

Nonlinear Analysis of Interference of Three Surface Strip Footing by Finite Element Method

by

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Introduction

In practice footings are rarely isolated and they do interfere with each other depending on their relative positions. Bearing capacity, total and differential settlements, tilts and heave and failure pattern are some of the parameters which need thorough investigations in respect of their interference problems. The interference of two surface footings has been attempted in the past by several research workers, both by experimental and analytical methods. The problem of interference of three surface footings remain somewhat different from the problem of two footings due to presence of central footing. Such types of problems are also encountered at many places in civil engineering constructions such as grain storage godowns, loaded areas, runway strips, culvert foundations etc.

The analysis of interference between neighbouring foundations received a momentum after the publication of theoretical and experimental investigations of Stuart and Hanna (1961). Further the problem was also studied by Stuart (1962), Biarez (1963), Mandel (1963-65), West and Stuart (1965), Rao (1965), Amir (1967), Karandikar (1968), Dimbicki and Koll (1971), Sigh, Punmia and Ohri (1973), Swymi and Agarwal (1974), Khadilkar and Varma (1977) etc. The authors in general investigated the problem with reference to strip foundation and that also in the light of a single bearing capacity theory. A few had also analysed square and rectangular foundations giving efficiency factors for bearing capacity and settlements. It is ascertained that neighbouring foundations influence each other until a certain distance between them has been attained and that the ultimate load of each foundation is different from that of the individual foundations. Thus in the area of interference of shallow foundations parameters such as *individual behaviour and group behaviour* in respect of bearing capacity, settlements etc., is of prime importance. Further interfering foundations are found to give rise to tilts and heave even under the uniform vertical loading.

Statement of Problem

In order to investigate the interference of three surface strip footings two important parameters of study namely rigidity of footing and their spacing which significantly affect the behaviour, have been considered. In

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the analysis, three footings (each 20 cm width) are assumed to be uniformly and equally loaded and spaced at equal distances from each other. Finite element, nonlinear, stress dependent and inelastic behaviour of the cohesionless soil mass and the incremental method of loading has been used in the analysis. The problem is studied in four different parts and the details of the same have been given in Table 1. For the investigations a rough interface has been assumed. At the end a typical case of field size footing (width 1 m) has also been analysed.

Material Characterisation

In the present problem, the behaviour of cohesionless soil is approximated by using hyperbolic simulation. Equation 1 gives the usual constitutive relation for plane strain condition.

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} M_B + M_D & M_B - M_D & 0 \\ & M_B + M_D & 0 \\ S_{YM} & & M_D \end{bmatrix} \begin{bmatrix} \epsilon_y \\ \epsilon_x \\ \gamma_{xy} \end{bmatrix} \quad \dots (1)$$

where

M_B —Bulk modulus of soil.

M_D —Shear modulus of soil.

The values of these modulus can be calculated by using elastic constants and are given as below,

$$M_B = \frac{E}{2(1+\nu)(1-2\nu)} \quad \dots (2)$$

$$M_D = \frac{E}{2(1+\nu)} \quad \dots (3)$$

E = Young's modulus of elasticity

ν = Poisson's ratio.

Hyperbolic Parameters

The initial tangent modulus (zero shear stress) is assumed to vary with the confining pressure (minor principal stress σ_3) as follow :—

$$E_t = K \cdot P_a \left[\frac{\sigma_3}{P_a} \right]^n \quad \dots (4)$$

where

E_t = initial tangent modulus

P_a = atmospheric pressure expressed in unit of stress.

K, n = empirical curve fitting constants.

The tangent modulus decreases progressively along an assumed hyperbolic stress-strain curve which reaches an upper asymptotic limit of stress and is given by

$$E_t = \left[1 - \frac{R_f(1-\sin \phi)(\sigma_1 - \sigma_3)}{2C \cos \phi + 2\sigma_3 \sin \phi} \right]^2 \cdot E_l \quad \dots (5)$$

TABLE 1
Details of Case Studies

Sr. No.	Type of foundation	Spacing between foundations	Remarks
Part I Three Rigid Footings, freely connected.			
1	Rigid	$2 B$	} Incremental loading over footing material
2	Rigid	$3 B$	
3	Rigid	$4 B$	
	Rigid	$5 B$	
5	Rigid	$3.5B$	
6	Rigid	$4.5B$	
7	Rigid (Field size)	$2 B$	
Part II Three Flexible Footings, freely connected.			
8	Flexible	$2 B$	} Incremental nodal loads.
9	Flexible	$4 B$	
10	Flexible	$5 B$	
Part III Three perfectly Rigid footings, Rigidly connected.			
11	Perfectly Rigid	$2 B$	} Incremental nodal settlements.
12	Perfectly Rigid	$5 B$	
Part IV Single Footings.			
13	Rigid	—	As per serial No. 1
14	Flexible	—	As per serial No. 8
15	Perfectly Rigid	—	As per serial No. 11

B —Width of footing is assumed as 20 cm in the analysis, except for case at Sr. No. 7 where it is 1 m.

where

E_t = Tangent modulus of elasticity

R_f = Failure ratio.

ϕ = Angle of internal friction.

C = Cohesion.

σ_1, σ_3 = Principal stresses.

Parameters assumed in the investigation are given in Table 2.

