Lateral Load-Deflection Behaviour of Pile Groups

by

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Introduction

Earlier methods of analysis of predicting the behaviour of laterally loaded pile groups have generally employed the concept of subgrade reaction [Hrennikoff (1950), Vesic (1956), Matlock and Reese (1960) and Broms (1964)]. The subgrade modulus is either assumed constant or increasing linearly with depth from zero at ground surface. Mohan (1981) has spotlighted some of the important problems of research, design and testing of pile foundations under various conditions of loadings.

A number of tests on laterally loaded pile groups have been reported [Feagin (1953), Prakash (1961), Prakash and Saran (1967), Pise (1969), Davisson and Salley (1971), Oteo (1972), Monolieu et al (1977)]. The test results have been generally analysed using the subgrade concept, assuming a constant or variable soil modulus with depth, deflection dependent soil modulus, using group reduction factors determined experimentally or otherwise.

Recently, Poulos (1971a, 1971b, 1973 and 1975) has presented analytical solutions for laterally loaded single pile and pile groups considering the soil as elastic continuum. Analysis and results of Poulos (1971b) has opened a new spectrum in the study of pile groups subjected to lateral loads. It is very versatile and involves elastic properties of the soil, pile stiffness, length to diameter ratio, l/d, of a pile, pile spacing in a group, geometry and orientation of the group, direction of loading and pile head and tip conditions. He concluded that the above mentioned factors largely influence the deflection characteristics of pile groups. Systematic experimental investigation to study the effects of l/d, pile spacing, pile flexibility factor, geometry of the group, on the interaction factors and displacement ratios as given by Poulos (1971b) is lacking. It is needless to say that it is always advisable to substantiate the theoretical results by experimental studies.

Scope of the Study

In this investigation the load-deflection characteristics of pile groups, interaction factors, and displacement ratios have been experimentally studied in the light of Poulos' analysis (1971b, 1973). To optimize the study, three lengths of model piles have been used in 2-pile and 4-pile group combinations at different spacings for two orientations of loading. An attempt has also been made to predict the displacement ratios for 4-pile groups from the experimental values of interaction factors determined from model tests.

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Theoretical Analysis

Interaction Factor

Two-pile Group : The centre to centre spacing between the piles is S and the angle between the line joining the centres of piles and the direction of loading is designated as departure angle β as shown in Figure 1. H and M are the applied lateral load and moment per pile respectively in Fig. 1. It is convenient to express the additional displacement due to interaction between two piles by interaction factor, α, defined as Poulos (1971b).

α=additional displacement due to adjacent pile/displacement of pile due to its own loading ...(1)

Four-pile Group : Referring to Figure 2, the interaction factor, α_G, for a 4-pile group consisting of indentical, equally loaded piles, obtained by the principle of superposition, Poulos (1971b), i,e. the increase in displacement of a pile due to all the surrounding piles can be calculated by summing the increases in displacement due to each pile in turn, is given by

$$\alpha_G=\alpha_2+\alpha_3+\alpha_4 \qquad ...(2)$$

in which α_2, α_3 and α_4 are the values of the appropriate interaction factors for pile 1 due to piles 2, 3 and 4 for the respective spacings and values of β between each of these piles and pile 1 (Figure 2).

For a 2-pile or 4-pile group, when all piles in the group are identical and equally loaded, the ratio of the displacement of group y_{gG} to the displacement y_{gs} of a single pile carrying the same load as a pile in the group is

$$y_{gG}/y_{gs}=1+\alpha_G \qquad ...(3)$$

For a 2-pile group, $\alpha_G=\alpha$

Displacement Ratio

The displacement of a group may be conveniently expressed in terms of displacement ratio R, which is the ratio of the displacement of the pile group to the displacement of a single pile carrying the same average load which is the applied load per pile in the group, Poulos (1971b).

$$R=y_{gG}/y_{gs} \qquad ...(4)$$

From Equations 3 and 4,

$$R=1+\alpha_G \qquad ...(5)$$

The interaction factors, α, displacement ratios, R, are functions of L/d, pile spacing s, pile flexibity factor K_R, pile head condition, geometry of pile group and its orientation with direction of loading. Theoretical results assuming constant soil modulus, E_s, have been presented for different pile head conditions by Poulos (1971*b*).

$$K_R = E_p\, I_p/E_s\, L^4 \qquad ...(6)$$

$E_p\, I_p$ = pile stiffness and E_s = Young's modulus of soil.

