Estimation of the Angle of Internal Friction of Non-Cohesive Soils from Sounding Tests

by

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Introduetion

Soil parameters like the angle of internal friction, ϕ, relative density, I_D, modulus of compressibility E_S, etc. are determined either in the laboratory or in situ. In situ testing should be preferred than the laboratory testing for obvious reason that the results obtained by testing soils in their natural state are more reliable than the results obtained in the laboratory. In some instances it is very difficult to secure undisturbed samples. In fact there is not to date a reliable and satisfactory method for securing undisturbed samples of non-cohesive soils.

For more than three decades, sounding tests have been used widely for the purpose of soil identification. If judiciously used sounding tests also offer the possibility of obtaninig quantitative information on soil parameters. A method for estimating the modulus of compressibility from sounding tests has been presented elsewhere (Teferra, 1976*a*). In this paper results of static and dynamic penetrometers are analysed and statistical formulae are suggested to determine indirectly the angle of internal friction of non-cohesive soils.

Sounding Test

Sounding tests are commonly carried out by using quasi-static penetrometers—which hereafter will be called static penetrometers and dynamic penetrometers.

In the static penetrometer, the sounding device which has a diameter of 3.5 cm is pushed into the ground at a low speed (0.25 m/min). The point and total resistances are recorded at specified depths. Definite procedures exist for carrying out the test (Schultze and Muhs, 1967).

In the dynamic penetrometer, the device is pushed into the ground at specified depths by applying an impact load. The magnitude of the impact load, the falling height and the diameter of the penetrometer vary according to the type of penetrometer. The specified depths of penetration may be 13, 20, or 30 cm (Schultze and Muhs, 1967; Sanglerat, 1972).

The point resistance of a static penetrometer is designated by σ_s and is usually measured in kg/cm², while that of a dynamic penetrometer is designated by N_{10}, N_{20}, or N_{30}, in which N stands for number of blows and the indices 10, 20, and 30 represent the specified depths of penetration in cm.

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(*This paper was received in October 1983, and is open for discussion till the end of April 1984*).

In this paper the Dutch-Cone and Maihak Penetrometers are used to determine the static point resistance (Schultze and Muhs, 1967; Sanglerat, 1972). The dynamic resistance is measured by the number of blows necessary to penetrate the ground by 20 cm. The diameter of the penetrometer is 4.4 cm. The falling height is 50 cm with a mass of 50 kg. Typical test results of static and dynamic penetrometers for homogeneous sand are given in Fig. 1.

Limiting Depth

In homogeneous non-cohesive soils, field and laboratory test indicate that the point resistance increases generally with depth. From a certain point onwards, however, the resistance becomes constant (Fig. 1). The depth beyond which no substantial increase in point resistance is registered is called the limiting or critical depth.

Based on the bearing capacity theory of Meyerhof, de Beer (1963) gave a theoretical analysis of the limiting depth (Fig. 2). According to him, the limiting depth is reached when angle θ attains the value of π (Fig. 2*b*).

The limiting depth is expressed as

$$t_{c2} = d \tan \ (45 + \phi/2) \, e^{\pi \tan \phi} \qquad \text{...(1)}$$

or

$$\eta = \frac{t_{c2}}{d} = \tan (45+\phi/2) \ e^{\pi \tan \phi} \qquad \text{...(2)}$$

where

t_{c2} = limiting depth,

η = limiting depth coefficient,

ϕ = angle of internal friction, and

d = diameter of penetrometer.

The corresponding point resistance would then be :

$$\sigma_{sc} = \nu_t \tan^2 (45+\phi/2) \, e^{2\pi \tan \phi} \ .\gamma.t_{c2} \qquad \text{...(3)}$$

or

$$\sigma_{sc} = \nu_t.\gamma d \ \tan^3 (45+\phi/2) \ e^{3\pi \tan \phi} \qquad \text{...(4)}$$

where

ν_t = shape factor, and

γ = unit weight of the soil.

Even though the above equations are strictly valid for plastic materials (de Beer, 1963), one could use them in dense sands, since the deformation behaviour of dense sand approximately follows that of a plastic material. A closer inspection of the Equations (1) to (4) reveals that the angle of internal friction plays an important role in influencing the magnitude of the limiting depth and that of the point resistance.

