Analysis of Tunnels by Boundary Element Method

by

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Introduction

Many multipurpose river valley projects in India involve extensive use of tunnels. Prediction of deformation and stresses around the tunnels is essential in design, instrumentation and evaluation of performance of tunnels. Several closed form solutions are available for the prediction of stresses and deformations for regular geometric shapes of tunnels such as circular, rectangular and elliptic and for idealized conditions of geological media, viz. homogeneous, isotropic, linear elastic and infinite conditions. In practice, tunnels seldom have such regular shapes. The functional and construction requirements require adoption of tunnel shapes which are horse-shoe type, D-type, four arc type etc. Further, the geological media often have discontinuities such as joints and faults and rarely satisfy the idealized conditions on which the closed form solutions are based. The use of Finite Element Method (FEM) is best suited for these class of problems. However, it requires the solution of large systems of simultaneous equations in unknowns associated with nodal points distributed throughout the domain and clearly the preparation of input data can be very time consuming, tedious and expensive.

To get reasonably accurate results from the analysis, it is imperative to use realistic input data regarding the discontinuities in the geological media, material properties, insitu stress conditions etc. In view of the difficulties and the cost involved in acquiring such data, a parametric study may have to be carried out for a range of values of material properties and insitu stress conditions. In addition, for selecting a suitable shape of tunnel at a particular site, several shapes may have to be tried. Under these circumstances, the Boundary Element Method (BEM), in which only the boundaries of the domain are to be discretised, can be used because of its following advantages over the Finite Element Method :

(*i*) Less time and effort required to prepare input data,

(*ii*) Less computer storage, and

(*iii*) Less computer time.

Banerjee and Butterfield (1977) pointed out the distinct advantages of the BEM over the FEM. However' the BEM has certain restrictions in incorporating complex material behaviour and discontinuities in the field.

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Boundary Element Method

Boundary Element Methods (BEM) are classified as direct and indirect (Brebbia and Butterfield, 1978). The formulations for these differ in the procedures used to construct relationships between the traction and displacements on an excavation surface.

Fig. 1 represents problem to be analysed. The cross-section of the surface S is that of a long tunnel excavation. This is inscribed in an infinite elastic medium. At a particular location, it is subjected to either imposed traction components t_x, t_z or imposed displacements components u_x, u_z. These quantities represent the induced tractions as displacements on surface S to simulate excavation of the opening. It is required that a solution should be obtained for stresses and displacement in the medium surrounding the surface S which satisfy the equations of equilibrium, stress-strain relationships, etc. for the material and which satisfy the imposed boundary conditions on the surface S.

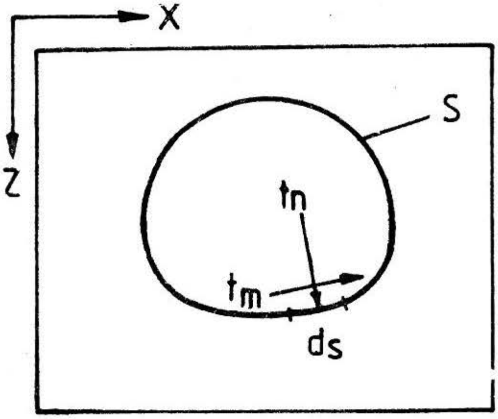


FIGURE 1 Surfaces *S* Inscribed in an Infinite Continuum and Subject to Imposed Tractions

In any boundary element formulation, the solution procedure involve dividing the surface S into a set of discrete boundary elements. Solution to problems using the Boundary Element Method is based on the superposition of stress and displacement induced by selected singularities. A knowledge of the fundamental singular solution of the problem is, therefore, required; for example, the basic particular analytical solution in elastostatics, is that of a concentrated point load in an infinite space (Kelvin's problem). The procedure then is to determine the unknown values of tractions and displacements on each element from the known surface values.

In the case of direct formulation, a system of simultaneous integral equations is obtained in terms of known boundary values of traction or displacement. The unknown in these equations are the remaining boundary tractions or displacements. These equations are solved numerically after making certain assumptions about the way in which these tractions or displacements are distributed over small areas of the boundary.

In the indirect formulation, the procedure is achieved by relating the surface tractions and displacements through a set of fictitious quantities. The distribution of these tractions or displacements required to describe the problem have no physical significance as regards the actual problem.

Indirect Boundary Element Method

The objective in the Indirect formulation is to find suitable approximations to the distribution of singularities which when applied to the problem boundaries, produce the known surface values of traction or displacements. In this method the discretised integral equations are formulated in terms of fictitious distributions of the singular (fundamental) solution of the field equations. After obtaining these fictitious quantities the stresses and displacements can be obtained at any point in the medium.

Fig. 2 shows the cross-section of the surface S of a long excavation

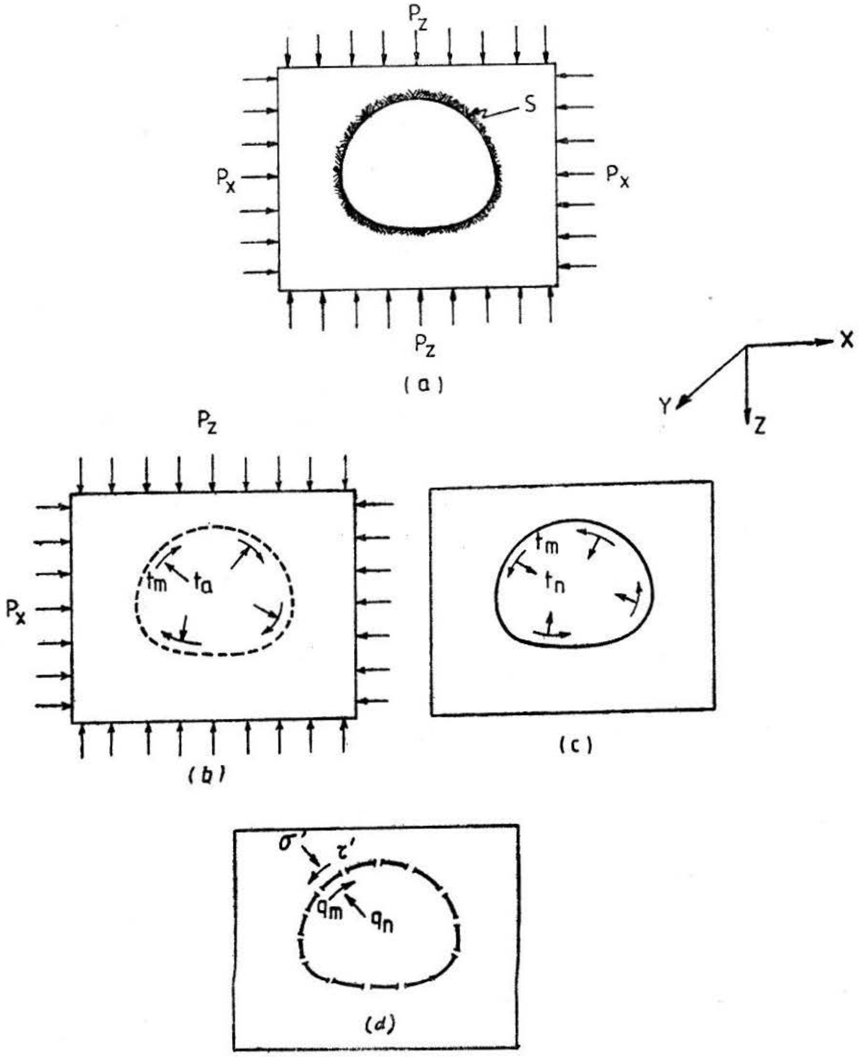


FIGURE 2 (*a*) **Problem to be Solved**
(*b*) **Traction on Potential Boundary due to Field Stresses Before Excavation**
(*c*) **Negative Traction (induced) Representing Effect of Excavation**
(*d*) **Infinite Plate, Fictitious Forces and Stresses on Elements of Imaginary Surface**

inscribed in an infinite elastic medium, subject to plane strain condition of loading. Consider a point force (of components t_n, t_m) acting perpendicular to and distributed uniformly over a small area ds of the surface S. The intensity of these fictitious forces at any point j is charaterised by Q_{mj}, Q_{nj} which are required to satisfy the boundary conditions on S.

