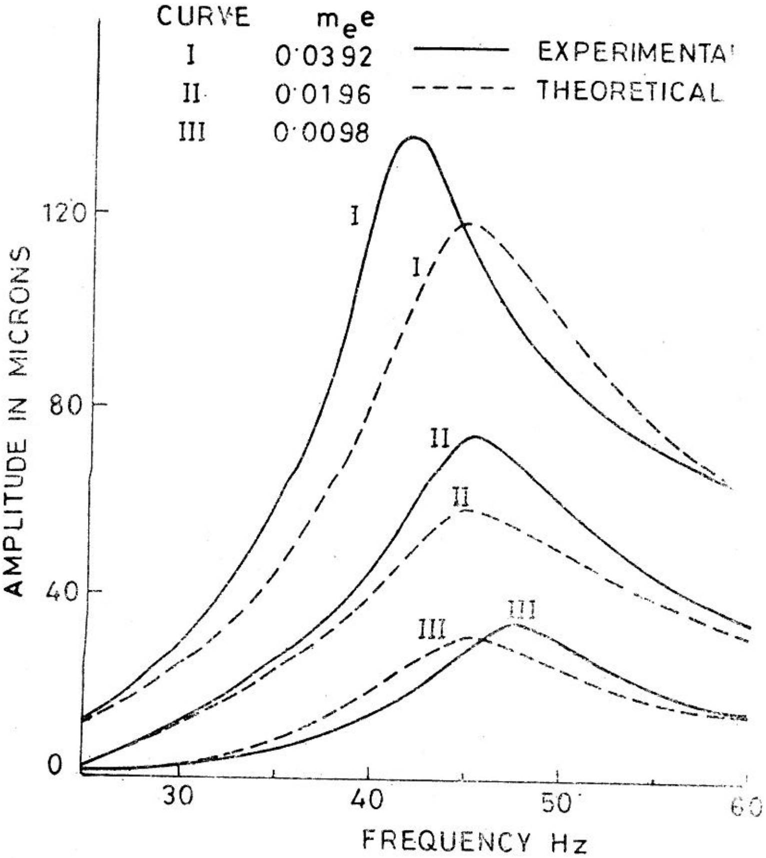

FIGURE 3 Comparison of Experimental and Theoretical Response.

theoretical and experimental response curves are plotted. The three sets of curves are presented for three different values of eccentric mass moment of excitation $m_e.e$. The nonlinearity of experimental curves is demonstrated by the shifting of resonant frequencies. Theoretical curves do not show such nonlinearity. It is observed that the agreement between experimental and theoretical curves is very good. The theoretical resonant amplitudes are somewhat less than those obtained from experiments. The agreement between the predicted and experimental resonant frequency is very satisfactory.

Some response curves of tapered piles for different values of angle of taper δ and wave velocity ratio V_s/V_c, V_c being the compressional wave velocity in pile, are shown in Fig. 4. It is observed that with increase