TABLE 2

Material Characterisation for Continuum

Type of soil	Relative density	Angle of friction (ϕ) Degrees	Failure ratio (R_f)	Curve fitting constant		Unload reload modulus (K_{ur})	Density of soil (γ) $\text{kg/cm}^3 \times 10^{-3}$	Coefficient of earth pressure at rest (K_0)
				(K)	(n)			
Cohesionless soil	100 percent dense-silica sand	36.5	0.91	2000	0.54	2120	0.17	0.5

Incremental Method

Initial stresses are, first introduced in the soil for the condition at rest. The nonlinear stress dependent and inelastic behaviour of the soil is represented in the analysis by using incremental method, wherein the stress components of the element are accumulated at the end of each load step and from the resulting principal stresses the tangent moduli for the successive load increment are computed after ascertaining the strength criteria based on Mohr-Coulomb failure condition.

The inelasticity of soil behaviour is considered in the analysis by adopting the appropriate unload and reload modulus for elements where the major principal stress decreases for the progressive load increments. The modulus is calculated from Equation 6 until the element develops a value of major principal stress which exceeds the corresponding value prior to unloading.

$$E_{ur} = K_{ur} \left[\frac{\sigma_3}{P_a} \right]^n \quad \dots (6)$$

where

E_{ur} = Unload-reload modulus

K_{ur} = corresponding modulus number,

Finite Element Idealisation

Figure 1 shows finite element idealisation for the footings and soil (half portion) for a typical case of investigation (rigid footings spaced at $2B$ *e/c*). Usual boundary conditions i.e. right hand side boundary nodes (due to the axis of symmetry) and left side boundary nodes (at a distance of $5.5B$ from the centre line of outer footing) have been kept on rollers and bottom boundary nodes (at a distance of $5B$ from the surface) are assumed as fixed. Three different rigidities of footings namely flexible, rigid and perfectly rigid have been used in the analysis. A quadrilateral finite element with four constant strain triangles (4 C.S.T.) is adopted to represent soil and the footing material. The number of elements and nodes adopted in each problem depend on spacing of footings. Maximum number of elements and nodes used in a problem with a spacing $S = 5B$ and are 351, 391 respectively. The ultimate bearing capacity of an isolated footing of width 20 cm and resting on the surface of cohesionless soil (Table 2) works out to be 0.85 kg/cm². In order to obtain the performance of an interfering footing a maximum load intensity of 1.4 kg/cm² is attained over the footing by adopting several load steps in this non-linear analysis.

A large amount of data has been obtained from the results of various cases studied and given in Table 1 in respect of contact stresses, load settlement characteristics continuum stresses and settlements, surface profile of continuum, tilts and heave etc. Due to limitations of space only selected results have been reported here along with suitable discussion thereon.

Contact Stresses

The normalised contact stress in the soil corresponding to this non-linear stress dependent material characterisation under isolated and

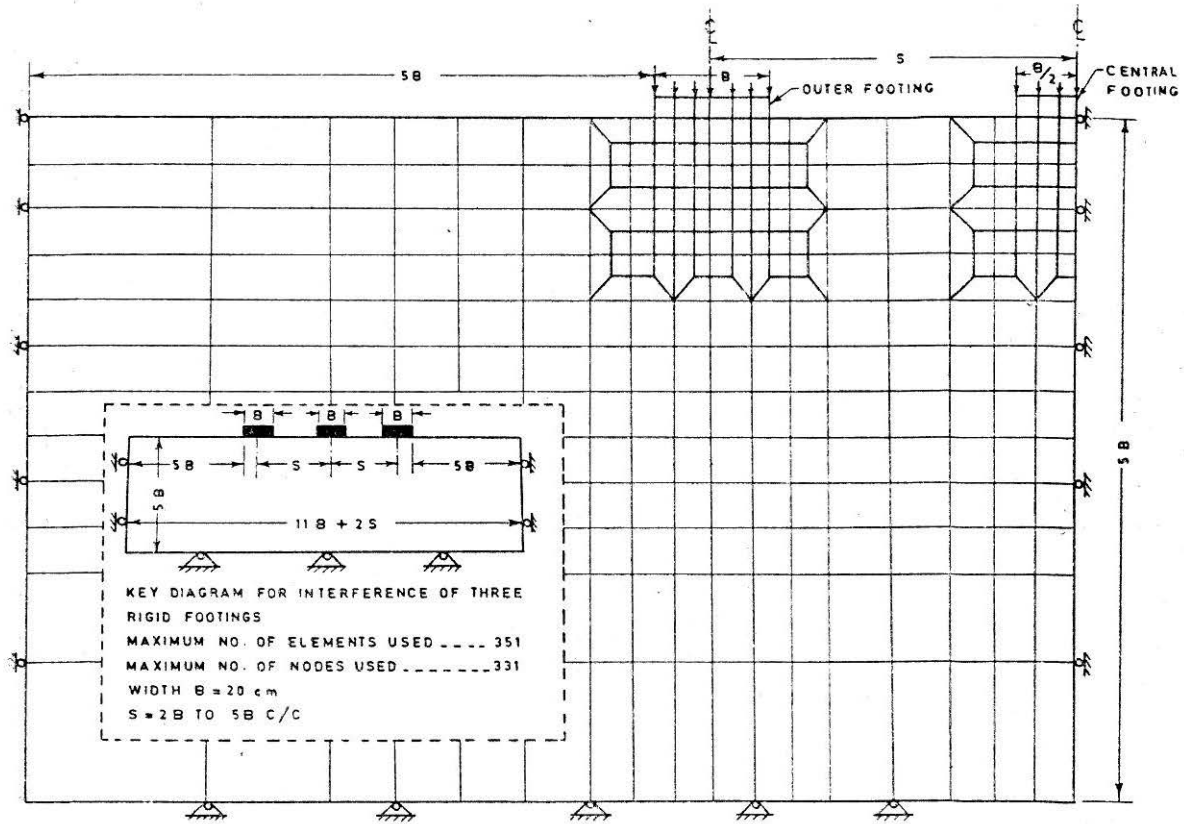


FIGURE 1 Finite element idealisation for interference of three single footings (rigid)