The stress and displacement components at any point i in the medium, due to singularity distributions are obtained in terms of θ_{mj} and θ_{nj} by superposition of the stresses and displacements induced by load increments on small element of the surface, ds. For example :

$$\sigma_{x_i} = \int_S \{B^{x_i}_{mj}\, Q_{mj} + B^{x_i}_{nj}\, Q_{nj}\}\, ds$$

$$u_{x_i} = \int_S \{T^{x_i}_{mj}\, Q_{mj} + T^{x_i}_{nj}\, Q_{nj}\}\, ds \qquad \ldots(1)$$

where the coefficients (Kernel functions) $B^{x_i}_{mj}$, $T^{x_i}_{nj}$ etc. are determined by the particular distributed singularities over the surface S.

If the surface S is divided into a set of K elements, each element (j) of which is subject to singularity intensities, the discretised forms of equations can be written as :

$$\sigma_{xi} = \sum_{j=1}^{K} \{B'_{m(i,j)}\, q_{mj} + B'_{n(i,j)}\, q_{nj}\}$$

$$u_{xi} = \sum_{j=1}^{K} \{T'_{m(i,j)}\, q_{mj} + T'_{n(i,j)}\, q_{nj}\} \qquad \ldots(2)$$

with similar expressions for σ_{zi}, T_{zxi}, u_{zi}. The coefficients $B'_{m(i,j)}$, $T'_{m\,(i,j)}$ etc. are obtained by integrating the functions $B^{x_i}_{mj}$, $T^{x_i}_{mj}$ etc. for the unit solutions over range of each element.

Eq. (2) may be expressed in matrix notations as

$$\sigma_{x_i} = \{B'\}\,\{q\}$$

$$u_{x_i} = \{T'\}\,\{q\} \qquad \ldots(3)$$

where $\{B'\}$, $\{T'\}$ are row vectors of order $2K$, $\{q\}$ is a column vector of order $2K$. For a properly posed problem, Eq. (3) provides sufficient information in principle to determine a set of element load intensities which satisfy the known boundary conditions. Once the set of element fictitious load intensities has been determined, it can be used by applying Eq. (2) to obtain stresses and displacements at any point in the medium.

Bray (1976) used live load singularities in an infinite medium to provide Kernel functions as shown in Eq. (2) assuming uniform loading of elements. A computer program was developed similar to one given by Bray in Hoek and Brown (1980) based on his work of 1976 on indirect boundary element formulation. The program can be used for the two dimensional analysis of tunnel in homogeneous. isotropic, linear elastic media under plane strain condition for the determination of stresses and displacements.

The Problem

For the purpose of analysis, a typical tunnel project in the Himalayan region was considered. The area of cross-section of this tunnel is 19.7 m^2. The overburden above the tunnel is of 70 m. The properties of rock are :

Uniaxial compressive strength	= 6800 t/m^2
Tensile strength	= 700 t/m^2
Young's modulus	= 4.7×10^6 t/m^2
Poisson's ratio	= 0.17
Unit weight	= 2.8 t/m^3
The constants m and s	= 10 and 1.0

The failure criterion (Hoek and Brown, 1980) used was

$$\sigma_1 = \sigma_3 + (m\sigma_c \ \sigma_3 + s\sigma_c^2)^{1/2} \qquad \ldots(4)$$

where, σ_1, σ_3 are major and minor principal stresses, σ_c is uniaxial compressive strength and m and s are the empirical constants.

Three in-situ state of stress conditions with stress ratios, $K_0 = 0.5$, 1.0 and 2.0 were selected for the study.

The effect of three shapes of tunnel *viz.*, D type, horse-shoe type and 4 arc type (Figs. 3, 4 and 5) with same area of cross-section was investigated. For a horse-shoe type tunnel, the effect of three width/height ratios was studied.

Analysis

For all the analyses, 36 boundary elements were used. Excavation of tunnel was simulated. Induced deformations, stresses and factor of safety were calculated along the boundary as well as in the surrounding rock.

Results and Discussions

Effect of shape

Fig. 3 shows the displaced shape of the D type tunnel in an enlarged scale (Shape I) for the three stress ratios $K_0 = 0.5$, 1.0 and 2.0. The effect of stress ratio is pronounced on the wall as compared to the crown and invert of the tunnel.

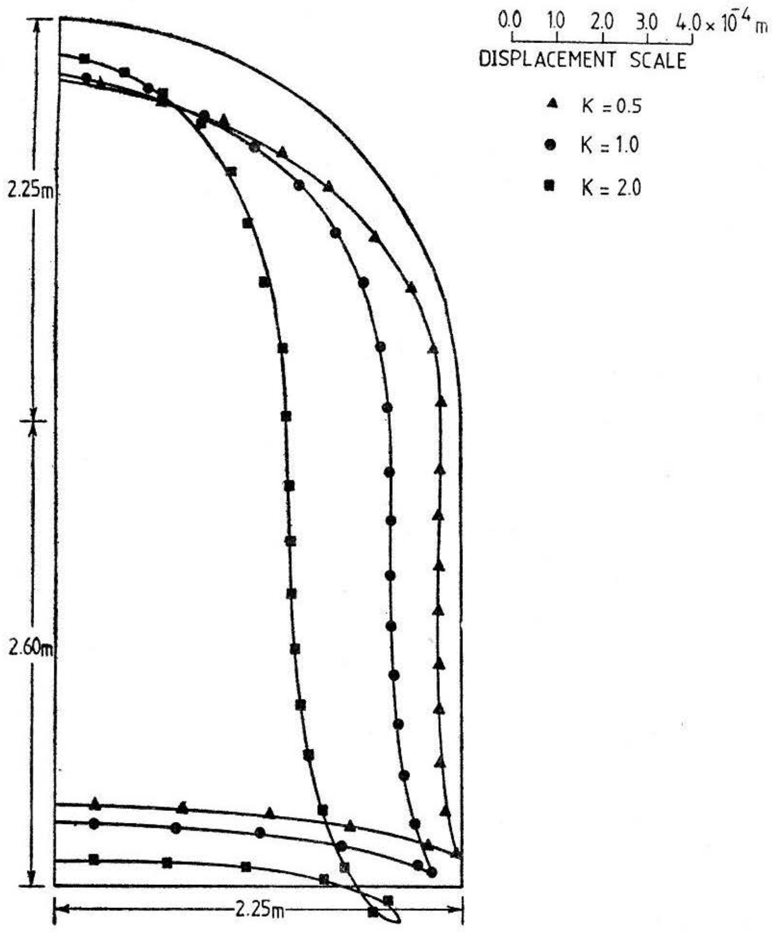


FIGURE 3 Displacements Along Boundary

Figs. 4 and 5 show the displaced shape of horse-shoe type (Shape II) and four arc type (Shape III) tunnels. The effect of stress ratio on the displacement pattern is similar to that observed for Shape I. In all the tunnel shapes, the displacement pattern is similar except at the bottom corner portion of the tunnel. In Shape III the nature of deformation is smooth as compared to those observed for the other two shapes.