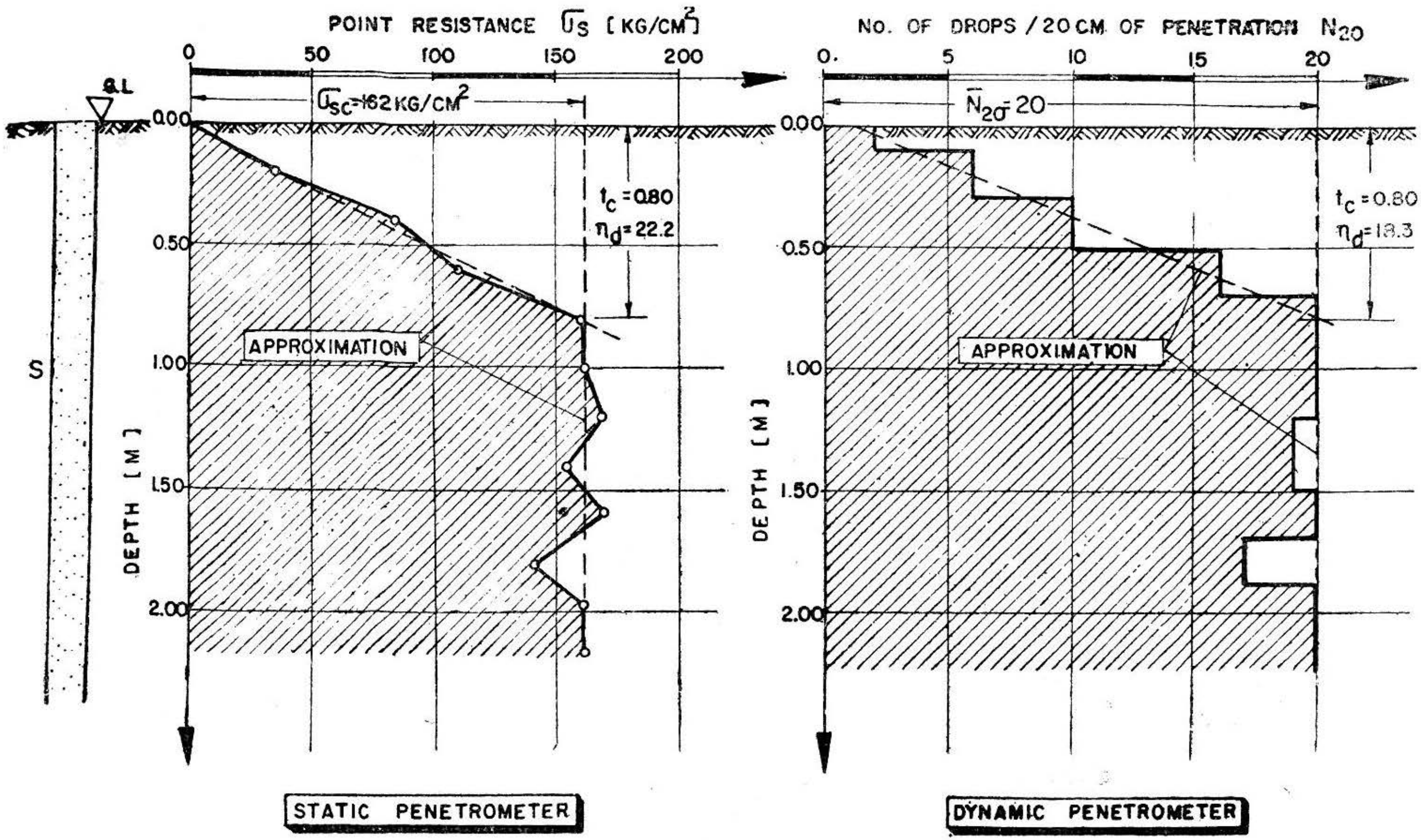


FIGURE 1 Typical Test Results of Penetration Tests for Homogeneous Sand

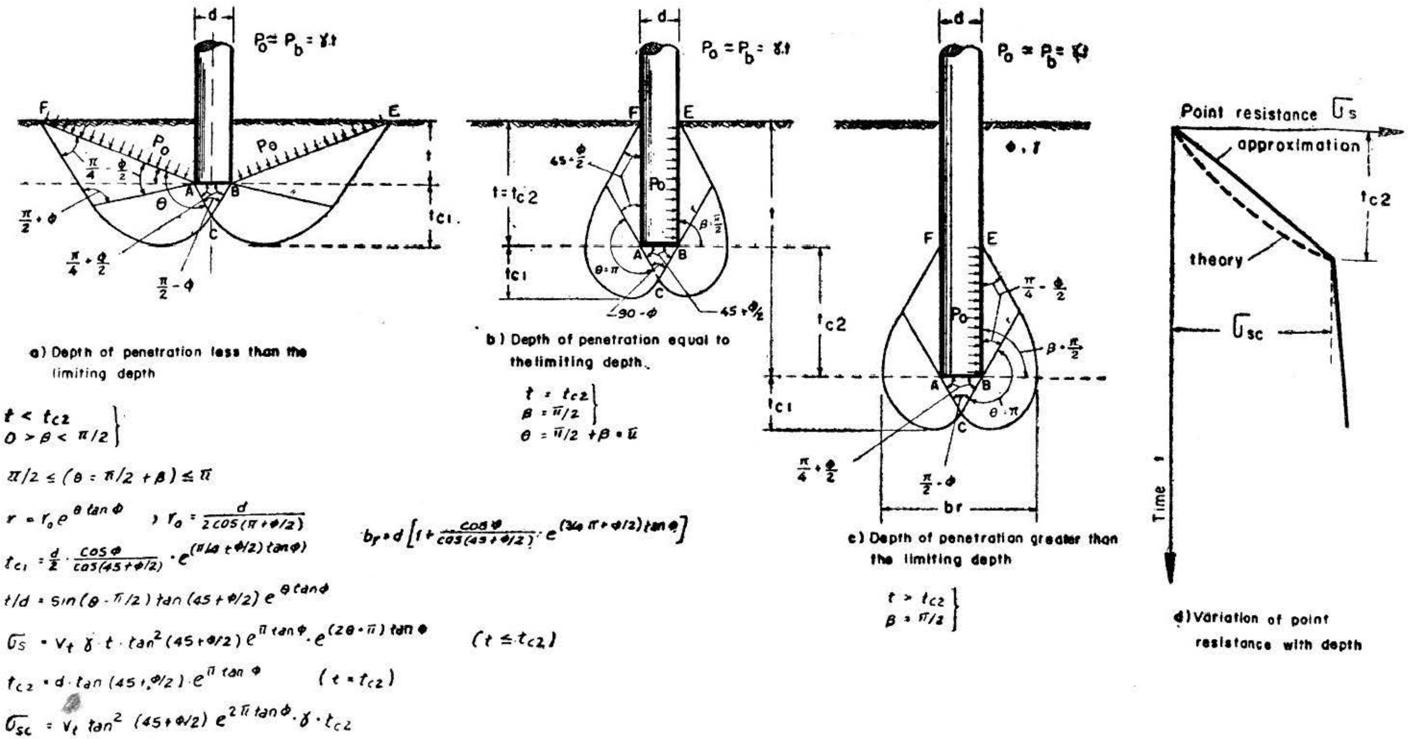


FIGURE 2 Development of Slip Surface During Penetration Test for a Homogeneous Plastic Material According to de Beer (1963)

For ground conditions which have layers of different densities, the variation of the point resistance will be influenced by the magnitude of the angle of internal friction of the respective stratum. At the boundary of two strata, there will be an additional or reduced point resistance depending upon the magnitude of the angle of internal friction of the neighbouring layer. The additional or reduced point resistance commences at a distance of t_{c1}. The limiting depth, t_c, in this case will be the sum of t_{c1} and t_{c2} (Fig. 3).