interfering footings spaced at various distances from each other and subjected to different loading stages are shown in Figure 2. These normalised stresses (at centre of gravity of element) are obtained by dividing contact normal stresses by the magnitude of the applied vertical intensity of loading. It has been found that for an isolated rigid footing the contact stresses are greater near the edge for smaller loads and decrease progressively as the load over the foundation increases. Influence of interference in the contact stress distribution beneath rigid footings has been shown for spacings $2B$ to $5B$ for the load intensity of 0.855 kg/cm^2 . From these diagrams it is observed that for an outer as well as central footing at closer spacing ($2B$), there is a tendency to develop more contract stresses (10 to 30 per cent) in the central part of footing than

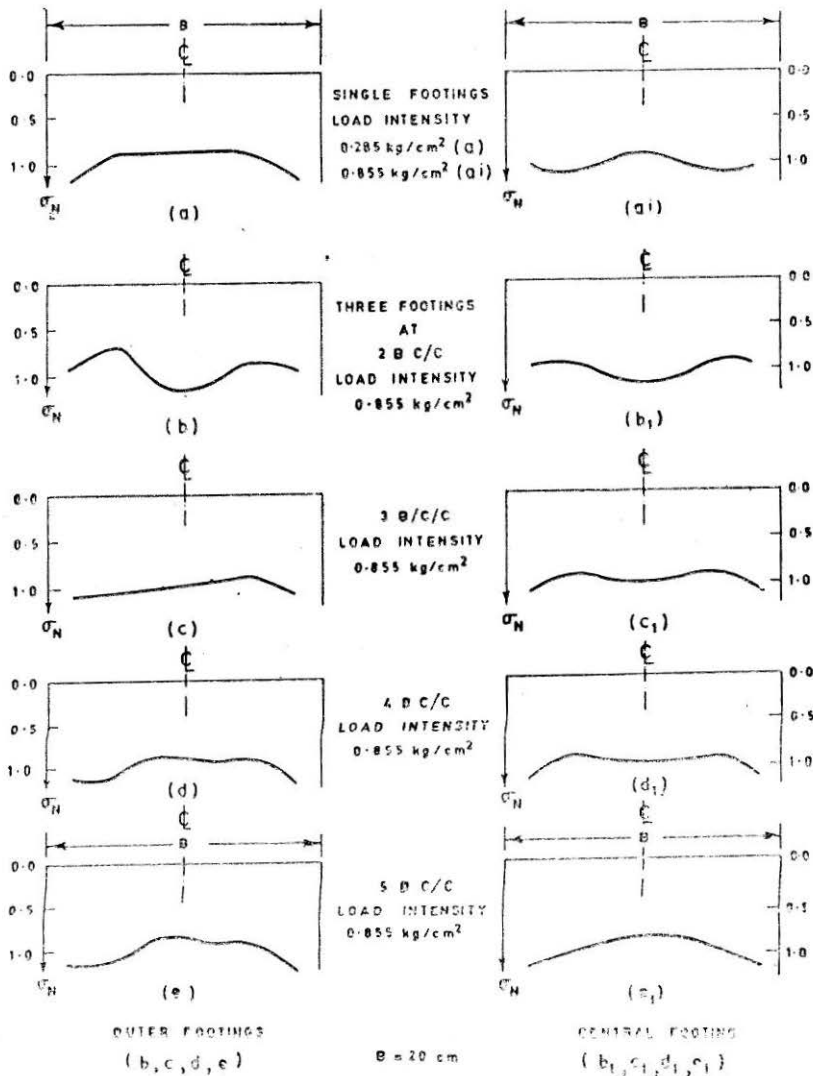


FIGURE 2 Normalised contact stresses, σ_n (rigid footings)

at the edges with more stresses are found on interfering side. This pattern gradually changes as the spacing between them increases (from $2B$ to $5B$). At a spacing of $5B$ c/c the edge stresses are more (15 to 20 per cent) than the stresses in the central part. Further it is observed that the interfering edge still shows stresses (5 to 7 per cent) than offside edge of footing. Similar trend in the distribution of pressure has been observed for perfectly rigid foundations (not shown). The effect of interference is more predominant on contract settlements of flexible foundations. Isolated flexible footings would show increasing magnitude of settlement from the edge to centre. Due to interference the increasing trend continues approximately upto $B/4$ distance from the edge beyond which it is practically same (not shown).

Load Settlement Characteristics

The load settlement diagrams obtained from the analysis for different cases of study are shown in Figures 3 and 4. For rigid footings the behaviour pattern of central and outer footing is consistent for spacings of $2B$ and $5B$ c/c in which the central footing is found to settle less due to the effects of interference from both sides, than outer footing. At a spacing of $4B$ c/c a reverse phenomenon is observed in which central footing is found to settle more than outer one for all loading stages. In order to confirm the results, additional investigations for $3.5B$ and $4.5B$ c/c spacings had been undertaken which showed gradual reversal in load settlement character from $3.5B$ to $4.5B$ spacings. Further it was observed it was observed that in a three strip-soil system the zones of tension are developed near the surface of continuum (Figures 6 and 7).