In Fig. 6 are shown the maximum principal stress as the ratio of vertical stress along the boundary of the *D* shaped tunnel for three stress ratios. As compared to the crown and invert portions, the wall portion of the tunnel is significantly affected by the variation of stress ratio. The bottom corner of the tunnel shows large stress concentration. Figs. 7 and 8 show the maximum principal stress along the boundary for the other two

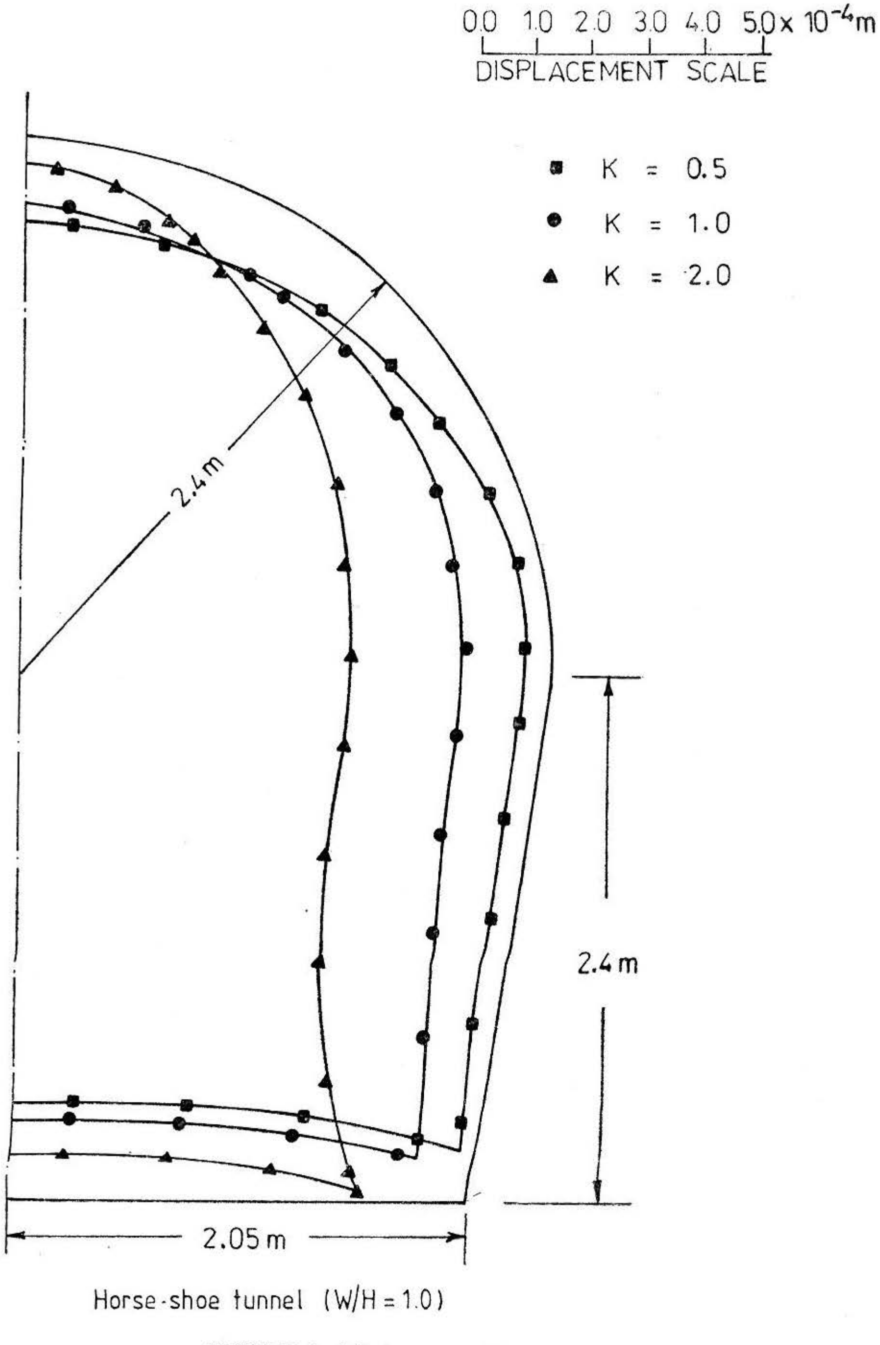
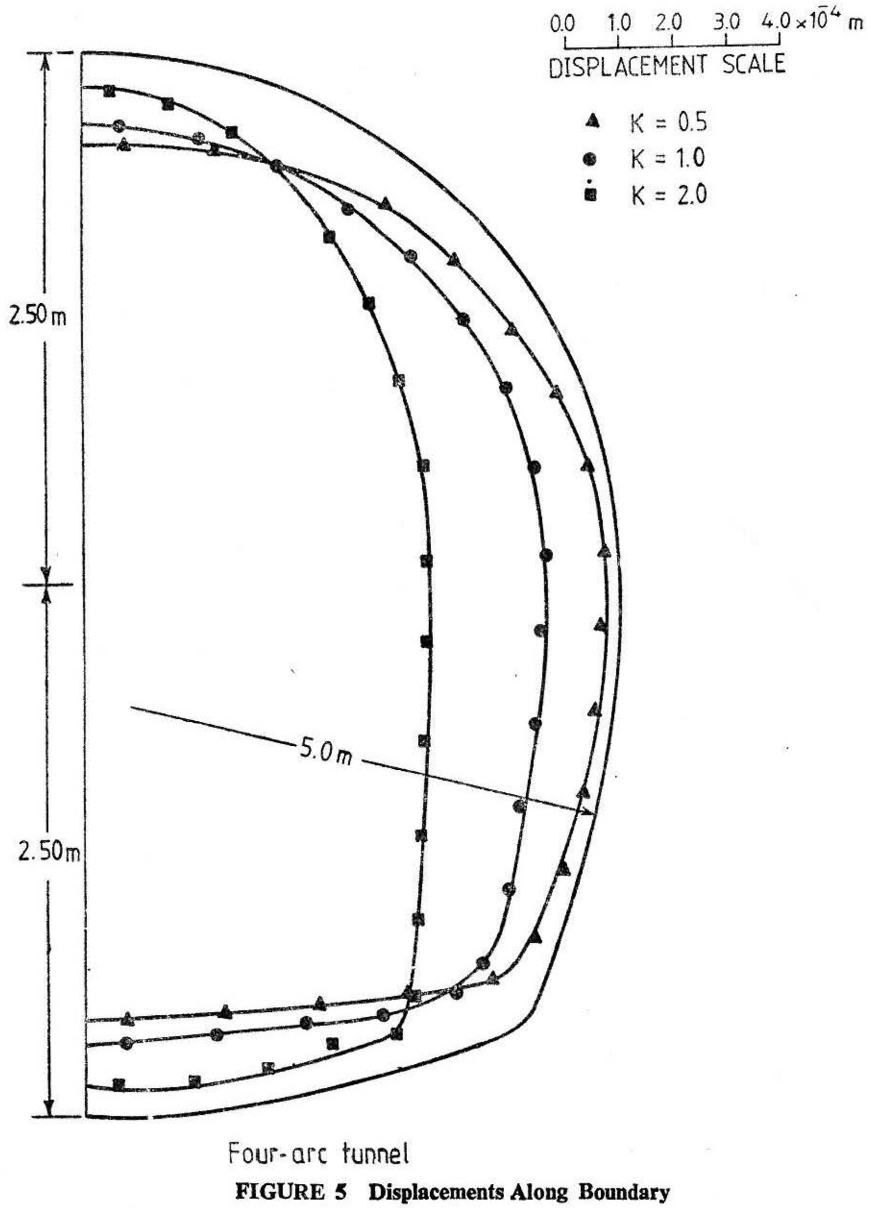


