Foundation Response to Horizontal Vibrations

by

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Introduction

RESONANT frequency and resonant amplitude are the important criteria involved in the design of foundation subjected to periodic loading. Proper estimation of these two quantities are necessary for the design of a foundation subjected to vibration. Elastic half-space model has been used by many Investigators (Sung; 1953 ; Bycroft, 1956 : Hsieh, 1962) to predict the dynamic response of foundations. Chase et al. (1965) and Richart and Whitman (1967) have shown the variation in the contact pressure distribution with frequency of vibration. Moore (1971) and Richart and Whitman (1967) showed the necessity for considering the effect of contact pressure distribution while predicting the dynamic response. Housner and Castellani (1969) have shown for a particular case, the significant variation in the dynamic response by considering the displacement at the centre of the footing, the average displacement and the weighted average displacement. Thus it is seen that the effect of the type of contact pressure distribution and the displaclemnet conditions are to be considered while predicting the dynamic response of footings Bycroft (1956) gave solutions for the dynamic response of a circular footing with rigid base pressure distribution, subjected to horizontal vibration considering the weighted average displacement. Sankaran et al. (1977) predicted the dynamic response of a machine foundation resting on soil surface and subjected to horizontal vibration using a lumped parameter model, considering radiation damping and viscous damping separately. It can be seen from literature, that for horizontal vibration, solutions are not available for different pressure distributions, (viz., rigid base, uniform and parabolic), and for all displacement conditions (viz. central, average and weighted average except that of Bycroft (1956). In this paper solutions have been obtained for the remaining cases.

The geometric peculiarity (i.e. semi-infinite) of the elastic half-space model led to the development of lumped paramater model to predict the dynamic response. Lysmer and Richart (1966) and Nagendra and Sridharan (1981) developed analog model for vertical vibration. Hall (1967) developed an analog model for a circular footing with rigid base pressure distribution subjected to horizontal vibration., considering weighted average displacement. Analog solutions are not available for all

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conditions of displacement and contact pressure distributions except for rigid base pressure distribution with weighted average displacement. Hence in this paper analog solutions have been obtained for the dynamic response of circular footings, subjected to frequency dependent and frequency independent excitations, taking in to consideration three types of contact pressure distributions and three types of displacement conditions. Though weighted average displacement is more appropriate for design purposes (Bycroft, 1956 ; Housner and Castellani, 1969 ; Richart et al., 1970) the other two displacement conditions have also been studied. The results have been presented in the form of tables and charts which could be readily used for design purposes. The results of the analog solutions have been compared with the results obtained from elastic half-space model and the agreement is found to be very good.

Analysis

Elastic Half-space Theory

Using elastic half-space theroy, assuming that the vertical displacement due to horizontal vibration is zero, Bycroft (1956) expressed the weighted average displacement (U_{WR}) of a circular footing with rigid base pressure distribution in the form

$$U_{WR} = \frac{Z}{4\pi G a^2} \int_0^\infty \left[\frac{x^2 - \alpha\,(x^2-k^2)^{1/2}}{k^2\,\alpha x} + \frac{1}{(x^2-k^2)^{1/2} x} \right] \sin^2 (xa)\; dx \tag{1}$$

where Z = amplitude of dynamic force; a = radius of footing; G = shear modulus of soil; $\alpha = (x^2 - h^2)^{\frac{1}{2}}$; $h^2 = \rho\omega^2/(\lambda+2G)$; λ = Lame's constant; $k^2 = \rho\omega^2/G$; ρ = mass density of soil and ω = exciting frequency. Similar expressions for the central displacement. (U_c), average displacement (U_a), and weighted average displacement U_w), for the three types of contact pressure distributions (viz. rigid base, uniform and parabolic), in general can be written as

$$U_c = \frac{Z}{2\pi G a} \int_0^\infty \left[\frac{x^2 - \alpha(x^2-k^2)^{1/2}}{k^2 \,.\, \alpha} + \frac{1}{(x^2-k^2)^{1/2}} \right] M(xa)\; dx \qquad \ldots(2)$$

$$U_a = \frac{Z}{2\pi G a} \int_0^\infty \left[\frac{x^2 - \alpha(x^2-k^2)^{1/2}}{k^2 \,.\, \alpha} + \frac{1}{(x^2-k^2)^{1/2}} \right] \frac{M(xa) \,.\, 2 \,.\, J_1(xa)}{xa}\; dx \qquad \ldots(3)$$

$$U_w = \frac{Z}{2\pi G a} \int_0^\infty \left[\frac{x^2 - \alpha(x^2-k^2)^{1/2}}{k^2 \,.\, \alpha} + \frac{1}{(x^2-k^2)^{1/2}} \right] \frac{2[M(xa)]^2}{xa}\; dx \qquad \ldots(4)$$

where $M(xa) = \sin(xa)/2$ for rigid base; $J_1(xa)$ for uniform and $4J_2(xa)/(xa)$ for parabolic contact pressure distributions respectively. By substituting $x = kn$, $ak = a_0$ and $h^2/k^2 = s^2$ in Equation 4 the weighted average displacement (U_w) can be written as

$$U_w = \frac{Z}{2\pi Ga} \int_0^\infty \left[\frac{n^2}{(n^2-s^2)^{1/2}} + \frac{2n^2}{(n^2-1)^{1/2}} \right] \frac{2[M(na_0)]^2}{na_0}\, dn \qquad \ldots(5)$$

where $a_0 = (\omega a(\rho/G)^{1/2})$ is called nondimensional frequency factor. Splitting the integral and separating the real and imaginary parts, U_w can be written as

$$U_w = \frac{Z}{2\pi Ga} \left[\int_s^1 \frac{n^2}{(n^2-s^2)^{1/2}} + \int_1^\infty \frac{2-n^2}{(n^2-1)^{1/2}} - i \left\{ \int_0^\infty \frac{n^2}{(s^2-n^2)^{1/2}} + \int_0^1 \frac{2-n^2}{(1-n^2)^{1/2}} \right\} \right] \frac{2[M(na_0)]^2}{na_0}\, dn \qquad \ldots(6)$$

Representing the real and imaginary parts by f_1 and f_2, which are called 'displacement functions' Equation 6 is written in the form

$$U_w = \frac{Z}{Ga}(f_1 + if_2) \qquad \ldots(7)$$

Following the procedure of Reissner (1936), the value of displacement function f_1 is evaluated as follows. From Equation 6 and 7, the expression for f_1 can be written as

$$f_1 = \frac{1}{2\pi} \left[\int_s^\infty \frac{n^2}{n^2-s^2)^{1/2}} + \int_1^\infty \frac{2-n^2}{(n^2-1)^{1/2}} \right] \frac{2[M(na_0)]^2}{na_0} dn \qquad \ldots(8)$$