Due to closer spacings of foundations such tensile zones are not effectively developed in the portion of the continuum between two foundations. As the centre to centre distance between the foundations increases there is tendency to form such tensile stress zones between the two footing portions which might not allow the effect of interference on adjacent foundation. At a spacing of $4B$ a larger zone of tensile stress going to a greater depth is developed between the foundations which may be affecting the stress deformation characteristic and may lead to more settlements of central footing alone as is observed in the present case (Figure 4).

Dotted lines of load settlement curves in Figure 3 (upper part) refer to the behaviour of field size footing of 1 m width. Due to increase in size of foundation the settlements are found to increase however the trend of settlement curves for outer and central footings has maintained the pattern similar to the model foundations ($B=20$ cm). The difference in the two settlements is not much probably because of the confining pressure has been accounted in the nonlinear soil used for these investigations.

Interference Augmentation Factors

From the load settlement characteristics, it is possible to calculate interference augmentation factor related to load carrying capacity of foundations based on certain predetermined settlement criterion. Thus If γ_g is the inference augmentation factor of the group based on certain

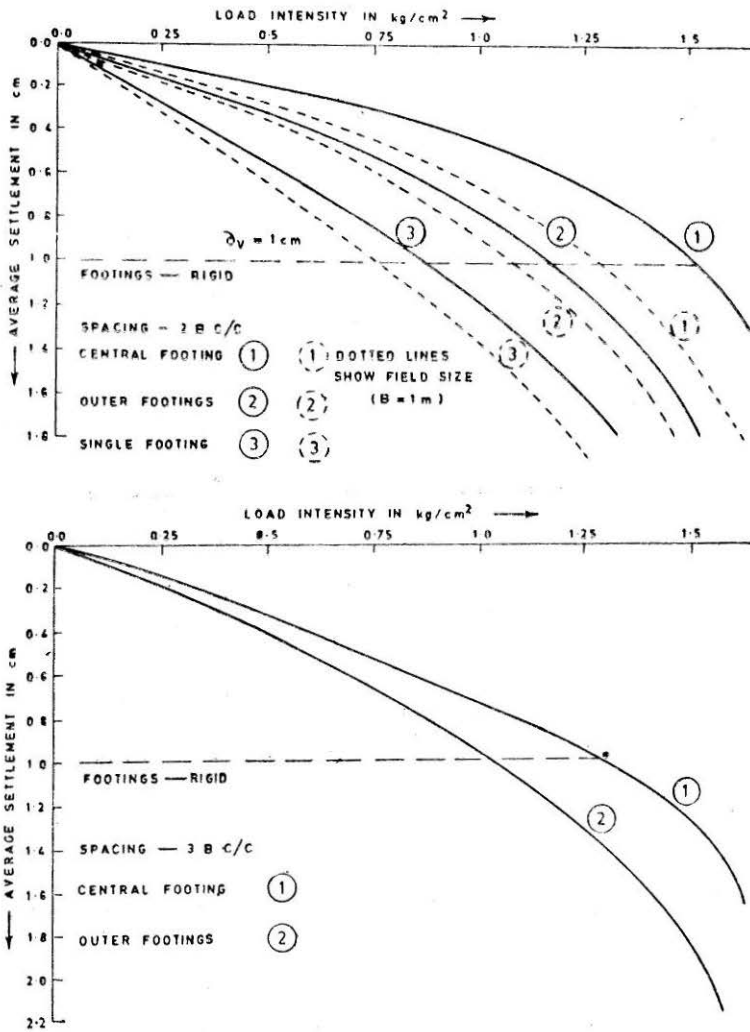


FIGURE 3 Load settlement characteristics (footings-rigid-spacings 2B, 3B, c/c)

arbitrary settlement value of foundation (say 5 per cent width of footing = 1 cm.) the same may expressed as

$$\eta_g = \frac{\text{load carrying capacity of the group (cummulative)}}{\text{number of footings} \times \text{load carrying capacity of single footing}}$$

Group augmentation factors for three different types of footings have been presented in Figure 5. Augmentation factors for rigid foundations have been reported by some of earlier research workers and the present values show satisfactory comparison with them. Further from the graph (Figure 5) it may be clearly seen that the rigidity of footing plays an important part in the behaviour of isolated and group of foundations.

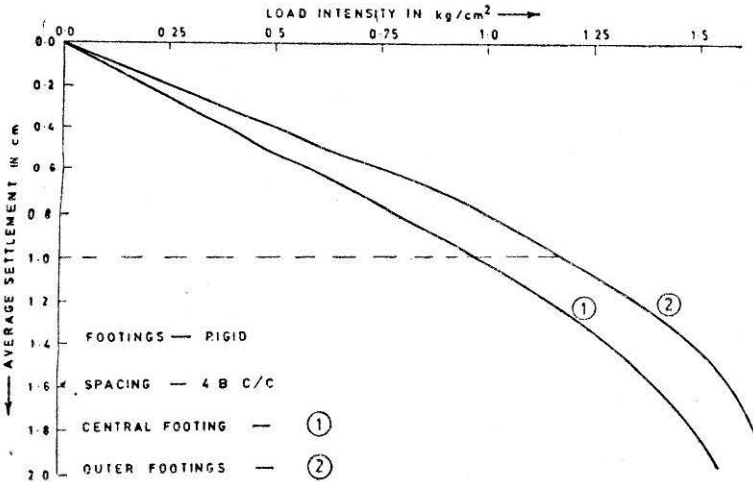


FIGURE 4 Load settlement characteristics (footing-rigid-spacing 4B c/c)