FIGURE 4 Displacements Along Boundary



Four-arc tunnel

FIGURE 5 Displacements Along Boundary

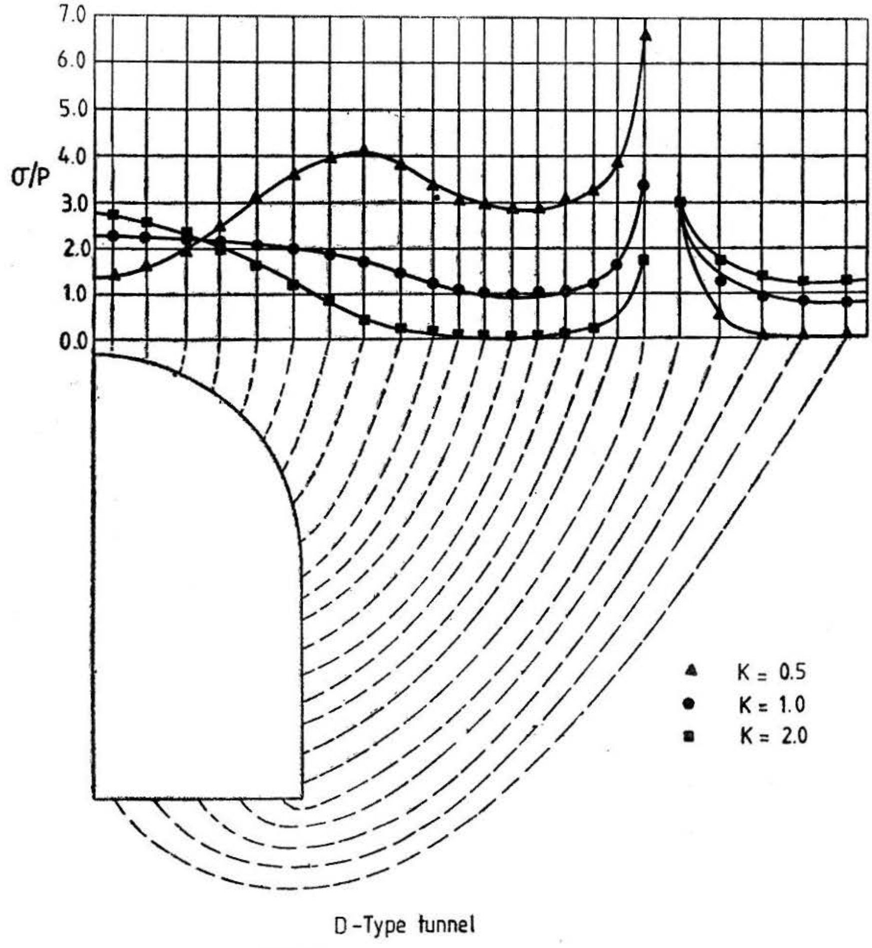


FIGURE 6 Stresses Along Boundary

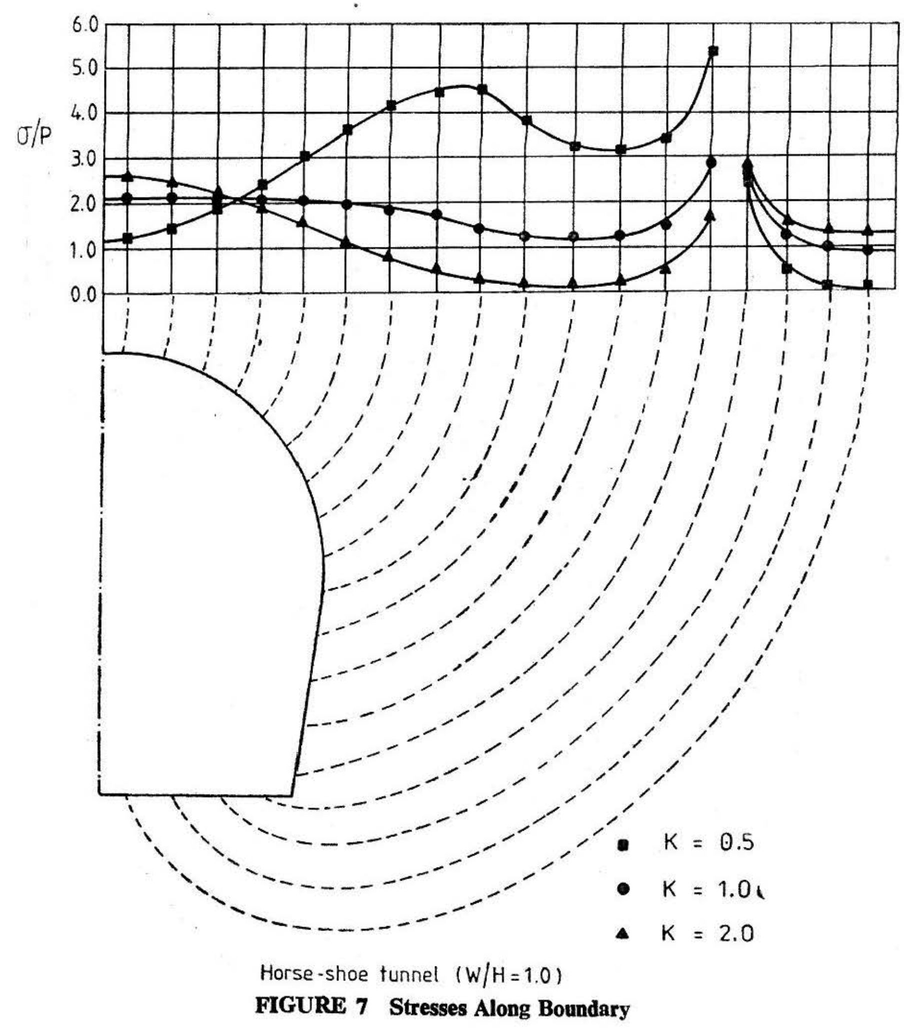


FIGURE 7 Stresses Along Boundary

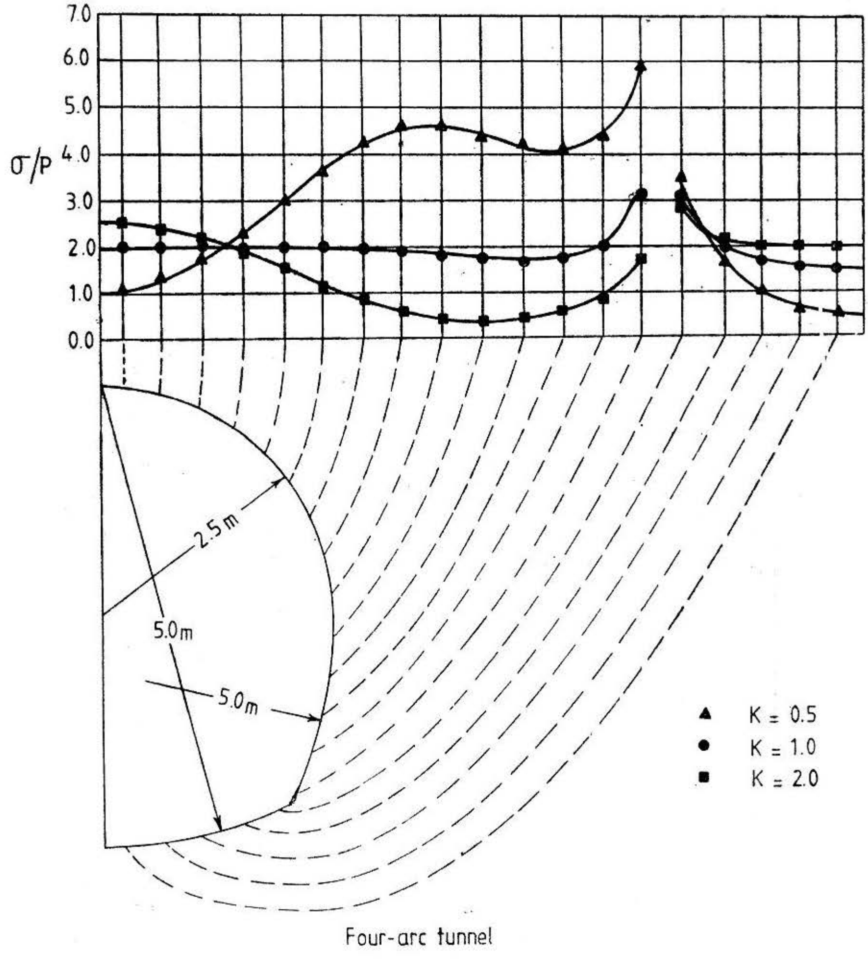


FIGURE 8 Stresses Along Boundary

tunnel shapes. It may be observed that the effect of stress ratio is similar in these shapes also. At the crown portion the stress is less for these shapes as compared to the *D* type shaped tunnel whereas it is more at invert and sidewall portions. The four arc shaped tunnel tends to give a more uniform stress.

Fig. 9 shows the factor of safety along the boundary of the *D* shaped tunnel for the three stress ratios. The effect of stress ratio is more pronounced in the straight wall and invert as compared to the crown of the tunnel. In Figs. 10 and 11 are shown the factors of safety along the boundary for the other two shapes. Horse-shoe shaped tunnel shows behaviour similar to that of the first one. In the four arc type, however, the effect of stress ratio is maximum at the crown and invert portions as compared to the sidewall. This shape indicates more uniform behaviour than the other two shapes.

