Effect of Assumed Boundary Conditions on the Computed Stresses for A Concrete Gravity Dam

by

D.K. Sehgal*

R. Natarajan**

S.M.A. Kazimi***

Introduction

In solving the dam problem by finite element method, assumption of zero displacements (Zienkiewicz and Cheung, 1964, 1965; Varshney, 1974, 1975) is usually made at the extreme boundary of dam foundation. Generally this boundary is assumed at a distance of two to four times the base width of the dam. This is an approximate but easy method. A more accurate (Bonaldi, Monaco and Fanelli, 1975) but tedious method is to prescribe Boussinesq (Frocht, 1971; Fung, 1965; Timoshenko and Goodier, 1970) like displacements at boundary.

In the present study, efforts are made to find out the effects of (*i*) extent of domain and (*ii*) method of prescribing displacements at the boundary on computed stresses from a finite element solution of concrete gravity dam.

Method of Prescribing Boussinesq like Displacements :

Boussinesq's solution makes use of the well-known Airy's stress-functions. The final results for a concentrated inclined force P acting on a horizontal straight boundary of a semi-infinite plate (Fig. 1) are given by equation 1. Here at any point at a distance r from the point of application of load, the compressive stress in radial direction is given by

$$\sigma_r = -\frac{2P}{\pi}\frac{Cos\,\theta}{r} \qquad \ldots(1)$$

Where θ is the angle between the line of action of force and radial direction. The thickness of the plate is taken as unity.

The corresponding radial and tangential displacements are given by

$$U_r = -\frac{2P}{\pi E}\,\text{Cos}\theta \log r - \frac{(1-\nu)P}{\pi E}\theta\,\text{Sin}\theta + A\,\text{Sin}\theta + B\,\text{Cos}\theta \qquad (2)$$

* Lecturer,
** Professor,
*** Professor,
Applied Mechanics Department, I.I.T., Delhi-110016, India.

(This technical note was received in January, 1983 and is open for discussion till the end of December, 1983).

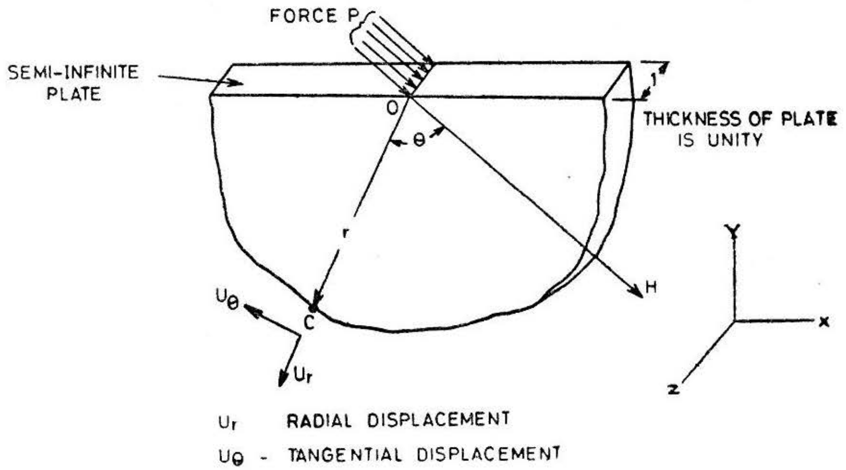

FIGURE 1 Inclined Force in a Semi-Infinite Plate

$$U_\theta = \frac{2\nu P}{\pi E}\,\text{Sin}\,\theta + \frac{2P}{\pi E}\log r\,\text{Sin}\,\theta - \frac{(1-\nu)P}{\pi E}\,\theta\,\text{Cos}\theta$$

$$+\frac{(1-\nu)P}{\pi E}\,\text{Sin}\theta + A\,\text{Cos}\theta - B\,\text{Sin}\theta + Cr \qquad \ldots(3)$$

where E is the modulus of elasticity and ν is the Poisson's ratio. A, B and C are constants of integration. For determining the integration constants, it is assumed that the constraints of semi-infinite plate are such that the points on the OH line (line of action of force) have no lateral displacements. Therefore, $U_\theta = 0$ for $\theta = 0$ and from equation 3, it can be found that $A = 0$ and $C = 0$.

