Short Communication

Seepage under a Dam on Sloping Soil with Discontinuities in Slopes

by

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Introduction

Herein, the influence on the flow characteristics of the discontinuities in the slopes overlying and underlying the porous medium of finite depth has been studied. Such geometry may occur in several field situations. One example is a major project undertaken by the Government of Karnataka, namely, the Chakra dam. It is being constructed across the Chakra river situated in the valley adjoining the Linganamakki catchment area to divert the river water to the Sharavathi valley to augment power supply for the Kudremukh Iron Ore Project.

Analysis

Fig 1 (*a*) shows the flow domain in the physical plane, for which $Z = x + iy$. The region ABCDEFGA is transformed onto the lower half of the auxiliary t plane, (Fig 1 (*c*)), using Schwarz-Christoffel transformation. The relationship for the mapping is obtained as follows

$$Z = M \int \frac{dt}{t^{1-\rho}\,(t-\alpha)^{1-\sigma}\,(t-\gamma)\,(t-\delta)^{\sigma-1}\,(t-1)^{\rho-1}} + N \qquad \ldots(1)$$

in which M and N are arbitrary constants, the vertices of the polygon ABCDEFGA in the Z plane being mapped onto $-\infty$, 0, α, β, γ, δ, 1, ∞ respectively, in the t plane.

Integrating Equation (1) for $0 < t < \alpha$ [Gradshteyn and Ryzhik, 1965] and noting that $Z = Z_c$ at $t = \alpha$, B_1 (See Fig 1) can be obtained as follows.

$$B_1 = |Z_c| = |M| \; | (-1)^{\rho} \; \alpha^{\rho+\sigma-1} \, \gamma^{-1} \, \delta^{1-\sigma} \sum_{n=0}^{n=\infty} A_n \, B(n+\rho, \sigma).$$

$$F_1 (n+\rho,\; \rho-1,\, 1;\, n+\rho+\sigma;\, \alpha,\, \frac{\alpha}{\gamma}) \,| \qquad \ldots(2)$$

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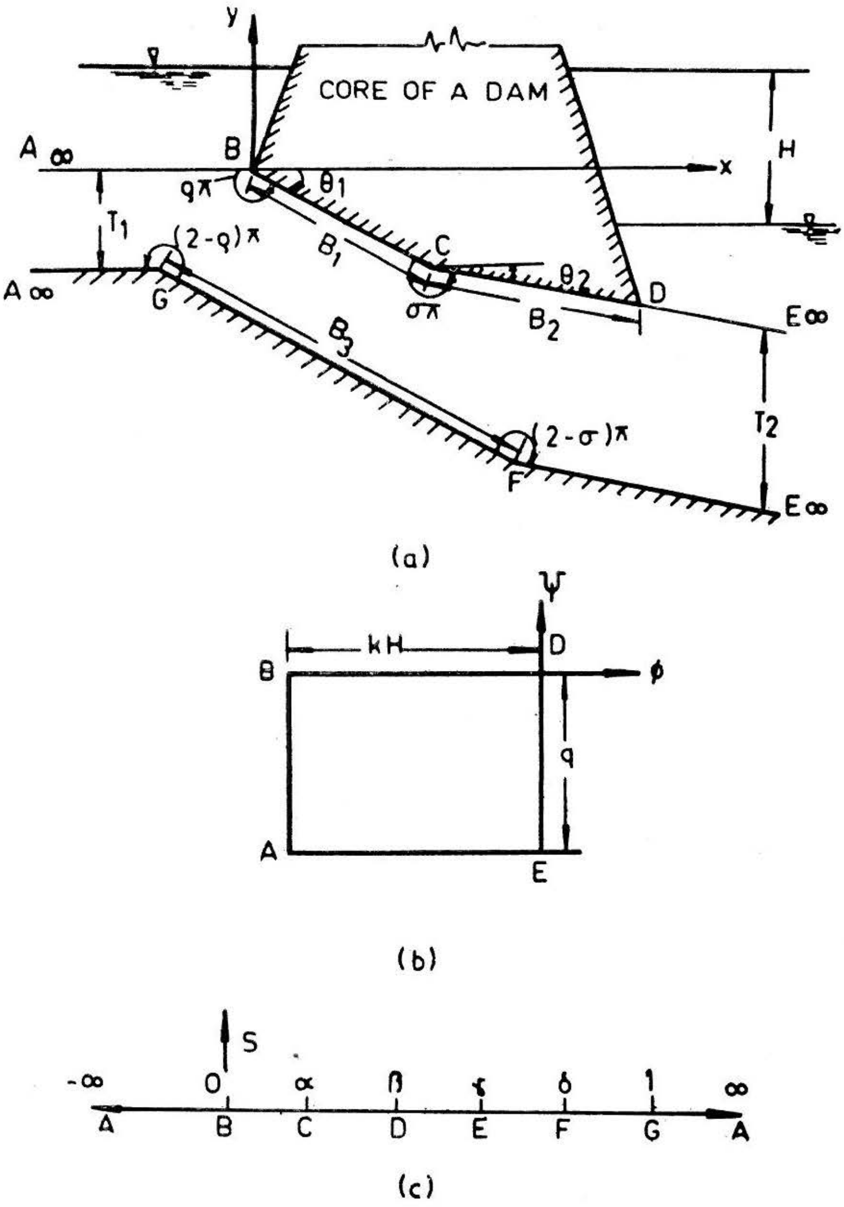