Equation 8 can be simplified as

$$f_1 = \frac{1}{2\pi} \int_s^\infty \frac{n^2}{(n^2-s^2)^{1/2}} + \frac{2[M(na_0)]^2}{na_0} dn$$

$$+ \frac{1}{2\pi} \int_1^\infty \left[\frac{n^2}{(n^2-s^2)^{1/2}} + \frac{2n^2}{(n^2-1)^{1/2}} \right] \frac{2[M(na_0)]^2}{na_0}\, dn \qquad \ldots(9)$$

The value of the first integral is small when compared to that of second and can be neglected. Expanding the terms $(n^2-s^2)^{-1/2}$ and $(n-1)^{-1/2}$ in series form and simplifying Equation 9 can be written as

$$f_1 = \frac{1}{2\pi} \int_1^\infty \left[n \left(1 + \frac{s^2}{2n^2} + \frac{3s^4}{8n^4} + \frac{5s^6}{16n^6} + \ldots \right) \right.$$

$$- n\left(1 - \frac{3}{2n^2} - \frac{5}{1n^4} - \frac{7}{16n^6} - \dots\right)\Big]\frac{2\,[M\,(na_0)]^2}{na_0}\;dn \quad \dots (10)$$

On further simplification f_1 can be written as

$$f_1 = \frac{1}{2\pi}\int_1^\infty \left[\frac{1}{n}\sum_{\nu=0}^{\infty} b_\nu\; n^{-2\nu}\right] - \frac{2[M\,(na_g)]^2}{na_0} \quad \dots (11)$$

where $b_0 = (s^2+3)/2$, $b_1 = (3s^4+5)/8$, $b_2 = (5s^6+7/16$, etc. The values of b_ν ($\nu = 0, 1, 2$) for four values of Poisson's ratio, μ $(= 1-1/2\,(1-s^2))$. viz. 0.00, 0.25, 0.33 and 0.5. are presented in Table 1. Substituting

TABLE 1

Values of b_ν for Different Poisson's Ratios

μ	s^2	b_0	b_1	b_2
0	1/2	7/4	23/32	61/128
1/4	1/3	5/3	3/3	97/216
1/3	1/4	13/8	83/128	453/1024
1/2	0	3/2	5/8	7/16

$ma_0 = t$ and interchanging the summation and integration Equation 11 can be written as

$$f_1 = \frac{1}{2\pi}\sum_{\nu=0}^{\infty} b_\nu\;\; G_\nu\;\; a_0^{2\nu} \quad \dots (12)$$

where $G_\nu = \int_1^\infty \frac{2\,[M\,(t)]^2}{t_{2\nu+2}}\,dt$. Approximating this integral to be between the limits 0 and ∞, and substituting the value of $M(t)$ for various pressure distributions the value of integrals are evaluated for $\nu = 0, 1, 2$, (Watson, 1966), and are presented in Table 2. The values of G_ν for average displacement have also been presented in Table 2. It is seen from Tables 1 and 2 that the value of b_ν is a function of Poisson's ratio only and G_ν is a function of type of displacement condition and pressure distribution only.

From Equations 6 and 7, the expression for f_2 for weighted average displacement condition can be written as

$$f_2 = \frac{-1}{2\pi}\left[\int_0^s \frac{n^2}{(s^2-n^2)^{1/2}} + \int_0^1 \frac{2-n^2}{(1-n^2)^{1/2}}\right]\frac{2\,[M\,(na_0)]^2}{na_0}\;dn \quad \dots (13)$$

TABLE 2

Value of G_ν for Different Displacement Conditions and Contact Pressure Distributions

Displacement condition	Contact pressure distribution	G_0	G_1	G_2
+Central	Rigid base	$\pi/4$	$-\pi/8$	$\pi/96$
	Uniform	1	$-1/3$	$1/45$
	Parabolic	$4/3$	$-4/15$	$4/315$
Average	Rigid base	$\pi/4$	$-5\pi/32$	$7\pi/256$
	Uniform	$\frac{8}{3\pi}$	$\frac{-64}{45\pi}$	$\frac{1024}{4725\pi}$
	Parabolic	$\frac{128}{45\pi}$	$\frac{-2048}{1575\pi}$	$\frac{16384}{99295\pi}$
Weighted average	Rigid base	$\pi/4$	$-\pi/6$	$\pi/30$
	Uniform	$\frac{8}{3\pi}$	$\frac{964}{45\pi}$	$\frac{1024}{4725\pi}$
	Parabolic	$\frac{1024}{315\pi}$	$\frac{-16384}{14175\pi}$	$\frac{131072}{1091475\pi}$

+ Values of G_x for central displacement are taken from Sung (1953)

Substituting the value of $M\ (na_0)$ for various pressure distributions in series form and simplifying f_2 can be written as

$$f_2 = -\frac{1}{2\pi}\left[\int_0^s \frac{n^2}{2(s^2-n^2)^{1/2}}\ n\ dn + \int_0^1 \frac{2-n^2}{2\ (1-n^2)^{1/2}}\ n\ dn\right](X_1).\ a_0$$

$$-\frac{1}{2\pi}\left[\int_0^s \frac{n^2}{2(s^2-n^2)^{1/2}} n^3\ dn + \int_0^1 \frac{2-n^2}{2\ (1-n^2)^{1/2}} n^3\ dn\right](X_2).\ a_0^3$$

$$-\frac{1}{2\pi}\left[\int_0^s \frac{n^2}{2(s^2-n^2)^{1/2}} n^5\ dn + \int_0^1 \frac{2-n^2}{2\ (1-n^2)^{1/2}} n^5\ dn\right](X_3).\ a_0^5$$

$$- \ldots \qquad \ldots (14)$$

On integration, f_2 can be represented by

$$f_2 = -\frac{1}{4\pi}\left[\left(\frac{2}{3}\ s^2 + \frac{4}{3}\right)(X_1)\ a_0 + \left(\frac{8}{15}\ s^5 + \frac{4}{5}\right)(X_2)\ a_0^5\right.$$

$$\left. + \left(\frac{16}{35}\ s^7 + \frac{64}{105}\right)(X_3)\ a_0^6 + \ldots\right] \qquad \ldots (15)$$

where the values of X_1, X_2 and X_3 for different pressure distributions and displacement conditions are presented in Table 3.