To find out the constant B, it can be assumed that a point on OH line and at a distance d from the point O does not move along the line of action of force.

Then from equation 2,

$$B = \frac{2P}{\pi E}\log d$$

Now by putting the values of constants A, B and C and arranging the terms, equations 2 and 3 reduce to,

$$U_r = \frac{2P}{\pi E}\text{Cos}\theta\log\frac{d}{r} - \frac{(1-\nu)P}{\pi E}\theta\,\text{Sin}\,\theta \qquad \ldots(4)$$

$$U_\theta = \frac{P}{\pi E}\text{Sin}\theta\,(1+\nu) - \frac{2P}{\pi E}\text{Sin}\theta\log\frac{d}{r} - \frac{(1-\nu)P}{\pi E}\,\theta\,\text{Cos}\theta \qquad \ldots(5)$$

Once the displacement components, U_r and U_θ are known, the displacements along x and y direction U_x and U_y can be found out easily.

For prescribing Boussinesq's displacements at dam boundary the following procedure is adopted :

(*i*) Resultant (R_V) of total vertical forces acting on dam proper is found out.

(*ii*) Resultant (R_H) of total horizontal forces acting on dam proper is found out.

(*iii*) Net resultant (R) of the two forces R_V and R_H is found out. The point (Q) where it cuts the dam base is determined (Fig. 2). Now displacements U_r and U_θ at nodes on external boundary of dam foundation due to a force R can be found out, using equations 4 and 5. Once U_r and U_θ at a node are known, displacement U_x and U_y along cartesian axes can be found out.

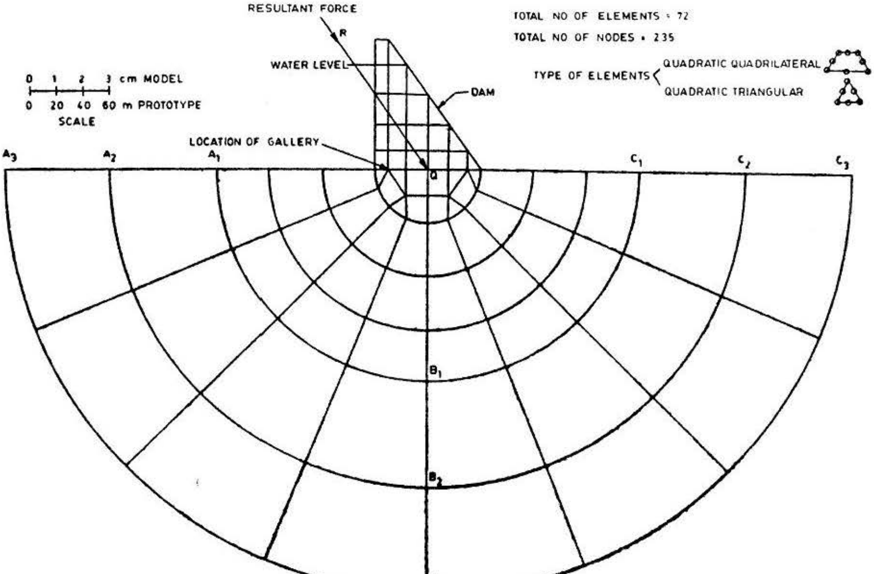

FIGURE 2 Finte Element Discretization of the Dam

The Computer Program :

For the analysis of gravity dams, an efficient computer program 'Dam 2*D*' is developed taking into consideration the actual loading conditions of the dam. For 2*D* analysis combined eight noded quadratic quadrilateral and six noded quadratic triangular type of isoparametric elements can be used. The program can take into consideration forces due to static water pressure, dead weight of dam, water and foundation, hydrodynamic water pressure, uplift pressure, silt pressure and equivalent static earthquake loads. For economising computer memory space and for the reduction of computer time the following steps have been taken :

(*i*) Intermediate results are stored on scratch tapes ;

(*ii*) Frontal solution technique (Irons, 1970) is used to solve large number of equations;

(*iii*) At various stages of computation, the data in the blank common, which has been processed and is no longer required is overwritten.

