Short Communication

Interference between Surface Footing on Purely Cohesive Soil

by

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Introduction

The problem of determination of ultimate bearing capacity of a strip footing on semi-infinite soil mass has been investigated by many investigators (Fellenius 1929, Terzaghi 1943, Meyerhof 1951, Sheild 1954, Brinch Hansen 1961, Sokolovskii 1965, Chen and Dividson 1973.) using the methods of limit equilibrium, slipline and limit analysis. The limit equilibrium method adopted for $\phi = 0$ soil assumes a semi-circular section of soil mass which fails by rotation about its centre located at the corner of the footing. It has been possible to predict the failure load and the geometry of the failure surface by taking moments of all the forces about the centre of rotation and equating them to zero. Button's (1953) analysis shows that the magnitude of the ultimate bearing capacity is 5.51 times the cohesion for a surface footing in which case the centre of the failure surface lies not at the corner of the footing, but at a radial distance of. 1.0881 times the width from the outer corner of the footing and the arc makes an angle of 2.33 radians at the centre (Figure 1). Chen's (1975) analysis of the same problem indicates that, using the upper bound method of limit analysis, the ultimate bearing capacity is found to be 5.52 times the cohesion and the critical circle makes an angle of 2.33 radians at the centre. Further it is seen that the centre of this critical circle lies above the inner corner of the footing. Hence Button's analysis, which is similar to Fellenius solution obtained from the method of limit equilibrium, agrees with the Chen's solution obtained by the method of limit analysis.

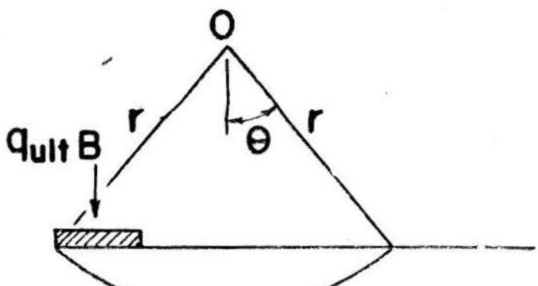

FIGURE 1 Single Footing

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Studies conducted by Stuart (1962), Mandel (1963,1965) and several other investigators reveal that the load carrying capacity of a footing is affected by the presence of adjacent footing (s). In order to investigate into the problem of determining the ultimate bearing capacity of a strip footing on the surface of a purely cohesive soil mass when another strip footing carrying certain load is located nearby, the following assumptions are made.

(1) The slip surface is an arc of a circle having its centre at a radial distance of r from the outer corner of the footing. This arc makes a central angle of 2θ.

(2) The failure one being symmetrical about the centre of the circle, vertical line passing through its centre of gravity passes through the centre of the circle. Therefore the weight of the soil mass within this zone does not cause any moment about this centre.

(3) Soil mass on the outer side of the footing loaded to failure is prevented from heaving and consequent failure. (A wall footing with additional construction for flooring on one side represents such a situation.)

(4) Shear stress along the rupture surface is uniform and equal to cohesion in magnitude.

(5) The adjacent footing carrying a load of p/unit area acts as a surcharge only. It does not develop any failure surface of its own.

The conventional method of limit equilibrium has been adopted here to th obtain the solution.

Analysis

Case I

Both the footings are no one side of the centre of the circular arc. (Figure 2)

Taking moment about 0,

$$q_{ult}\, B\,(r \sin\theta - B/2) + pB\,(r \sin\theta - B/2 - SB) = \int_0^{2\theta} c\, r^2\, d\theta \qquad \ldots (1)$$