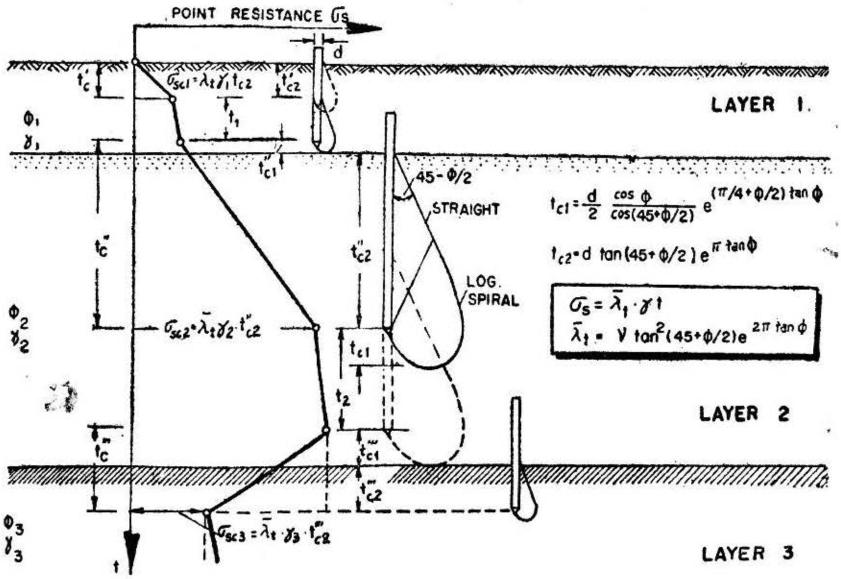


FIGURE 3 Point Resistance in a Layered Soil Profile with Different Densities according to Schultze (1970).

Based on the above theoretical equations, one may arrive at the conclusion that for static and dynamic penetrometers the relationship between the limiting depth coefficient and point resistance may be expressed by the following exponential functions (Teferra, 1976 *b*):

$$\eta_s = a_s\,(\sigma_{sc})^{b_s} \qquad \ldots(5)$$

$$\eta_d = a_d\,(N_{20})^{b_d} \qquad \ldots(6)$$

Penetration test results of non-cohesive soils, in which the limiting depth and the corresponding point resistance are clearly identifiable, were collected and by the application of Equations (5) and (6) were statistically correlated (Teferra, 1976 *b*). The results of the regression analyses are given below:

(*i*) For static penetrometer:

$$\log \eta_s = 0.508 + 0.407 \log \bar{\sigma}_{sc} \pm 0.085 \qquad \ldots(7)$$

(*ii*) For dynamic penetrometer:

$$\log \eta_d = 0.839 + 0.296 \log \bar{N}_{20} \pm 0.024 \qquad \ldots(8)$$

where

$$\eta_s = \frac{t_c}{d} = \text{limiting depth coefficient for the static penetrometer,}$$

$\eta_d = \frac{t_c}{d}$ = limiting depth coefficient for the dynamic penetrometer,

$\bar{\sigma}_{sc} = \frac{\sigma_{sc}}{\sigma_{so}}$,

σ_{so} = unit point resistance, and

$\bar{N}_{20} = N_{20}$ at the limiting depth.

Relationship Between Relative Density and Penetration Test Results

Static and dynamic penetration test results are directly affected by the relative density. For compacted materials, the penetration test results are considerably higher than for a less compacted material. Based on experimental observations and theoretical analysis, the relationship that exists between relative density, point resistance and overburden pressure may be expressed in the following general equations for the static and dynamic penetrometers (Teferra, 1976 *b*).

For static penetrometer :

$$I_D = a_O + m_1 \log \bar{\sigma}_S - m_1 \log \bar{\sigma}_O \qquad \text{...(9)}$$

For dynamic penetrometer :

$$I_D = c_O + m_2 \log \bar{N}_{20} - m_2 \log \bar{\sigma}_O \qquad \text{...(10)}$$

where

$\bar{\sigma}_S = \frac{\sigma_S}{\sigma_{SO}}$,

σ_S = point resistance for static penetrometer,

$\bar{\sigma}_O = \frac{\sigma_O}{\sigma_{SO}}$,

σ_O = overburden pressure equal to $\gamma\, t$,

t = depth of penetration, and

a_O, c_O, m_1, m_2 = statistical coefficients.

For conditions below the limiting depth, as explained earlier, the overburden pressure does not have an effect on the point resistance. As a result the last expressions of Eqs. (9) and (10) will be independent of the depth t and thus attain constant values. Introducing the limiting depth into Eqs. (9) and (10) one obtains general equations for the relative density below the limiting depth (Teferra, 1976 *b*).