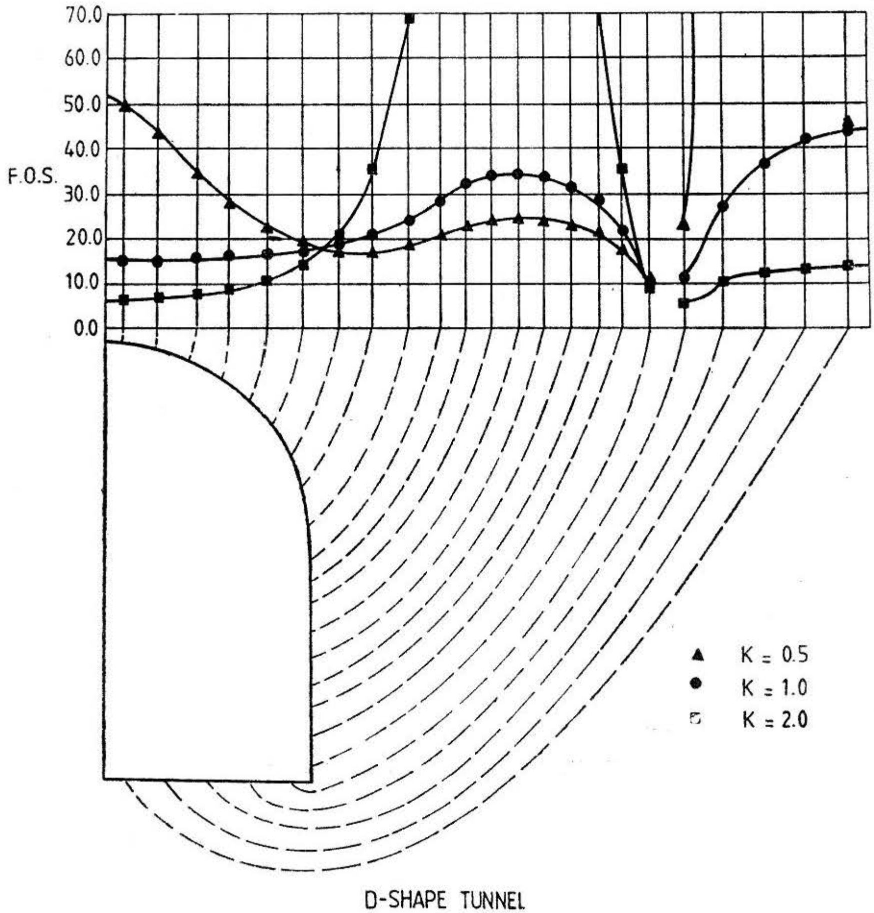


FIGURE 9 Factor of Safety Along Boundary

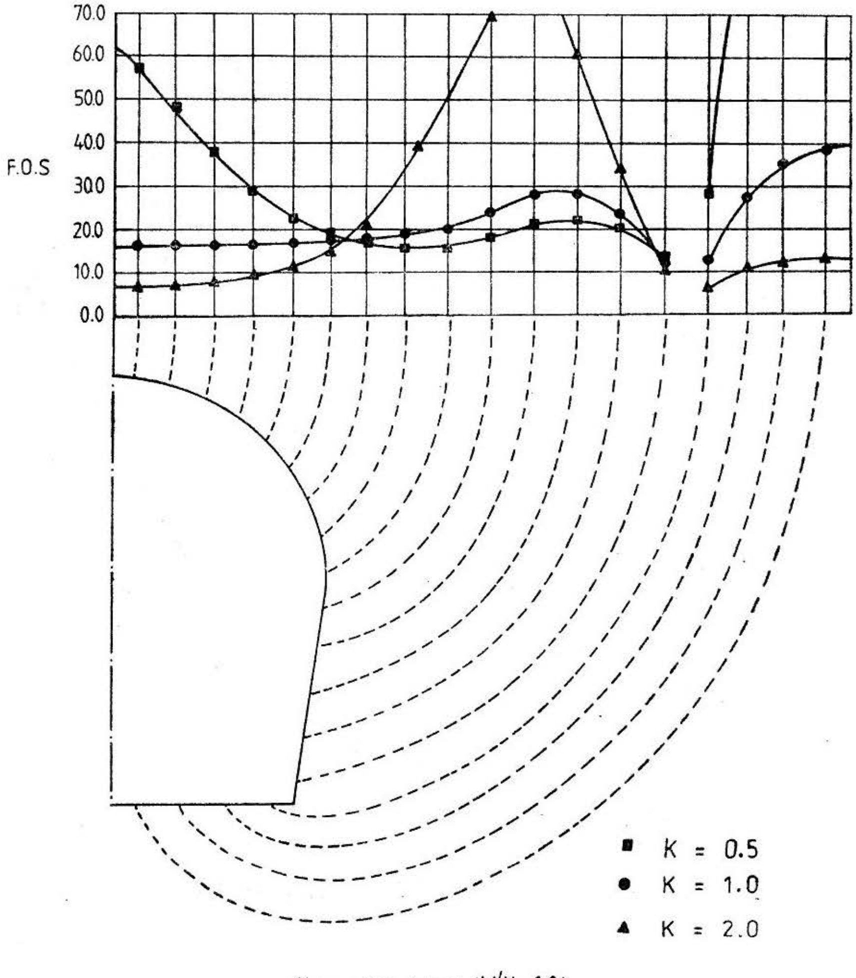


FIGURE 10 Factor of Safety Along Boundary

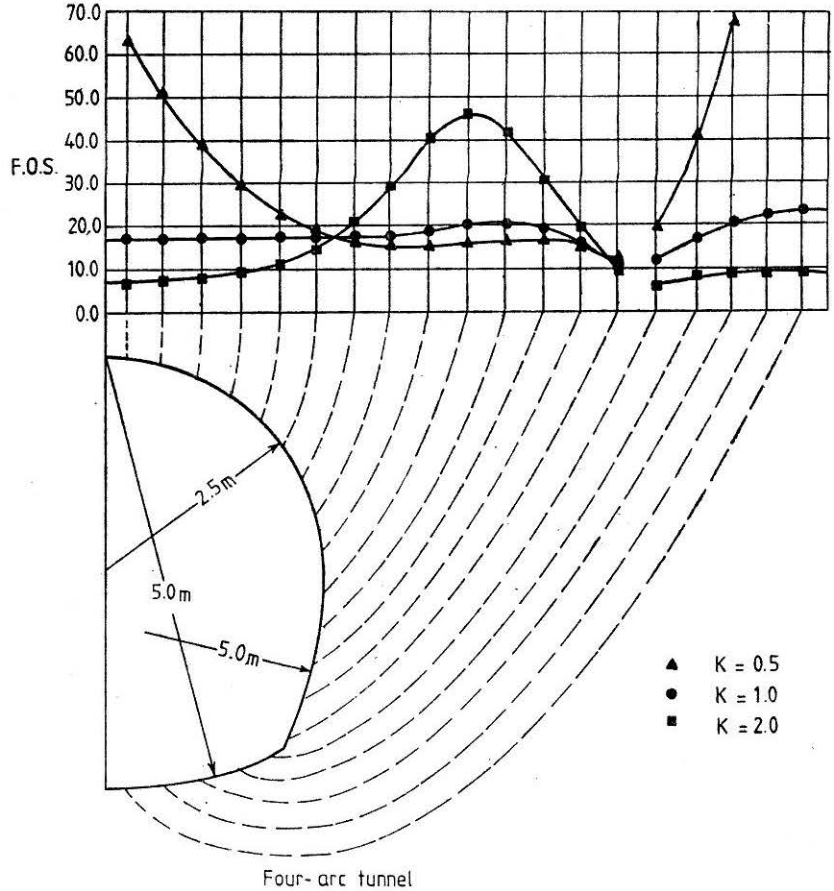


FIGURE 11 Factor of Safety Along Boundary

Effect of width/height ratio

Figs. 12, 4 and 13 show the deformed shape of the horse-shoe shaped tunnel with width/height ratios 0.73, 1.00 and 1.28 for the stress ratios 0.5, 1.0 and 2.0. The effect of stress ratio is similar in all cases. But width/height ratio has significant effect at the crown and invert portion of the tunnel.