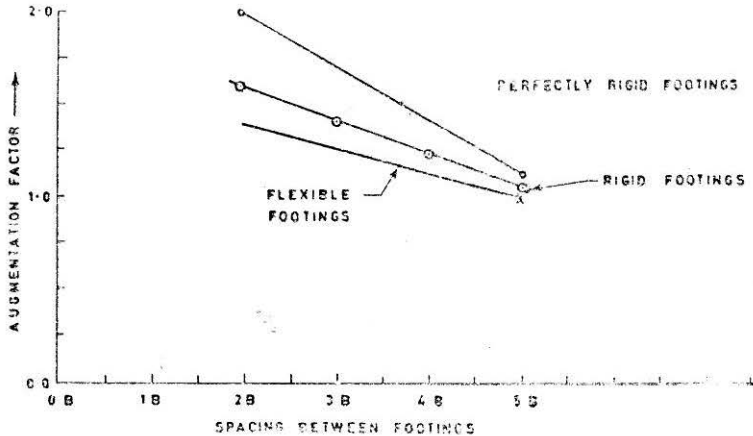
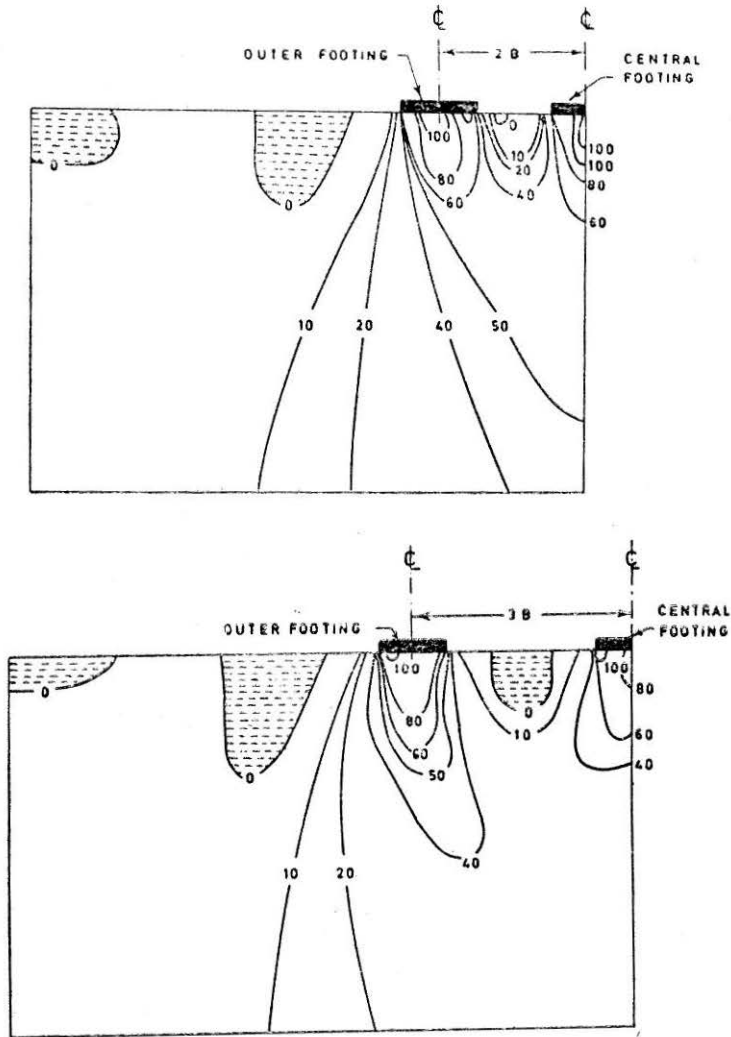


FIGURE 5 Performance curves for group behaviour

Interference Augmentation Factors for Individual Footing

The interference augmentation factors discussed above are mainly for the entire system. As the load settlement characteristics of central and outer footing show significant difference in the behaviour, it may be worthwhile to find out augmentation factors for the central and outer footing. It is observed that interference augmentation factors for individual footings depend on three factors namely type, location and centre to centre distance between foundations. Maximum value of these is 1.98 for central footing of perfectly rigid foundations spaced at 2B centre to centre and minimum value is 1.03 for the outer footing of flexible foundations spaced at 5B centre to centre for a predetermined total settlement of 1 cm (5 per cent width of footing). It may be interesting to note interrelation of load carrying capacity of single footing of different rigidity and is 1 : 1.5 : 1.55 for flexible, rigid and perfectly rigid foundations. This suggests that perfectly rigid foundations show better response to load carrying capacity of foundation obtained from the settlement criterion.