For static penetrometer :

$$I_D = b_O + b_1 \log \bar{\sigma}_{SC} \qquad \text{...(11)}$$

For dynamic penetrometer :

$$I_D = d_O + d_1 \log \bar{N}_{20} \qquad \text{...(12)}$$

where

b_O, b_1, d_O, d_1 are statistical coefficients.

Data from sounding tests on non-cohesive soils with different grain size distribution ranges (from 10 to 0.06 mm) were collected (Teferra, 1974). The relative density, the overburden pressure and the point resistance at specified depths were systematically compiled. Using the functions of Eqs. (9) to (12) statistical analyses were conducted the results of which are given below (Teferra, 1976).

For static penetrometer :

$$I_D = -0.260+0.340 \log \sigma_S - 0.340 \log \sigma_O \text{ for } t<t_C \quad \text{... (13)}$$

$$I_D = 0.310+0.200 \log \sigma_{SC} \text{ for } t>t_C \quad \text{... (14)}$$

For dynamic penetrometer :

$$I_D = -0.145+0.385 \log N_{20} - 0.384 \log \sigma_O \text{ for } t<t_C \quad \text{... (15)}$$

$$I_D = 0.340+0.270 \log N_{20} \text{ for } t>t_C \quad \text{... (16)}$$

Estimation of the Angle of Internal Friction

As indicated above, one is in a position to determine the relative density using the results of penetration tests. If the relative density is known, one may obtain the void ratio at the natural state from the well known definition of relative density.

$$I_D = \frac{e_{max} - e}{e_{max} - e_{min}} \quad \text{... (17)}$$

where

e_{max} = void ratio at the loosest state,

e_{min} = void ratio at the densest state, and

e = natural void ratio.

From Equation (17) one may write

$$e = e_{max}\,(1-I_D)+I_D\,e_{min} \quad \text{... (18)}$$

For non-cohesive soils which are not very densely compacted, the following empirical formula exists (Schultze, 1968) :

$$\cot \phi = a.e+b \quad \text{... (19)}$$

where a and b are coefficients which are functions of grain size distribution (Teferra, 1975).

With the help of Equation (19) one is in a position to estimate the angle of internal friction. One may conveniently present the procedure of arriving at the angle of internal friction as shown in Table 1.

Assessment of the Empirical Equations

The accuracy of the suggested method was assessed by using a non-cohesive soil which was not included in the statistical evaluation. The soil is sand with little gravel ($<10\%$) having a uniformity coefficient D_{60}/D_{10} $=2.42$ and a coefficient of curvature $= (D^2_{30})/D_{60}.D_{10} = 0.94$. The maximum and minimum void ratios as determined by DIN standard (Schultze and Muhs, 1967) are 0.772 and 0.438, respectively (Melzer, 1968). The specific gravity of the solid constituents is 2.66. The angle of internal friction is determined from consolidated drained triaxial tests.

Void ratio, overburden pressure, relative density and angle of internal friction together with point resistances were determined at specified depths (Melzer, 1968). Table 2 gives the measured and calculated values of the soil parameters. In calculating the relative density, the equation above the limiting depth is used since the limiting depth is not clearly indicated in the experiment. The measured and calculated values of the angle of internal friction are shown in Fig. 4. The static penetrometer appears to give better result

