Short Communications

Graphical Method for Dynamic Earth Pressure of Cohesionless Soils

by

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Introduction

Coulomb's theory of earth pressure behind a retaining wall is based on the following assumptions :

(*i*) Deformation condition is satisfied,

(*ii*) Soils is cohesionless, and

(*iii*) Sliding occurs along a rupture surface.

A graphical construction to determine the lateral earth pressure according to Coulomb's theory was given by Culman. Mononobe (1929) and Okabe (1929) were the first to modify the Coulomb's expression for incorporating the effect of inertia forces. They gave expressions both for active and passive cases. Kapila (1962) as reffered by Shamsher Prakash et al (1979) gave a graphical construction incorporating the *intertia* forces for the failure wedge.

In the present paper, a graphical method is suggested for determining the earth pressure of cohesionless soils considering the inertia forces based on Rebhan's construction. The method presented here is simpler than the earlier methods.

Method of Construction

(*a*) *Active case*

1. Draw a dimensional sketch of the retaining wall, (Fig. 1).

2. Draw BM at an angle (ϕ-θ) with the horizontal in upward direction.

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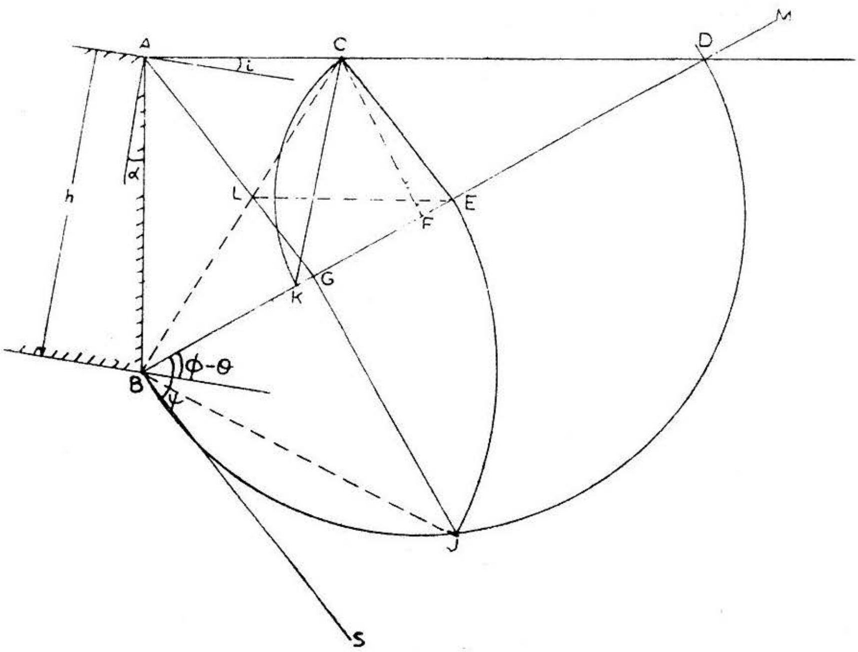

FIGURE 1 Method of Construction for Active Case

3. Draw BS at an angle $\psi = 90-(\alpha+\delta+\theta)$ with BM
4. With BD as diameter, draw a semicircle.
5. Draw AG parallel to BS cutting BM at G.
6. Draw GJ perpendicular to BD, cutting the semicircle at J.
7. With B as centre and BJ as radius, draw an arc JE, cutting BD at E.
8. Draw EC parallel to BS or AG cutting AD at C.
9. *With E as centre and EC as radius, draw EK cutting BD at K.*
10. Join CK.

Total dynamic active earth pressure $P_{A(dyn)}$ is given by

$$P_{A\,(dyn)} = \left(\frac{1 \pm a_v}{\cos\theta}\right) \cdot \gamma \cdot \text{ Area of triangle } CKE.$$

Proof :- From similar triangles AGD and CED

$$\frac{CE}{CD} = \frac{AG}{AD}$$

$$\text{or } CE^2 = \left(\frac{AG}{AD} \cdot CD\right)^2 \qquad ...(1)$$

From triangle GAB.