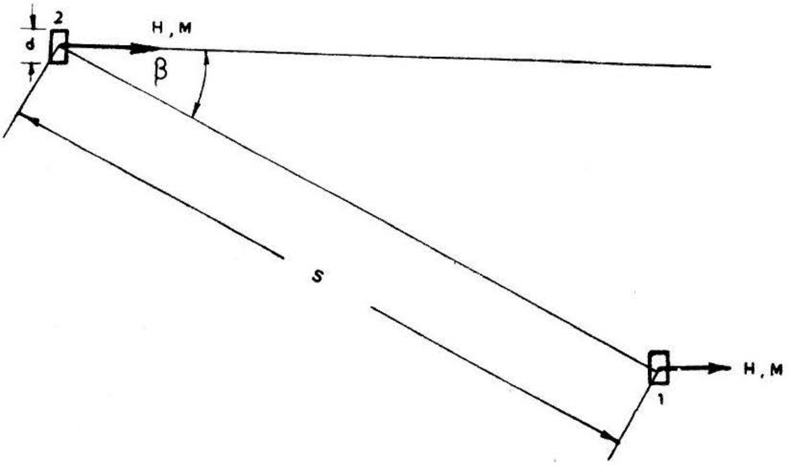

FIGURE 1 Two Pile group with direction of loading

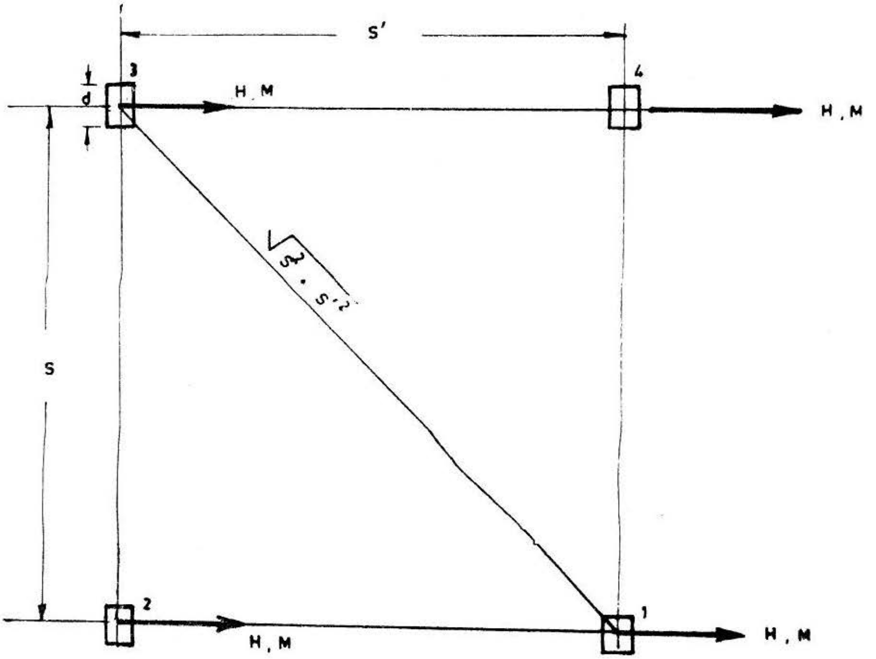

FIGURE 2 Four Pile group with direction of loading

Experimental Procedure and Test Programme

Soil

Dry Ennore (Madras) sand, having $D_{60} = 0.7$ mm and $D_{10} = 0.62$ mm, of uniformity coefficient 1.1, relative density $\simeq$ 40 per cent and angle of internal friction $\phi = 37°$, was used as a foundation medium in a tank 91.4 cm (36 inch) long, 76.2 cm (30 inch) wide, and 91.4 cm (36 inch) deep.

Pile

Aluminium alloy tubes 1.9 cm (0.75 inch) outside diameter, 0.088 cm (0.035 inch) wall thickness and $E_p I_p = 161{,}000$ kg/cm^2 were used as model piles. The lengths of piles were 28.6 cm (11.25 inch), 47.7 cm (18.75 inch), and 76.2 cm (30 inch) corresponding to L/d ratios of 15, 25 and 40 respectively.

Pile Caps

Sets of longitudinally split-pile caps of aluminium alloy were fabricated. They varied in length to accommodate 2 piles in a row at desired spacing. 3.2 cm (1.25 inch) long and 1.9 cm (0.75 inch) diameter holes in the cap housed the piles.

Test Conditions

Series A **2**-pile groups at spacings of 2, 4, 6 and 9 times the diameter of a pile and a direction of loading along the line joining the axes of the piles, i.e. $\beta = 0°$.

Series B 2-pile groups at spacings of 2, 4, 6 and 9 times the diameter of a pile and direction of loading perpendicular to and passing through the centre of the line joining the axes of the pile in the group, i.e. $\beta = 90°$.

Series C 4-pile groups at spacing of $3d \times 3d$, $3d \times 6d$, $6d \times 3d$ and $6d \times 6d$. The nomenclature of a group along with the direction of loading is shown in Figure 5.