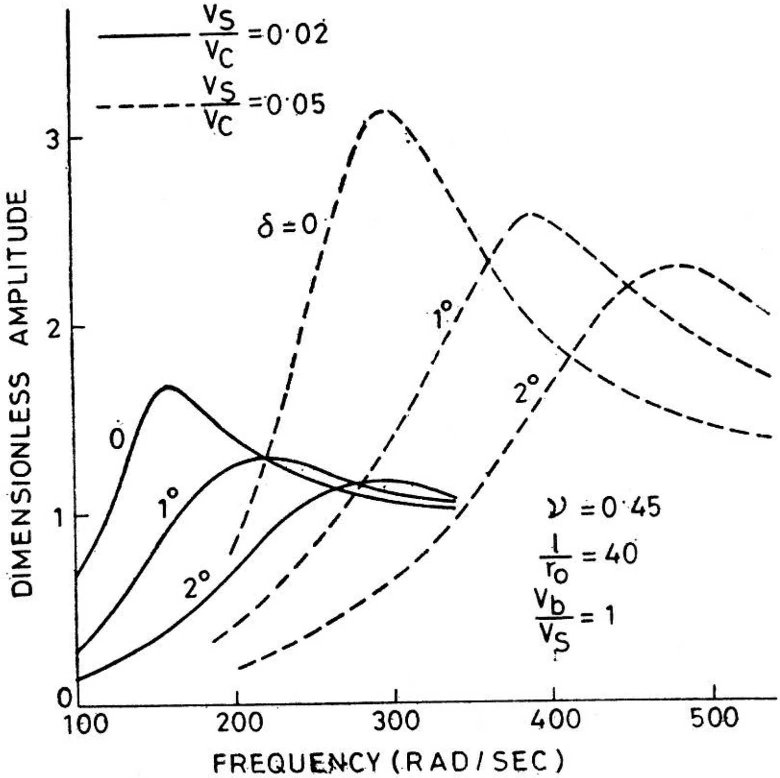

FIGURE. 4 Effect of Wave Velocity Ratio on Vertical Dynamic Response of Footing Supported by Tapered Pile.

of angle of taper, resonant frequency increases and resonant amplitude decreases at any particular relative stiffness (V_s/V_c) of soil. But both resonant amplitude and resonant frequency increase with increase of relative stiffness of soil at any particular angle of taper of the pile. Similar response curves of floating and end-bearing piles with different angles of taper are shown in Fig. 5. In all cases, the volume of the tapered piles and average radius r_o are kept constant. It is found that for both floating and end-bearing piles, resonant frequency increases and resonant amplitude decreases with the increase of angle of taper of pile. But for any particular value of angle of taper, the resonant frequency and resonant amplitude of end-bearing piles are much higher than that of a floating tapered pile.

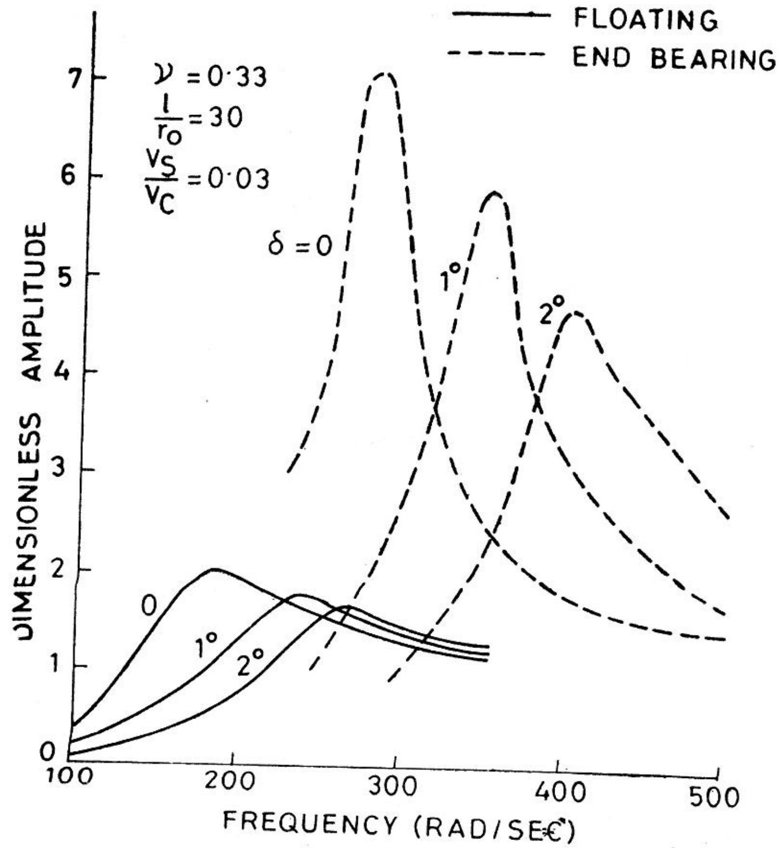

FIGURE. 5 Effect of Relaxation of Pile Tip on Vertical Dynamic Response of Footing Supported by Tapered Pile.