TABLE 1

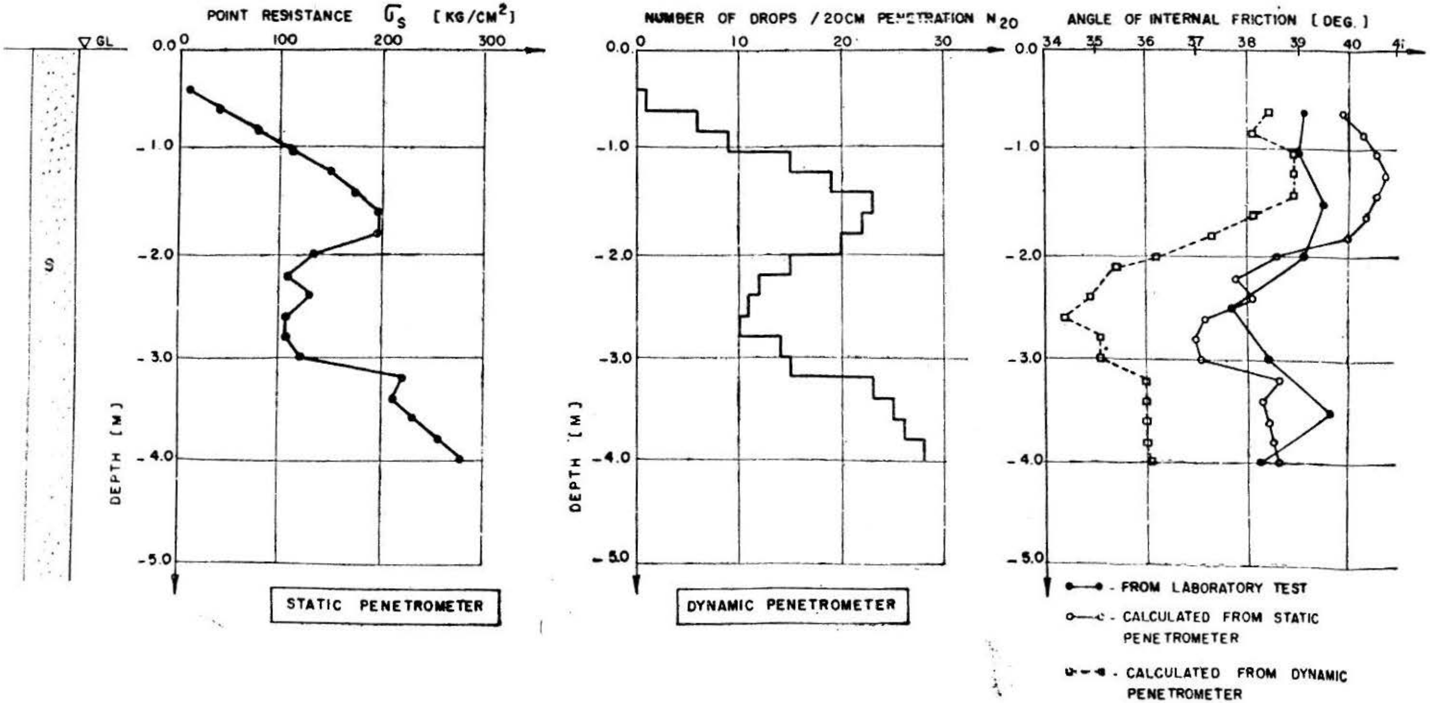
Determination of Angle of Internal Friction from Sounding Tests

		Relative Density from Sounding Tests	Angle of Internal Friction
Equations above the limiting depth	Static Penetrometer	$I_D = -0{\cdot}260+0.340 \log \sigma_S$ $-0.340 \log \bar{\sigma}_O$	$\cot \phi = a.e+b$
	Dynamic Penetrometer	$I_D = -0.145+0.385 \log N_{20}$ $-0.385 \log \bar{\sigma}_O$	$a = 2.135+0.097\ D_{85}/D_{15}$
Equations below the limiting depth	Static Penetrometer	$I_D = 0.310+0.200 \log \bar{\sigma}_{SC}$	$b = 0.845-0.398\ a$
	Dynamic Penetrometer	$I_D = 0.340+0.270 \log \bar{N}_{20}$	D_{85} = grain-size at $P = 85\%$ D_{15} = grain-size at $P = 15\%$

TABLE 2

Sounding Tests Results Together with Measured and Calculated Values of Angle of Internal Friction Using the Test Material of Melzer (1968)