Loading : Lateral loads, H, ranging from 1.13 kg (2.5 *lb*) to 5.66 kg (12.5 lb) per pile in stages were applied to the pile groups. The horizontal displacements were measured by dial gauges.

Testing Procedure

In general the test procedure for all groups was same though slight variations were involved due to varying number and arrangement of piles in the group. Procedure for a 4-pile group is described briefly as a typical case.

Four piles were arranged in the split-pile cap pieces at required spacings. The four pile cap pieces were tightened by nuts and studs. The pile cap with piles in position was supported on two rails, running across the width of the empty tank nearly in the centre. The verticality of piles was ensured by checking the horizontality of the pile cap. Sand was poured manually by a hopper of slot opening 0.31 cm (0.125 inch).

It was moved horizontally in the tank such that the opening is always at a distance of about 7.6 cm (3 inch) to 10.2 cm (4 inch) from the sand surface. This technique gave fairly well reproducible density of sand in reasonable amount of time, Pise (1969). When the piles were sufficiently embedded in sand, pile cap pieces as well as supporting rails were gently removed. Sand pouring was continued. Leaving 3.2 cm (1.25 inch) of piles exposed for insertion in the pile cap, it was discontinued. Pile cap pieces were carefully inserted and tightened. Care was taken to see that the pile cap did not rest on sand surface.

The pulley used for applying lateral loads was arranged in the centre line of the group. The lateral load was applied through a wire rope passing over a pulley. The wire rope was connected to the pile cap at one end at ground surface and to a loading pan at the other end. The horizontal displacement of the pile cap was measured by a dial gauge.

Analysis of Test Results

Young's Modulus of Soil :

To determine the Young's modulus of soil E_s, set of triaxial compression tests were conducted on sand at the density of test. The initial tangent moduli, E_i, of soil at different confining pressures were evaluated from Kondner and Zelasko's (1963) hyperbolic stress-strain relationship. Duncan and Chang (1970) has suggested a non-linear relation for the variation of initial tangent modulus, E_i, with initial confining pressure σ_o. Plot of E_i versus σ_o was made as shown in Figure 3. For the tests under

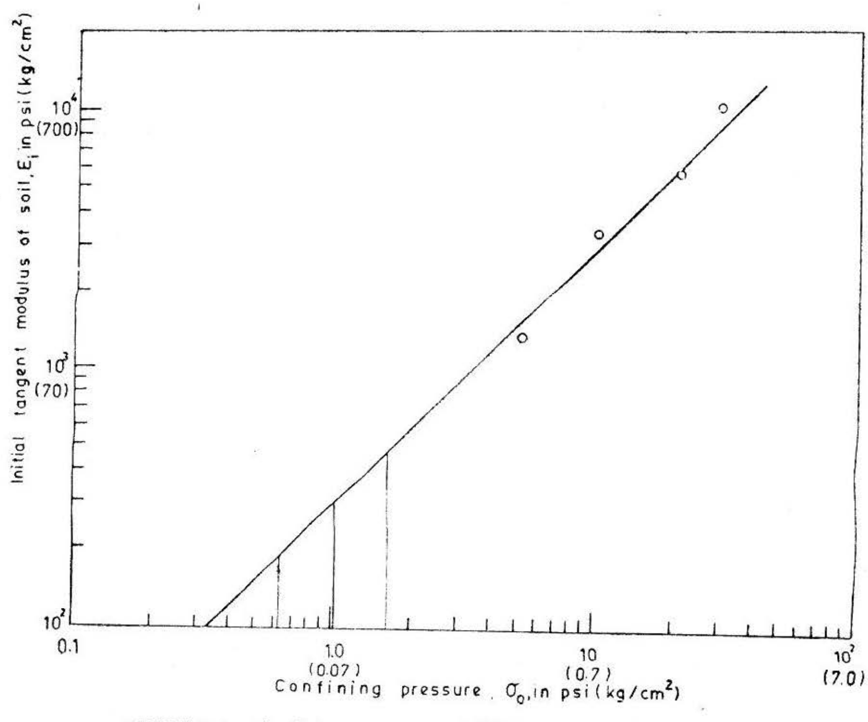

FIGURE 3 Confining pressure vs initial tangent modulus of Soil

review, the overburden pressures at the tips of the piles were 0.044 kg/cm² (0.63 psi), 0.074 kg/cm² (1.05 psi) and 0.118 kg/cm² (1.68 psi) for $L/d =$ 15, 25 and 40 respectively. E_i-values corresponding to these pressures were 12.6 kg/cm² (180 psi), 19.7 kg/cm² (280 psi) and 29.5 kg/cm² (420 psi) respectively from Figure 3.