FIGURE 1 Steps of Conformal Mapping

(*a*) z **plane**

(*b*) w **plane**

(*c*) t **plane.**

in which

$$A_n = \frac{(\sigma-1)(\sigma)\ldots\ldots(\sigma-1+\overline{n-1})}{n}\left(\frac{a}{\delta}\right)^n ; n = 1,2\ldots$$

and $A_o = 1$.

$B(\quad) =$ Complete beta function

$F_1(\quad) =$ Hypergeometric function of two variables.

Integrating Eq. (1) for $a < t < \gamma$ and noting that $Z = Z_D$ at $t = \beta$, B_2 (See Fig. 1) can be obtained as

$$B_2 = | Z_D - Z_c | = | (-1)^{1-\rho-\sigma} M\gamma^{\rho-1} (\gamma-a)^{\sigma-1} (\delta-a)^{1-\sigma} (1-a)^{1-\rho} \sum_{n=0}^{n=\infty} B_n (\gamma-a)^n \sum_{p=0}^{n=\infty} C_p I_p | \quad \ldots(3)$$

in which

$$B_n = \frac{(1-\rho)(2-\rho)\ldots\ldots\ldots(n-\rho)}{n!}\frac{1}{\gamma^n} \quad n = 1,2\ldots$$

$$B_o = 1 ;$$

$$C_p = C'_p C''_0 + C'_{p-1} C''_1 + \ldots\ldots + C'_1 C''_{p-1} + C'_0 C''_p$$
$$p = 0, 1, 2, \ldots\ldots$$

$$C'_p = \frac{(\sigma-1)(\sigma)(\sigma+1)\ldots\ldots(\sigma-1-\overline{p-1})}{p}\left(\frac{\gamma-a}{\delta-a}\right)^p$$
$$p = 1,2,\ldots$$

$$C''_p = \frac{(-1)^p (1-\rho)(-\rho)\ldots\ldots(1-\rho-\overline{p-1})}{p}\left(\frac{\gamma-a}{1-a}\right)^p$$
$$p = 1,2,\ldots$$

$$C'_0 = 1 ;$$

$$C''_0 = 1 ;$$

$$I_p = \sum_{r=0}^{r=\infty} \frac{\left(\frac{t-a}{\gamma-a}\right)^{p+\sigma+r}}{p+\sigma+r} ; r = 0, 1, 2\ldots\ldots$$
for $n=0$

$= B_r (p+\sigma, n)$ for $n > 0$;

$B_r(\quad) =$ incomplete beta function.

Again, integrating Eq. (1) for $\delta < t < 1$ and introducing the condition $Z = Z_F$ at $t = \delta$, B_3 (See Fig. 1) is obtained as

$$B_3 = | Z_F - Z_G | = | (-1)^{-\rho} M (1-\alpha)^{\sigma-1} (1-\gamma)^{-1} (1-\delta)^{3-\rho-\sigma}$$

$$\sum_{n=0}^{n=\infty} D_n \; B\,(n+2-\rho, 2-\sigma).$$

$$F_1\,(n+2-\rho, 1-\sigma, 1; n+4-\rho-\sigma; \frac{1-\delta}{1-\alpha}; \frac{1-\delta}{1-\gamma}) \,| \qquad \ldots(4)$$

in which

$$D_n = \frac{(1-\rho)\,(2-\rho)\ldots\ldots\ldots(n-\rho)}{n!}\,(1-\delta)^n :$$

$$n = 1,2,\ldots\ldots$$

$$D_o = 1.$$

As t passes around a semicircle of small radius at point E in the t plane the corresponding change in Z is given by

$$-T_2 \operatorname{Cos} \theta_2 \,[\operatorname{Sin} \theta_2 + i \operatorname{Cos} \theta_2]$$