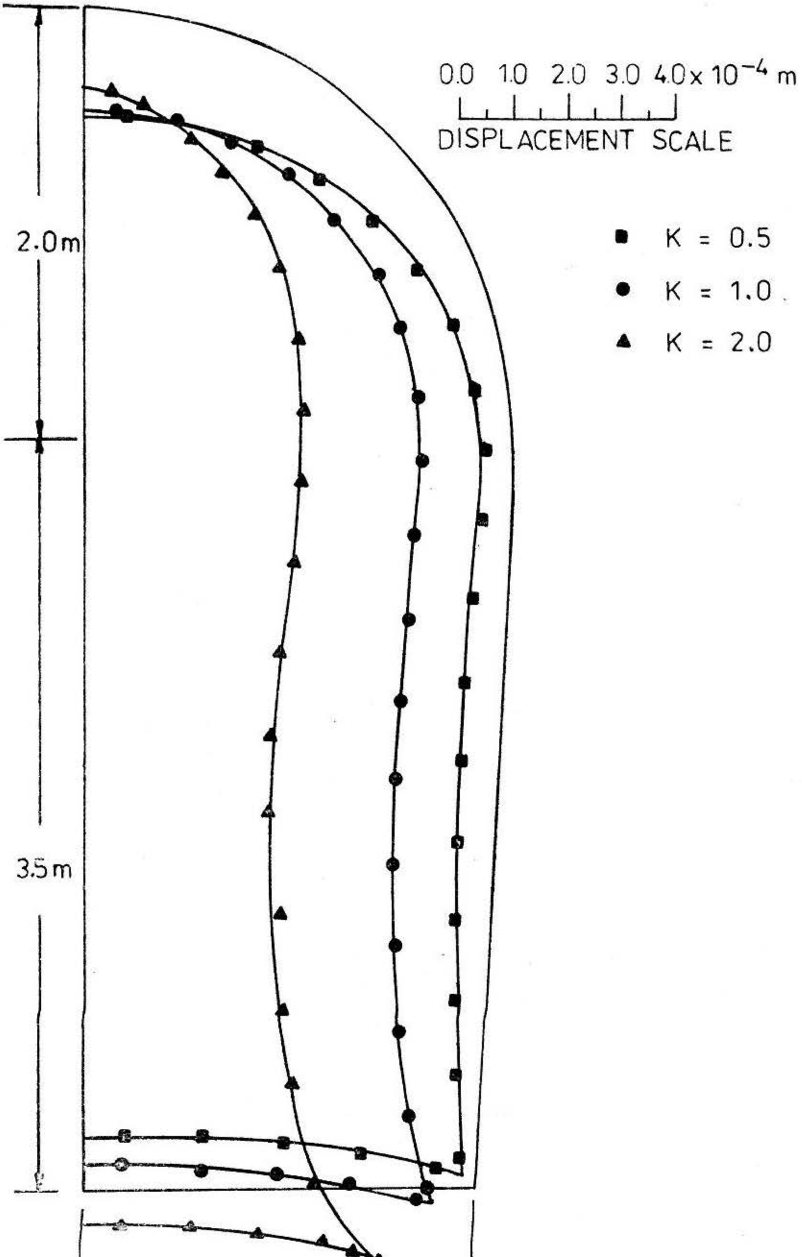


FIGURE 12 Displacements Along Boundary

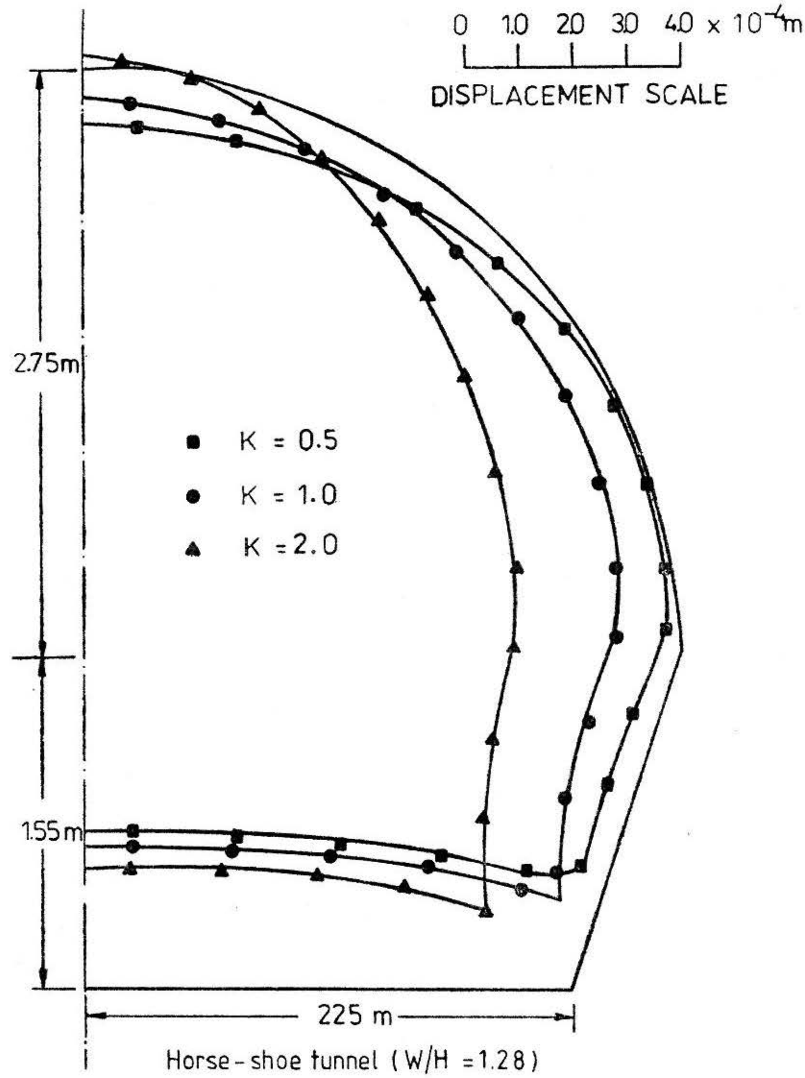
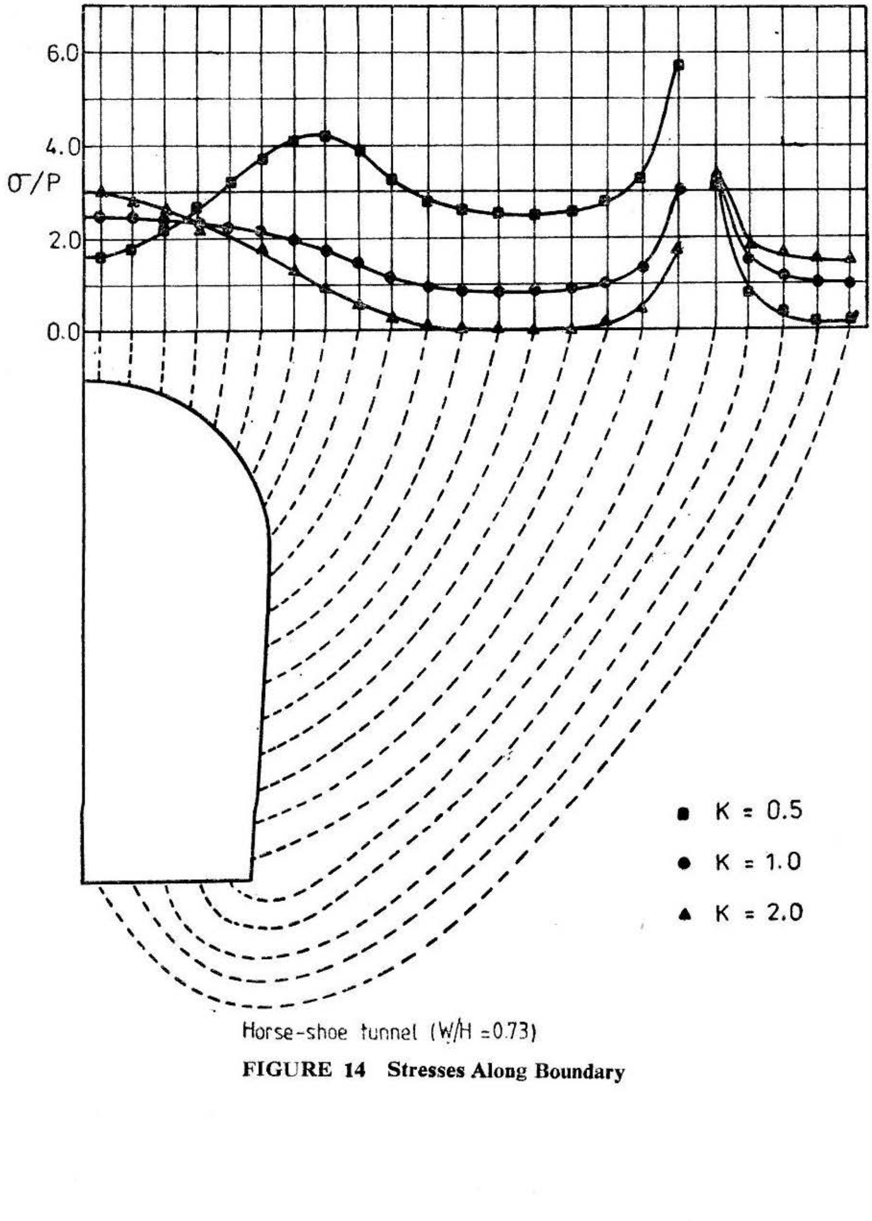


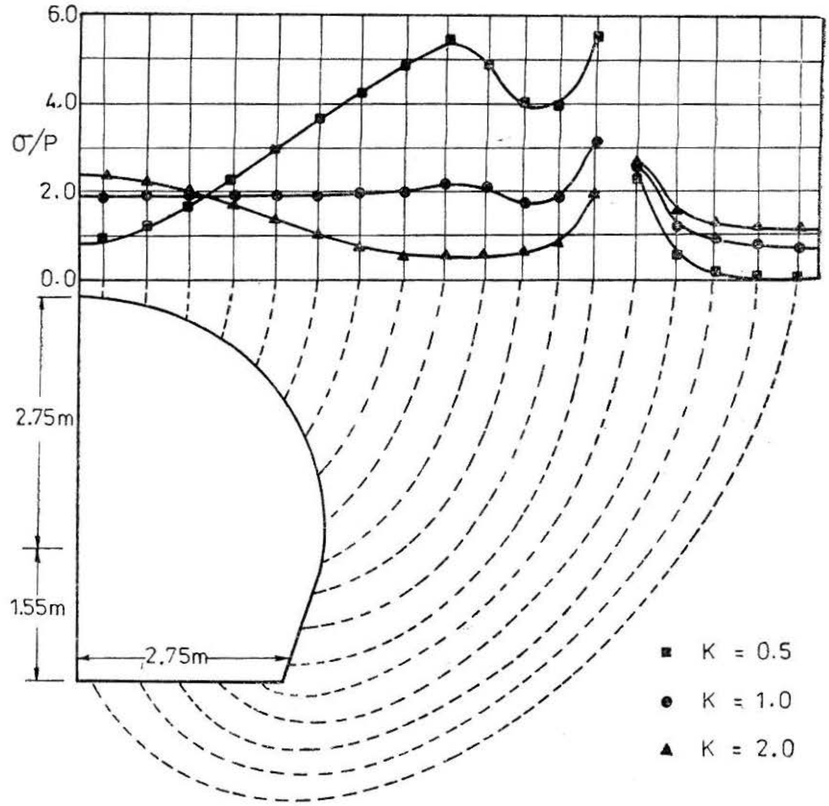
FIGURE 13 Boundary Displacements

Figs. 14, 7 and 15 show major principal stress variation along the boundary of the tunnel for the three cases. The effect of stress ratio is similar in all cases. As the width/height ratio increases the stress at crown and invert portion decreases and the stress in the sidewall portion increases.



