Maximum Density of Cohesionless Soils

by

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Introduction

Probably, the most important index to the behaviour of cohesionless soils is relative density (D_r). It is widely used in seismic studies such as for foundations in cohesionless soils and embankments subjected to earthquake and other vibrations. The susceptibility of a given cohesionless soil to liquefaction is estimated to a high degree by its relative density. It is expressed with respect to its maximum and minimum densities obtained. The computation of relative density basically involves three different density (maximum, minimum and insitu) parameters. However, the maximum density is of very much interest since it is closely related to the relative density. Also, any error in the determination of maximum density greatly affects the accuracy of relative density (Tavenas and La Rochelle, 1972).

Previous Work

A brief literature survey shows that more than thirty different methods have been proposed for the determination of maximum density and most of them recommend the use of vibration in one form or another. However, there is a wide difference in these recommendations as to the type and intensity of vibrations which should be employed. Also, the method used to determine the maximum density plays an important role on the computation of relative density.

Selig (1963) used a vertical shake table with a variable frequency and amplitude for compaction of sand. The maximum frequency obtained was 250 Hz. He concluded that for a unique correlation between density and vibration, only two vibration parameters are required and the best two are *acceleration and frequency. D'Appolonia and D'*Appolonia (1967) used a vertical vibrating table with variable amplitude (0-2.54 mm) and frequency (10-60 Hz), which produced unidirectional harmonic motions. The conclusion drawn by them is that acceleration is the primary factor for obtaining the maximum density of a given cohesionless soil. Das (1969) determined the maximum density of four sands employing a horizontal shake table with three different sizes of mould. The frequency range was 0-20 Hz and the amplitude range was between 0 and 10 mm. Tavenas and La Rochelle (1972) carried out maximum density tests on five sands using the ASTM-D-2049-64T specifications. This code recommended a fixed

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amplitude of 0.635 mm and a frequency of 60 Hz. The maximum density values for all the soils achieved were at different amplitudes which were greater than the ASTM provisions. Further, the ASTM standards specify a constant value of amplitude for all the soils irrespective of their grain size, shape, etc. More or less, the ASTM-D-2049-69 and IS: 2720 (Part XIV)—1968 standards recommend the same specifications. Nevertheless, these codes do not provide a clear-cut specification regarding the vibration parameters which can be employed to get the maximum density value for a particular soil.

Therefore, a universally recognized and standard (simplest and most practical) method is essential for the determination of maximum density and the lack of such a method hampers the field compaction control. As can be seen, the codes do not point out the appropriate vibration parameters to be used for achieving the maximum density The apparatus and the test procedures required for obtaining the maximum density are left to the discretion of the user. Therefore, it was felt desirable to simplify the procedure without sacrificing the accuracy.

The important parameters affecting the maximum density of a given soil can be grouped into

1. Soil characteristics
   (*a*) gradation of the soil,
   (*b*) grain size,
   (*c*) grain shape.
2. Vibration characteristics
   (*a*) acceleration,
   (*b*) frequency,
   (*c*) displacement,
   (*d*) type of vibration,
   (*e*) duration of vibration,
   (*f*) surcharge during vibration.

The factors considered in this study include particle size distribution, mean diameter of soils, particle shape and vibration parameters. Many tests have been carried out to study these factors and the mean values are presented herein.

Scope of Work

The following materials in which the fines (passing through 0.075 mm sieve) not more than 12 per cent were selected for the purpose of investigation.

(*a*) Sample A-poorly graded fine sand with sub-rounded particles.

(*b*) Sample B-poorly graded fine sand with sub-angular particles.

(*c*) Sample C-well graded sand with rounded to sub-rounded particles.

(*d*) Sample D-poorly graded coarse sand with angular particles.

(*e*) Sample E-poorly graded gravel with well rounded particles.

For all testing work the procedure recommended in IS: 2720 (Part XIV)—1968 has been adopted. Fundamental considerations and the brief review of existing literature show that acceleration is a primary factor causing compaction of granular materials. Thus, in the present work only acceleration has been varied.

Apparatus

The commercially available vertical shake table used in the present study consists of a cushioned steel vibrating deck about 76.2 cm on which moulds could be mounted. A vibrator is fixed at the bottom of the deck and is driven by an alternate current motor through a belt drive. The deck has been mounted on the top of the table through four spring supports.