$$\frac{AG}{AB}=\frac{Sin\,(90+\alpha-\phi+\theta)}{Sin\,(90-(\alpha+\theta+\delta))}$$

$$\text{or } \frac{AG}{AB}=\frac{Cos\,(\phi-\alpha-\theta)}{Cos\,(\alpha+\theta+\delta)} \qquad ...(2)$$

Also

$$\frac{AD}{CD}=\frac{AC+CD}{CD}=\left(\frac{AC}{CD}\right)+1$$

But $AC = LE$

$$\text{Therefore } \frac{AD}{CD}=\frac{LE}{CD}+1$$

From triangles BCD and BLE

$$\frac{LE}{CD}=\frac{BE}{BD}$$

$$\text{or } \frac{AD}{CD}=\frac{BE}{BD}+1 \qquad ...(3)$$

But from the properties of a circle,

$$BG\times GD = GJ^2$$

or $BG\,(BG+GD) = BJ^2 = BE^2$

$$\text{or } \frac{BG}{BE}=\frac{BE}{BD} \qquad ...(4)$$

$\therefore$ from equations (3) and (4)

$$\frac{AD}{CD}=\sqrt{\frac{BG}{BD}}+1 \qquad ...(5)$$

In triangle GAB.

$$\frac{BG}{AB}=\frac{Sin\,(\phi+\delta)}{Cos\,(\alpha+\theta+\delta)}$$

$$\text{or } BG = AB\,\frac{Sin\,(\phi+\delta)}{Cos\,(\alpha+\delta+\theta)} \qquad ...(6)$$

From triangle ABD,

$$\frac{BD}{AB}=\frac{Sin\,(90-\alpha+i)}{Sin\,(\phi-\theta-i)}$$

$$\text{or } \frac{BD}{AB}=\frac{Cos\,(\alpha-i)}{Sin\,(\phi-\theta-i)}$$

$$\text{or } BD = \frac{AB\ Cos\ (\alpha - i)}{Sin\ (\phi - \theta - i)} \qquad \ldots(7)$$

Substituting the values from equations (2) and (5) in equation (1), we get.

$$CE^2 = (AB)^2\ \frac{Cos^2\ (\phi - \alpha - \theta)}{Cos^2\ (\alpha + \theta + \delta)} \left[\frac{1}{\sqrt{\frac{BG}{BD}} + 1} \right]^2$$

And from equation (6) and (7)

$$CE^2 = (AB)^2\ \frac{Cos^2\ (\phi - \alpha - \theta)}{Cos^2\ (\alpha + \theta + \delta)} \left[1 + \sqrt{\frac{Sin\ (\phi + \delta)\ Sin\ (\phi - \theta - i)}{Cos\ (\alpha + \theta + \delta)\ Cos\ (\alpha - i)}} \right]^{-2}$$

Therefore, the total dynamic active earth pressure

$$P_{A\ (dyn)} = \frac{1 \pm a_v}{Cos\ \theta} \cdot \gamma \cdot \text{ Area of triangle } CKE.$$

$$= \frac{1 \pm a_v}{Cos\ \theta} \cdot \gamma \cdot \tfrac{1}{2} \cdot CF.LE$$

$$= \frac{1 \pm a_v}{Cos\ \theta} \cdot \gamma \cdot \tfrac{1}{2}\ CE^2\ Sin\ \psi$$

Rearranging we get.

$$\therefore P_A\ (_{dyn}) = \tfrac{1}{2} \cdot \gamma \cdot h^2\ \frac{(1 \pm a_v)}{Cos\ \theta\ Cos^2 \alpha}\ \frac{Cos^2\ (\phi - \alpha - \theta)}{Cos\ (\alpha + \theta + \delta)} \cdot$$

$$\left[1 + \sqrt{\frac{Sin\ (\phi + \delta)\ Sin\ (\phi - \theta - i)}{Cos\ (\alpha + \theta + \delta)\ Cos\ (\alpha - i)}} \right]^{-2}$$

which is the same as the expression given by Mononobe and Okabe.