The value of f_1 for $a_0 = 0$, represents the factor Ga/k, where K is the static stiffness coefficient. Table 4 presents the values of static stiffness

TABLE 3

Value of the Coefficients X_1, X_2, X_3 for Different Displacement Conditions and Contact Pressure Distributions

Displacement condition	Contact pressure distribution	X_1	X_2	X_3
Central	Rigid base	1	−1/6	1/120
	Uniform	1	−1/8	1/192
	Parabolic	1	−1/12	1/384
Average	Rigid base	1	−7/24	11/320
	Uniform	1	−1/4	5/192
	Parabolic	1	−5/24	7/384
Weighted Average	Rigid base	1	−1/3	2/45
	Uniform	1	−1/4	5/192
	Parabolic	1	−1/6	7/576

TABLE 4

Values of Stiffness Coefficients

Displacement condition	Contact pressure distribution		
	Rigid base	Uniform	Parabolic
Central	$\frac{32(1-\mu)Ga}{7-8\mu}$ (Bycroft, 1)	$\frac{8\pi(1-\mu)Ga}{7-8\mu}$ (Authors)	$\frac{6\pi(1-\mu)Ga}{7-8\mu}$ (Authors)
Average	$\frac{32(1-\mu)Ga}{7-8\mu}$ (Bycroft, 1)	$\frac{3\pi^2(1-\mu)Ga}{7-8\mu}$ (Authors)	$\frac{45\pi^2(1-\mu)Ga}{16(7-8\mu)}$ (Authors)
Weighted average	$\frac{32(1-\mu)Ga}{7-8\mu}$ (Bycroft, 1)	$\frac{3\pi^2(1-\mu)Ga}{7-8\mu}$ (Authors)	$\frac{315\pi^2(1-\mu)Ga}{128(7-8\mu)}$ (Authors)

coefficients as a function of Poisson's ratio for various types of contact pressure distributions and displacement conditions.

Lysmer and Richart (1966) showed that the displacement functions f_1 and f_2 for rigid base pressure distribution under vertical vibration, when multiplied by a factor $4/(1-\mu)$ becomes almost equal and independent of Poisson's ratio. Similarly the f_1 and f_2 values obtained from Equation 12 and 15, for diffierent displacement conditions and pressure distributions, when multiplied by the factor K/Ga (corresponding K as listed in Table 4 are found to be almost equal and independent of Poisson's ratio. The average of the values of f_1K/Ga and f_2K/Ga for different Poisson's ratios represented by 'modified displacement functions'. F_1 and F_2 (Nagendra and Sridharan, 1981). The values of F_1 and F_2 for different conditions of pressure distributions and displacement conditions are presented in Table 5.

By definition, the resonant amplitude, A is related to the maximum magnification factor M_{xrm} and M_{xm} by the expressions

TABLE 5

Modified Displacement Functions, F_1 and F_2

$$F_1 = Y_1 - Y_2 a_0^2 + Y_3 a_0^4 \; ; \quad -F_2 = Z_1 a_0 - Z_2 a_0^3 + Z_3 a_0^5$$

Type of displacement	Type of contact pressure distribution	Y_1	Y_2	Y_3
Central	Rigid base	1.000000	0.203303	0.011518
	Uniform	1.000000	0.135535	0.006142
	Parabolic	1.000000	0.081321	0.002633
Average	Rigid base	1.000001	0.254127	0.030233
	Uniform	1.000004	0.216856	0.022463
	Parabolic	1.000000	0.185878	0.015977
Weighted average	Rigid base	1.000000	0.271070	0.036857
	Uniform	1.000000	0.216156	0.022463
	Parabolic	1.090000	0.144571	0.010210
		Z_1	Z_2	Z_3
Central	Rigid base	0.562494	0.054309	0.002025
	Uniform	0.441782	0.031992	0.000994
	Parabolic	0.331336	0.015995	0.000372
Average	Rigid base	0.562494	0.095043	0.008351
	Uniform	0.520463	0.075377	0.005855
	Parabolic	0.487933	0.058889	0.003841
Weighted average	Rigid base	0.562494	0.108221	0.010798
	Uniform	0.520463	0.075377	0.005155
	Parabolic	0.426941	0.041222	0.002241

$$A = \frac{m_0 e}{m} M_{Xrm} \qquad \ldots (16)$$

and

$$A = \frac{Z}{K} M_{Xm} \qquad \ldots (17)$$

where m = mass of the footing, m_0 = eccentric mass and e = eccentricity, for frequency dependent and frequency independent excitations respectively. The resonant frequency, f_r is related to the resonant frequency factor a_{0m} by the expression

$$f_r = \frac{1}{2\pi} \frac{a_{0m}}{a} \left(\frac{G}{\rho}\right)^{1/2} \qquad \ldots (18)$$

The maximum magnification factor, M_{Xrm} and M_{Xm} and the corresponding frequency factors a_{0rm} and a_{0m} are found from the expressions for magnification factors M_{xr} and M_x, which are of the form

$$M_{xr} = B_x\, a_0^2 \left[\frac{F_1^2 + F_2^2}{(1 - B_x\, a_0^2\, F_1)^2 + (B_x\, a_0^2\, F_2)^2} \right]^{1/2} \quad \dots (19)$$

$$M_x = \left[\frac{F_1^2 + F_2^2}{(1 - B_x\, a_0^2\, F_1)^2 + (B_x\, a_0^2\, F_2)^2} \right]^{1/2} \quad \dots (20)$$

where modified mass ratio, $B_x = bGa/K$, and b = mass-ratio ($= m/\rho\ a^3$), for various values of B_x from 0.25 to 10.0. Table 6 and 7 presents the values of M_{Xrm} and M_{Xm} for various values of B_x. The corresponding values of frequency factors a_{0mr} and a_{0m} are presented in Tables 8 and 9. Figures 1 and 2 present the variation of maximum magnification factor for

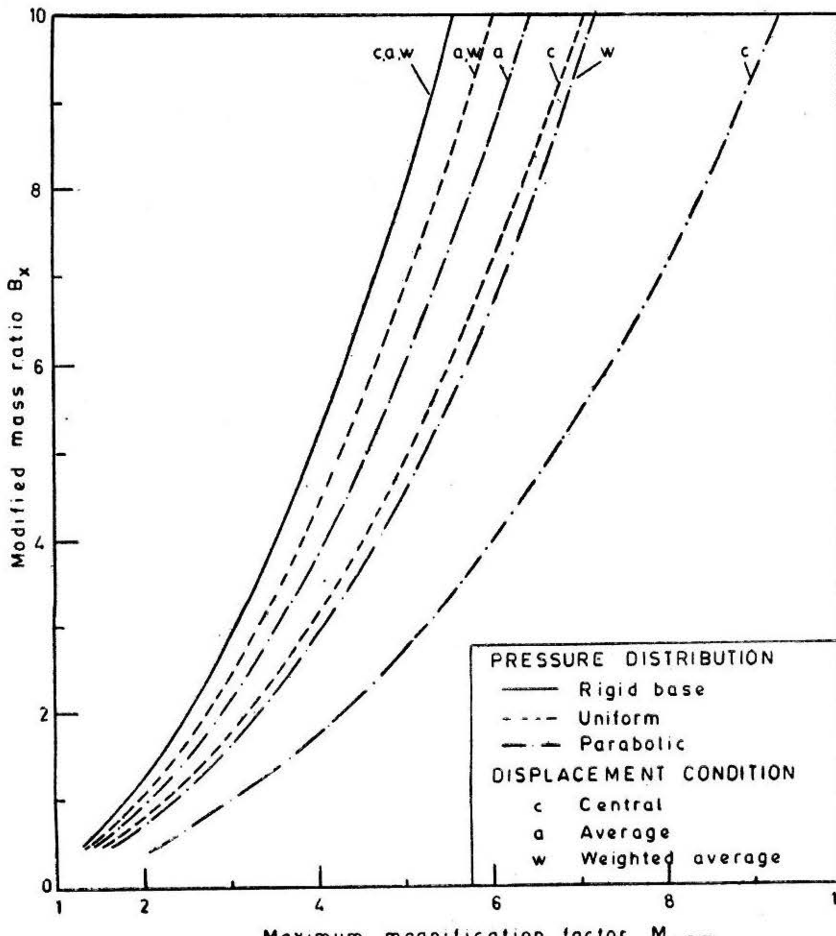

FIGURE 1 **Variation of maximum magnification factor M_{xrm} with modified mass ratio B_x—Frequeneay dependant excitation**