No arrangement has been provided to vary any of the four vibration parameters (acceleration, velocity, displacement or amplitude and frequency) but with a fixed frequency of 56 Hz. The vibrator is of rotating mass type consisting of two pulleys with eccentric masses. The vibrator generated a total centrifugal force of 480 kg. The net weight of the vibrating deck, guides and springs is 45.40 kg. The details of the apparatus and the eccentric mass system are shown in Figs. 1 and 2, respectively.

Measurement of Acceleration

Acceleration has been recorded at the bottom of the vibrator by fixing an acceleration pick-up and whose output was amplified through a universal amplifier and obtained on an ink-writing oscillograph. Recording acceleration at the bottom of the vibrator was significant since this location was lying along the line passing through the centre of gravity of the top deck.

Also, free vibration test was conducted to calculate the natural frequency (=15.44 Hz) of the vibrating system using impact hammer method.

Modification

The apparatus is capable of producing (without any modification) a constant acceleration of 0.55 g. Arrangement has been made to vary the acceleration from 0.55 g onwards by increasing the eccentric mass which was bolted as an additional mass on either pulleys. The details of modification are also shown in Fig. 2. The different masses employed and the acceleration generated thereby are given in Table 1.

Soils Tested

Particle size distribution of the samples used are shown in Fig. 3. The soils fall between the range of 12 per cent fines (maximum amount passing through 75 microns) and 40 mm (maximum size). Properties such as angle of friction (ϕ) using direct shear boxes at D_r = 50 per cent and specific gravity (G) were determined in the laboratory and these values are shown in Table 2.

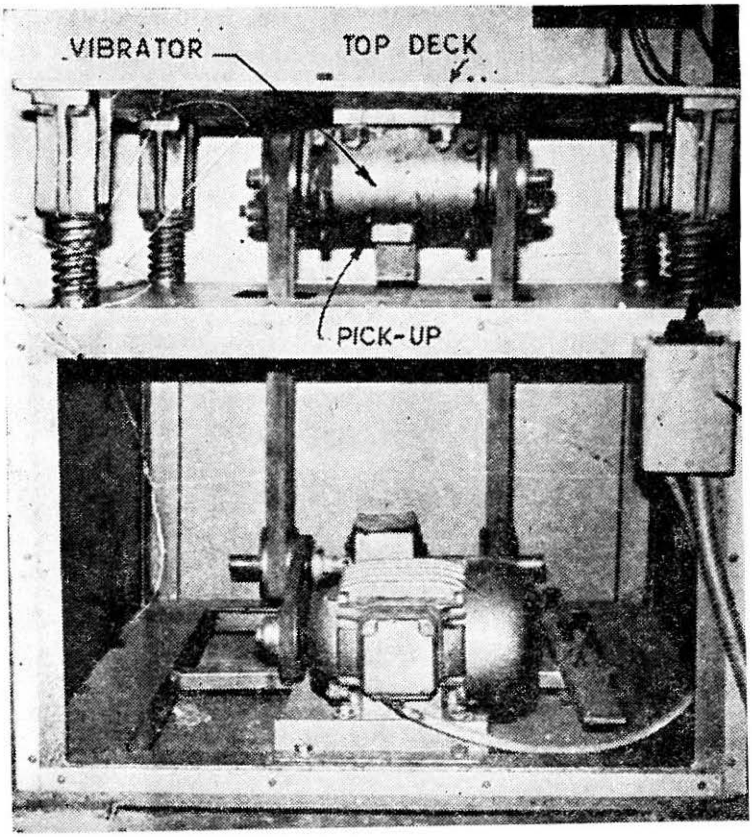
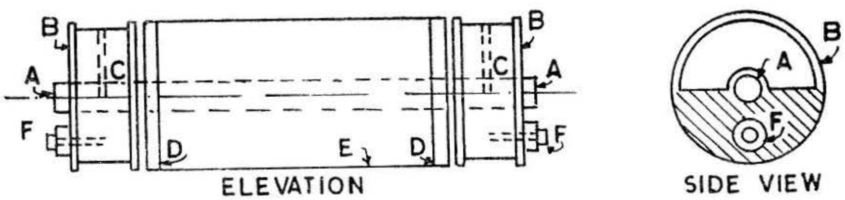