(b) Passive Case

1. Draw a dimensional sketch of the retaining wall, showing the backfill line (Fig. 2).
2. Draw *BM* at an angle of $(\phi - \theta)$ with horizontal, downward in direction.
3. Draw *BS* at an angle of $\psi = (90 + \theta + \delta - \alpha)$ with *BM*.
4. Extend *MB* and *PA* backwards cutting each other at *D*.
5. With *BD* as diameter, draw a semi-circle.
6. Through *A* draw a line parallel to *BS* cutting *BD* at *G*.
7. Draw *GJ* perpendicular to *BD* cutting the semicircle at *J*.
8. With *B* as centre and *BJ* as radius, draw *BE* cutting *BM* at *E*.

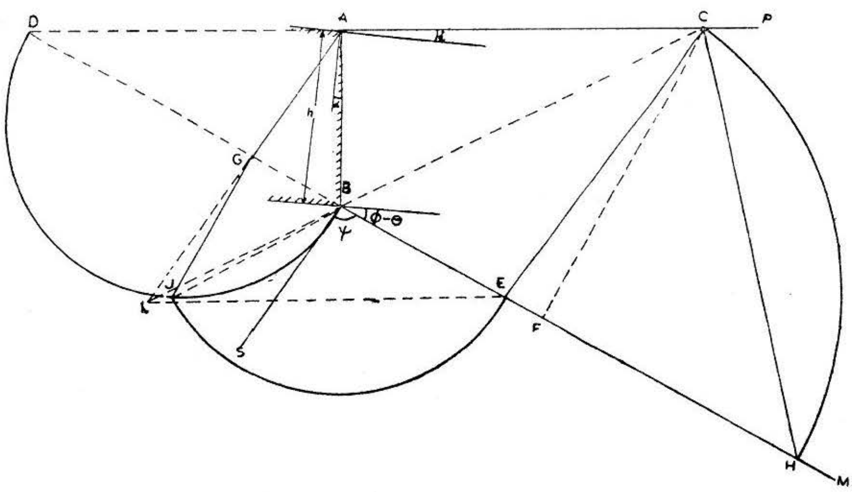

FIGURE 2 Method of Construction for Passive Case

9. Draw EC parallel to BS cutting AP at C.
10. With E as centre and EC as radius, draw EH cutting BM at H.
11. Join CH.

Calculation :

Dynamic passive pressure

$$P_{P\,(dyn)} = \frac{1 \pm a_v}{Cos\ \theta} \cdot \gamma \cdot \text{Area of triangle } CEK.$$

Proof :

Draw CF perpendicular to BM.

From the properties of a circle.

$$BG \times GD = GJ^2$$

$$\text{or } BG \times GD + BG^2 = GJ^2 + BG^2$$

$$\text{or } \frac{BG}{BE} = \frac{BE}{BD} \quad \ldots(8)$$

In similar triangles AGD and CED

$$\frac{CE}{CD} = \frac{AG}{AD}$$

$$\text{or } CE^2 = \left(\frac{AG}{AD}\, CD\right)^2 \quad \ldots(9)$$

from triangle GAB

$$\frac{AG}{AB} = \frac{Sin\,(90-\alpha-\phi+\theta)}{Sin\,(\pi-\psi)}$$

$$\text{or } \frac{AG}{AB} = \frac{Cos\,(\alpha+\phi-\theta)}{Cos\,(\theta+\delta-\alpha)}$$

$$\text{or } AG = AB\,\frac{Cos\,(\alpha+\phi-\theta)}{Cos\,(\phi+\delta-\alpha)} \qquad ...(10)$$

$$\text{Also } \frac{AD}{CD} = \frac{CD-AC}{CD} = 1-\frac{AC}{CD}$$

$$\text{But } AC = EL$$

$$\text{And } \frac{EL}{CD} = \frac{BE}{BD}$$

$$\text{or } \frac{AD}{CD} = 1-\frac{BE}{BD} \qquad ...(11)$$

From equation (8)

$$\frac{BE}{BD} = \frac{BG}{BE}$$

$$\text{or } \frac{BE^2}{BD^2} = BD \times \frac{BG}{BD^2}$$

$$\text{or } \frac{BE}{BD} = \sqrt{\frac{BG}{BD}} \qquad ...(12)$$

From equations (11) and (12)

$$\frac{AD}{CD} = 1-\sqrt{\frac{BG}{BD}} \qquad ..(13)$$

In triangle ABG

$$\frac{BG}{AB} = \frac{Sin\,(\phi+\delta)}{Sin\,(\pi-\psi)}$$

$$\frac{BG}{AB} = \frac{Sin\,(\phi+\delta)}{Sin\,(90+\theta+\delta-\alpha)}$$

$$\text{or } BG = AB\,\frac{Sin\,(\phi+\delta)}{Cos\,(\theta+\delta-\alpha)} \qquad ...(14)$$