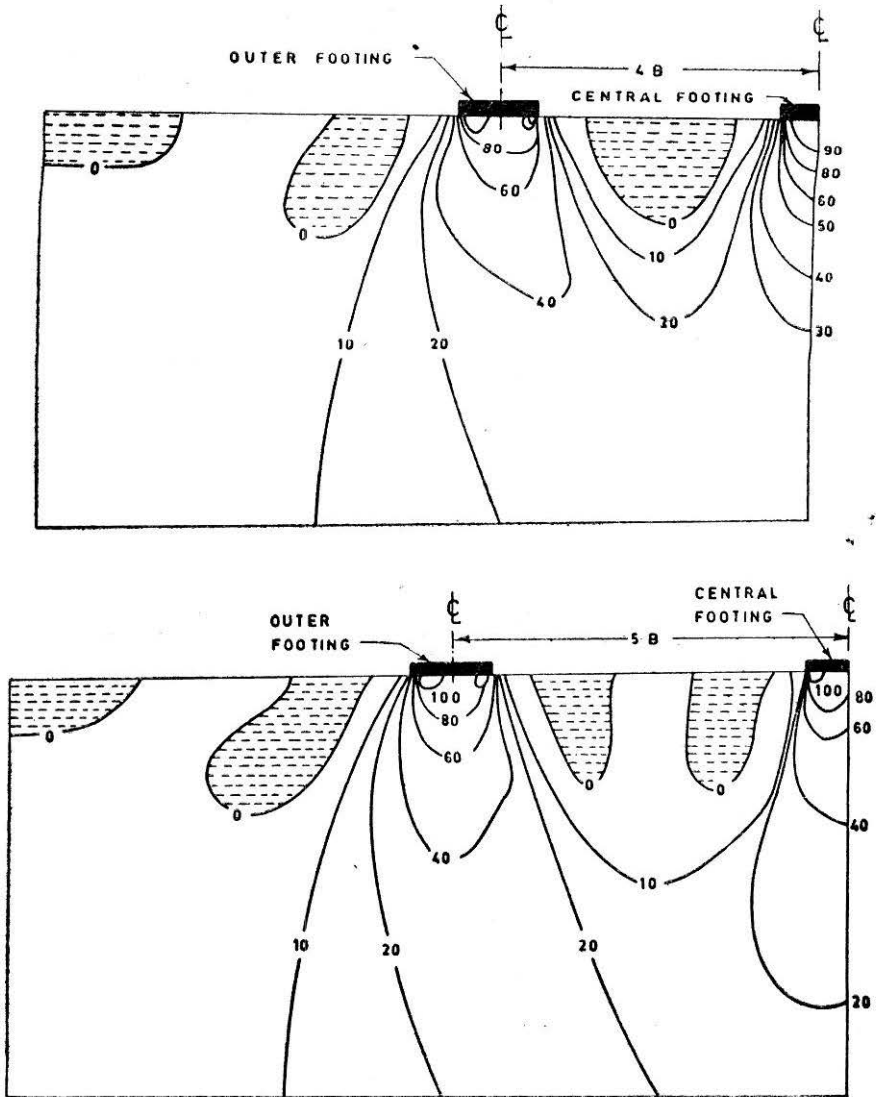


- (1) LOAD INTENSITY OVER FOOTINGS ----- 0.855 kg/cm^2
 (2) CONTOUR MAGNITUDE INDICATES PERCENTAGE NORMALISED STRESSES
 (3) SHADED PORTIONS SHOW THE ZONES OF TENSION

FIGURE 6. Normalised vertical stresses, σ_{ZN} (rigid footings spaced at $2B$ and $3B$ c/c)

Tilts in Footings

Although there is an indication of improved values of load settlement characteristics of interfering footing, based on average settlement criterion, it is observed from the analysis that interfering foundations at small spacings are associated with tilts. Table 3 shows the development of tilt for outer footing for various cases of investigation. The absolute magnitude of tilt as well as inclination of the footing expressed as slope value have also been presented for comparison. It may be seen that in general the magnitude of tilts are greater for smaller spacings for the same intensity of



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FIGURE 7 Normalised vertical stress, σ_{zN} (rigid footings spaced $4Bc/c$)

applied load. Further the titles are found for the outer footing only and is mainly on the off side of interference. The phenomenon of tilt is probably due to the fact that the edge of foundation towards the interfering side is experiencing the reduced settlements than other side, to influence of interference which mainly depends on spacing between the footing and the intensity of load over the system. Bracketted numerical values in the table for the spacing $2B$ centre to centre refer to the field

TABLE 3
Tilt for Outer Footing

Spacing between footing	Settlement in Centimeter		Absolute magnitude of tilt.	Inclination or slope
	Off side edge of footing	Interfering edge of footing		
2 B	0.7036 (0.7994)	0.5196 (0.6321)	-0.1840 -0.1673	1:1090 (1:6000)
3 B	0.8413	0.8003	-0.0410	1:4850
3.5 B	0.7657	0.7476	-0.0180	1:1110
4 B	0.7475	0.7375	-0.0100	1:2000
4.5 B	0.8359	0.8274	-0.0085	1:2340
5 B	0.8860	0.8810	-0.0050	1:4000

Note: (a) Load intensity over the footing is 0.855 kg/cm²

(b) Tilt on left side is treated as negative.

size ($A=1$ m wide) footings. As compared with the increase in the size of foundation the differential settlement does not increase in the same proportion and therefore the value of tilt expressed as slope decreases. It may however be noted that the direction to tilt has been maintained on left hand side as per model size footing.

Differential Settlement

In a problem of three strip footings resting on the surface of sand, if the foundations are mutually free to settle under the load over the system, a differential settlement between centre line points of outer and central footing would take place. Table 4 shows the differential settlements between central and outer footing for two typical incremental loading stages over the system. Total settlement affects factors such as access and services of structure however the differential settlement is closely linked with the causes governing damage to a structure. Skempton and Mac Donald (1956) suggested their recommendations based on settlement case records of large number of buildings which showed signs of damage. They have prescribed as a criterion for damage, the angular distortion d/L where d is the differential settlement between adjacent supports and L is the distance between supports. It was concluded that a value of d/L greater than 1/300 would cause cracking in walls and value greater than 1/150 would cause structural distress.

TABLE 4

Differential Settlements Between Outer and Central Footings

Spacing between footings	Differential settlements for the load intensity of	
	0.57 kg/cm ²	0.85 kg/cm ²
2 B	+0.245 (1:165)	+0.267 (1:155)
2 B (Field)	+0.511	+0.112
3 B	+0.098	+0.137
3.5 B	-0.074	-0.086
4 B	-0.152	-0.223
4.5 B	+0.079	+0.118
5 B	+0.012 (1:8200)	+0.016 (1:6200)

Note: (a) +ve sign indicates that outer footing settles more than central footing and -ve sign for the reverse.