FIGURE 1 Relative Density Appartus

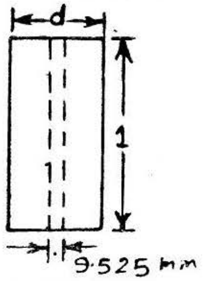


A — SHAFT
B — PULLEY
C — CLAMP
D — COUPLING
E — CASING
F — ARRANGEMENT TO INCREASE ACCELERATION

FIGURE 2 Vibrator with Modifications

TABLE 1

Details of Mass and Acceleration

Eccentric mass increment %	Acceleration g	Details of mass: Mass (gm/cm) x (sec)²	Details of mass: Diameter d (cm)	Details of mass: Length l (cm)	Remarks
0	0.55	—	—	—	
13	1.00	0.0854	2.6	2.0	
25	1.10	0.1642	2.6	3.5	
35	1.20	0.2299	2.6	3.5	
45	1.30	0.2955	—	—	
51	1.37	0.3349	—	—	

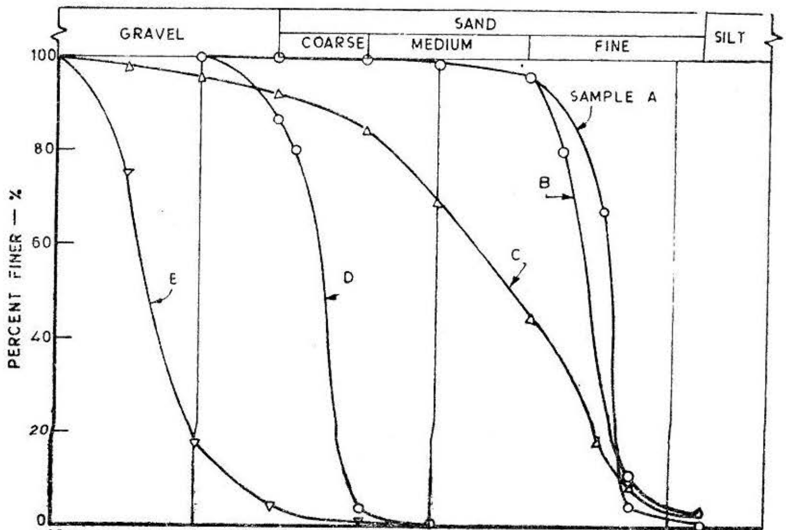


FIGURE 3 Grain Size Distribution Curves

Test Procedure

As stated earlier the maximum density tests were conducted as per the specifications given in IS:2720 (Part XIV)—1968. A brief description of the methods is given below.

TABLE 2

Different Properties of the Tested Soils

Sample	A	B	C	D	E
C_c	0.91	1.173	0.8	0.9	1.164
C_u	1.214	1.803	5.0	1.08	1.28
D_{50} (mm)	0.179	0.223	0.51	2.108	15.00
G	2.564	2.647	2.730	2.703	2.72
ϕ (deg)	27	29	33	38	36
Classification (IS:1498-1970)	SP	SP	SW	SP	GP
Major mineral component	Quartz	Quartz	Silica	Silica	Silica
Particle shape	Rounded	Sub angular and sub rounded	Rounded to sub rounded	Angular	Well rounded

Dry Method

In this method the soil was filled in the small mould (3000 cc) and the surcharge base plate was placed gently at the top level of the soil. The initial dial gauge reading was taken at six positions on the surcharge base plate and the mean value was calculated. The mould with the soil and the surcharge base plate was held firmly on the top deck with screw bolts. The dead weight surcharge was placed over the surcharge base plate and the outer collar was fixed in position and vibrated for a period of 8 minutes using the steady-state response of the shake table since the difference between natural frequency and forcing frequency of the system is very large.

The final dial gauge reading was taken as before. Acceleration had been varied between 0.55 g to 1.20 g for samples A, B, C and E whereas, for sample D alone the range was from 0.55 g to 1.35 g.

Wet Method

Here the mould was first fixed on the deck and filled with fully saturated sample. A small amount of free water was allowed to accumulate on the surface of the soil during filling. During and just after filling, the mould was vibrated for a total period of 6 minutes. During the final minutes of vibration, any water appearing above the soil surface was removed.

After 6 minutes of vibration the top surface was levelled off and vibrated for 8 minutes with all the accessories (surcharge base plate, surcharge weight, etc.) in position. The sample was kept for drying in an oven and the maximum density was computed. As mentioned in the dry method, the initial and final dial gauge readings were determined and the same acceleration range was used. At least five tests in dry method and three tests in wet method have been carried out for determining the maximum density at each step of acceleration for each soil.

Besides the maximum density tests, the adequacy of the available frequency (56 *Hz* fixed by the manufacturer) had been verified using a speed control unit and a direct current motor by varying the frequency on sample A alone.

Test Results and Discussions

The maximum density values as obtained above and the vibration parameters employed are given in Table 3.