Pile Flexibility Factor, K_R

For sandy soils, it is usually assumed that E_s approximately increases linearly with depth from zero at the surface. Average values of E_i have been taken as E_s for the soil, and they have been used to evaluate the K_R-values of the piles. For $L/d = 15$, 25 and 40, the E_s-values are 6.3 kg/cm² (90 psi), 9.85 kg/cm² (140 psi) and 14.75 kg/cm² (210 psi) and corresponding K_R-values are 4×10^{-2}. 3.45×10^{-3}, and 3.2×10^{-4} respectively. The E_s-values have also been backfigured from the load deflection diagram of the 2-pile groups later, and their influence on the results have been discussed there.

Ultimate Lateral Load Capacity of a Single Pile

The ultimate lateral load capacity of a single pile, H_{us}, has been evaluated by using the analysis of Poulos (1973). The ultimate lateral resistances, H_{us}, of piles having $L/d = 15$, 25 and 40 have been theoretically found to be 3.58 kg (7.9 lb), 9.8 kg (21.6 lb) and 25.4 kg (56 lb) per pile respectively.

Prediction of Interaction Factors and Displacement ratios

Linear Range of Load-displacement Diagrams : Load versus displacement diagrams for all the series are shown in Figures 4 to 5. It is seen that all the groups exhibit similar load-displacement behaviour. For all the curves, the slope of the initial portion of the curve does not change significantly upto a certain load and it is, therefore, reasonable to assume a linear load-displacement response upto this load. The limiting values of these loads, referred here as H_1, appear to be in the neighbourhood of 1.13 kg (2.5 lb), 2.27 kg (5 lb) and 3.4 kg (7.5 lb) per pile for $L/d = 15$, 25 and 40 respectively for all group combinations. The corresponding dimensionless load ratios, H_1/H_{us} along with the theoretical values of Poulos (1973) have been recorded in Table 1.

Instead of smooth load-displacement curves (Figures 4 and 5), best fit lines were made to pass through the data points (lines not shown) upto the loads H_1, the maximum deviation in the deflection given by curves at

TABLE 1

Dimensionless Load Ratios, H_1/H_{us}

L/d	15	25	40
K_R	4×10^{-2}	3.45×10^{-3}	3.2×10^{-4}
Experimental value of H_1/H_{us}	0.31	0.23	0.13
Poulos' (1973) Single pile value, H_1/H_{us}	0.33	0.25	0.12

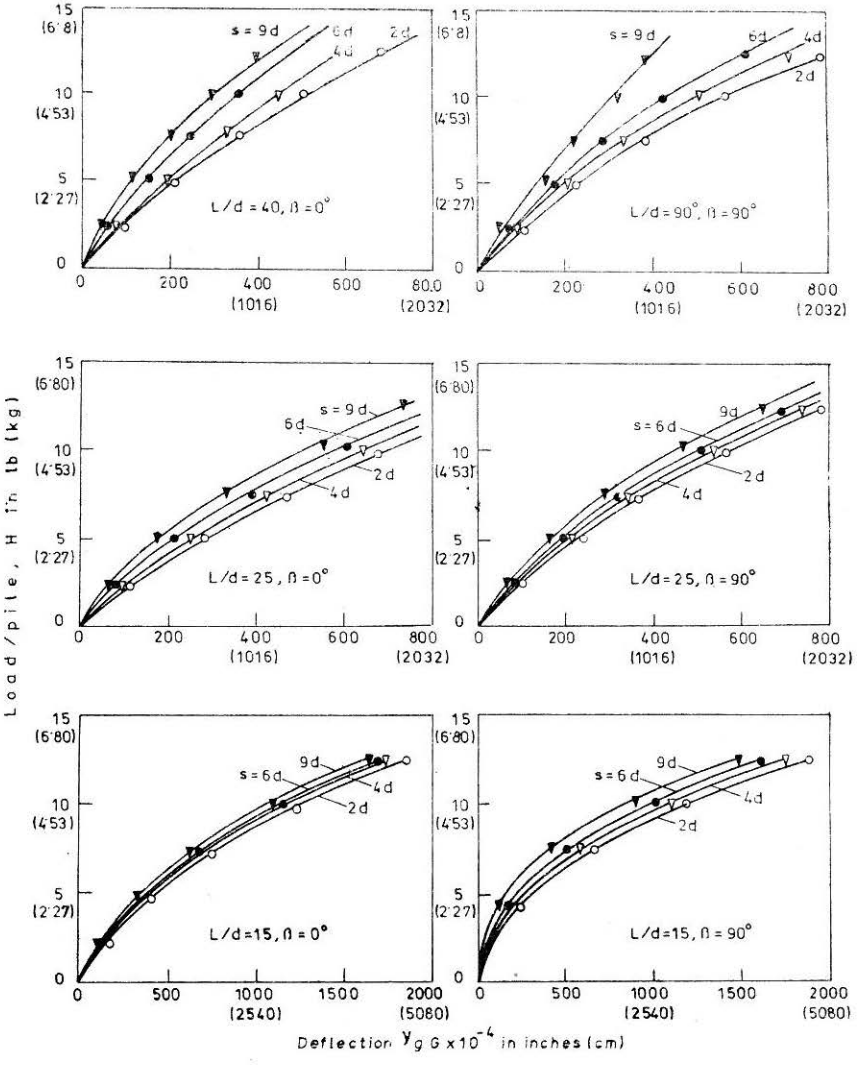

FIGURE 4 Displacement for pile groups (Series A and B)

H_1, from straight lines, was within 0 to 8 per cent for all groups. Therefore, the limiting values of the loads H_1, taken above are accurate enough to the extent of the discrepancy exhibited.