The Case Study

Basic geometry of the 100 m high concrete gravity dam chosen for the present study is shown in Fig. 2. The finite element discretization is also shown in the same figure. The whole dam is divided into 72 elements with 235 nodes.

For different dam locations, a great variation for the ratio of modulus of elasticity of concrete and rock material exists. Finite element technique is capable of incorporating these variations with ease. However, for a general study like the present one, it is thought suitable to use the same elastic constants for rock and concrete; a normal design hypothesis.

In the present case study, six different cases have been studied as given below :

First Case : Foundation domain is considered upto a distance of twice the base width of dam (upto Arc $A_1B_1C_1$ Fig. 2). Zero displacements are prescribed at boundary.

Second Case : Foundation domain is considered upto a distance of three times the base width of dam (upto Arc $A_2B_2C_2$ Fig. 2). Zero displacements are prescribed at boundary.

Third Case : Foundation domain is considered upto a distance of four times the base width of dam (upto Arc $A_3B_3C_3$ Fig. 2). Zero displacements are prescribed at boundary.

Fourth Case : Similar to Case 1+Boussinesq's displacements at boundary $A_1B_1C_1$ (Fig. 2).

Fifth Case : Similar to Case 2+Boussinesq's displacements at boundary $A_2B_2C_2$ (Fig. 2).

Sixth Case : Similar to Case 3+Boussinesq's displacements at boundary $A_3B_3C_3$ (Fig. 2).

Forces due to the following effects are considered in the analysis :

(*i*) Hydrostatic force of water

(*ii*) Dead weight of dam proper, foundation and water

(*iii*) Uplift in dam proper and in foundation

(*iv*) Hydrodynamic force of water

(*v*) Equivalent static earthquake (inertia) forces

Discussion of the Results :

In order to compare the stresses for six different cases thirty two points of interest are chosen in the domain of the dam, as shown in Fig. 3. Principal stresses are computed at these points and tabulated as shown in Table 1. Percentage difference of the stresses with respect to case 6 (most accurate case) is calculated for each point.

Comparison of the results for various cases reveals the following facts :

(*i*) Values of the stresses for the points above the dam base are not much effected either by the extent of domain or by the values of prescribed deflections.

(*ii*) At the heel of the dam a maximum difference of 30 per cent in major principal stress (σ_1) is noted. Probably the heel of the dam is the most sensitive and critical point in the dam. At this point, extent of domain as well as prescribed displacements have marked effect on tensile stress developed. For case 6 (the most accurate one) tensile stress developed is less than that for other cases. This