Fig. 4 shows the acceleration—density relationship for sample A, obtained using both dry and wet methods. Similarly, samples B, C, D and E also exhibited the same trend. It can be seen in all the cases that there exists an optimum value of acceleration, may be, because as the acceleration increases beyond this value, the imparted energy to the soil is more and the soil particles tend to separate during vibrarion and settle into a looser state when vibration is halted (Selig, 1963).

The acceleration—density relationships for all the samples are shown in Fig. 5. Samples A, B, C and E yielded the maximum density at an acceleration of 1.0 *g* and sample D alone yielded at 1.20 *g*. This may be due to greater influence of some important parameters and hence a shake table wherein acceleration could be varied is inevitable.

Further, it is seen that for samples A, B, D and E the maximum density is obtained using the dry method whereas, for sample C it is obtained in the wet method.

TABLE 3

Maximum Density and Vibratory Parameters

Sample	Optimum acceleration (*g*)	Frequency (*Hz*)	Maximum density (g/cc) Method	
			Dry	Wet
A	1.0	56	1.720	1.652
B	1.0	56	1.740	1.680
C	1.0	56	1.958	1.985
D	1.0	56	1.774	1.675
E	1.0	56	1.832	1.814

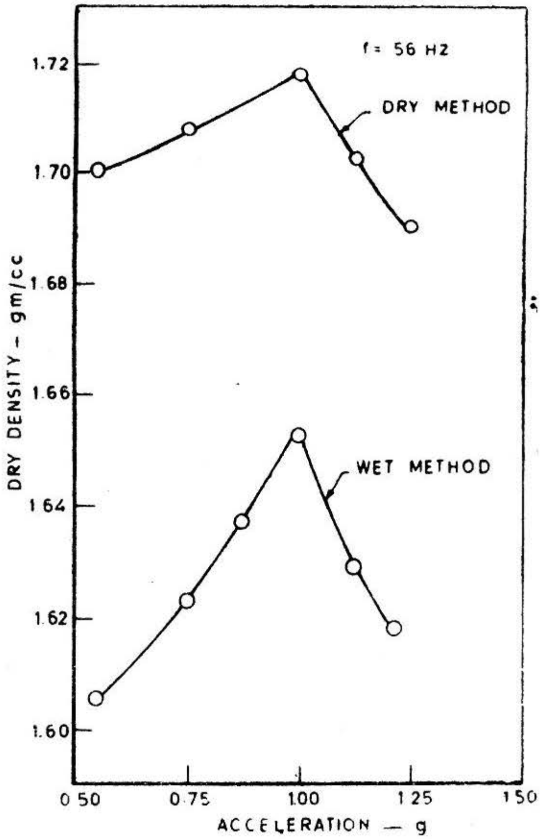


FIGURE 4 Acceleration vs Dry Density for Sample A

Effect of C_u on Optimum Acceleration

The relationship between optimum acceleration and uniformity coefficient is depicted in Fig. 6. From this plot it can be concluded that the grain size distribution of particles is less significant at the optimum acceleration.

Effect of D_{50} on Optimum Acceleration

Fig. 7 is a plot between optimum acceleration and mean diameter of particles which indicates that the size of particle has less effect on optimum acceleration.

Effect of Particle Shape on Acceleration

Fig. 8 shows the relatiohship between angle of friction and optimum acceleration. It can be noticed that as the angle of friction increases the optimum acceleration also increases. This leads to the conclusion that particles which are angular in shape will require larger values of acceleration to yield the maximum density value.