Single Pile Head Deflection : From the observations of rotations of the pile caps as well as pile head moments, Sengupta (1981), it was found that the pile heads in different groups exhibited a partial fixity of 20 to 35 per cent. It was practically impossible to test a single pile at the partial pile head fixity of the group and, there, an indirect method of predicting the single pile head deflection described below has been employed.

The displacements at the loads H_1, for different spacings and conditions have been read from Figure 4 (Series A and B). They have been plotted

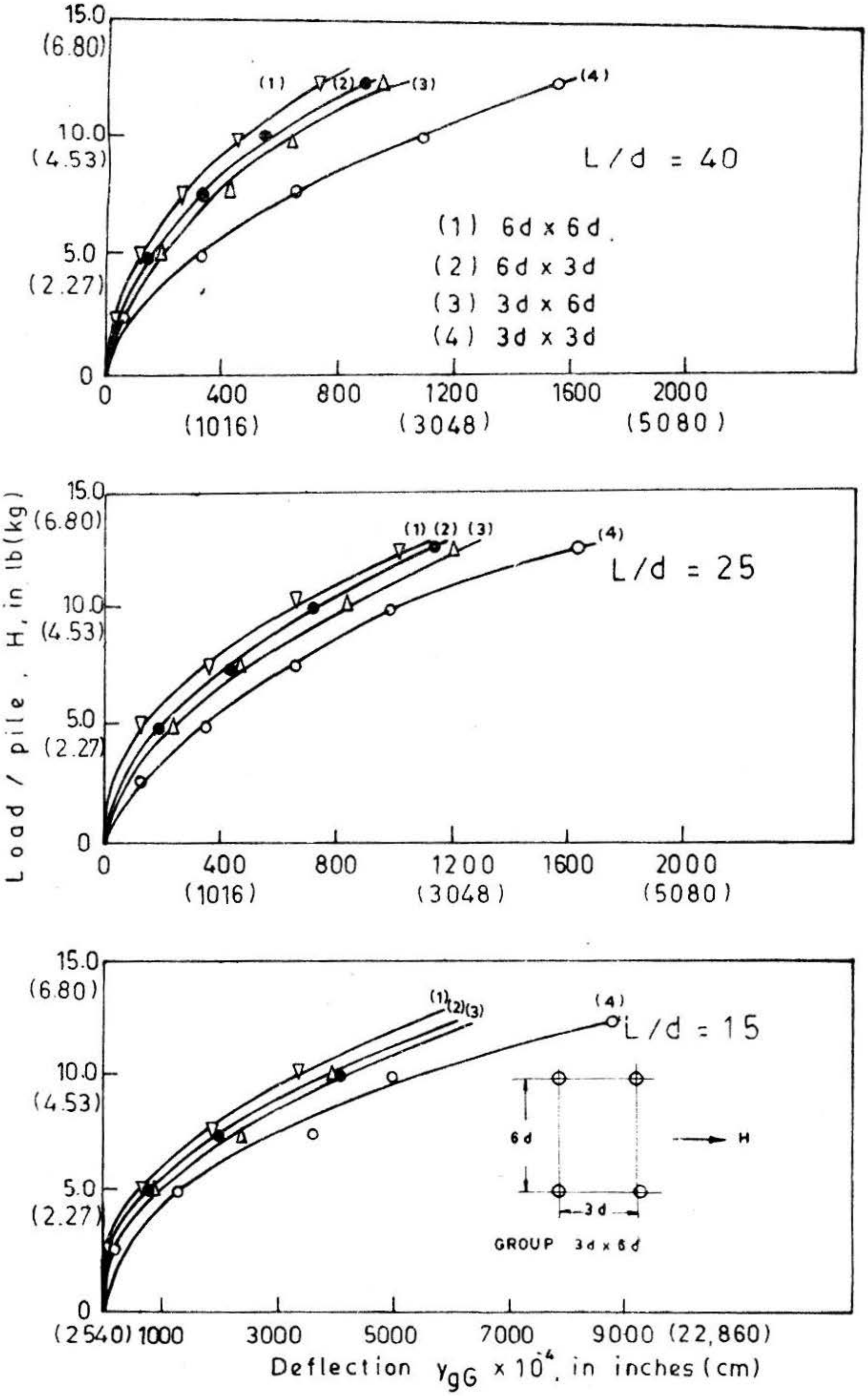

FIGURE 5 Load vs displacement for pile groups (Series C)

against spacing, s, in Figure 6. It is seen that the displacement versus spacing curves have tendency to become asymptotic to the s-axis. After extending the curves beyond the maximum range of spacings used in the tests i.e. $s = 9d$, it is seen that the curves practically become parallel to the s-axis at a spacing of approximately $10d$. The displacement corresponding to spacing $10d$ is taken as single pile deflection for that condition of test.