The variations of dimensionless stiffness and damping parameters of tapered piles with dimensionless frequency a_o, slenderness ratio l/r_o, the ratio of shear wave velocities of soil below and above pile tip, and relative stiffness of soil V_s/V_c are shown in Figures 6 to 9. The stiffness increases and damping decreases with increase of angle of taper at any frequency (Fig. 6), whereas at any angle of taper, the stiffness parameter decreases slowly with frequency factor upto $a_o \approx 0.8$ and then increases. But at any angle of taper, damping increases with the frequency factor.

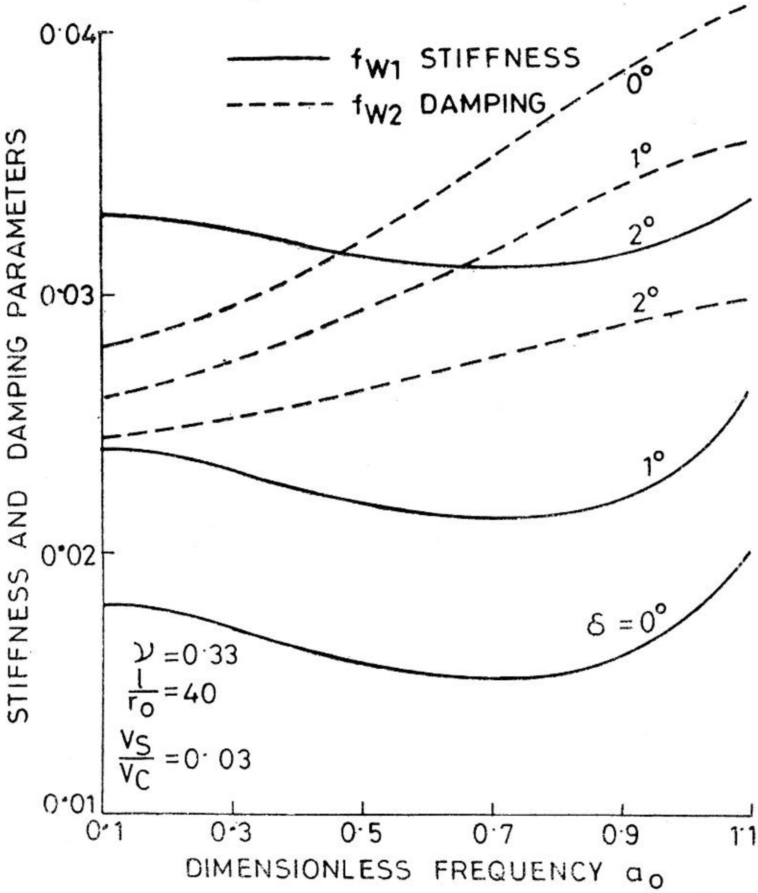

FIGURE. 6 Variations of Stiffness and Damping Parameters with Dimensionless Frequency.

The stiffness parameter in general increases (Fig. 7) with both slenderness ratio and the angle of taper. But damping increases with slenderness ratio upto $l/r_o = 30$ and then decreases. In all cases damping decreases with the increase of angle of taper. Again stiffness decreases and damping increases (Fig. 8) with lack of fixity of pile tip i.e. with decrease of V_b/V_s. The increase of stiffness and decrease of damping with the increase of angle of taper of pile are also represented in Fig 9. It is noted that with the increase of relative stiffness of soil, both stiffness and damping of a tapered pile in increase.