(b) The settlements reported here are along the centre line.

The maximum angular distortion for spacing of 2B centre to centre and at the load intensity of 0.85 kg/cm² has been observed to be 1:55 and is minimum at the spacing of 5 B which has magnitude of 1:6200. Earlier results indicate the need to reduce or avoid differential settlements. The differential settlement may be reduced or avoided if the superstructure connections of the foundations are assumed. Two typical cases (Table 1, No. 11, 12) in which perfectly rigid connections of superstructure have been assumed, here the differential settlements are absent but in turn the portion between the footings is observed to heave up.

A reverse phenomenon in differential settlement is also observed for 4B spacing in which central footing at the ultimate load shows maximum differential settlement (0.223 cm) with central footing settling more than outer footing. There is gradual change over as would be seen clearly from the Table 4. For the field size footings spaced at 2B centre to centre the differential settlements are also shown in Table 4. It may be seen that the magnitude of differential settlement has been reduced because of increase in linear distance of size as well as of spacing and increase in settlement due to size are governed by different law.

Continuum Stresses

The distribution of normalised vertical stresses (σ_{ZN}) developed in the entire continuum have been shown in Figures 6 and 7 for some of the cases of investigations. The normalised stresses are obtained by dividing the induced stresses at the centre of gravity respective elements in the continuum by the applied intensity of load over the footing ($q = 0.855 \text{ kg/cm}^2$).

It may be seen that when spacing between the two footing is close ($2B$ or $3B$ centre to centre) due to interference the portion between and under three footing is subjected to higher compressive stresses. However, as the spacing increases the tensile zones are developed near the surface and between the footings. This is probably because of the fact that, as centre to centre distance increases failure curves try to come to the surface from both the sides and give rise to tension zones.

These tension zones change the behaviour pattern of the system in respect of total and differential settlements as reported earlier. Further, it is observed that even at a spacing of $5B$ centre to centre the effect of interference continues but is comparatively less significant.

Continuum Resultant Settlements

It is possible to present an overall picture of the resultant nodal settlements developed in the continuum (Figure 8). Arrows indicating directional nodal resultant settlements at the respective nodal points of the continuum have been shown in Figure 8. From these diagrams, it is observed that entire portion of the continuum below a depth of 1.5 to $2B$ from the surface has a tendency to move laterally on left hand side of centre line. This would naturally restrict the movement of soil below the central footing to a limited extent due to nonlinear elastic properties of soil. As a result of this central portion of three strip soil system may offer better response in respect of stress deformation characteristics as is seen earlier.

It is also possible to show approximate failure pattern developed in the continuum for incremental loading over the foundation soil system. Failure surfaces of rough based foundations on the surface of cohesionless soils have been predicted in theoretical analysis and it may be seen that the failure surfaces developed with the help of finite element method are similar to those assumed in theoretical analysis. Further in finite element analysis an incremental as well as a detailed behaviour of various regions of movements in the continuum can be obtained easily.

Surface Heave

Due to the load over the foundations, the original surface profile of the continuum is disturbed and the zones of surface heave and downward settlements are formed. Figure 9 shows details of surface profile of the continuum for the three sequential incremental loading over rigid, flexible and absolutely rigid foundation for some typical cases of investigations. ($2B$ centre to centre).