Interaction Factors : The interaction factors, α, for series A and B are evaluated by using Figure 6 and Equation 3. They are recorded in Table 2 along with the theoretical results of Poulos (1971*b*). Typical results for $L/d = 40$ are presented in Figure 7.

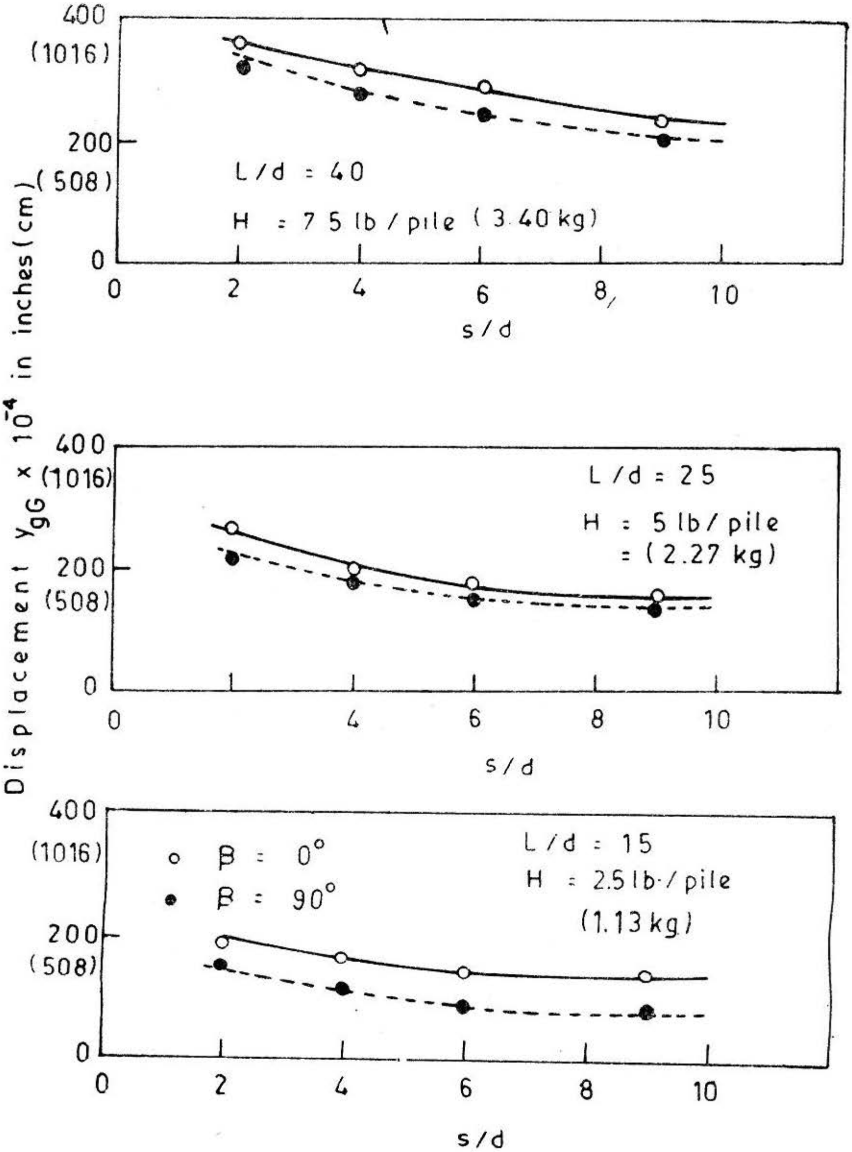

FIGURE 6 Displacement vs S/d (Series A and B)

TABLE 2

Interaction Factors for Deflection, α

Test condition	Spacing	$2d$	$4d$	$6d$	$9d$
$L/d = 15$,	Experimental value	0.72	0.48	0.23	0.12
$= 0°$	Poulos' value (1971*b*)	0.58	0.41	0.30	0.22
$L/d = 15$,	Experimental value	0.72	0.52	0.23	0.08
$= 90°$	Poulos' value (1971*b*)	0.37	0.26	0.17	0.13
$L/d = 25$,	Experimental value	0.67	0.40	0.15	0.07
$= 0°$	Poulos' value (1971*b*)	0.55	0.39	0.26	0.20
$L/d = 25$,	Experimental value	0.55	0.30	0.09	0.01
$= 90°$	Poulos' value (1971*b*)	0.37	0.23	0.15	0.10