TABLE 6

Value of B_x and M_{psm}—Frequency Dependent Excitation

B_x	Values of M_{xrm}								
	C			*A*			*W*		
	B	*U*	*P*	*R*	*U*	*P*	*R*	*U*	*P*
0.25	1.15	1.10	1 48	1.31	1.28	1.24	1.38	1.28	1.24
0.5	1.23	1.57	2.11	1.26	1.35	1.44	1.30	1.35	1.64
0.75	1.52	1.94	2.56	1.52	1.65	1.76	1.52	1.65	2.01
1.0	1.76	2.24	3.00	1.76	1.90	2.03	1.76	1.90	2.33
1.5	2.16	2.75	3.68	2.16	2.34	2.49	2.16	2.34	2.86
2.0	2.50	3.18	4.26	2.50	2.70	2.88	2.49	2.70	3.30
3.0	3.07	3.91	5.22	3.06	3.31	3.53	3.06	3.31	4.05
4.0	3.54	4.51	6.02	3.54	3.82	4.08	3.54	3.82	4.67
5.0	3.97	5.05	6.73	3.96	4.29	4.57	3.96	4.29	5.23
6.0	4.35	5.54	7.38	4.34	4.70	5.01	4.34	4.74	5.73
7.0	4.70	5.98	7.97	4.69	5.08	5.41	4.69	5.08	6.19
8.0	4.98	6.31	8.36	4.98	5.38	5.72	4.99	5.38	6.52
10.0	7.58	7.08	9.36	5.60	6.04	6.44	5.50	6.04	7.30

Displacement condition
C—Central
A—Average
W—Weighted average

Contact Pressure Distribution
R—Rigid base
U—Uniform
P—Parabolic

TABLE 7

Values of B_x and M_{xm}—Frequency Independent Excitation

B_x	Values of M_{xm}								
	C			*A*			*W*		
	R	*U*	*P*	*R*	*U*	*P*	*R*	*U*	*P*
0.25	1.23	1.37	1.66	1.13	1.19	1.24	1.11	1.19	1.35
0.5	1.51	1.77	2,24	1.41	1.50	1.59	1.38	1.50	1.77
0.75	1.75	2.10	2,70	1.66	1.77	1.89	1.63	1.77	2.12
1.0	1.96	2.38	3.09	1.88	2.01	2.14	1.85	2.01	2.42
1.5	2.32	2.87	3.75	2.25	2.42	2.59	2.23	2.42	2.93
2.0	2.64	3.28	4.32	2.58	2.78	2.29	2.56	2.78	3.36
3.0	3.18	3.98	5.27	3.13	3.38	3.60	3.11	3.38	4.09
4.0	3.64	3.58	6.03	3.59	3.88	4.14	3.58	3.88	4.72
5.0	4.04	5.11	6.74	4.01	4.33	4.62	4.00	4.33	5.27
6.0	4.43	5.57	7.32	4.39	4.74	5.05	4.38	4.74	5.73
7.0	5.77	5.99	7.88	4.73	5.10	5.43	4.72	5 10	6.16
8.0	5.08	6.44	8.53	5.06	5.46	5.83	5.05	5.46	6.65
10.0	5.67	7.09	9.18	5 62	6.06	6.44	5.61	6.06	7.29

Displacement condition
C—Central
A—Average
W—Weight average

Contact Pressure Distribution
R—Rigid base
U—Uniform
P—Parabolic

TABLE 8

Values of B_x and a_{0m}—Frequency Dependent Excitation

Values of a_{0m}

B_x	C			A			W		
	R	U	P	R	U	P	R	U	P
0·25	3.93	2.19	2.11	2.64	2.74	2.92	2.49	2.74	2.82
0.5	1.48	1.47	1.45	1.64	1.57	1.53	1.91	1.57	1.50
0.75	1.19	1.18	1.18	1.24	1.22	1.21	1.26	1.22	1.20
1.0	1.02	1.02	1.01	1.05	1.04	1.04	1.06	1.04	1.03
1.5	0.83	0.83	0.82	0.84	0.84	0.84	0.85	0.84	0.83
2.0	0.72	0.71	0.71	0.72	0.72	0.72	0.73	0.72	0.72
3.0	0.58	0.58	0.58	0.59	0.59	0.58	0.59	0.59	0.58
4.0	0.50	0.50	0.50	0.51	0.50	0.50	0.51	0.50	0.50
5.0	0.45	0.45	0.45	0.45	0.45	0.45	0.45	0.45	0.45
6.0	0.41	0.41	0.41	0.41	0.41	0.41	0.41	0.41	0.41
7.0	0.38	0.38	0.38	0.38	0.38	0.38	0.38	0.38	0.39
8.0	0.36	0.35	0.35	0.36	2.36	0.36	0.36	0.36	0.35
10.0	0.32	0.32	0.32	0.32	0.32	0.32	0.32	0.32	0.32

Displacement condition
C—Central
A—Average
W—Weighted average

Contact Pressure Distribution
R—Rigid base
U—Uniform
P—Parabolic

TABLE 9

Value of B_x and a_{0m}—Frequency Independent Excitation

Valves of a_{0m}

B_x	C			A			W		
	R	U	P	R	U	P	R	U	P
0.25	1.28	1.48	1.69	1.22	1.34	1.40	1.19	1.33	1.53
0.5	1.11	1.21	1.30	1.11	1.16	1.18	1.12	1.16	1.24
0.75	0.98	1.04	1.09	0,99	1.01	1.03	0.99	1.01	1.06
1.0	0.88	0.92	0.96	0.89	0.91	0.92	0.89	0.91	0.94
1.5	0.75	0.77	0.79	0.76	0.76	0.77	0.76	0.76	0.78
2.0	0.66	0.68	0.69	0.67	0.67	0.68	0.67	0.67	0.68
3.0	0.55	0.56	0.57	0.56	0.56	0.56	0.56	0.56	0.56
4.0	0.48	0.49	0.49	0.49	0.49	0.49	0.49	0.49	0.49
5.0	0.44	0.44	0.44	0.44	0.44	0.44	0.44	0.44	0.44
6.0	0.40	0 40	0.41	0.40	0.40	0.40	0.40	0.40	0.40
7.0	0.37	0.37	0.38	0.37	0.37	0.37	0.37	0.37	0.37
8.0	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35	0.35
10.0	0.31	0.31	0.31	0.31	0.31	0.32	0.31	0.31	0.31