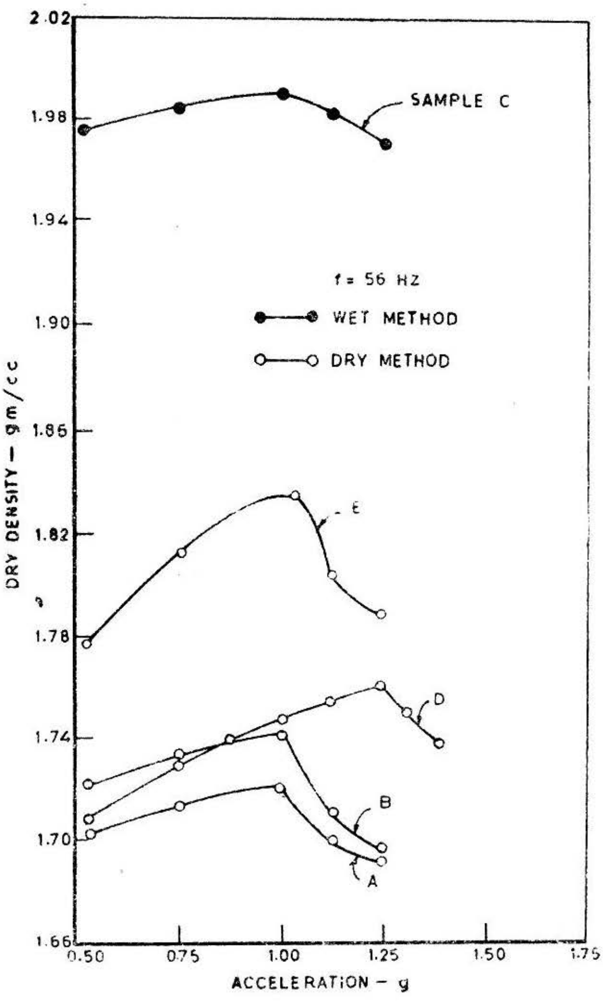


FIGURE 5 **Acceleration Dry Density Relationship for Samples A-F**

Adequacy of the Existing Frequency (56 Hz)

The test on effect of frequency (Fig. 9) on density (with an increased eccentric mass corresponding to the maximum density observed during the constant frequency tests) show that when the frequency is beyond 50 Hz, the maximum density value tends to remain constant at this frequency and beyond. Thus the available frequency (as fixed by the manufacturer, 56 Hz) is suitable for carrying out maximum density tests. However, this was done for only one soil (sample **A**) and tests on other samples are also anticipated to yield identical results.

From the number of tests carried out it has been observed that the maximum density values obtained in the current investigation are higher than those reported by Das (1969) on samples A and B by an amount 3·02 per cent and 2.99 per cent, respectively. Since Das had carried out the tests

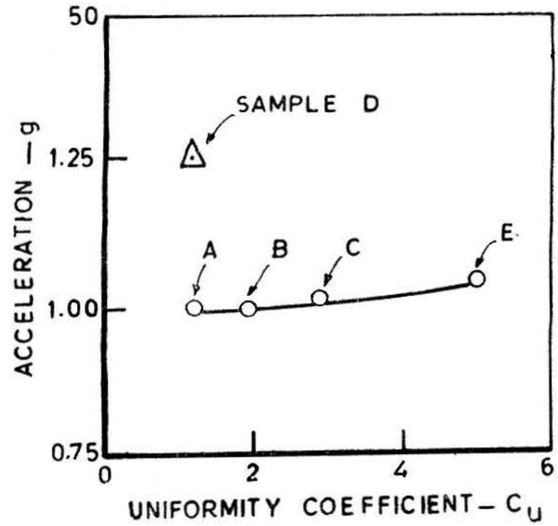


FIGURE 6 Effect of C_u on Acceleration

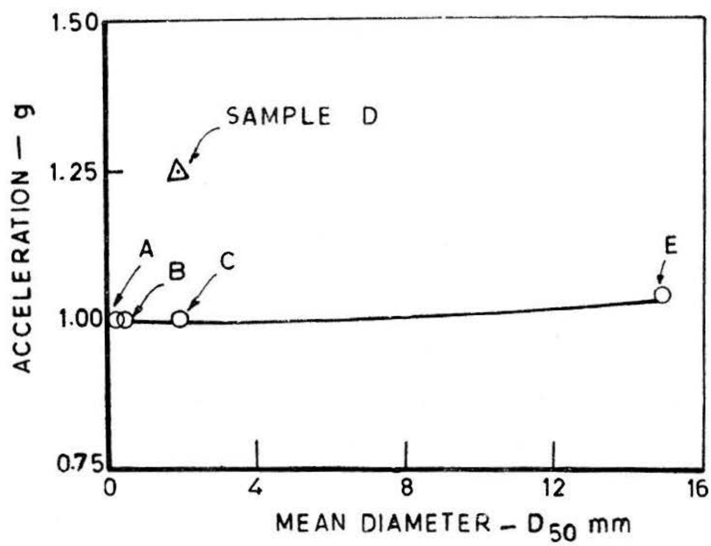


FIGURE 7 Effect of D_{50} on Acceleration

on a horizontal shake table without any surcharge, which might have led to lower values of densities. Also, samples C, D and E were not tested by any of the previous investigators thus comparison on density values could not be made.