Horse-shoe tunnel (W/H = 0.73)

FIGURE 14 Stresses Along Boundary



Horse-shoe tunnel (W/H = 1.28)

FIGURE 15 Stresses Along Boundary

Figs. 16, 10 and 17 show the variation of factor of safety along the boundary of the tunnel. With the increase in width/height ratio, there is increase in factor of safety at the crown and invert of the tunnel and decrease in the value in the sidewall. The tunnel with higher width/height ratio show better behaviour for all the stress ratios.

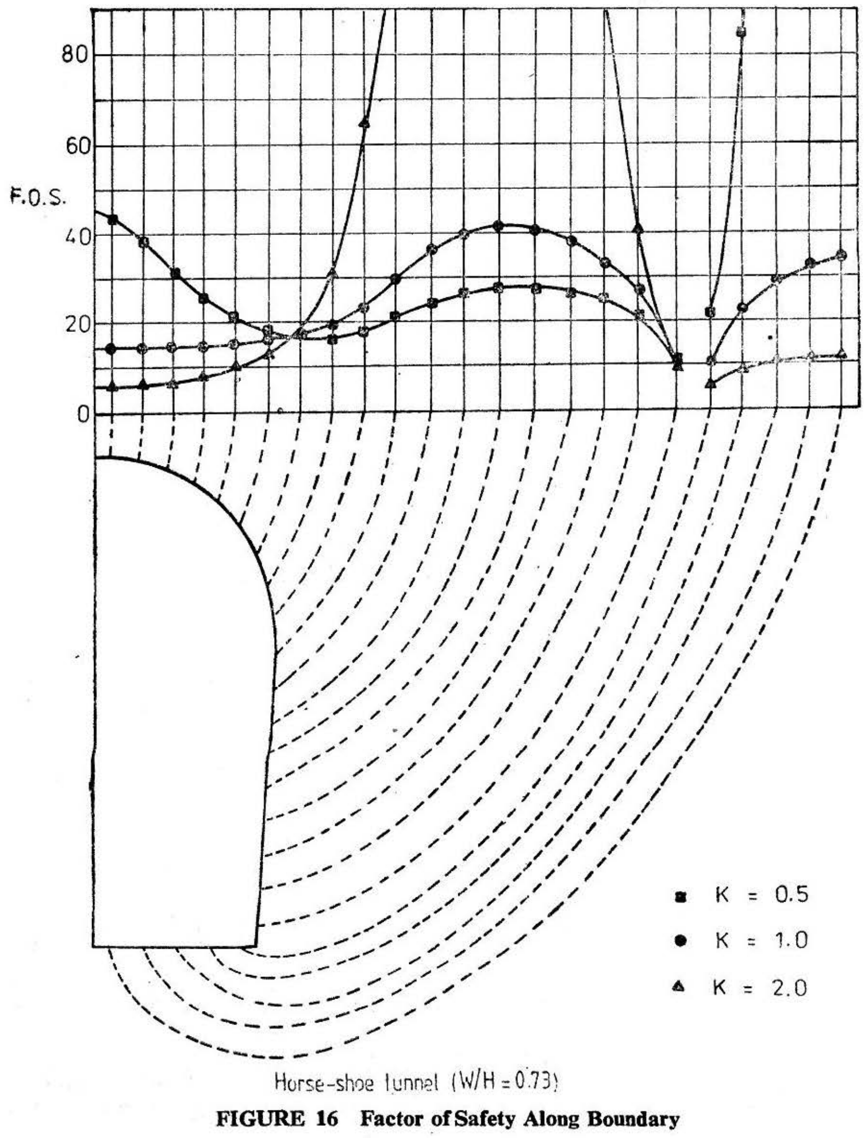


FIGURE 16 Factor of Safety Along Boundary

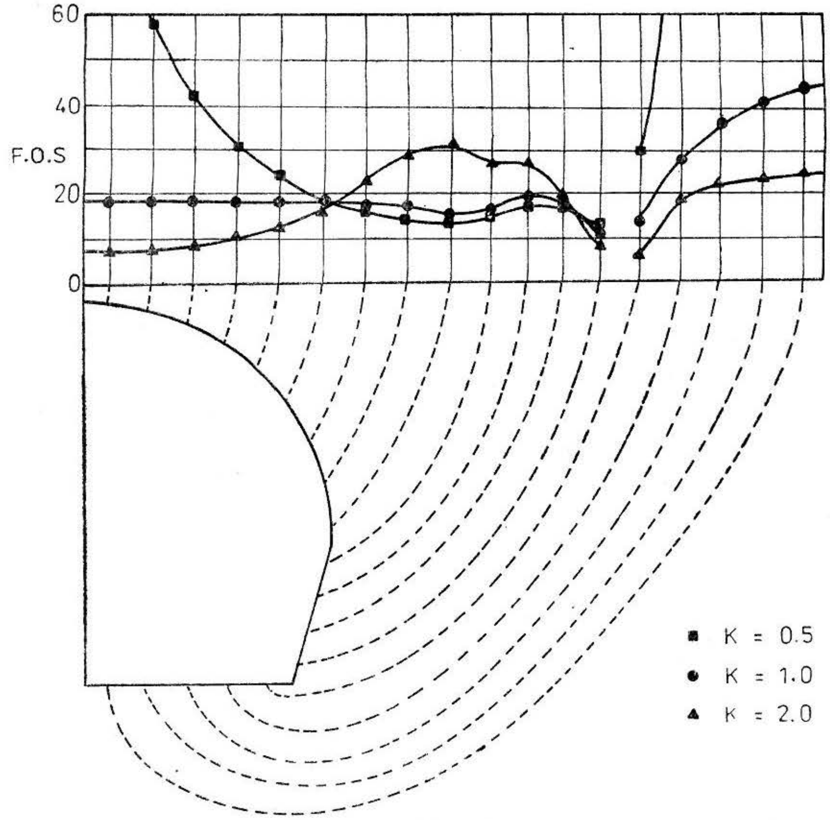


FIGURE 17 Factor of Safety Along Boundary

Conclusions

The Boundary Element Method with particular reference to indirect formulation is presented. The use of BEM in tunnel openings is illustrated by the study of the effect of tunnel shapes and the effect of width/height ratio on horse-shoe tunnel.

Of the three tunnel shapes, *viz.*, *D* type, horse-shoe and 4 arc, the four arc shaped tunnel indicate better behaviour. In the case of horse-shoe shaped tunnel, it is observed larger the width/height ratio, better is the performance of the tunnel.

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