Displacement condition
C—Central
A—Average
W—Weighted average

Contact Pressure Distrbution
R—Rigid base
U—Uniform
P—Parabolic

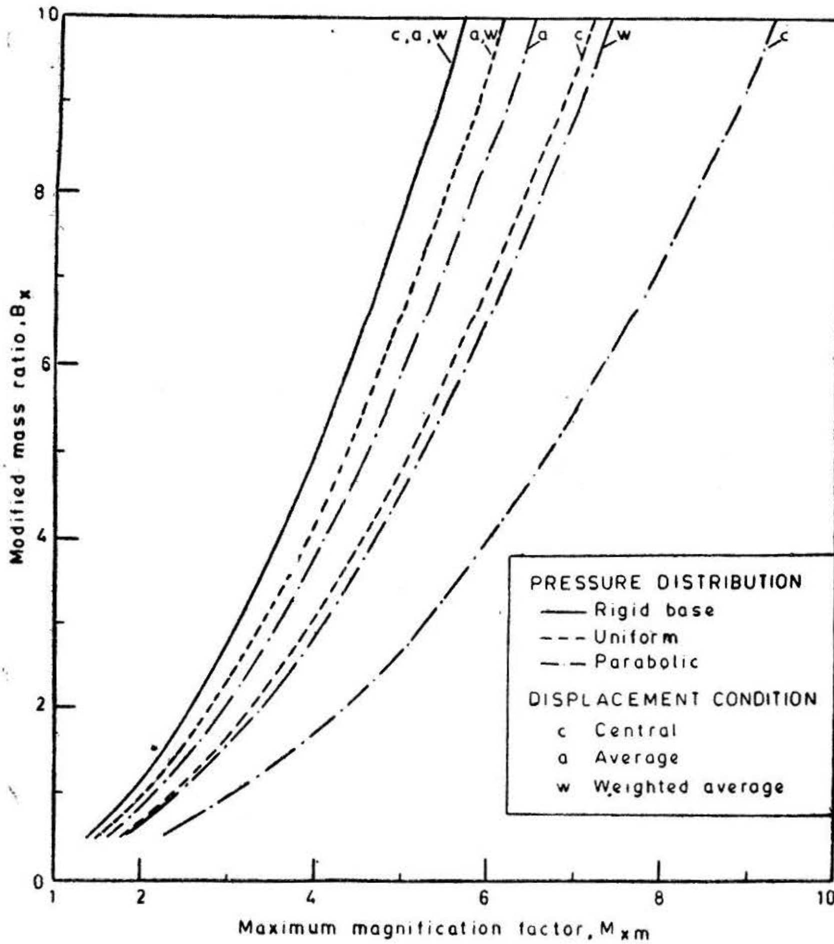

FIGURE 2 **Variation of maximum magnification factor M_{xm} with modified mass ratio B_x frequency independent excitation**

frequency dependent excitation and frequently independent excitations respectively with modified mass ratio B_x. It is observed from Figures 1 and 2 and Tables 6 and 7 that the results for rigid base pressure distribution is essentially same for all the 3 types of displacement conditions as it ought to be. For uniform pressure distribution, the results for average and weighted average displacement conditions are same. From Tables 8 and 9 it is seen that the value of frequency factor at responance is same for all conditions for B_x values greater than 1.0. The variation of frequency factor at resonance with B_x is shown Figures 3 and 4.

Analog Model

Analog model has been proposed by Lysmer and Richart (1966) for its simplicity and easy use. Analog solution is available only for rigid base pressure distribution, weighted average displacement condition (Hall 1967).

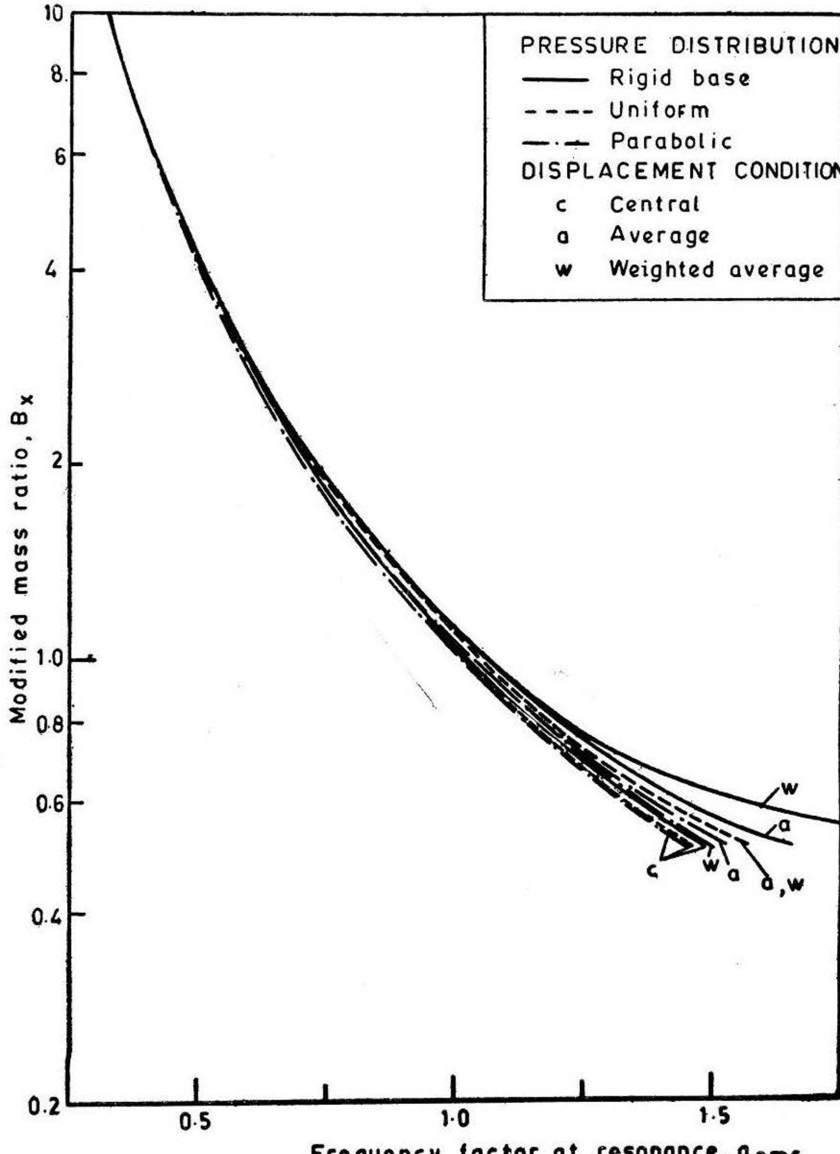

FIGURE 3 Variation of frequency factor at resonance, a_{0mr} wlth modified mass ratio B_x frequency dependent excitation