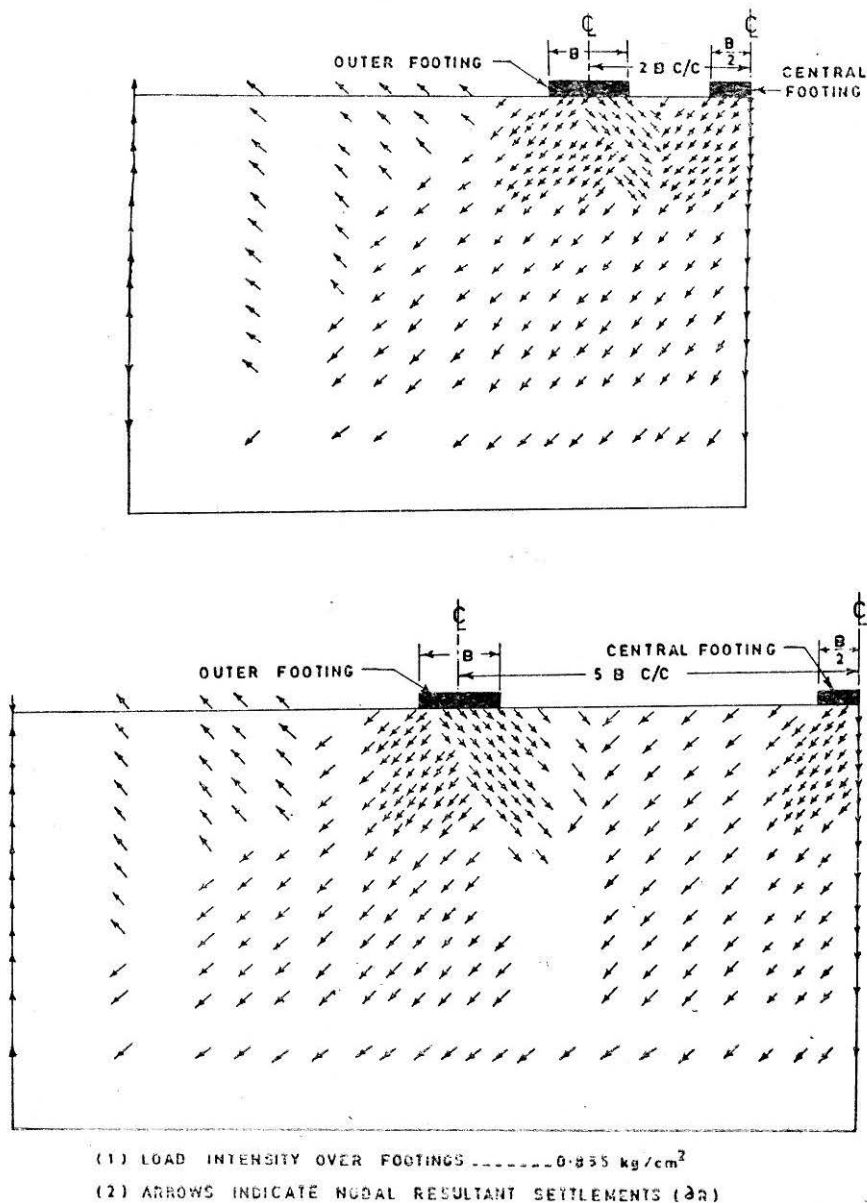


FIGURE 8 Nodal resultant settlement d_R (rigid footings spaced at $2B$, $5B$ c/c)

It may be seen that for the rigid foundations, the heave is taking place on the left side and is beyond a distance of about $0.75 B$ from the non-interfering edge of outer foundations. However, the portion of the continuum between the foundation ($2S+B$) and some portion ($0.75 B$) on either side of footings show the downward movement.

The heave pattern for perfectly rigid foundations superimposed in Figure 9b shows symmetrical heave pattern as tilts are not possible in these

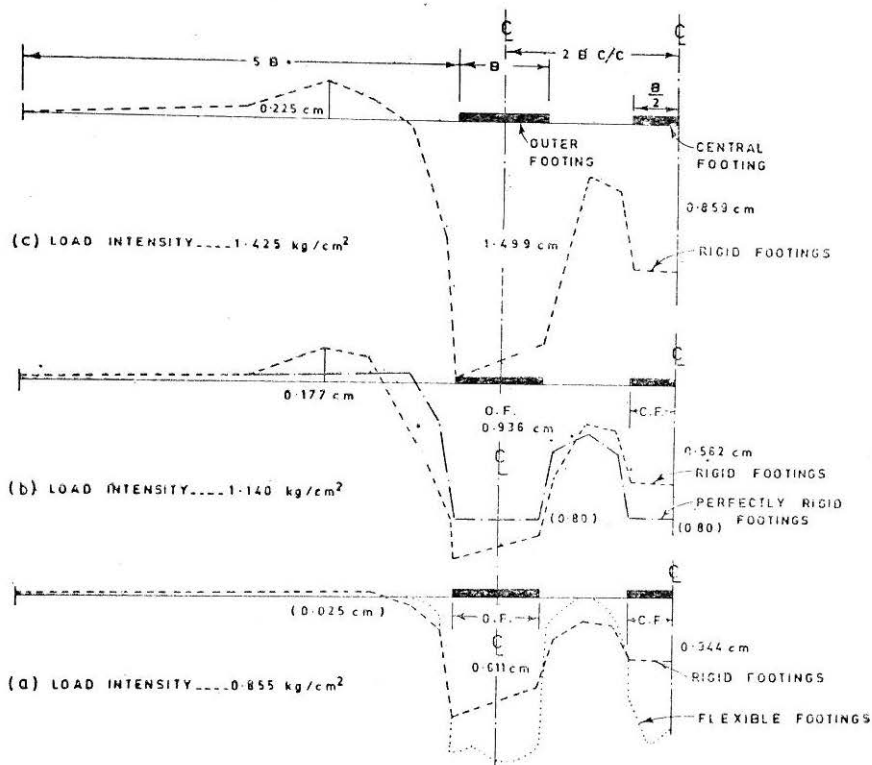


FIGURE 9 Surface profile of continuum

foundations. Behaviour of flexible foundations shown in Figure 9a, it may be concluded that the flexible foundations do not produce any significant heave.

Conclusions

Nonlinear and incremental analysis as reported here provides a very good understanding of the influence of the interference on the behaviour of three strip-soil system. Based on the investigations, the following conclusions are drawn.

Contact stresses of unsymmetrical and symmetrical patterns are observed for outer and central footings respectively. Due to interferences edge contact stresses are found to be lesser than at central portion for both the footings. This pattern reverses as the spacing is increased. Contact stresses for central footing are found to be lesser than corresponding outer footings.

Based on the average settlement criterion, the interfering footings indicate increase in their load carrying capacity and the same may be incorporated as interference augmentation factor in the design of interfering foundations to achieve economy, due to reduction in size of footings.

interfering foundations are associated with stress concentration at smaller spacings which would result in tilt for the outer footing and differential settlements between outer and central footings.

From finite element nonlinear analysis, it is possible to develop failure surfaces as per theoretical assumptions for a progressively incremental loading. An interesting pattern of heave and downward settlements in the continuum have also been observed due to interference.

Zones of tension have been found to be developed near the surface and on either side of footings. These zones are found to extend sideways and downwards depending on increase in spacing and thereby affect the behaviour pattern.

Increase in rigidity of footing from flexible to perfectly rigid conditions is found to improve load settlement characteristics and thereby augmentation factor. Perfectly rigid foundations may avoid tilt and differential settlements but it is found to develop heaving in the continuum between footings.

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