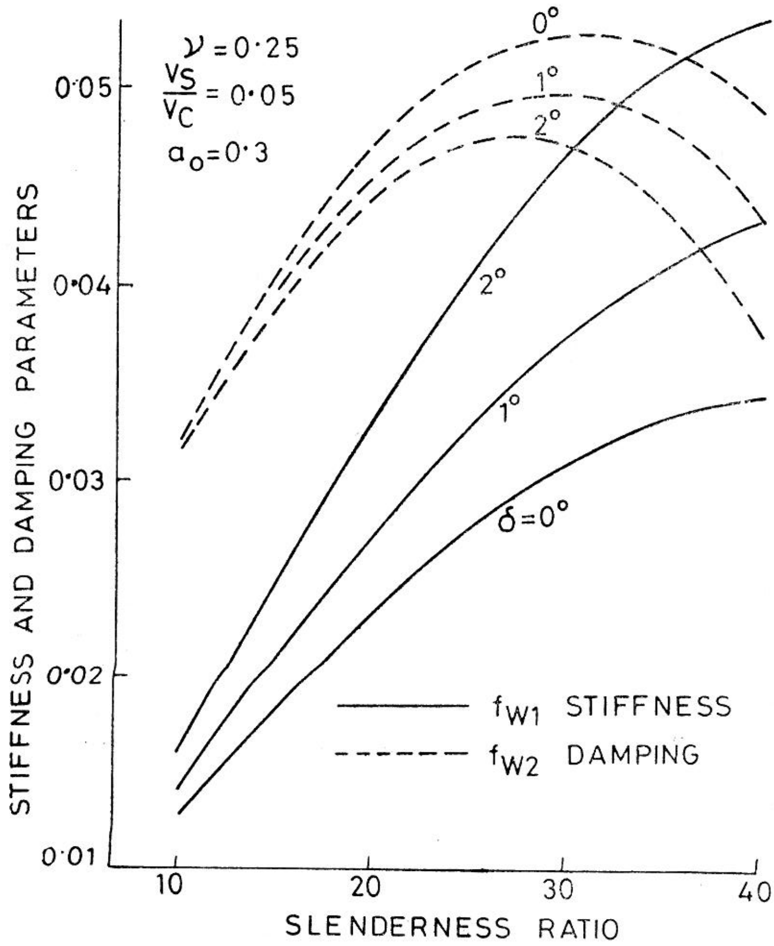

FIGURE. 7 Variations of Stiffness and Damping Parameters with Slenderness Ratio.