Displacement Ratio : For predicting displacement ratio, R, of a group (Series C), the average deflection at a spacing of $10d$, for 2-pile group with $\beta = 0°$ and $90°$ conditions (Figure 6), have been taken as single pile deflection. The displacement ratios, R, are then predicted by using Figure 5 and Equation 4. They have also been predicted from Equation 5 using the interaction factors, predicted from experimental results of 2-pile groups, from Series A and B tests. Both of the results along with the theoretical results of Poulos (1971*b*) are recorded in Table 3. Typical results for square pile groups of four piles for $L/d = 40$ are shown in Figure 8.

Young's Modulus of soil, E_s, from Load-Deflection Diagrams

To compare the E_s-values used earlier from the triaxial compression test results with that obtained from the load-deflection diagrams, a procedure similar to Poulos' (1975) has been employed.

Using the K_R-values recorded in Table 1, elastic influence factors, I_{yH}, for deflection given by Poulos (1971*a*), assuming a pile head fixity of 25 per cent, and considering the pile head deflection of a 2-pile group at $s = 10d$ and $\beta = 90°$ condition, at loads H_1, as the single pile head deflection, the backfigured values of E_s, from Equation $y_{g_1} = (I_{yH} . H_1)/(E_s . L)$, have been found to be 7.75 kg/cm² (110 psi), 9.15 kg/cm² (130 psi), and 10.20) kg/cm² (145 psi) for the L/d ratios 15, 25 and 40 respectively. With these values of E_s, the K_R-values, given in Table 1, modify to 3.27×10^{-2}, 3.72×10^{-3} and 4.63×10^{-4} for the respective L/d ratios. The above procedure can be repeated for modified sets of K_R-values which will result in new sets of E_s values.

TABLE 3
Displacement Ratios, R, for 4-Pile Groups

L/d		$3d \times 3d$	$3d \times 6d$	$6d \times 3d$	$6d \times 6d$
15	Experimental value	3.25	2.20	1.80	1.55
	Value from experimental interaction factors	2.62	1.90	2.00	1.50
	Poulos' value (1971*b*)	2.15	1.94	1.89	1.70
25	Experimental value	2.65	1.80	1.55	1.26
	Value from experimental interaction factors	2.24	1.83	1.70	1.28
	Poulos' value (1971*b*)	2.07	1.78	1.75	1.56
40	Experimental value	2.80	1.92	1.55	1.25
	Value from experimental interaction factors	2 36	1.90	1.87	1.50
	Poulos' value (1971*b*)	2.06	1.62	1.60	1.52

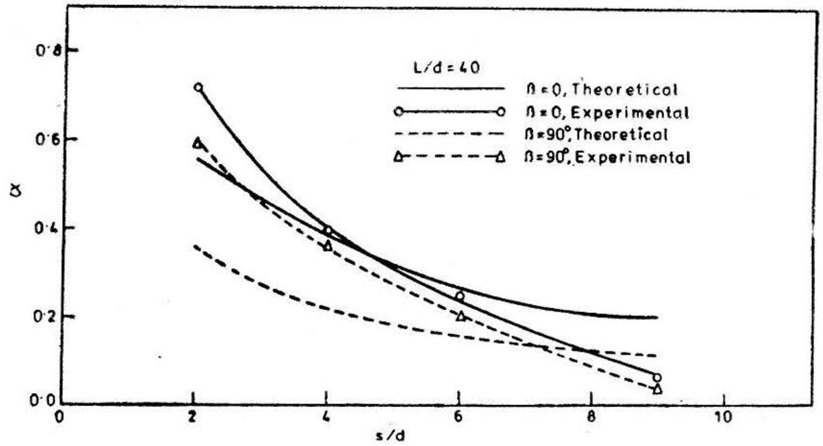

FIG.7. COMPARISON OF INTERACTION FACTORS, α'