For other remaining cases, of type of pressure distribution and displacement conditions, analog solutions have been obtained in the following paragraphs based on the procedures of Lysner and Richart (1966) and Nagendra and Sridharan (1981). Lysmer and Richart (1966) expressed the analog parameters for stiffness, K', and damping, C, as

$$K' = \frac{F_1}{F_1^2 + F_2^2} K = k_1 K \qquad \text{... (21)}$$

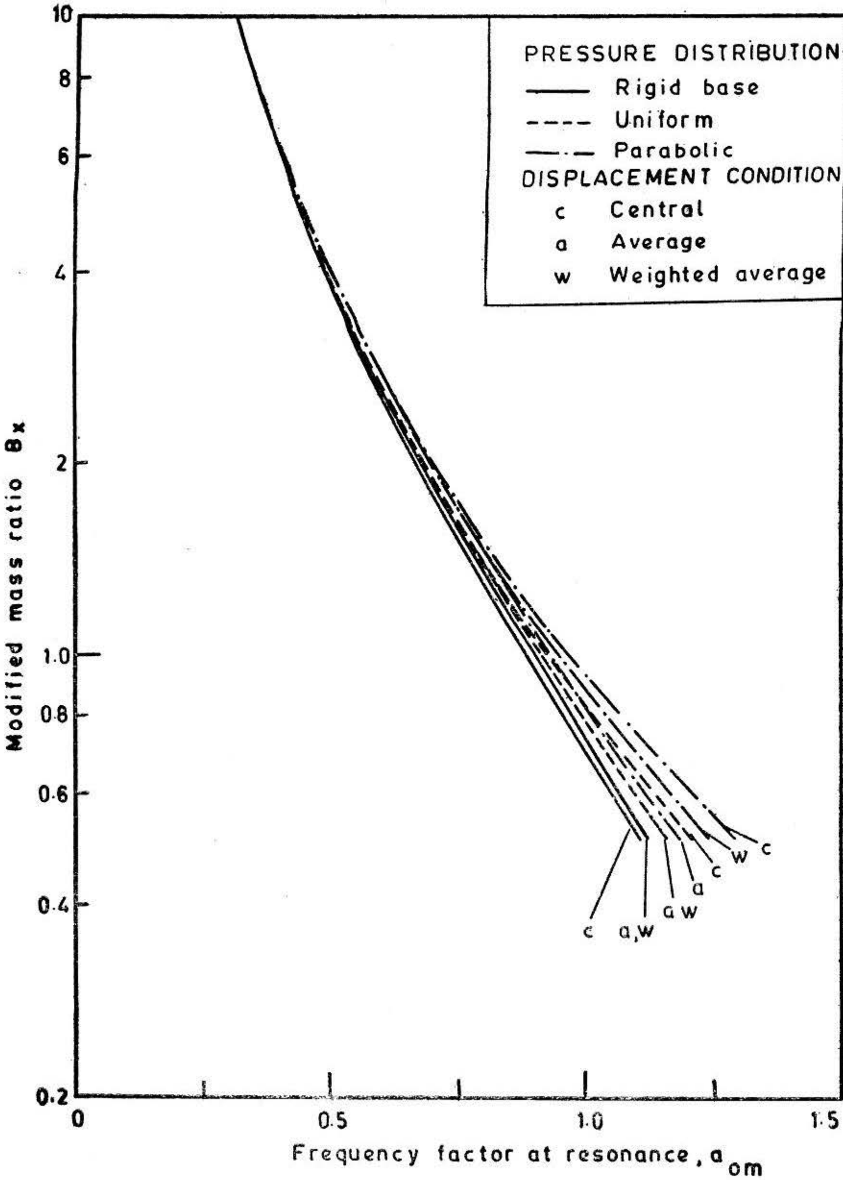

FIGURE 4 Variation of frequency factor at resonance a_{om}, with modified mass ratio B_x frequency independent excitation

and

$$C = \frac{-F_2/a_0}{F_1^2 + F_2^2}\ k.\,a.\left(\frac{\rho}{G}\right)^{1/2} = C_1\, K\, a\left(\frac{\rho}{G}\right)^{1/2} \qquad \ldots (22)$$

where K is of the static stiffness coefficient. To satisfy the static condition, assuming k_1 to be equal to 1, the analog parameter for stiffness can be written as (Table 4)

$$K' = \frac{3\pi^2 (1-\mu)\, Ga}{7-8\mu} \qquad \text{... (23)}$$

Since k_1 is assumed to be equal to 1, from Equation 21 the damping coefficients, C, can be written in the form

$$C = \frac{-F_2}{a_0 F_1} \; \frac{3\pi^2 (1-\mu)}{7-8\mu} \; a^2 (G\rho)^{1/2} \qquad \text{... (24)}$$

From the values of F_1 and F_2 presented in Table 5, the expression for $-F_2/a_0 F_1$ can be determined analytically as

$$\frac{-F_2}{a_0 F_1} = 0.520 + 0.38\, a_0^2 + 0.002\, a_0^4 \qquad \text{... (25)}$$

To suggest a constant value to damping coefficient (Lysmer and Richart, 1966), the value of $-F_2/a_0 F_2$ is taken as the average over the range $0.3 < a_0 < 0.6$ and is found to be equal to 0.528, the damping coefficient for the analog parameter can be written as

$$C = \frac{C_k (1-\mu) (G\rho)^{1/2} a^2}{(7-8\mu)} = \frac{1.584\pi^2 (1-\mu) (G\rho)^{1/2} a^2}{(7-8\mu)} \qquad \text{... (26)}$$

Thus the analog model for a footing resting on an elastic half-space with uniform pressure distribution can be written in the form

$$m\ddot{x} + \frac{1.584\pi^2(1-\mu) (G\rho)^{1/2} a^2}{(7-8\mu)} + \frac{3\pi^2(1-\mu)\, Ga}{(7-8\mu)} X = Q(t) \qquad \text{... (27)}$$

where X = acceleration, X = velocity, X = displacement and $Q(t)$ = forcing function.