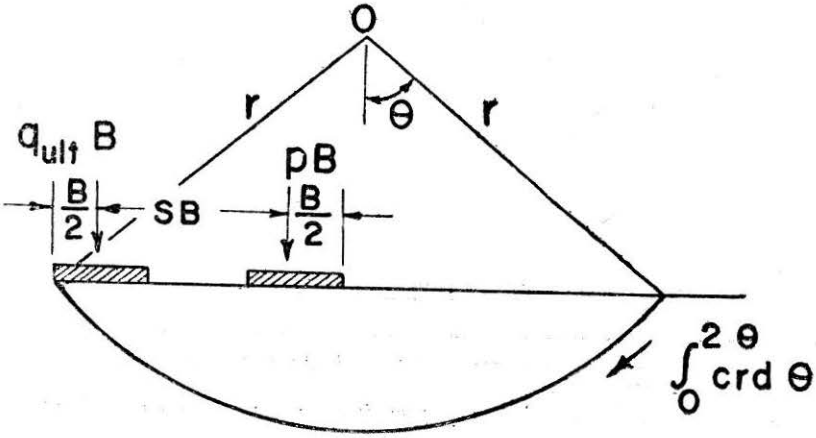

FIGURE 2 Case I Two footings

where,

q_{ult} = ultimate bearing capacity of footing,

B = width of footings,

r = radius of circular rupture surface,

θ = half of the angle subtended by the rupture surface at its centre,

p = load intensity on the adjacent footing,

S = Spacing between the footings/width of the footing,

and c = cohesion.

On simplification,

$$q_{ult} = \frac{2\,c\,R^2\,\theta - pR\sin\theta + 0.5\,p + S\,p}{R\sin\theta - 0.5} \qquad \dots (2)$$

where $R = r/B$ ... (3)

Since $\theta = 0$, $q_{ult} = c\,N_c$,

where N_c is the bearing capacity factor.

or,

$$N_c = \frac{2\,R^2\,\theta - A\,R\sin\theta + 0.5\,A + S\,A}{R\sin\theta - 0.5} \qquad \dots (4)$$

where $A = p/c$ ... (5)

In order to determine the values of R and θ for the minimum value of N_c, i.e. for the most critical arc, the expression for N_c is differentiated with respect to R and θ separately and equated to zero.

$$\partial N_c/\partial R = 0 \qquad \dots (6)$$

On simplification

$$R = \frac{1 \pm \sqrt{1 + A\,S\sin^2\theta/\theta}}{2\sin\theta} \qquad \dots (7)$$

Also $\partial N_c/\partial\theta = 0$ ... (8)

On simplification.

$$R = \frac{1 \pm \sqrt{1 + 8\,S\,A\cos\theta\,(\sin\theta - \cos\theta)}}{4\,(\sin\theta - \theta\cos\theta)} \qquad \dots (9)$$

The magnitudes of R and θ for the most critical arc are found out by computing the magnitudes of R for different values of ϕ using Equations 7 and 9, and plotting the same to determine the values of R and θ satisfying both the equations.

Validity of the solution

For both the footings to remain on one side of the centre of the circular arc,

$$S\,B < r\sin\theta - B/2 \qquad \dots (10)$$

or

$$S < R\sin\theta - 0.5 \qquad \dots (11)$$

Hence the values of R and θ satisfying the relationship as stated in Equation 11 are only considered for determining the value of N_c.

Case II

The footings are on either side of the centre of the circular arc. (Figure 3)

Taking moment about O,

$$q_{ult}\ B\ (r \sin\theta - B/2) \int_0^{2\theta} c\, r^2\, d\,\theta + p\ B\ (SB + B/2 - r \sin\theta) \quad \text{... (12)}$$

or, on simplification,

$$q_{ult} = \frac{2\ c\ R^2\ \theta - P\ R \sin\theta + 0.5\ P + S\ p}{R \sin\theta - 0.5} \quad \text{... (13)}$$

Since, for $\theta = o$, $q_{ult} = c\ N_c$

or

$$N_c = \frac{2\ R^2\ \theta - A\ R \sin\theta + 0.5\ A + S\ A}{R \sin\theta - 0.5} \quad \text{... (14)}$$

It is seen that the expression for N_c in both the cases, i.e. Equations 4 and 14 are similar. Hence, for Case II, the expression for R satisfying $\partial N_c / \partial R = 0$ and $\partial N_c / \partial \theta = 0$ are also similar to Equations 7 and 9 respectively.

Validity of the solution

In order that the footings shall not remain on one side of the centre,

$$S\ B > r \sin\theta - B/2 \quad \text{... (15)}$$

or

$$S > R \sin\theta - 0.5 \quad \text{... (16)}$$

Case III

When there is no load on the adjacent footing.