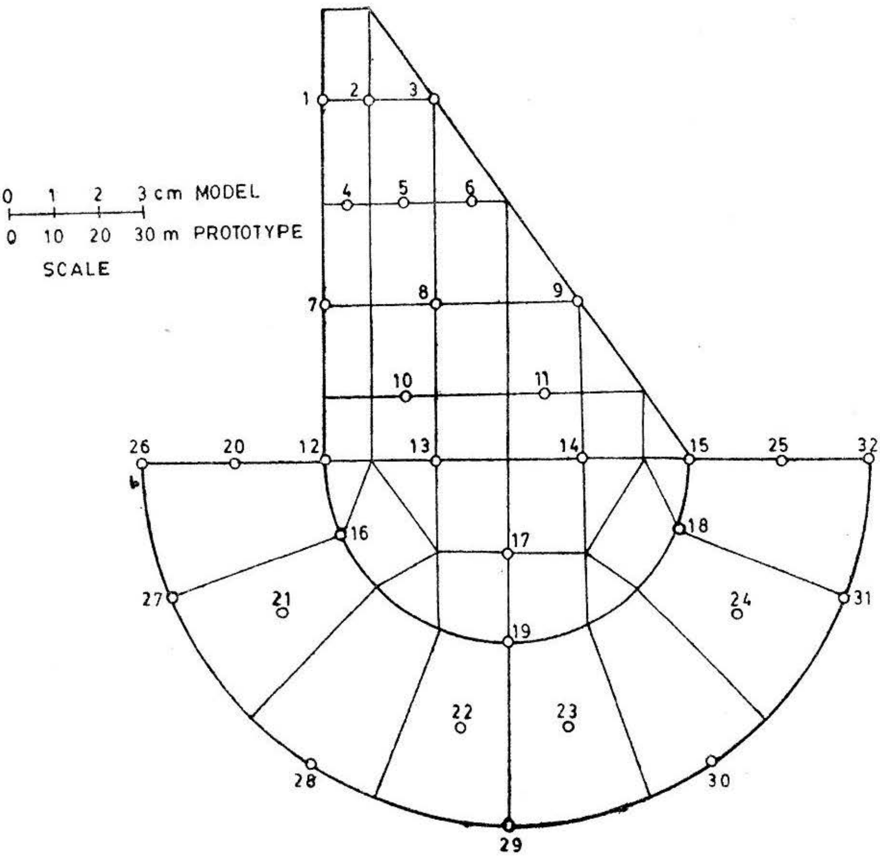

FIGURE 3 Selected Points on Dam for Comparison of Stresses

TABLE 1

Principal Stresses at Various Points of Dam

Case / Point	Principal stresses (kg/cm²)	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Max, percentage Difference w. r. t. case 6
1.	σ_1	−0.8	−0.8	−0.8	−0.8	−0.8	−0.8	0.0
	σ_2	−3.0	−3.0	−3.0	−3.0	−3.0	−3.0	0.0
2.	σ_1	−0.9	−0.9	−0.9	−0.9	−0.9	−0.9	0.0
	σ_2	−3.0	−3.0	−3.0	−3.0	−3.0	−3.0	0.0
3.	σ_1	−0.8	−0.8	−0.8	−0.8	−0.8	−0.8	0.0
	σ_2	−2.8	−2.8	−2.8	−2.8	−2.8	−2.8	0.0
4.	σ_1	−1.1	−1.1	−1.1	−1.1	−1.1	−1.1	0.0
	σ_2	−5.0	−5.0	−5.0	−5.0	−5.0	−5.0	0.0
5.	σ_1	−1.2	−1.2	−1.2	−1.2	−1.2	−1.2	0.0
	σ_2	−4.8	−4.8	−4.8	−4.8	−4.8	−4.8	0.0
6.	σ_1	−1.0	−1.0	−1.0	−1.0	−1.0	−1.0	0.0
	σ_2	−4.5	−4.5	−4.5	−4.5	−4.5	−4.5	0.0
7.	σ_1	−1.8	−1.8	−1.8	−1.8	−1.8	−1.8	0.0
	σ_2	−6.0	−6,0	−6.0	−6.0	−6.0	−6.0	0.0
8.	σ_1	−1.9	−1.9	−1.9	−1.9	−1.9	−1.9	0.0
	σ_2	−6.2	−6.2	−6.2	−6.2	−6.2	−6.2	0.0
9.	σ_1	−1.0	−1.0	−1.0	−1.0	−1.0	−1.0	0.0
	σ_2	−7.9	−7.8	−7.7	−7.6	−7.6	−7.6	3.9
10.	σ_1	−2.1	−2.1	−2.1	−2.2	−2.2	−2.2	4.5
	σ_2	−6.8	−6.8	−6.8	−6.8	−6.8	−6.8	0.0
11.	σ_1	−1.5	−1.5	−1.5	−1.5	−1.5	−1.5	0.0
	σ_2	−9.5	−9.5	−9.5	−9.5	−9.5	−9.5	0.0
12.	σ_1	+5.2	+4.9	+4.7	+4.4	+4.2	+4.0	30.0
(Heel)	σ_2	+0.5	+0.5	+0.5	+0.4	+0.4	+0.4	25.0
13.	σ_1	−3.6	−3.9	−4.2	−4.6	−4.6	−4.6	21.7
	σ_2	−8.0	−8.0	−8.0	−8.0	−8.0	−8.0	0.0
14.	σ_1	−5.8	−5.9	−6.0	−6.2	−6.2	−6.2	6.5
	σ_2	−11.6	−12.0	−12.5	−13.0	−13.0	−13.0	10.8
15	σ_1	−5.8	−6.2	−6.4	−7.0	−7.0	−7.0	17.1
(Toe)	σ_2	−30.0	−33.0	−35.0	−39.0	−39.0	−39.0	23.0
16.	σ_1	−1.8	−1.9	−2.0	−2.1	−2.2	−2.3	21.7
	σ_2	−6.7	−6.8	−6.9	−7.0	−7.0	−7.0	4.3
17.	σ_1	−6.8	−7.2	−7.8	−8.5	−8.5	−8.5	20.0
	σ_2	−10.0	−10.0	−10.0	−10.0	−10.0	−10.0	0.0