From this condition,

$$T_2 = \frac{-i\,(-1)^{1-\rho-\sigma}\,\pi\, M\gamma^{\rho-1}\,(\gamma-\alpha)^{\sigma-1}\,(\delta-\gamma)^{1-\sigma}\,(1-\gamma)^{1-\rho}}{(\operatorname{Cos}\theta_2 \operatorname{Sin}\theta_2 + i \operatorname{Cos}^2\theta_2)} \ldots(5)$$

From the condition at point A,

$$T_1 = -M\pi \qquad \ldots(6)$$

For the case $T_1 = T_2 = T$, eliminating T from Eqs. (5) and (6).

$$\delta = \gamma + (\gamma-\alpha) \left(\frac{\gamma}{1-\gamma}\right)^{\frac{1-\rho}{1-\sigma}} [\operatorname{Cos}^2\theta_2 \operatorname{Cos}(\sigma+\rho)\pi$$

$$-\operatorname{Cos}\theta_2 \operatorname{Sin}\theta_2 \operatorname{Sin}(\rho+\sigma)\,\pi]^{\frac{1}{1-\sigma}} \qquad \ldots(7)$$

For this case with points G and F being vertically below point B and C, respectively, B_1 and B_3 are equal for the angular relations shown in Fig 1 (*a*). The unknowns α, γ, and δ are obtained by using Eqs. (2), (4) and (7) for the given physical quantities B_1, B_3 and T. β which corresponds to B_2 is obtained from Eq. (3).

Mapping the Complex Potential Plane

In the complex potential plane (w plane) the boundaries of the flow region from a rectangle (Fig. 1(b)). The complex potential, w. is defined as

$$w = \phi + i\,\Psi$$

in which, the velocity potential, $\phi = -k\left(\frac{p_l}{\gamma_w} + y\right) + C_1$

k = coefficient of permeability,

p_i = pressure at the point being considered with γ as coordinats,

γ_w = unit weight of water and

Ψ = stream function

C_1 = constant.

The mapping of the w plane onto the lower half of the t plane, according to Schwarz-Christoffel transformation is given by

$$w = M' \int \frac{dt}{t^{1/2}\,(t-\beta)^{1/2}\,(t-\gamma)^{1/2}} + N' \qquad \ldots(8)$$

in which M' and N' are constants.

Using the conditions $w = O$ at $t = \beta$ and $w = iq$ at $t = \gamma$ in Eq. (8), the quantity of seepage in non dimensional form is obtained as

$$\frac{q}{kH} = \frac{K\left(\dfrac{\sqrt{\lambda-\beta}}{\gamma}\right)}{K\left(\dfrac{\sqrt{\beta}}{\gamma}\right)} \qquad \ldots(9)$$

in which $K(\ )$ = complete elleptic integral of first kind.

Exit Gradient

The exit gradient, I_E, perpendicular to the downstream boundary is given by

$$I_E = \frac{1}{k}\left(\frac{dw}{dt}\,\frac{dt}{dz}\right)(Cos\theta_3 + iSin\theta_3) \qquad \ldots(10)$$

in which θ_3 is the angle made by a streamline with the x axis at exit point. $\frac{dz}{dt}$ and $\frac{dw}{dt}$ are obtained from Eq. (1) and (8). Hence, Eq. (11) becomes

$$\left| \frac{I_E T}{H} \right| = \left| \frac{\pi\sqrt{\gamma}}{2k\left(\sqrt{\dfrac{\beta}{\gamma}}\right)} \, \frac{t^{\frac{1}{2}-\rho}\,(t-\alpha)^{1-\sigma}\,(\gamma-t)^{\frac{1}{2}}\,(\delta-t)^{\sigma-1}\,(1-t)^{\rho-1}}{(t-\beta)^{\frac{1}{2}}} \right| \qquad \ldots(11)$$

To check the results obtained, finite difference technique is used and the resulting simultaneous equations are solved by Successive Over Relaxation method.