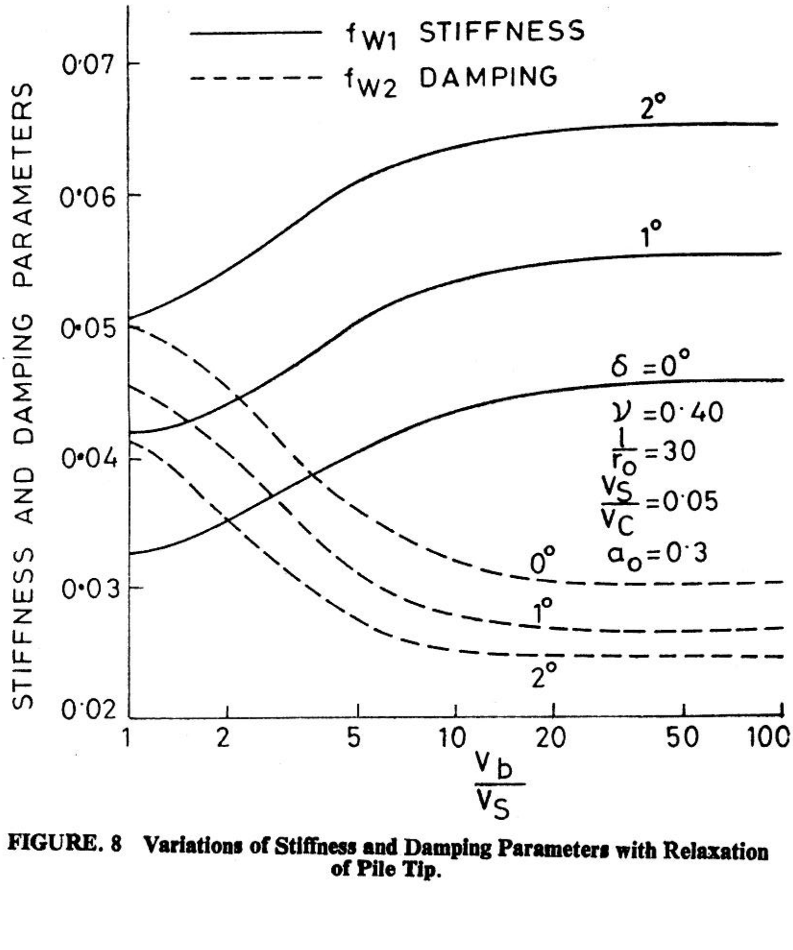

FIGURE. 8 Variations of Stiffness and Damping Parameters with Relaxation of Pile Tip.

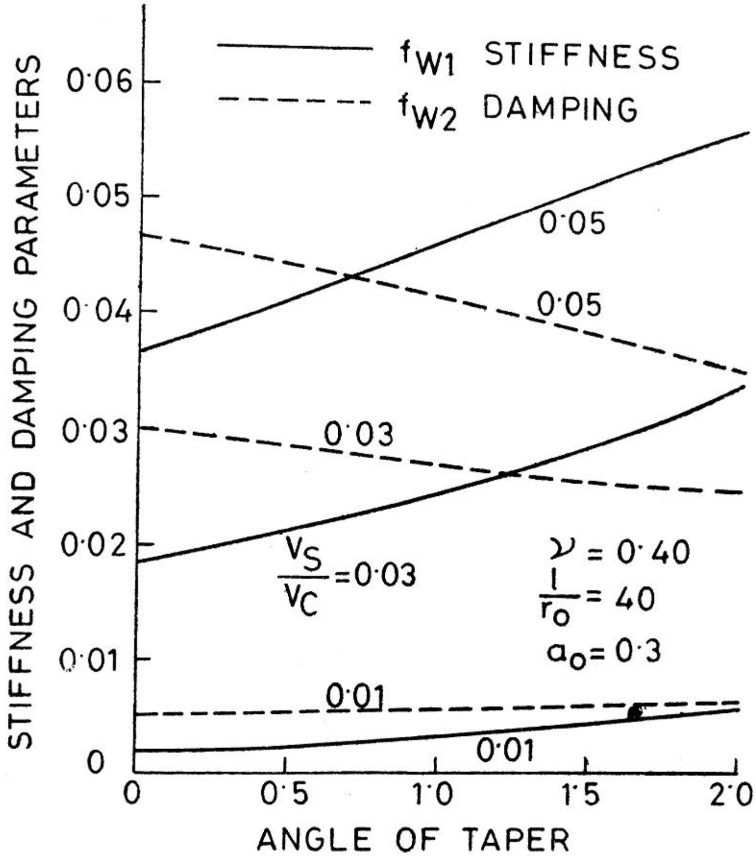

FIGURE. 9 Variations of Stiffness and Damping Parameters with Angle of Taper and Relative Stiffness of Soil.

Conclusion

From the analysis and discussion of the results presented in this paper, it may be concluded that the proposed simple and approximate method predicts quite satisfactorily the dynamic response characteristics of a tapered pile. Use of a tapered pile is always recommendable for increasing the resonant frequency and decreasing the resonant amplitude of a pile-supported footing. In general stiffness increases and damping decreases with the increase of angle of taper of a pile. But at the same time these parameters depend on several other factors like frequency factor, slenderness ratio, relaxation of the pile tip and the relative stiffness of soil. Stiffness always decrease with decrease of slenderness ratio, lack of fixity of pile tip and decrease of relative stiffness of soil for a pile with a particular angle of taper; whereas damping increases with lack of fixity of pile tip, decreases with decrease of relative stiffness of soil and increases with slenderness rato upto $l/r_o = 30$ and then decreases.

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