Using the two analog parameters (Equations 23 and 26), the following parameters of a mass-spring-dashpot model have been evaluated

$$\text{Damping factor, } D = \frac{C}{2\,(K_m)^{1/2}} = D_k B_x^{-\frac{1}{2}} = 0.264\, B_x^{-\frac{1}{2}} \qquad \text{... (28)}$$

The variation of D withe B_x is shown in Figure 5. Magnification factors at resonance for frequency dependent excitation, M_{xrm}, and for frequency independent excitation, M_{xrm} can be written as

$$M_{xrm} = M_{xrm} = \frac{1}{2D\,(1-D^2)^{1/2}} = \frac{B_x}{0.528\,(B_x - 0.07)^{1/2}} \qquad \text{... (29)}$$

The values of M_{xrm} and M_{rm} for various of B_x are obtained from Equation 29. Figure 6 compares the values of M_{xrm} obtained from elastic half-space theory and analog model and the results are found to be in good agreement. Resonant frequency factor for frequency dependent excitation. a_{omr} and for frequency independent excitation a_{om}, can be written as

$$a_{omr} = \left(\frac{K\rho}{mG}\right)^{1/2} a \; \frac{1}{(1-2D^2)^{1/2}} = \frac{1}{(B_x - 0.139)^{1/2}} \qquad \text{... (31)}$$

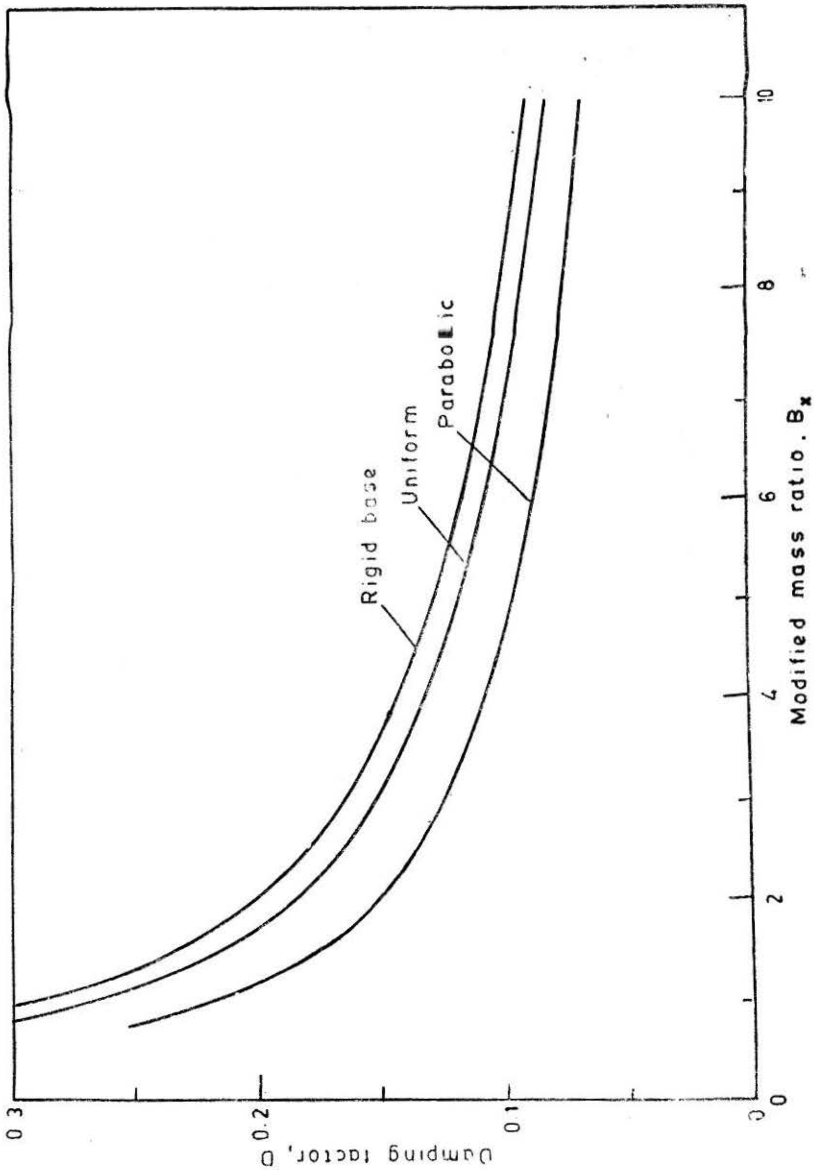

FIGURE 5 Variation of damping factor, D with modified mars ratio, B_x

and

$$a_{om} = \left(\frac{K\rho}{mG}\right)^{1/2} a\,(1-2D^2)^{1/2} = \frac{(B_x-0.139)^{1/2})}{B_x}$$

The values of the frequency factor at resonance, a_{omr}, obtained from Equation 30 are compared with that of elastic half-space theory in Figure 7 and found to be in good agreement.

Following the procedure explained above, using the values of the stiffness coefficients (Table 4) and modified displacement function F^1 and

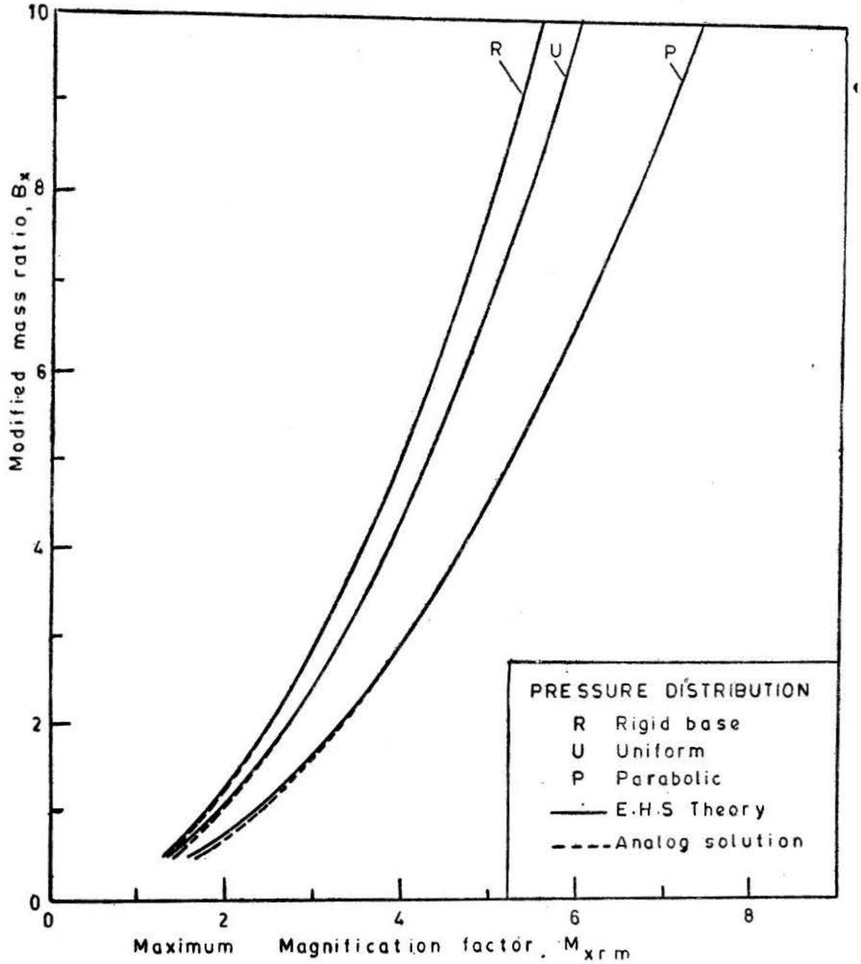

FIGURE 6 Comparison of the variation of M_{vrm} with B_x obtained from elastic half space theory and analog model