The results of the analytical method were also checked for a particular case by electrical analog method.

Results and Discussions

In the present analysis. points G and F in the physical plane (z plane) are assumed to be directly below points B and C, respectively ; T_1 and T_2

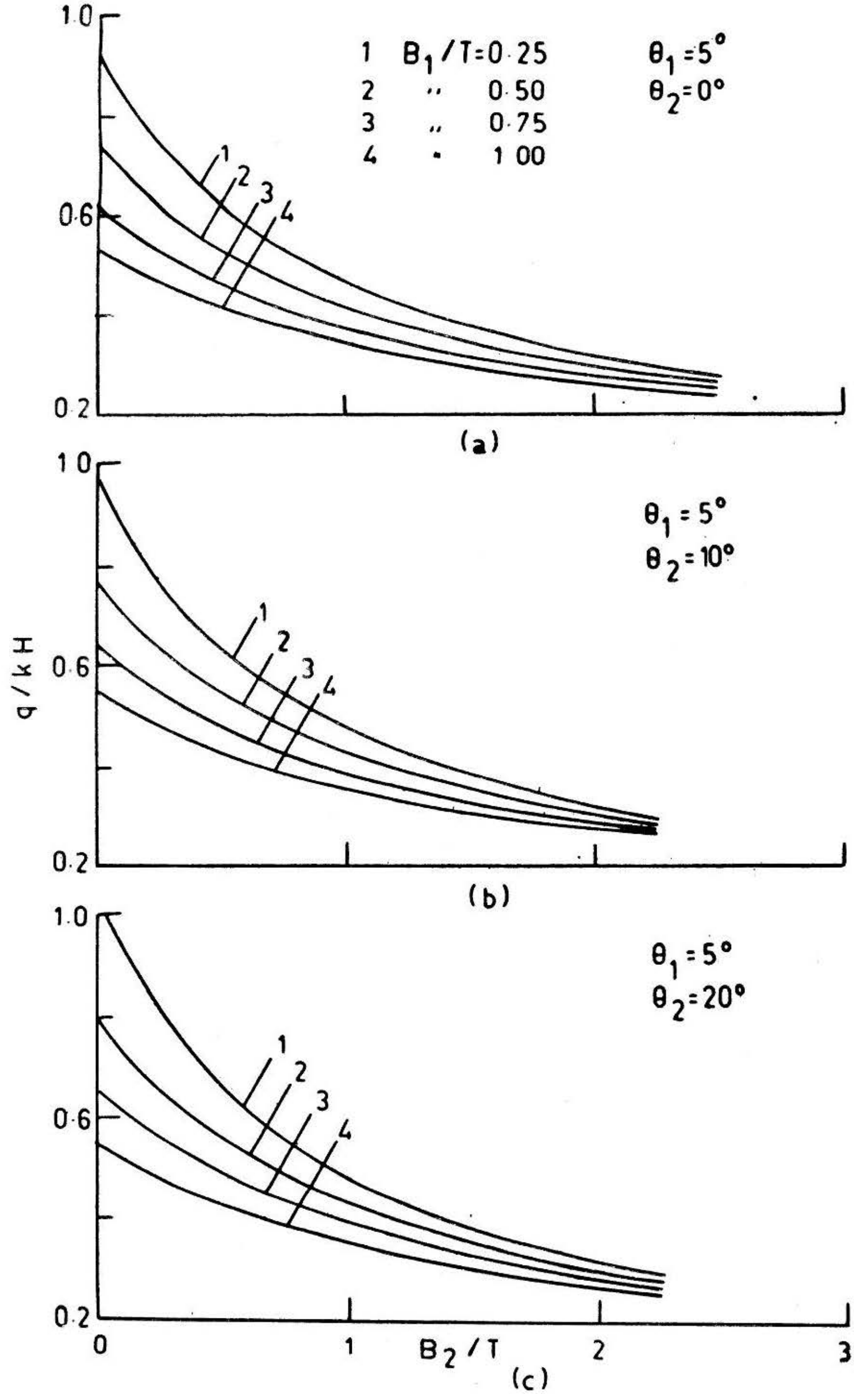

FIGURE 2 Variation of q/kH with B_2/T for $\theta_1 = 5°$

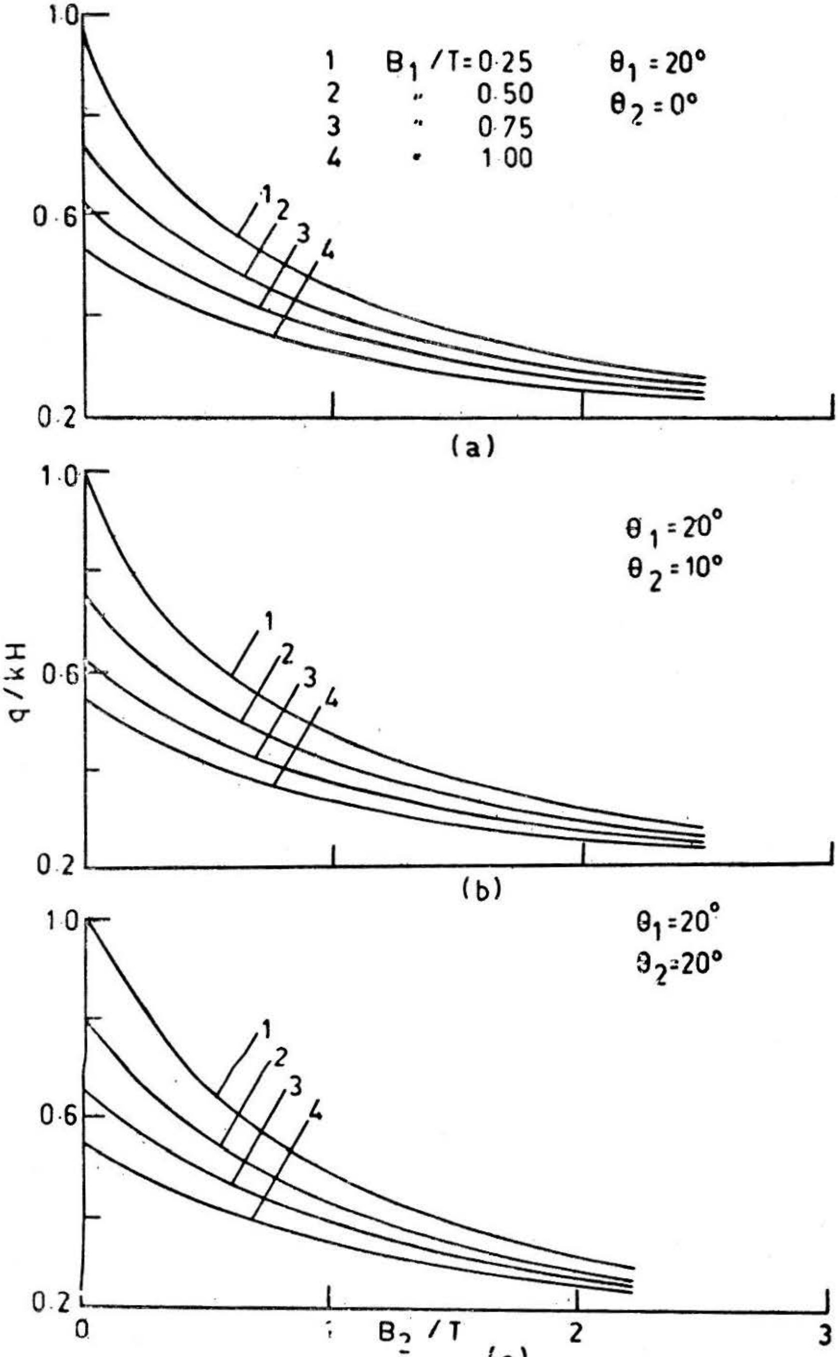

FIGURE 3 Variation of qHk with B_2/T for $\theta_1 = 20°$

are assumed to be equal. Knowing the physical parameters H, k, B_2, B_1 T, ρ and σ, and the transformation relations, the information regarding quantity of seepage and distribution of exit gradient can be obtained. The variables, α, β and γ, in t plane were obtained by solving Eq. (2), (3) uations and (4).

Numerical results are presented for quantity of seepage and exit gradient distribution in nondimensional form for the ranges of θ_1 from 5° to 20° and θ_2 from 0° to 20°.