The foregoing observations reveal that for the soils for which the relative density term is applicable, the soil characteristics such as grain size and uniformity coefficient are less significant and on the other hand the

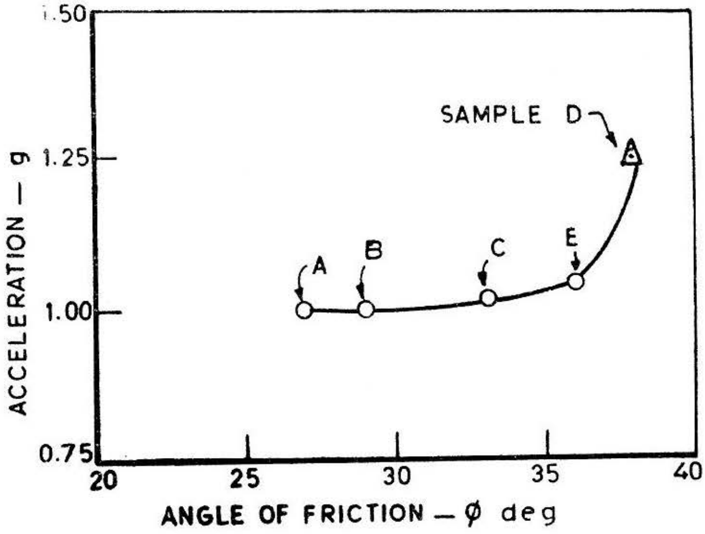


FIGURE 8 Effect of ϕ on Acceleration

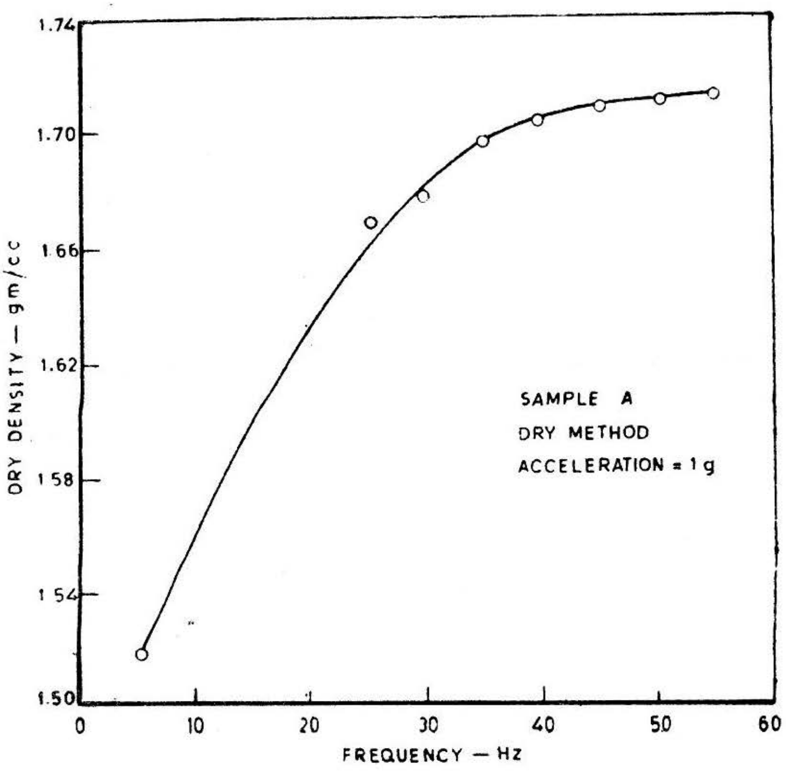


FIGURE 9 Variation in Density with Frequency

grain shape as represented by the angle of friction (at D_r=50 per cent) has a definite bearing on the determination of maximum density of any cohesionless soil. More important than all is the existence of an optimum value of acceleration which has been found to fall within the range of 1.0 g to 1.35 g. Therefore, it is felt essential to make all shake tables to have the provision for varying the acceleration on a larger range. Further, the individual acceleration-density relationship should be obtained for a particular cohesionless soil from which the maximum density value is readily picked up.

For all the tests carried out, only the steady-state vibration has been employed since the natural frequency of the table (for no additional mass) is very low compared to the forcing frequency of the system. Therefore, the resonance condition would not be of much significant in densifying a given soil. As mentioned earlier, there is no provision to alter the forcing frequency of the table thus densification could not be done at resonance conditions of 1.0 g to 1.35 g. Therefore, it is felt essential to have all apparatus to be fabricated with the provisions for obtaining the acceleration in the above range and all soil samples should be tested so as to get the individual acceleration density relationship. From this the maximum density of the given cohesionless soil should be picked up.