TABLE 10

Values of C_k : $C = \dfrac{C_k.(1-\mu)(G\rho)^{1/2}a^2}{7-8\mu}$

Displacement condition	Values of C_k		
	Contact pressure distribution		
	Rigid base	Uniform	Parabolic
Central	18.432	3.684π	2.004π
Average	18.304	$1.584\pi^3$	$1.395\pi^2$
Weighted average	18.304	$1.584\pi^3$	$1.063\pi^3$

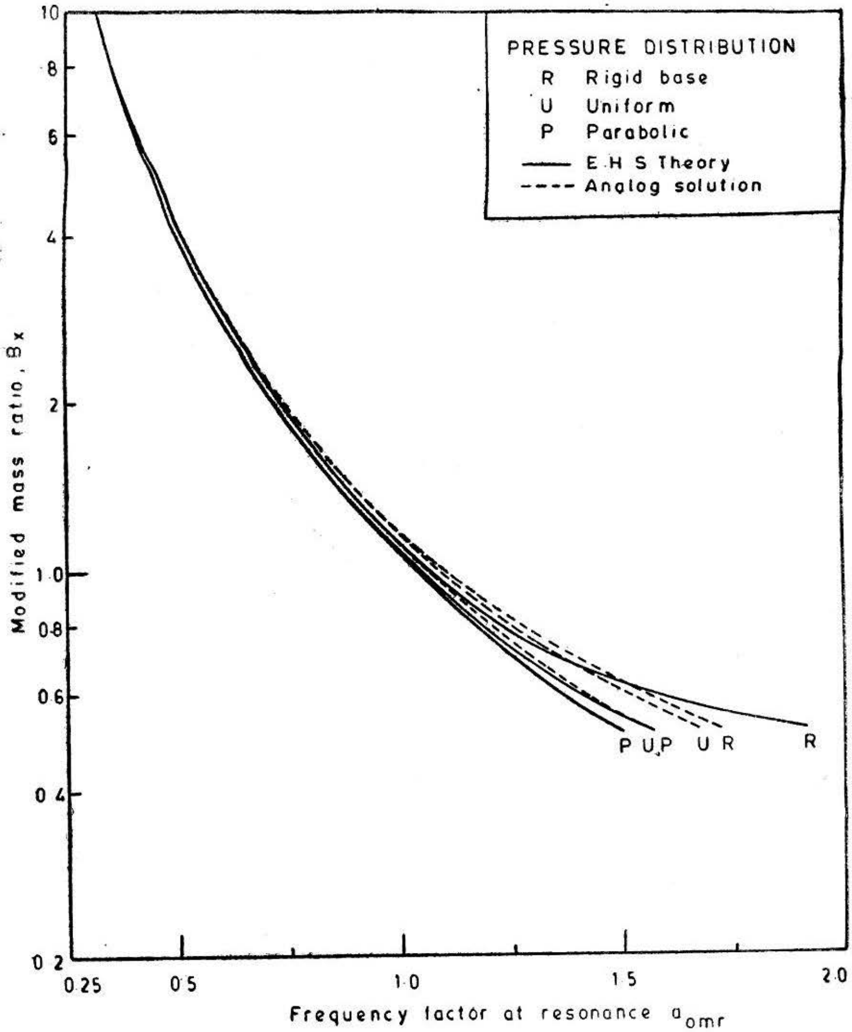

FIGURE 7 Comparison of the variation of a_{omr} with B_x obtained from elastic half-space theory and analog model

TABLE 11

Values of D_k : Damping factor $D = D_k . B^{-1/2}$

Displacement condition	Values of D_k		
	Contact Pressure Distribution		
	Rigid base	Uniform	Parabolic
Central	0.288	0.244	0.167
Average	0.286	0.264	0.248
Weighted average	0.286	0.264	0.216

F_2 (Table 5), the analog parameters for different types of pressure distribution and displacement conditions have been developed. The damping coefficients and damping factors evaluated have been presented in Tables 10 and 11 respectively. The values of C_k and D_k obtained by the above procedure for rigid base—weighted average displacement condition are found to be 18.304 and 0.286 as against the values of 18.4 and 0.288 obtained by Hall (1967).

Conclusions

Solution based on elastic half space theory for prediction of displacement amplitude and frequency at resonance for horizontal mode of vibration is available for rigid base pressure distribution—weighted average displacement conditions only. For remaining type of pressure distributions (parabolic and uniform) and displacement conditions (central, average and weighted average), solutions based on elastic half stress have been obtained and presented for ready use in designs.

Analog solutions have also been obtained for the above cases and it is seen that there is good agreement between analog model and elastic half space model. Values of stiffness coefficients and damping coefficients have been obtained and tabulated for ready use.

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Notations

A	= Resonant amplitude
a	= Radius of the footing
a_o	= non-dimensional frequency factor
a_{om}	= nondimensional frequency factor at resonance frequency independent excitation
a_{omr}	= Non-dimensional frequency factor at resonance-frequency dependent excitation
B_x	= Modified mass ratio
b	= Mass ratio
b_ν	= Constant
C	= Damping coefficient
D	= Damping factor
e	= Eccentricity
F_1, F_2	= Modified displacement functions
f_1, f_2	= Displacement functions
f_r	= Resonant frequency
G	= Shear modulus
G_ν	= Constant
h	= $h^2 = \rho\omega^2/(\lambda+2G)$
i	= $(-1)^{1/2}$
J_1	= Bessel function of first kind—first order
J_2	= Bessel function of first kind—second order
K	= Static stiffness coefficient
k	= $k^2 = \rho\omega^2/G$
k_1	= $F_1/(F_1^2+F_2^2)$
m	= Mass of footing
m_e	= Eccentric mass
M_x	= Non dimensional magnification factor for frequency independent excitation
M_{xm}	= Non dimensional maximum magnification factor for frequency independent excitation
M_{xr}	= Non dimensional magnification factor for frequency dependent excitation
M_{xrm}	= Non dimensional maximum magnification factor for frequency dependent excitation
n	= Variable
P	= Parabolic pressure distribution

R	=	Rigid base pressure distribution
s	=	$s^2 = h^2/k^2$
t	=	variable
U	=	Uniform pressure distribution
U_c	=	Central displacement
U_a	=	Average displacement
U_w	=	Weighted average displacement
U_{wR}	=	Weighted average displacement—rigid base contact pressure distribution
X_1, X_2, X_3	=	Constants
x	=	Variable
Z	=	Amplitude of dynamic force
α	=	$(x^2—h^2)^{1/2}$
λ	=	Lame's constant
ν	=	Constant
μ	=	Poisson's ratio
π	=	3,1415926
ρ	=	Mass density of soil
ω	=	Exciting frequency