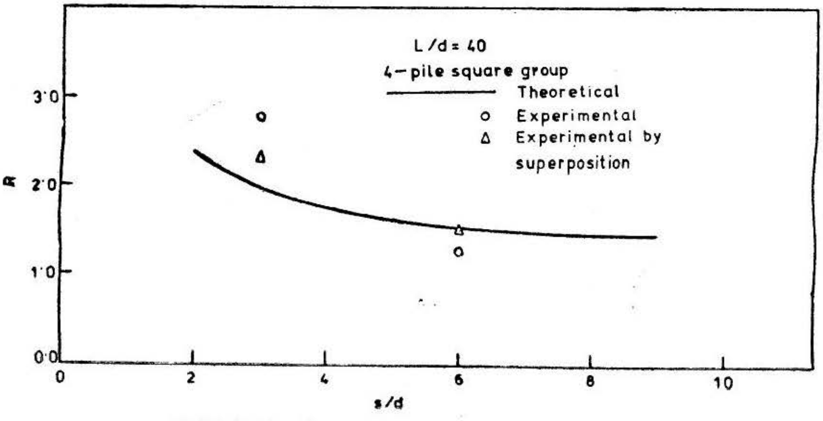

FIGURE 8 Comparison of displacement ratios R

But, due to the small change in K_R-values, insignificant change will occur in E_S, and, therefore, on the theoretical results of Poulos (1971 b, 1973) incorporated here. Whereas, the experimental findings remain unchanged. It is, therefore, concluded that the earlier values of E_S taken from triaxial compression test results are quite adequate.

Discussion of Results

Load-Deflection Characteristics :

The load-deflection relationships of pile groups (Figures 4 and 5), may be reasonably considered linear upto the dimensionless load ratios given in Table 1. Comparison of these load ratios, with the theoretical single pile values of Poulos (1973) shows a good measure of agreement. Beyond this load range, the load-deflection diagrams are non-linear.

It is seen that for 2-pile groups (Figure 4), the lateral resistance of group to deflection increases with increase in spacing. This is also evident from Figure 6. From the typical results shown in Figure 6, it is noted that at same spacing a 2-pile group with $\beta = 0°$ offers less resistance than $\beta = 90°$ group. Also, at higher loads, although results are not shown, generally above trend was noticed.

From Figure 5, it is observed that 4-pile group of $6d \times 6d$ spacing is more resistant to deflection than $3d \times 3d$ group and load-deflcetion curves of $6d \times 3d$ and $3d \times 6d$ groups lie within the curve of these groups.

Interaction Factors, α :

From the typical results shown in figure 7, it is seen that interaction factors as determined from experimental results decrease with increasing spacing and are greater for $\beta = 0°$ than for $\beta = 90°$. The qualitative trend of experimental results is analogous to the theory, Poulos (1971b), however, they differ numerically. Experimental interaction factors are more than the theoretical values at closer spacing and less at larger spacings, i.e. the theory tends to underestimate them at closer spacing and overestimate them at larger spacing. Similar trend is observed at other L/d ratios which can be seen from Table 2.

Displacement Ratios, R :

From the typical results shown in figure. 8, it is seen that the experimental values of displacement ratios, R, are more than the theoretical values of Poulos (1971b) at closer spacing and less at larger spacing. Similar trend is noted for rectangular groups also. The displacement ratio, R, predicted from the experimental values of interaction factors by superposition, are generally less than the experimental values at closer spacing and more at larger spacing, however, they are more closer to the theoretical values. Similar trend is noted at other L/d ratios which can be seen from Table 3.

Summary and Conclusions

The load versus displacement diagrams for the groups may reasonably be considered linear upto a dimensionless load ratio H_1/H_{us}, which is a function of L/d ratio. The values of H_1/H_{us} for the groups having $L/d=15$, 25 and 40 have been found to be 0.31, 0.23 and 0.13 respectively. They are in good agreement with the single pile values of Poulos (1973).

At same spacing, a 2-pile group with direction of loading along the line joining the axes of the piles ($\beta = 0°$) is less resistant to deflection than the one with loading at right angle to this line ($\beta = 90°$).

A 4-pile square group of larger spacing is more resistant than that of smaller spacing. A rectangular group of four piles is more resistant when the longer side is parallel to the direction of loading.

Quantitative estimates of the interaction factors, α, and displacement ratios, R, from experimental results indicate that at closer spacing, the theory underestimates them, whereas, at larger spacing it overestimates them.

Although, Poulos (1971b) has evaluated the interaction factors, α, and displacement ratios, R, on the assumption of constant soil modulus E_s, strictly valid for clay soils, and questionable for sandy soils, the measure of agreement between the theoretical and experimental values of α and R appears encouraging.

Acknowledgements

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Notations

d = diameter of a pile

E_i = initial tangent modulus of soil

E_p = Young's modulus of pile material

E_s = Young's modulus of soil

H = applied lateral load per pile

H_1 = limiting lateral load per pile upto which the load-deflection diagram is linear

H_{us} = ultimate lateral load of a single pile

I_p = moment of inertia of a pile

K_R = $E_p I_p / E_s L^4$ = pile flexibility factor

L = length of a pile

R = displacement ratio

S = centre to centre spacing between piles

y_gG = displacement of a group

y_{gs} = displacement of a single pile

α = two-pile interaction factor for deflection

α_G = interaction factor = ratio of increase in displacement due to equally loaded adjacent piles

β = departure angle = angle between direction of applied loading and the line joining the axes of the piles.

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