Studies on Number of Tests Required

Statistical analysis

To estimate the sufficient number of tests at the maximum density state that will lead to the accurate computation of relative density, statistical analysis data are given between the number of tests and standard deviation (σ_D) in Table 4. When the number of tests is four and more than four the standard deviation value tends to remain constant, Hence for all practical purposes four tests at the maximum density state will suffice.

TABLE 4

Variation Between Number of Tests and σ_D

No. of tests	Std. deviation in density (g/cc)
2	0.0074
3	0.0065
4	0.0061
9	0.0057

Criteria to Select the Method of Vibration

Even a small percent of error in determining maximum density values is greatly magnified on the computation of relative density (Suppiah and Nandakumaran, 1982). Therefore, to decide the method of vibration either dry or wet, the following criteria should be used. If the wet method gives higher values than that of the dry method by an amount of 0.5 percent, then the wet method should be used for obtaining the maximum density values. If not, the dry method should be used. And whichever method yields the highest density value, a minimum of four tests should be repeated so as to minimise the standard devitation.

Further Observations

Since, the surcharge base plate handle is being removed to give access to place the surcharge weight, the base plate hole should be covered (Fig. 10) wiih a suitable plugging arrangement. If uncovered, this hole permit soils to come up and settle on the top of the base plate, of the order of 100 to 200 gm and this amount of soil will be included in the computation of maximum density. Due to this mistake the calculated value of maximum density will be larger than the actual one (more the weight of soil and lesser the volume). Therefore, it is recommended that the soil below the surcharge base plate should only be taken into account after virbration for the computation of maximum density value even after the plugging arrangement.

Secondly, to reduce undesirable impact between the collar and the surcharge weight, a rubber packing should be tied-up at the top limb of the surcharge weight (Figs. 11 and 12). And due to this impact the table does not vibrate as a single-degree-of-freedom system.

Suggested Method for Obtaining Maximum Density

The discussions so far made show that optimum acceleration is an important factor to obtain the maximum density for any given soil. Also, accelerations greater or lesser than the optimum value do not aid to achieve the maximum density values.