In triangle BDA

$$\frac{BD}{AB} = \frac{Sin\,(90+\alpha-i)}{Sin\,(\phi-\theta+i)}$$

$$\text{or } BD = AB\,\frac{Cos\,(\alpha-i)}{Sin\,(\phi+i-\theta)} \qquad ...(14)$$

Putting the values from equations (10) and (13) in equation (9)

$$CE^2 = \frac{Cos^2(\alpha+\phi-\theta)}{Cos^2(\theta+\delta-\alpha)} AB^2 \left[1-\sqrt{\frac{BG}{BD}}\right]^{-2}$$

And from equations (7) and (8)

$$CE^2 = \frac{AB^2\, Cos^2(\alpha+\phi-\theta)}{Cos^2(\theta+\delta-\alpha)} \left[1-\sqrt{\frac{Sin(\phi+\delta)}{Cos(\alpha-i)}\,\frac{Sin(\phi+i-\theta)}{Cos(\theta+\delta-\alpha)}}\right]^{-2}$$

Dynamic Passive pressure

$$P_P(dyn) = \frac{1\pm a_v}{Cos\,\theta}\cdot\gamma\cdot\tfrac{1}{2}\,CF\times CE$$

$$= \tfrac{1}{2}\cdot\frac{1\pm a_v}{Cos\,\theta}\cdot\gamma\cdot CE^2\,Sin\,\psi$$

$$= \tfrac{1}{2}\cdot\gamma\cdot\frac{(1\pm av)}{Cos\,\theta}\,AB^2\,\frac{Cos(\theta+\delta-\alpha)}{Cos^2(\theta+\delta-\alpha)}\cdot Cos^2(\alpha+\phi-\theta)$$

$$\cdot\left[1-\sqrt{\frac{Sin(\phi+\delta)\,Sin(\phi+i-\theta)}{Cos(\alpha-i)\,Cos(\theta+\delta-\alpha)}}\right]^{-2}$$

$$= \tfrac{1}{2}\cdot\gamma\cdot\frac{(1\pm a_v)}{Cos\,\theta}\,AB^2\,\frac{Cos^2(\alpha+\phi-\theta)}{Cos(\theta+\delta-\alpha)}\cdot$$

$$\cdot\left[1-\sqrt{\frac{Sin(\phi+\delta)\,Sin(\phi+i-\theta)}{Cos(\alpha-i)\,Cos(\theta+\delta-\alpha)}}\right]^{-2}$$

$$Since\ AB^2 = \frac{h^2}{Cos^2\alpha}$$

$$\therefore\ P_P(dyn) = \tfrac{1}{2}\cdot\gamma\cdot h^2\,\frac{(1\pm av)}{Cos\,\theta\,Cos^2\alpha}\cdot\frac{Cos^2(\alpha+\phi-\theta)}{Cos(\theta+\delta-\alpha)}\cdot$$

$$\left[1-\sqrt{\frac{Sin(\phi+\delta)\,Sin(\phi+i-\theta)}{Cos(\alpha-i)\,Cos(\theta+\delta-\alpha)}}\right]^{-2}$$

Which is the same expression as given by Mononobe and Okabe.

Conclusion :

A graphical method based on Rebhan's construction is given for determining the earth pressure of cohesionless soils, considering the effect of inertia force. Rebhan's construction is limited to uniform deposits of cohesionless soils only and as such method is also limited for use in the case of uniform deposits of cohesionless soils only.

References

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Notations

i = angle of backfill with horizontal

α = angle of wall with vertical

δ = angle of wall friction

ϕ = angle of internal friction

h = height of wall

α_h = horizontal seismic coefficient

α_v = vertical seismic coefficient

γ = density of backfill

$\theta = \tan^{-1} \frac{\alpha_h}{1 \pm \alpha_v}$