(Continued)

TABLE 1 (*Continued*)

18.	σ_1	−6.6	−7.0	−7.7	−8.5	−8.5	−8.5	22.3
	σ_2	−13.1	−13.7	−14.3	−16.0	−16.0	−16.0	18.1
19.	σ_1	−7.6	−8.2	−8.5	−9.5	−9.5	−9.5	20.0
	σ_2	−13.5	−13.5	−13.5	−13.5	−13.5	−13.5	0.0
20.	σ_1	+1.9	+2.0	+2.1	+2.1	+2.1	+2.1	9.5
	σ_2	−1.4	−1.4	−1.4	−1.5	−1.5	−1.5	6.7
21.	σ_1	−0.5	−0.5	−0.5	−0.6	−0.6	−0.6	16.7
	σ_2	−7.0	−7.0	−7.0	−7.5	−7.5	−7.5	7.2
22.	σ_1	−6.9	−7.0	−7.1	−7.2	−7.2	−7.2	4.2
	σ_2	−14.0	−14.0	−14.0	−14.0	−14.0	−14.0	0.0
23.	σ_1	−8.8	−9.2	−9.6	−10.0	−10.2	−10.4	15.4
	σ_2	−15.0	−15.0	−15.0	−14.8	−14.8	−14.8	1.3
24.	σ_1	−6.0	−6.2	−6.3	−6.4	−6.5	−6.6	9.0
	σ_2	−14.0	−14.5	−15.0	−15.5	−15.8	−16.0	12.5
25.	σ_1	−4.0	−4.0	−4.0	−4.0	−4.0	−4.0	0.0
	σ_2	−13.2	−14.0	−14.8	−16.0	−16.0	−16.0	17.5
26.	σ_1	+2.0	+1.9	+1.9	+1.8	+1.7	+1.6	25.0
	σ_2	−2.0	−2.1	−2.2	−2.3	−2.4	−2.5	20.0
27.	σ_1	+0.5	+0.5	+0.5	+0.6	+0.6	+0.6	16.7
	σ_2	−3.8	−4.0	−4.1	−4.2	−4.2	−4.2	9.5
28.	σ_1	−1.7	−1.8	−1.9	−2.0	−2.0	−2.0	15.0
	σ_2	−10.0	−10.0	−10.0	−10.0	−10.0	−10.0	0.0
29.	σ_1	−6.8	−7.0	−7.2	−7.5	−7.6	−7.7	11.6
	σ_2	−15.0	−15.0	−15.0	−14.8	−14.8	−14.8	1.3
30.	σ_1	−7.5	−7.7	−7.9	−8.2	−8.4	−8.6	12.8
	σ_2	−16.4	−16.5	−16.6	−17.0	−17.0	−17.0	3.5
31.	σ_1	−3.0	−4.0	−4.1	−4.3	−4.3	−4.3	9.3
	σ_2	−13.9	−13.5	−14.0	−16.0	−16.4	−16.8	22.6
32.	σ_1	+0.8	+0.8	+0.8	+0.9	+0.9	+0.9	11.1
	σ_2	−9.5	−10.0	−10.5	−12.0	−12.0	−12.0	20.8

shows that we are on the safer side, while considering smaller extent of foundation domain, for approximate F.E.M. solutions, and should not cause undue worry to the designer.

(*iii*) At the toe of the dam, a maximum difference of 23 per cent in minor principal stress (σ_2) is noted. For the cases 4, 5 and 6 (where Boussinesq's displacements are prescribed), magnitude of compressive stresses is same and higher than that for rest of the cases. Hence, extent of domain has little influence on the magnitude of the stresses for the cases where Boussinesq's displacements are prescribed. However, for the cases 1, 2 and 3 (where zero displacements are prescribed), extent of domain influence the results significantly. There small domain considered gives less magnitude of the compressive stresses, hence we are on unsafe side and have to be careful.