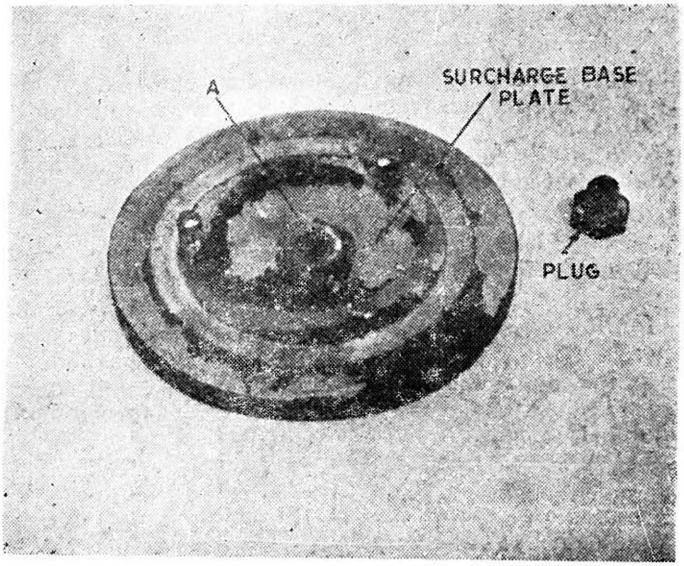


FIGURE 10 Surcharge Base Plate and Plug

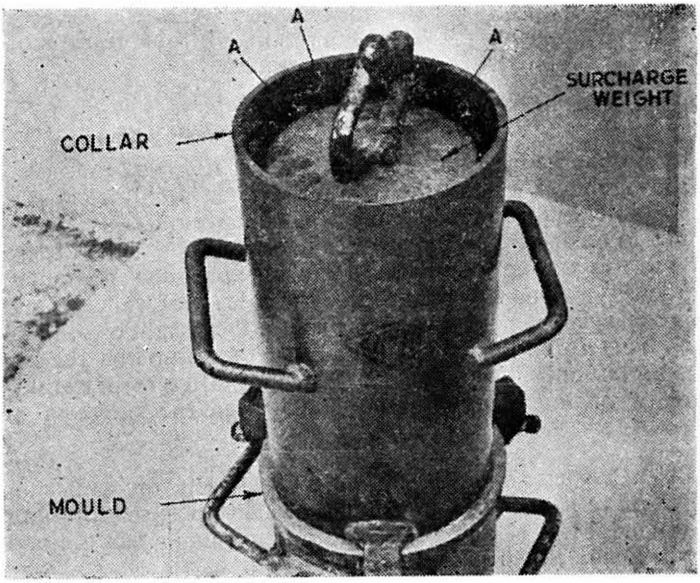


FIGURE 11 **Surcharge Weight and Collar in Position**

The available relative density apparatus in the country does not yield the true maximum density for any given soil, but a lower value at a lower acceleration (0.55 g). Therefore, to obtain the acceleration-density relationship for any given soil the apparatus should be modified so that the true maximum density values can be achieved.

From the detailed studies carried out on five different soils, except for angular soils, the maximum density values can be obtained at an acceleration 1.0 g. For angular soil larger acceleration than 1.0 g should be employed in four increments or more, upto 1.5 g or more and the acceleration—density relationship be established from which the maximum density is feasible. At the optimum acceleration for a given soil, one test on each using the dry method and the wet method should be conducted and percentage different in density values be computed.

If the wet method yields larger value than that of the dry method by an amount 0.5 per cent, then the wet method should be used for obtaining the maximum density. If not the dry method should be adopted. Whichever method gives the highest density value (as per the above specification), using that method at least four tests should be repeated.

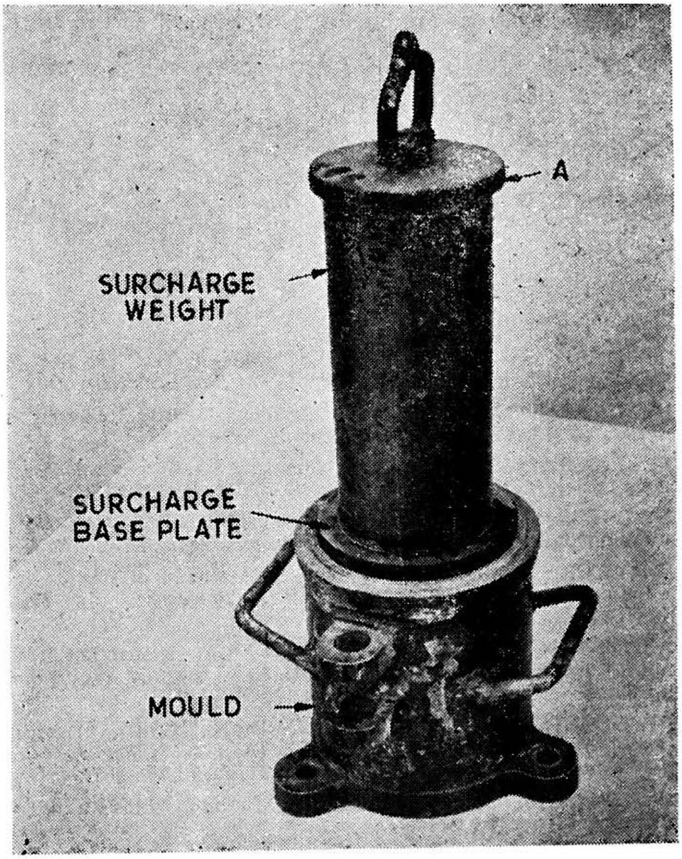


FIGURE 12 Surcharge Weight Without Collar

Conclusions

The following conclusions are arrived at from the present investigation :

1. Cohesionless soils should be vibrated at optimum accelerations alone to get the maximum density values. Vibrating at greater or lesser than the optimum value will not yield the true maximum density for any given soil.

2. Most soils but for angular shaped ones, the maximum densities can be obtained at an acceleration 1.0 g. For angular soils the acceleration should be varied beyond 1.0 g to get the maximum density.

3. For any given soil, the true maximum density can be obtained using the present apparatus by varying only one vibration parameter, viz,, acceleration, whereas, previous investigators have recommended a combination of any two vibration parameters.

4. The shape of particles is more significant on the optimum acceleration than the grain size distribution and the grain size.

5. A minimum of four tests for determining the maximum density is required.

Acknowledgement

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Notations

C_c = Coefficient of curvature

C_u = Uniformity coefficient

D_{50} = Mean diameter

D_r = Relative density (or Density index)

d = Diameter

f = Frequency

G = Specific gravity of soil

g = Acceleration due to gravity

L = Length

γ_{dmax} = Maximum dry density

ϕ = Angle of friction

σ_D = Standard deviation