Figs. 2 and 3 illustrate the relation between q/kH and B_2/T. It is observed from the curves that at constant values of θ_1 and θ_2, the reduction id q/kH with increase in B_2/T is relatively more at smaller values of B_1/T than when B_1/T is very large.

Comparison of curves for q/kH versus B_2/T for different sets of θ_1 and θ_2 indicates that the discontinuities in slopes do not have much influence on the seepage quantity. However, an increase in θ_1, for given values of θ_2, B_1/T and B_2/T, causes a small decrease in q/kH, whereas, an increase in θ_2 for given values of θ_1, B_1/T and B_2/T causes a small increase in q/kH. For instance, for given $\theta_2 = 10°$, $B_1/T = 0.5$ and $B_2/T = 0.5$ the corresponding values of q/kH for $\theta_1 = 5°$ and 20° are 0.5484 and 0.5317, i.e., the percentage decrease in q/kH is 3.05; while for given $\theta_1 = 10°$ and for the same values of B_1/T and B_2/T, q/kH increases from 0.5244 to 0.5518 when θ_2 changes from 5° to 20°, i.e., the percentage increase in q/kH is 5.23.

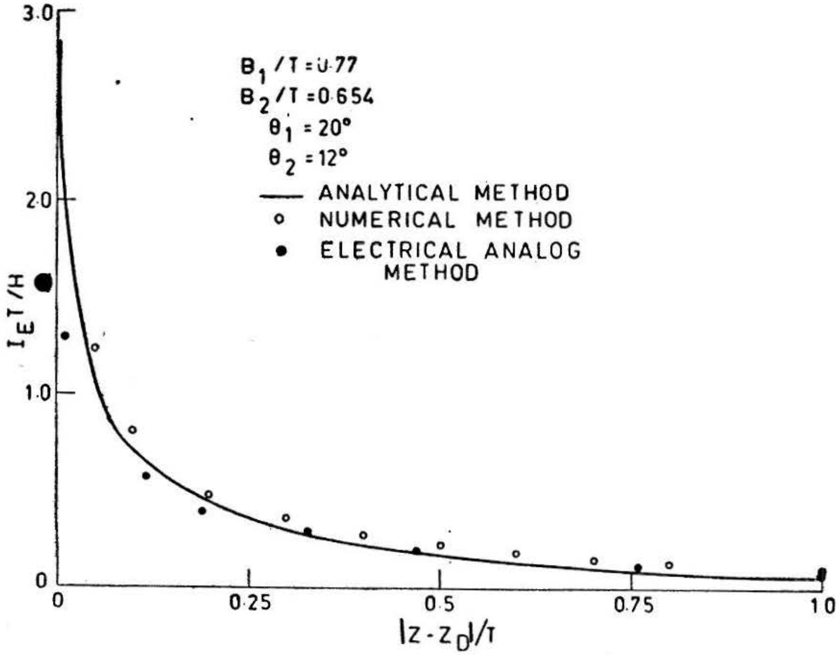

FIGURE 4 Comparison of Results of Analytical, Experimental and Numerical Methods for Distribution of Exit Gradient

When $\theta_1 = \theta_2 = 0°$ and for $B_1/T = 0.5$ and $B_2/T = 0.5$, the obtained value of $q/kH = 0.53375$ compares well with Pavlovsky's solution ($q/kH = 0.53307$) for same conditions (vide Harr, 1962).

Fig. 4 shows the comparison of results for exit gradient distribution obtained under similar conditions, from analytical method, numerical method and electrical analogy method. It is seen from Fig. 8, that the results of analtical method check satisfactorily with the results of numerical method and those of electrical analogy method.

Conclusions

An analytical solution has been presented for the problem of seepage under a dam located on sloping soil with discontinuities in slopes. Numerical results are presented for quantity of seepage and the exit gradient for θ_1 and θ_2 values between 5° and 20°. For this range. it is observed that an increase in θ_1 causes a decrease in q/kH while an increase in θ_2 causes an inerease in q/kH; but, however, the increase or dccrease in q/kH is found to be small. Thus, the discontinuities in slopes do not considerably influence the quantity of seepage.

Further, the results obtained from numerical method and electrical analog method compare well with the results of analytical method.

References

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HARR, M.E., (1962), "*Groundwater and Seepage*". McGraw-Hiil Book Company Inc, New York.