$(p = 0)$

In the Equations 4 and 14, by substituting O in place of A $(=p/c)$, the equations reduce to

$$N_c = \frac{2\ R^2\ \theta}{R \sin\theta = 0.5} \quad \text{... 17)}$$

The following numerical values are taken into account to study (i) the

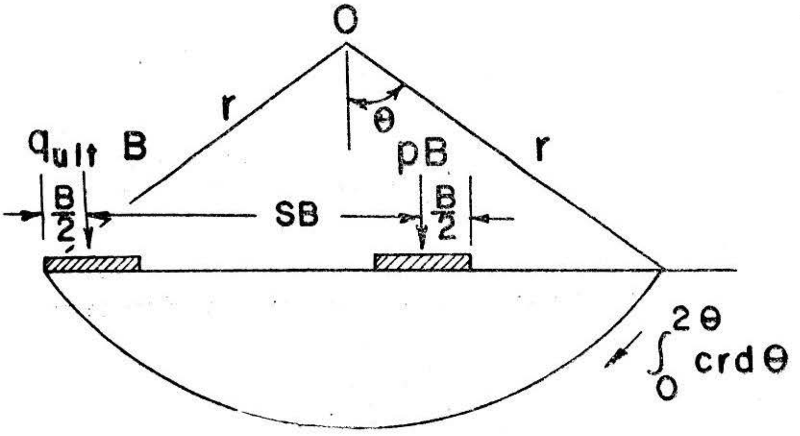

FIGURE 3 Case II Two footings

change in the geometry of the failure surface, (ii) the nature of variation in the magnitude of N_c due to the presence of adjacent footing carrying certain load, and (iii) change in the magnitude of N_c due to the presence of an unloaded adjacent footing,

$$c = 0.28 \text{ kg/cm}^2$$
$$p = 1.50 \text{ kg/cm}^2$$
$$S = 2.00 \text{ to } 5.50 \text{ at an interval of } 0.50.$$

From the calculated values of R at various θ values using the Equations 7 and 9 the values of R and θ satisfying Equation 6 and 8 at different footing spacings are computed. The magnitudes of N_c at different footing spacings are calculated using Equations 4. Considering the limitations as stated in Equations 11 and 16, it is seen that only at a spacing of 2.00 B c/c, the conditions for case I is satisfied. In the rest of the spacings case II is satisfied.

The effect of an unloaded adjacent footing on the bearing capacity factor is determined by comparing the magnitude of N_c using the Equation 17 keeping the geometry of rupture surface same as in case of a loaded adjacent footing. A factor designated as—Interference Factor is calculated as the ratio of N_c for $p \neq 0$ and N_c for $p = 0$. The calculated values of R critical, θ critical, N_c ($p \neq 0$), N_c ($p = 0$) and Interference factor for different footing spacings have been presented in Table 1.

TABLE 1

Computed Magnitudes of Bearing Capacity and Interference Factor for Various Footing Spacings

S	R_{cr}	ϕ_{cr}	N_c $p = 1.50$	N_c $p = 0$	Interer. factor	Remarks
2.00	2.7253	1.19	8.6291	8.7155	0.9901	Case I
2.50	2.9675	1.19	9.8780	9.3059	1.0615	
3.00	3.1876	1.19	11.0129	9.8478	1.1183	
3.50	3.3903	1.19	12.0601	10.3504	1.1652	
4.00	3.5805	1.19	13.0373	10.8239	1.2045	Case II
4.50	3.7588	1.19	13.9570	11.2693	1.2384	
5.00	3.9279	1.19	14.8282	11.6941	1.2729	
5.50	4.0892	1.19	15.6578	12.0966	1.2941	

Conclusion

The following conclusions are drawn from the analysis

The geometry of the rupture surface is dependant on load intensity of adjacent footing, cohesion, and spacing between the footings.

With the increase in spacing between the footings, the magnitude of $R_{critical}$ increases, while the central angle of the rupture surface ($2\ \theta_{critical}$) remains constant.

The locus of the centre of the circular rupture surface is a straight line passing through the outer corner of the footing and inclined at an angle of $\theta_{critical}$ with vertical.

As spacing between the footings increases, the adjacent footing assisting failure at closer spacing, resists failure and increases the ultimate bearing capacity. This is evident from the increase in the magnitude of Interference factor from less than unity to greater than that as spacing increases.

The magnitude of N_c for $p = 0$ case is more than 5.51 in all footing spacings investigated in this analysis. This indicates that the unloaded adjacent footing resists the movement of failure zone, thereby increasing the load carrying capacity of the footing loaded to failure. However, the effect of an unloaded footing on the geometry of failure zone needs further investigation.

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