Depth	Results of Laboratory Investigation						Calculated Value with $a = 2.521$; $b = 0.158$; $e_{max} = 0.772$ and $e_{min} = 0.438$							
							From Static Penetrometer				From Dynamic Penetrometer			
t m	e	σ_O kg/cm²	I_D	ϕ deg	σ_S kg/cm²	N_{20}	I_D	e	cot ϕ	ϕ deg	I_D	e	cot ϕ	ϕ deg
−0.40	—	—	—	—	9	1	—	—	—	—	—	—	—	—
−0.60	0.570	0.06	0.664	39.1	40	6	0.700	0.538	1.198	39.9	0.625	0.563	1.261	38.4
−0.80	0.570	0.10	0.664	—	78	9	0.723	0.531	1.181	40.3	0.607	0.569	1.276	38.1
−1.0	0.575	0.13	0.649	39.0	114	15	0.741	0.525	1.166	40.6	0.649	0.555	1.241	38.9
−1.20	0.577	0.16	0.644	—	150	19	0.750	0.522	1.158	40.8	0.654	0.554	1.239	38.9
−1.40	0.570	0.20	0.664	—	176	23	0.741	0.525	1.166	40.6	0.648	0.555	1.241	38.9
−1.50	—	—	—	39.5	—	—	—	—	—	—	—	—	—	—
−1.60	0.558	0.24	0.599	—	199	22	0.732	0.528	1.173	40.4	0.611	0.568	1.274	38.1
−1.80	0.553	0.27	0.713	—	199	20	0.705	0.536	1.193	40.0	0.563	0.584	1.314	37.3
−2.00	0.570	0.31	0.664	39.1	134	15	0.634	0.560	1.254	38.6	0.504	0.604	1.365	36.2
−2.20	0.577	0.34	0.664	—	109	12	0.592	0.574	1.289	37.8	0.451	0.621	1.408	35.4
−2.40	0.567	0.37	0.570	—	133	11	0.609	0.569	1.276	38.1	0.422	0.631	1.433	34.9
−2.50	—	—	—	37.7	=	—	—	—	—	—	—	—	—	—
−2.60	0.570	0.42	0.561	—	109	10	0.561	0.585	1.317	37.2	0.385	0.643	1.463	34.4
−2.80	0.575	0.45	0.545	—	108	14	0.549	0.589	1.327	37.0	0.430	0.628	1.425	35.1
−3.00	0.555	0.48	0.606	38.4	120	15	0.555	0.587	1.322	37.1	0.431	0.628	1.425	35.1
−3.20	0.520	0.52	0.712	—	222	23	0.634	0.560	1.254	38.6	0.489	0.609	1.377	36.0
−3.40	0.529	0.55	0.685	—	212	25	0.619	0.565	1.266	38.3	0.491	0.608	1.375	36.0
−3.50	—	—	—	39.6	—	—	—	—	—	—	—	—	—	—
−3.60	0.529	0.58	0.685	—	233	26	0.626	0.563	1.261	38.4	0.491	0.609	1.377	36.0
−3.80	0.538	0.62	0.685	—	255	28	0.629	0.562	1.259	38.5	0.492	0.608	1.375	36.0
−4.00	0.565	0.65	0.604	38.2	277	30	0.634	0.560	1.254	38.6	0.496	0.606	1.370	36.1
	$\phi_{mean} = 38.8$ Stand. deviation $= 0.7$ $n = 8$						$\phi_{mean} = 38.9$ Stand. deviation $= 1.3$ $n = 18$				$\phi_{mean} = 36.7$ Stand. deviation $= 1.5$ $n = 18$			



a) VARIATION OF THE POINT RESISTANCE   b) VARIATION OF NUMBER OF DROPS   c) ANGLE OF INTERNAL FRICTION

FIGURE 4 Measured and Calculated Values of the Angle of Internal Friction

than the dynamic penetrometer. In this connection it would be worthwhile to mention that since the determination of e_{max} and e_{min} is not internationally standardized one would obtain different values of e_{max} and e_{min} for the same material depending upon the method applied. However, since a large amount of data is obtainable from penetrometers, it would enable one to arrive at a statistically ascertained value of the angle of internal friction. In the present example the mean values for the angle of internal friction according to the test results of the static and dynamic penetrometers are 31.9° and 36.7°, respectively. However, if one combines the test data of both penetrometers, the mean value of the angle of internal friction would be 37.8°. The mean value of the angle of internal friction from actual test is 38.8°. The maximum variation of the calculated mean value from the measured mean value is 2°.

The strength of the above equations should be checked with other non-cohesive soils. One may even adjust the coefficients of the statistical equations if additional data are available.

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