(*iv*) At some points magnitude of stresses is quite low. Due to this, a small variation of stress among the different cases, causes a high percentage difference. However, little importance should be attached to such higher percentage difference in low stress zones. For example, minor principal stress at heel (difference 25 per cent), at point 16 (difference 21.7 per cent), at point 21 (difference 16.7 per cent), at point 26 (difference 25 per cent) and at point 27 (difference 16.7 per cent). Similarly errors in major principal stress at heel (difference 25 per cent) and at point 26 (difference 20 per cent) etc., are of little importance from a designer's point of view.

(*v*) Comparing the results for different cases, it has been noted that for the points lying towards heel side, variation in major principal stress was much more than that in minor principal stress and for the points lying towards toe side, variation in minor principal stress was much more than that in major principal stress.

(*vi*) For the cases where Boussinesq's displacements are prescribed (case 4, 5 and 6), the percentage variation in the magnitude of stresses is found to be very small in comparison to that for cases where zero displacements are prescribed (Cases 1, 2 and 3). This was expected because Boussinesq's equations give higher displacements at the boundary of a smaller domain (which becomes zero at infinity only). This automatically reduces the error due to smaller domain. This compensating built-in mechanism is absent if zero displacements at the boundary are assumed.

Conclusion :

Both, extent of domain as well as method of prescribing boundary displacements, effect the computed stresses from a finite element analysis to a great extent. For very accurate results, the foundation domain should be considered as large as possible and Boussinesq's displacements should be prescribed at the boundary.

However for all practical purposes, very large domain need not be considered. Because this will increase the computer time as well as the time and labour required for the preparation of basic finite element data. Similarly prescribing the Boussinesq's displacements at the boundary is a very time consuming and tedious work. However if Boussinesq's dlsplacements in a tabular form would be available for ready reference, a lot of labour can be saved. With Boussinesq's displacements prescribed at boundary, extent of domain has less effect on the computed stresses. In such a case smaller domain can be considered for all practical purposes and hence computer time can be saved.

Finally, there are lot of factors on which depend the choice of domain and prescribed boundary conditions. These factors are labour involved, manual time, computer and accuracy of the results required etc. For the analysis of preliminary designs, smaller extent of domain with zero boundary conditions can be considered, provided the analyst keeps himself aware of the magnitude of inaccuracies involved and makes suitable allowances. The present study is meant for that purpose, and may be of help to designers using the finite element method for gravity dams.

References

BONALDI, P., MONACO, A.D., and FANELLI, M (1975): "Concrete Dam Problems : An Outline of Numerical Analysis", *Proc. Symp. on 'Criteria and Assumptions for Numerical Analysis of Dam'*, Swansea, U.K., pp 27-108.

FROCHT, M.M. (1961) : "*Photoelasticity Vol II*," John. Wiley, N. York.

FUNG, Y.C. (1965) : "*Foundations of Solid Mechanics*", Prentice-Hall, Englewood Cliffs, N.J.

IRONS, B.M· (1970) : "A Front Solution Program", *Int. J. Num. Meth. Engg.*; 2, pp 5-12.

TIMOSHENKO, S.P. and GOODIER, J.N. (1970), "*Theory of Elasticity*", Mc Graw-Hill, N. York.

VARSHNEY, R.S. and BHARAT SINGH (1974) : "Effect of Fault in the Foundation on Dam Behaviour", *J. of Irrigation and Power India*, pp 451-467.

VARSHNEY, R. S. (1975) : "Stresses in the Layered Half Plane Below a Gravity Dam", *Int. J. of Rock Mechanics and Mining Sciences*, Vol. 12, pp 159-166.

ZIENKIEWICZ, O.C. and CHEUNG, Y.K. (1964) : "Buttress Dams on Complex Rock Foundations", *Water Power*, 16, pp 193-198.

ZIENKIEWICZ, O.C. and CHEUNG, Y.K. (1965) : "Stresses in Buttress Dams", *Water Power*, 17, pp 69-75.